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DEFORMATION OF A MONOCLINIC ELASTIC HALF-SPACE BY A LONG INCLINED STRIKE-SLIP FAULT

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ABSTRACT

Closed-form analytical expression for the horizontal displacement due to a long inclined strike-slip fault situated in a monoclinic elastic half-space is obtained. The fault is of infinite length in the strike-direction and of finite width in the down-dip-direction. The effect of the anisotropy on the displacement field is also studied.

KEYWORDS: Deformation, Half-space, Monoclinic, Strike-Slip

INTRODUCTION

The deformation of an isotropic elastic half-space by a long strike-slip fault has been studied very extensively (e.g., Maruyama, 1966; Savage, 1980). Pan (1989) formulated the problem of the deformation of a transversely isotropic multilayered half-space by a dislocation source in terms of layer matrices. Garg et al. (1996) obtained an analytical solution for the deformation of an orthotropic layered half-space caused by a long strike-slip fault. Ting (1995) derived the Green's functions for a line force and a screw dislocation for the anti-plane deformation of a monoclinic elastic medium consisting of a single half-space or two half-spaces in welded contact. The calculation of the anti-plane deformation due to a line source in a monoclinic medium is much more difficult than the corresponding calculation for a source in an orthotropic medium because of the presence of the mixed derivatives in the equation of equilibrium in the former case (see Equation (10)).

In this paper, we use the results of Ting (1995) to obtain a closed-form analytical expression for the along-strike horizontal displacement caused by a long inclined strike-slip fault located in a monoclinic elastic half-space. It is shown that the width and the inclination of the image fault are different from the width and the inclination of the source fault placed in a monoclinic half-space. For an isotropic or an orthotropic half-space, the width and the inclination of the image fault and the source fault are the same.

As mentioned by Crampin (1989), monoclinic symmetry is the symmetry of two sets of non-orthogonal parallel cracks, where the plane of symmetry is perpendicular to the lines of intersection of the two sets of crack faces. Monoclinic symmetry of systems of cracks may be found near the surface of the Earth where lithostatic pressures have not closed cracks perpendicular to the maximum compressional stress.

BASIC EQUATIONS

In the absence of body forces, the equations of equilibrium are

$$\tau_{ij,j} = 0 \ (i = 1, 2, 3) \tag{1}$$

where τ_{ij} is the stress tensor, and the comma indicates differentiation with respect to the Cartesian co-ordinates (x_1, x_2, x_3). Summation over repeated indices is understood. If e_{ij} denotes the strain tensor and u_i the displacement vector, the strain displacement relations are

$$e_{ij} = (u_{i,j} + u_{j,i})/2 \tag{2}$$

The generalized Hooke's law for a homogeneous, anisotropic, elastic medium may be expressed in the form

$$\tau_{ij} = C_{ijks}\, e_{ks} = C_{ijks}\, u_{k,s} \tag{3}$$

where C_{ijks} are the elastic stiffnesses satisfying the symmetry relations

$$C_{ijks} = C_{jiks} = C_{ksij} \tag{4}$$

From Equations (1) and (3), the equations of equilibrium become

$$C_{ijks}\, u_{k,sj} = 0 \tag{5}$$

For a two-dimensional deformation in which the displacement components u_i are independent of x_3, Equation (5) yields

$$\begin{aligned} &C_{i111}u_{1,11} + C_{i212}u_{1,22} + \left(C_{i112} + C_{i211}\right)u_{1,12} + C_{i121}u_{2,11} + C_{i222}u_{2,22} \\ &+ \left(C_{i122} + C_{i221}\right)u_{2,12} + C_{i131}u_{3,11} + C_{i232}u_{3,22} + \left(C_{i132} + C_{i231}\right)C_{3,12} = 0 \end{aligned} \tag{6}$$

