ISET Journal of Earthquake Technology, Paper No. 434, Vol. 40, No. 2-4, June-December 2003, pp. 99-115

MODE ACCELERATION APPROACH FOR GENERATION OF FLOOR SPECTRA INCLUDING SOIL-STRUCTURE INTERACTION

Samit Ray Chaudhuri and Vinay K. Gupta

Department of Civil Engineering
IIT Kanpur, Kanpur – 208016

ABSTRACT

A mode acceleration formulation is presented for the transfer function of absolute acceleration response of a single-degree-of-freedom oscillator which is supported on a base-excited, classically damped and flexible-base primary system. The primary system response has been described in terms of fixed-base primary mode shapes, with the response in last few modes assumed to be pseudo-static. Base flexibility has been assumed to be described by complex-valued impedance functions, and the effects of kinematic interaction have been assumed to be negligible. The proposed formulation can be used within the framework of any power spectral density function-based random vibration approach to generate floor response spectra of desired level of confidence. A numerical study is carried out with the help of an example primary system and band-limited white noise to illustrate the proposed formulation. It has been shown that the proposed formulation gives very accurate estimates of floor response spectra if pseudo-static response is assumed to be in those primary modes only which are stiff enough to the base excitation. It has also been shown that neglecting soil-structure interaction may give too over-conservative or unconservative estimates, depending on the damping ratio, natural period, and location of the oscillator, energy distribution in the excitation process, and shear wave velocity of soil.

KEYWORDS: PSDF-Based Seismic Response, Floor Response Spectra, Mode Acceleration Approach, Soil-Structure Interaction

INTRODUCTION

In major industrial structures, such as nuclear power plants, secondary systems (e.g., electrical and mechanical equipment) often play a critical role in maintaining the operation or safety immediately after an earthquake. Hence, it is important to ensure the seismic safety of such systems when those are designed. It has been an accepted practice to check the seismic safety by generating floor response spectra for specified seismic hazard at the site of the primary system.

The conventional approach of generating floor response spectra (e.g., see Singh (1975, 1980a)) does not consider (i) dynamic interaction between the primary system and oscillator, and (ii) non-classical damping of the combined system. Dynamic interaction is significant when the oscillator is moderately heavy and is tuned to one of the natural frequencies of the primary system. Different energy absorption properties of the primary system and the oscillator lead to a non-classically damped combined system with complex-valued mode shapes, even though the primary system is assumed to be classically damped. Ignoring these effects often leads to overconservative designs, and, thus, a coupled analysis becomes necessary for accurate results in case of moderately heavy to heavy equipments. Coupled analysis approaches based on the state space approach of Foss (1958), e.g., those by Itoh (1973), Singh (1980b), Traill-Nash (1981), Igusa et al. (1984), Borino and Muscolino (1986), Veletsos and Ventura (1986) are not practical, and, thus, approximate approaches like perturbation approach and mode synthesis approach (see, for example, Sackman and Kelly (1979), Sackman et al. (1983), Igusa and Der Kiureghian (1985), Suarez and Singh (1987a, 1987b), and Perotti (1994)) using real eigenproperties of the primary system have been developed. Most of these approaches, though elegant, are applicable for light secondary systems and consider the combined system to be classically damped. Chen and Soong (1994) and Gupta (1997) have proposed simple approaches in which the modal properties of the combined system are not explicitly calculated.

The above approaches of floor spectra generation have not considered the effects of soil-structure interaction. Interaction of the foundation with the surrounding soil medium may alter the seismic response of the structure through alterations in the frequencies and damping of the structure-foundation system.

Values of the system frequencies are lowered depending on how stiff the structure is with respect to the soil. This study proposes a formulation for the absolute acceleration response transfer function of a single-degree-of-freedom (SDOF) oscillator which can then be used to generate stochastic floor response spectra within the framework of a power spectral density function (PSDF)-based random vibration approach. The oscillator is considered to be supported on a base-excited, classically damped and flexible-base primary system. The proposed formulation extends the fixed-base formulation of Gupta (1997) to include the effects of soil-structure interaction by using the sub-structure approach. The primary system response is thus expressed in terms of fixed-base primary modes, and the foundation and the primary system are treated as two separate dynamic units, with the interaction forces of equal magnitude acting in opposite directions on the two sub-systems. The force-deformation relationships and damping characteristics of the foundation are described by the complex-valued impedance functions which are obtained independently as functions of the properties of the layered soil medium and geometry and depth of embedment of the foundation (see, e.g., Wong and Luco (1978)). The primary system response in the last few (fixed-base) modes is approximated to be pseudo-static, thus making the determination of these modes to be unnecessary. This approach, called as the mode acceleration approach, allows us to consider dynamic response in those modes only which are not stiff enough to the excitation process. The proposed formulation is illustrated by generating floor response spectra in case of a 15-story shear building subjected to a band-limited white noise base excitation, for different values of shear wave velocity, number of dynamic modes, location of oscillator, and oscillator damping ratio.