From Equation (6), we note that the plane strain deformation

$$u_1 = u_1(x_1, x_2),\;\; u_2 = u_2(x_1, x_2),\;\; u_3 = 0 \tag{7}$$

and the anti-plane strain deformation

$$u_1 = u_2 = 0,\;\; u_3 = u_3(x_{1,}\;\; x_2) \tag{8}$$

are decoupled, provided (Ting, 1995)

$$C_{14} = C_{15} = C_{24} = C_{25} = C_{46} = C_{56} = 0 \tag{9}$$

In Equation (9), we have used the contracted Voigt notation for the stiffnesses C_{ijks} according to the scheme

$$11 \to 1,\, 22 \to 2,\;\; 33 \to 3,\;\; 23 \to 4,\;\; 13 \to 5,\;\; 12 \to 6.$$

The conditions, as in Equation (9), are satisfied by a monoclinic material with $x_3 = 0$ as the symmetry plane. However, Equation (9) represents a material more general than a monoclinic material, because the latter requires $C_{34} = C_{35} = 0$ also. In fact, C_{34} and C_{35} do not appear in Equation (6) at all. Assuming that the conditions, as in Equation (9), are satisfied, the only equation of equilibrium for anti-plane strain is

$$C_{55}u_{3,\,11} + 2C_{45}u_{3,\,12} + C_{44}u_{3,\,22} = 0 \tag{10}$$

From Equation (3), the non-zero stresses are given by

$$\begin{aligned} \tau_{31} &= C_{55}u_{3,1} + C_{45}u_{3,2} \\ \tau_{32} &= C_{45}u_{3,1} + C_{44}u_{3,2} \\ \tau_{33} &= C_{33}u_{3,1} + C_{34}u_{3,2} \end{aligned} \tag{11}$$

Thus, in general, $\tau_{33} \neq 0$. However, for a monoclinic material, $\tau_{33} = 0$. In the following, it will be assumed that the anisotropic material under discussion satisfies the relations, as in Equation (9).

LINE FORCE

As shown by Ting (1995), the solution of Equation (10) representing the displacement field due to a line force f per unit length parallel to the $x_3 -$axis acting in a homogeneous, anisotropic, infinite, elastic medium at the point $x_1 = 0,\; x_2 = d$ is

$$u_3 = -\frac{f}{2\pi m}\mathrm{Re}\,\ln\left(z - pd\right) \tag{12}$$

where Re denotes the real part and

$$z = x_1 + px_2$$
$$p = \left(-C_{45} + im\right)/C_{44}, \; i = \sqrt{-1} \tag{13}$$
$$m = \left(C_{44}C_{55} - C_{45}^2\right)^{1/2} > 0$$

The corresponding stresses are given by

$$\tau_{31} = -\phi_{,2}, \;\; \tau_{32} = \phi_{,1} \tag{14}$$

where

$$\phi = \frac{f}{2\pi} \mathrm{Im}\ \ln\left(z - pd\right) \tag{15}$$

and Im indicates the imaginary part.

For an isotropic body of rigidity μ, $C_{45} = 0$, $C_{44} = C_{55} = \mu$, $m = \mu$, $p = i$, and

$$u_3 = -\frac{f}{2\pi\mu} \mathrm{Re}\ \ln\left[x_1 + i\left(x_2 - d\right)\right]$$
$$\phi = \frac{f}{2\pi} \mathrm{Im}\ \ln\left[x_1 + i\left(x_2 - d\right)\right] \tag{16}$$

The elastic field due to a line force f placed at the point $x_1 = 0,\ x_2 = d$ of an anisotropic half-space $\left(x_2 \geq 0\right)$ is given by (Ting, 1995)

$$u_3 = -\frac{f}{2\pi m} \mathrm{Re}\ \ln\left[\left(z - pd\right)\left(z - \bar{p}d\right)\right]$$
$$\phi = \frac{f}{2\pi} \mathrm{Im}\ \ln\left[\left(z - pd\right)\left(z - \bar{p}d\right)\right] \tag{17}$$

where an overbar indicates complex conjugate. The solution, as in Equation (17), satisfies the boundary condition

$$\tau_{32} = 0 \text{ at } x_2 = 0 \tag{18}$$

STRIKE-SLIP FAULT

Taking the x_3-axis along the strike of the fault and the x_2-axis vertically downwards, the displacement field due to a long strike-slip fault of arbitrary orientation can be expressed as the line integral (Maruyama, 1966)

$$u_3\left(x\right) = \int_L \Delta u_3\left(\xi\right) G_{3k}^3\left(x,\xi\right) n_k\left(\xi\right) ds\left(\xi\right) \tag{19}$$

where $\Delta u_3\left(\xi\right)$ is the displacement discontinuity, n_k is the unit normal to the fault section *L*, and

$$G_{3k}^3\left(x,\xi\right) = C_{3k3s} \frac{\partial}{\partial \xi_s} G_3^3\left(x,\xi\right) \tag{20}$$

In Equation (20), $G_3^3\left(x,\xi\right)$ is the Green's function representing the displacement at the point $\left(x\right)$ in the x_3-direction due to a line force of unit magnitude acting at the point $\left(\xi\right)$ in the x_3-direction. From Equation (17), we have, for an anisotropic half-space,

$$\begin{aligned} G_3^3 &= -\frac{1}{2\pi m} \mathrm{Re}\ \ln\left[\left\{x_1 - \xi_1 + p\left(x_2 - \xi_2\right)\right\}\left\{x_1 - \xi_1 + px_2 - \bar{p}\xi_2\right\}\right] \\ &= -\frac{1}{2\pi m} \ln\left(\mathrm{RS}\right) \end{aligned} \tag{21}$$

where

$$
\begin{aligned}
&R^2 = \left[x_1 - \xi_1 + p_r\left(x_2 - \xi_2\right)\right]^2 + p_i^2\left(x_2 - \xi_2\right)^2, \\
&S^2 = \left[x_1 - \xi_1 + p_r\left(x_2 - \xi_2\right)\right]^2 + p_i^2\left(x_2 + \xi_2\right)^2, \\
&p = p_r + ip_i, \quad \bar{p} = p_r - ip_i, \\
&p_r = -C_{45}/C_{44}, \; p_i = m/C_{44} = \left(\frac{C_{55}}{C_{44}} - \frac{C_{45}^2}{C_{44}^2}\right)^{1/2}
\end{aligned}
\tag{22}
$$

Using the Voigt notation for the stiffnesses, Equations (19) and (20) yield

$$
u_3(x) = \int_{\mathrm{L}} b\left[\left(n_1C_{55} + n_2C_{45}\right)\frac{\partial}{\partial\xi_1} + \left(n_1C_{45} + n_2C_{44}\right)\frac{\partial}{\partial\xi_2}\right]G_3^3(x,\xi)\,\mathrm{ds} \tag{23}
$$

where $b = \Delta u_3$ is the displacement discontinuity. Inserting the expression for G_3^3 from (21), we find

$$
\begin{aligned}
u_3(x) = \frac{1}{2\pi m}\int_L b\Big[&\left(n_1C_{55} + n_2C_{45}\right)\left\{x_1 - \xi_1 + p_r\left(x_2 - \xi_2\right)\right\}\left(\frac{1}{R^2} + \frac{1}{S^2}\right) \\
&+\left(n_1C_{45} + n_2C_{44}\right)\Big\{p_r\left[x_1 - \xi_1 + p_r\left(x_2 - \xi_2\right)\right]\left(\frac{1}{R^2} + \frac{1}{S^2}\right) \\
&+p_i^2\left(\frac{x_2 - \xi_2}{R^2} - \frac{x_2 + \xi_2}{S^2}\right)\Big\}\Big]\mathrm{ds}
\end{aligned}
\tag{24}
$$

Consider a strike-slip fault of width L and infinite length along the strike (x_3) direction. Let d be the depth of the upper edge A of the fault. If (s, δ) are the polar coordinates of any point $Q(\xi_1, \xi_2)$ on the fault, we have (Figure 1)