PROPOSED FORMULATION

Let us consider the *n*-degree-of-freedom (DOF) "stick" model of a linear, classically damped primary system supporting a SDOF oscillator along its *p*th DOF. Let $\{X(t)\}$ denote the vector of displacements along the DOFs of the primary system relative to the foundation, and $u(t)$ denote the displacement of the oscillator relative to the *p*th primary DOF. The foundation is considered to be a rigid rectangular slab of negligible thickness and attached to the surface of a uniformly visco-elastic half-space. The foundation input motion is assumed to be same as the free-field ground motion, $\ddot{z}(t)$, and, thus, the free-field rocking and the effects of kinematic interaction are assumed to be negligible. Let the foundation undergo translation, $z_0(t)$, and rotation, $\theta_0(t)$, relative to the surrounding soil medium (see Figure 1). Thus, by using the sub-structure approach, the decoupled primary system may be considered as subjected to (i) base excitations, $(\ddot{z}(t)+\ddot{z}_0(t))$ and $\ddot{\theta}_0(t)$, (ii) interaction force, $v(t)$, acting at the attachment point of the oscillator, and (iii) to the interaction forces between the foundation and the half-space, i.e. to the base shear, $V_s(t)$, and the base moment, $M_s(t)$.

The equations of motion for the decoupled primary system may be written as

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K]\{X(t)\}=-[M]\{1\}(\ddot{z}(t)+\ddot{z}_0(t))-[M]\{h\}\ddot{\theta}_0(t)+\{f(t)\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$, respectively, represent the mass, damping and stiffness matrices of the primary system, $\{X(t)\}$ is measured relative to the foundation slab, $\{1\}$ denotes the unit vector, and the elements of vector, $\{h\}(=\{h_1\, h_2 ....h_n\}^T)$, denote the heights of the primary masses, M_1, M_2, ..., M_n, above the base. Since the oscillator is connected along the *p*th DOF, the *p*th element, $v(t)$, of $\{f(t)\}$ is the force applied by the oscillator on its support point, and the remaining $(n-1)$ elements are zeroes. $v(t)$ is given by $(C_s\dot{u}(t)+K_s u(t))$, where C_s and K_s, respectively, are the damping constant and stiffness of the element connecting the oscillator mass to the primary system along the *p*th DOF, and $u(t)$ is measured relative to the *p*th primary DOF. Let the primary system response be expanded in terms of the orthonormal mode shapes of the fixed-base system, i.e.