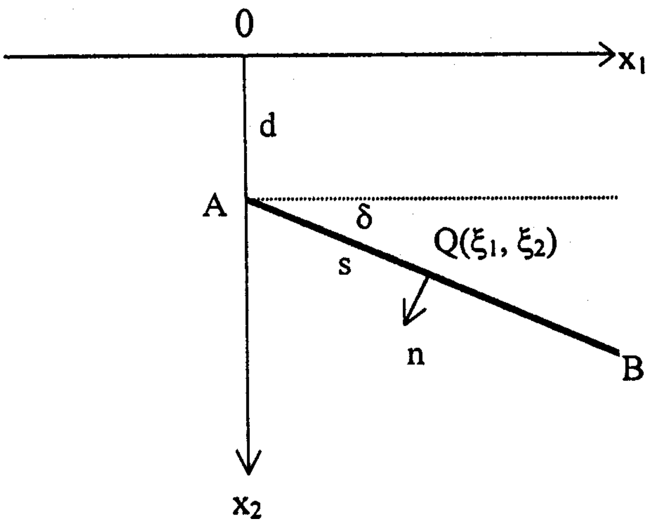

Fig. 1 Geometry of a long fault in a half-space (The fault is of infinite length in the strike (x_3) − direction. AB is the fault section by the x_1x_2 − plane which is also the plane of elastic symmetry of the monoclinic elastic half-space $x_2 \geq 0$, d is the depth of the upper edge A of the fault and δ the dip angle. (s, δ) denote the polar coordinates of any point $Q(\xi_1, \xi_2)$ on the fault.)

$$\begin{aligned} \xi_1 &= s\cos\delta, \quad \xi_2 = d + s\sin\delta \\ n_1 &= -\sin\delta, \quad n_2 = \cos\delta \end{aligned} \tag{25}$$

Using these values and (22), Equation (24) simplifies to

$$u_3(x) = \frac{\alpha}{2\pi}\int_0^L \left(\frac{Y_5}{R^2} - \frac{Y_6}{S^2}\right) b\,ds \tag{26}$$

where

$$\begin{aligned}
R^2 &= (A+\varepsilon\sin 2\delta)s^2 - 2\left[Cx_1 + B(x_2-d)\right]s + x_1^2 + \gamma(x_2-d)^2 + 2\varepsilon x_1(x_2-d) \\
&= \frac{1}{A+\varepsilon\sin 2\delta}\left\{\left[(A+\varepsilon\sin 2\delta)s - Cx_1 - B(x_2-d)\right]^2 + \alpha^2 Y_5^2\right\}, \\
S^2 &= (A+\varepsilon\sin 2\delta)s^2 - 2\left[C(x_1+2\varepsilon x_2) - B(x_2+d)\right]s + x_1^2 + \gamma(x_2+d)^2 + 2\varepsilon x_1(x_2-d) - 4\varepsilon^2 x_2 d \\
&= \frac{1}{A+\varepsilon\sin 2\delta}\left\{\left[(A+\varepsilon\sin 2\delta)s - C(x_1+2\varepsilon x_2) + B(x_2+d)\right]^2 + \alpha^2 Y_6^2\right\}, \\
Y_5 &= -x_1\sin\delta + (x_2-d)\cos\delta, \\
Y_6 &= (x_1+2\varepsilon x_2)\sin\delta + (x_2+d)\cos\delta, \\
\varepsilon &= p_r = -C_{45}/C_{44}, \quad \gamma = C_{55}/C_{44}, \\
\alpha &= p_i = \left(\gamma-\varepsilon^2\right)^{1/2}, \\
A &= \cos^2\delta + \gamma\sin^2\gamma, \\
B &= \varepsilon\cos\delta + \gamma\sin\delta, \\
C &= \cos\delta + \varepsilon\sin\delta
\end{aligned} \tag{27}$$

Assuming b to be constant over L and performing the integration in Equation (26), we obtain

$$\begin{aligned}
u_3(x) &= \frac{b}{2\pi}\tan^{-1}\left[\frac{(A+\varepsilon\, sin2\delta)L - Cx_1 - B(x_2-d)}{\alpha\left\{(x_2-d)\cos\delta - x_1\sin\delta\right\}}\right] \\
&+ \frac{b}{2\pi}\tan^{-1}\left[\frac{Cx_1 + B(x_2-d)}{\alpha\left\{(x_2-d)\cos\delta - x_1\sin\delta\right\}}\right] \\
&- \frac{b}{2\pi}\tan^{-1}\left[\frac{(A+\varepsilon\, sin2\delta)L - C(x_1+2\varepsilon x_2) + B(x_2+d)}{\alpha\left\{(x_1+2\varepsilon x_2)\sin\delta + (x_2+d)\cos\delta\right\}}\right] \\
&- \frac{b}{2\pi}\tan^{-1}\left[\frac{C(x_1+2\varepsilon x_2) - B(x_2+d)}{\alpha\left\{(x_1+2\varepsilon x_2)\sin\delta + (x_2+d)\cos\delta\right\}}\right]
\end{aligned} \tag{28}$$

For an orthotropic medium with the coordinate planes coinciding with the planes of symmetry, $\varepsilon = 0$ and, therefore, Equation (28) reduces to

$$u_3(x) = \frac{b}{2\pi}\tan^{-1}\left[\frac{AL - x_1\cos\delta - \gamma(x_2-d)\sin\delta}{\gamma^{1/2}\{(x_2-d)\cos\delta - x_1\sin\delta\}}\right] + \frac{b}{2\pi}\tan^{-1}\left[\frac{x_1\cos\delta + \gamma(x_2-d)\sin\delta}{\gamma^{1/2}\{(x_2-d)\cos\delta - x_1\sin\delta\}}\right]$$
$$-\frac{b}{2\pi}\tan^{-1}\left[\frac{AL - x_1\cos\delta + \gamma(x_2+d)\sin\delta}{\gamma^{1/2}\{x_1\sin\delta + (x_2+d)\cos\delta\}}\right] - \frac{b}{2\pi}\tan^{-1}\left[\frac{x_1\cos\delta - \gamma(x_2+d)\sin\delta}{\gamma^{1/2}\{x_1\sin\delta + (x_2+d)\cos\delta\}}\right] \quad (29)$$

For an isotropic material, $\varepsilon = 0$, $\gamma = 1$, and therefore, Equation (28) becomes

$$u_3(x) = \frac{b}{2\pi}\tan^{-1}\left[\frac{L - x_1\cos\delta - (x_2-d)\sin\delta}{(x_2-d)\cos\delta - x_1\sin\delta}\right] + \frac{b}{2\pi}\tan^{-1}\left[\frac{x_1\cos\delta + (x_2-d)\sin\delta}{(x_2-d)\cos\delta - x_1\sin\delta}\right]$$
$$-\frac{b}{2\pi}\tan^{-1}\left[\frac{L - x_1\cos\delta + (x_2+d)\sin\delta}{x_1\sin\delta + (x_2+d)\cos\delta}\right] - \frac{b}{2\pi}\tan^{-1}\left[\frac{x_1\cos\delta - (x_2+d)\sin\delta}{x_1\sin\delta + (x_2+d)\cos\delta}\right] \quad (30)$$

Equation (30) coincides with known results [see, e.g., Savage (1980) for the particular case of $\delta = 90^{\circ}$, and Singh and Rani (1996) for the particular case of $d = 0$].

The first two terms on the right-hand side of Equation (28) correspond to the source fault, and the remaining two terms correspond to the image fault (Figure 2). It can be shown that the image of the upper edge $A(0, d)$ of the source fault AB is not its mirror image $A_M(0, -d)$, but the point $A_I(2\varepsilon d, -d)$. Similarly, the image of the lower edge $B(L\cos\delta, d + L\sin\delta)$ is the point $B_I(L\cos\delta + 2\varepsilon(d + L\sin\delta), -d - L\sin\delta)$.

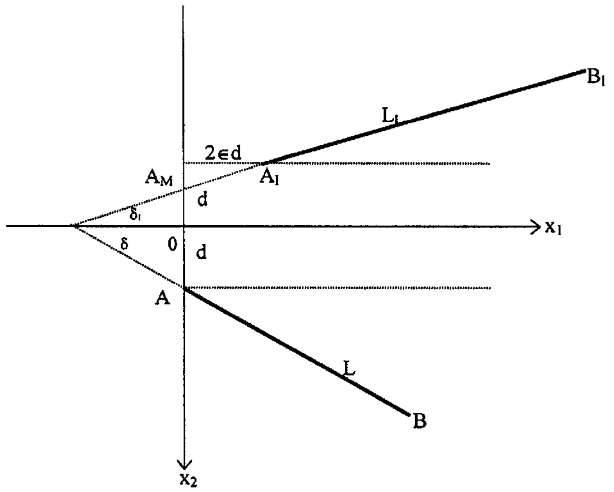

Fig. 2 Showing the source fault AB of width L and the image fault $A_I B_I$ of width L_I for a monoclinic half-space $x_2 \geq 0$ (The source fault makes an angle δ with the horizontal, and the image fault makes an angle δ_I with the horizontal in the opposite sense. The image fault in the case of a monoclinic half-space does not coincide with the mirror image of the source fault in the free boundary.)

The width of the image fault $A_I B_I$ is

$$L_I = L\left(1 + 4\varepsilon\sin\delta\cos\delta + 4\varepsilon^2\sin^2\delta\right)^{1/2} \quad (31)$$

and its inclination is given by

$$\cot\delta_I = \cot\delta + 2\varepsilon \quad (32)$$

However, for an orthotropic material, A_I coincides with A_M, and $L_I = L,\ \delta_1 = \delta.$ Figure 3 shows the image fault when the source fault is either horizontal or vertical. For a horizontal source fault, $L_I = L,\ \delta = 0$, and for a vertical source fault

$$L_I = L\left(1+4\varepsilon^2\right)^{1/2},\quad \cot\delta_1 = 2\varepsilon \tag{33}$$

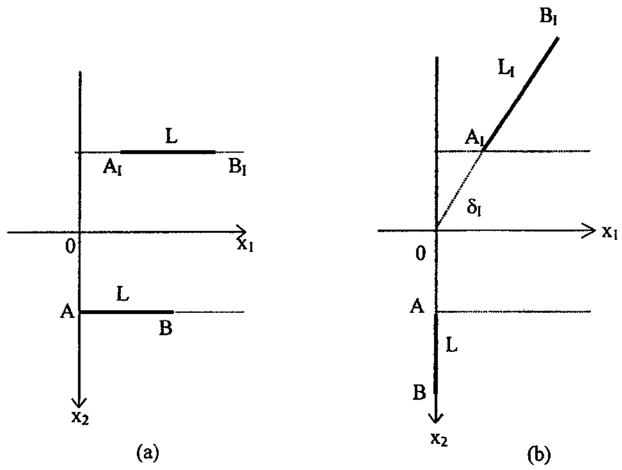

Fig. 3 Showing the image fault when the source fault is (a) parallel to the boundary, and (b) perpendicular to the boundary

In Figures 2 and 3, we have assumed that $\varepsilon > 0.$ For $\varepsilon < 0,$ these figures can be suitably modified.

NUMERICAL RESULTS

Figure 4 shows the variation of the amplification of the fault-width ratio (L_I/L) with the dip angle (δ) for three values of $\varepsilon = -C_{45}/C_{44}$, viz. $\varepsilon = 0, \pm 0.3$. $\varepsilon = 0$ corresponds to isotropic half-space. For a surface-breaking, vertical fault, the surface displacement is found from Equation (28) on taking $d = 0,\ \delta = \pi/2,\ x_2 = 0.$ We find

$$u_3 = \frac{b}{\pi}\left[\tan^{-1}\frac{\alpha x_1}{\gamma L - (\varepsilon x_1)} - \tan^{-1}\frac{\varepsilon}{\alpha} - \frac{\pi}{2}\operatorname{sgn}(x_1)\right] \tag{34}$$

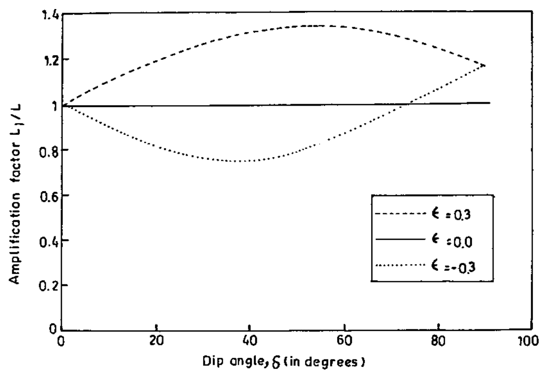

Fig. 4 Variation of the amplification factor L_I/L with the dip-angle for $\gamma = C_{55}/C_{44} = 1$ and for three values of the anisotropy parameter $\varepsilon = -C_{45}/C_{44}$, namely, $\varepsilon = 0,\ 0.3,\ -0.3$ ($\varepsilon = 0$ corresponds to the isotropic half-space)

Figure 5 shows the variation of the dimensionless horizontal surface displacement (u_3/b) with the dimensionless horizontal distance (x_1/L) from the fault-trace for $\gamma = 1$ and $\varepsilon = 0, \pm 0.3$. For the isotropic case $(\varepsilon = 0)$, the displacement is antisymmetric about the origin (fault-trace).

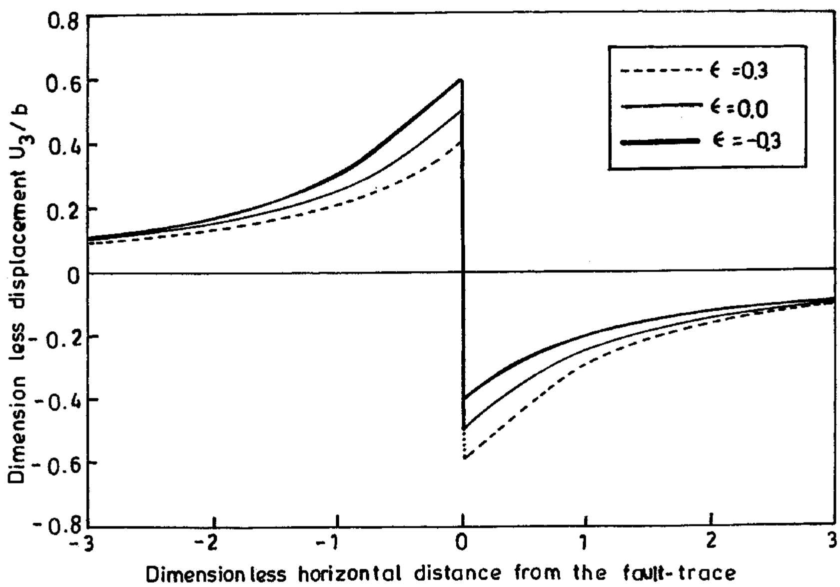

Fig. 5 Variation of the dimensionless along-strike surface displacement (u_3/b) with the dimensionless horizontal distance (x_1/L) from the upper edge of a surface-breaking vertical fault for $\gamma = 1$ and $\varepsilon = 0, +0.3, -0.3$

CONCLUSIONS

Study of the half-space deformation of an anisotropic half-space by internal sources is interesting from theoretical as well as practical point of view. In particular, it is useful to study the deformation of a monoclinic half-space by buried faults, since monoclinic symmetry of system of cracks is found near the surface of the Earth. To this end, we have solved the problem of a long inclined strike-slip fault in a monoclinic elastic half-space. It is remarkable that it is possible to find a closed form analytical solution for this complicated problem. The effect of anisotropy on the deformation field is significant. In the case of a long inclined strike-slip fault in an isotropic or orthotropic half-space, the width and the inclination of the image fault are the same as the width and the inclination, respectively, of the source fault. However, in the case of a monoclinic half-space, the width and the inclination of the image fault are different from the width and the inclination of the source fault. Further, in the case of a monoclinic half-space, while the image of a horizontal fault is a horizontal fault of equal width, the image of a vertical fault is not vertical and is of a different width. In the case of a surface-breaking long vertical strike-slip fault in an isotropic half-space, the surface displacement is antisymmetric about the fault-trace. However, in the case of a monoclinic half-space, the surface displacement is not antisymmetric about the fault-trace. Therefore, an examination of the degree of departure from the antisymmetry should give some idea about the anisotropy of the medium.

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