$$\{X(t)\}=[\phi]\{q(t)\} \tag{2}$$

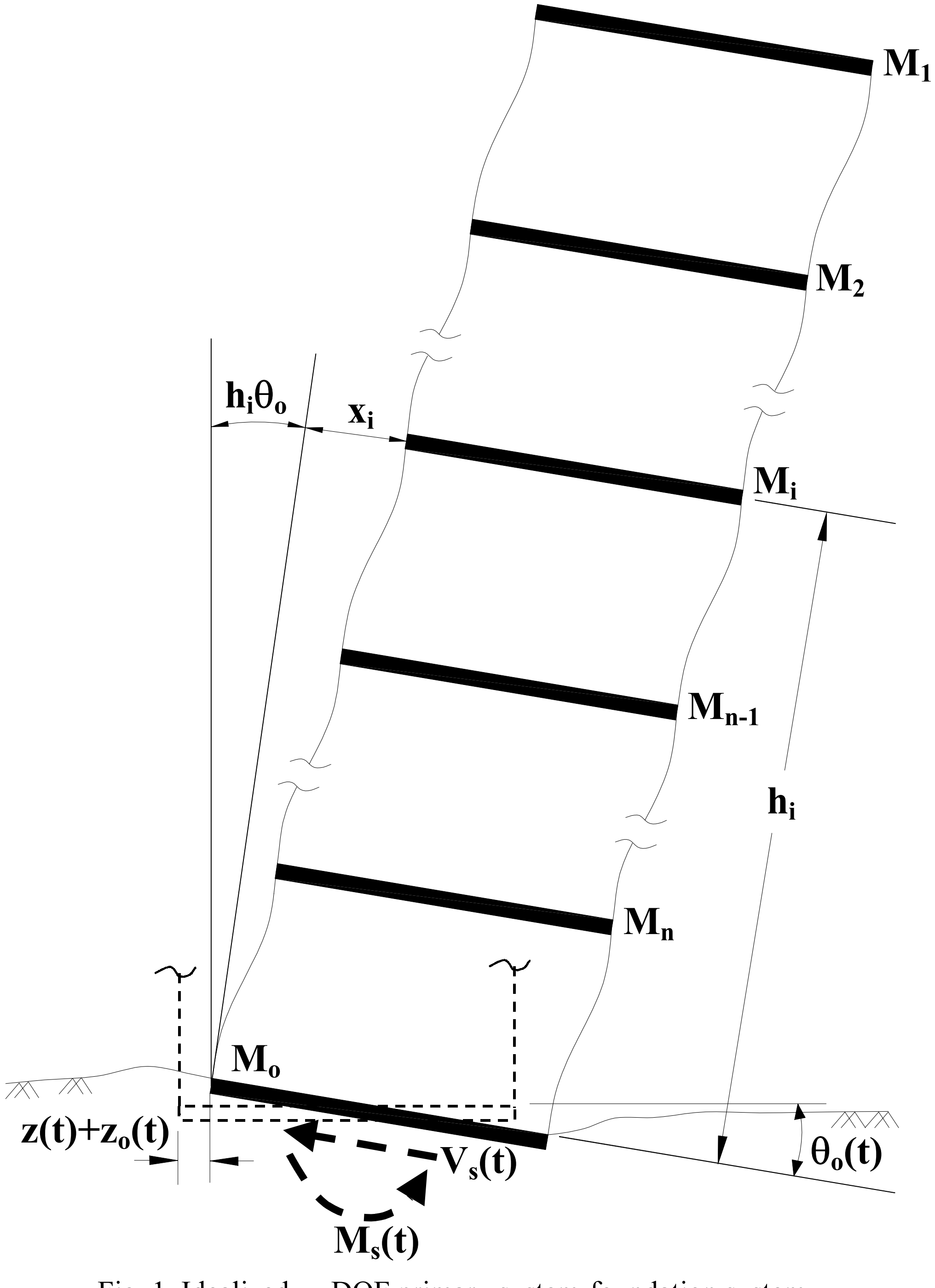

Fig. 1 Idealized n-DOF primary system-foundation system

where $[\phi]$ is the real-valued modal matrix of the fixed-base primary system, and $\{q(t)\}$ is the vector of normal coordinates. By using the orthogonality relationships, the decoupled equation for response in the rth primary mode can be written as

$$\ddot{q}_r(t) + 2\zeta_r\omega_r\dot{q}_r(t) + \omega_r^2 q_r(t) = -\alpha_r(\ddot{z}(t) + \ddot{z}_0(t)) - \gamma_r\ddot{\theta}_0(t) + \phi_p^{(r)}v(t);\ r = 1, 2, \ldots, n \tag{3}$$

where $\alpha_r (= \{\phi^{(r)}\}^T[M]\{1\})$ is the modal participation factor for the base translation, $\gamma_r (= \{\phi^{(r)}\}^T[M]\{h\})$ is the modal participation factor for the base rocking, ω_r and ζ_r respectively are the natural frequency and damping ratio, and $\phi_p^{(r)}$ is the pth element of mode shape, in the rth (fixed-base) mode of the primary system. On substituting the expression of $q_r(t)$, as in Equation (3), in Equation (2), the displacement response along the pth primary DOF can be expressed as

$$X_p(t)=\sum_{r=1}^{n}\frac{\phi_p^{(r)}}{\omega_r^2}\left[-\alpha_r\ddot{z}(t)-\alpha_r\ddot{z}_0(t)-\gamma_r\ddot{\theta}_0(t)+\phi_p^{(r)}v(t)\right]-\sum_{r=1}^{n}\frac{\phi_p^{(r)}}{\omega_r^2}[\ddot{q}_r(t)+2\zeta_r\omega_r\dot{q}_r(t)];\quad p=1,2,...,n \tag{4}$$

The first sum here represents the quasi-static part, $X_{ps}(t)$, and this can alternatively be obtained from Equation (1) by dropping the first two terms on the left hand side. The second part may be approximated by considering first $\hat{n}(<n)$ modes only, and, thus, $X_p(t)$ may be expressed in Fourier-transformed frequency domain as

$$X_p(\omega)=-\left[\sum_{i=1}^{n}F_{pi}M_i\right](\ddot{z}(\omega)+\ddot{z}_0(\omega))-\left[\sum_{i=1}^{n}F_{pi}M_ih_i\right]\ddot{\theta}_0(\omega)+F_{pp}v(\omega)+\sum_{r=1}^{\hat{n}}\frac{\phi_p^{(r)}}{\omega_r^2}\left[\omega^2-2i\zeta_r\omega\omega_r\right]q_r(\omega);\quad p=1,2,...,n \tag{5}$$

Here M_i is the *i*th diagonal element of the diagonal matrix $[M]$, and F_{pi} is the element corresponding to the *p*th row and the *i*th column of the flexibility matrix, $[F](=[K]^{-1})$ of the primary system. Further, $X_p(\omega)$, $\ddot{z}(\omega)$, $\ddot{z}_0(\omega)$, $\ddot{\theta}_0(\omega)$, $v(\omega)$, and $q_r(\omega)$ represent the Fourier transforms of $X_p(t)$, $\ddot{z}(t)$, $\ddot{z}_0(t)$, $\ddot{\theta}_0(t)$, $v(t)$, and $q_r(t)$, respectively. $q_r(\omega)$ is obtained on Fourier-transformation of Equation (3) as

$$q_r(\omega)=H_r(\omega)\left(-\alpha_r(\ddot{z}(\omega)+\ddot{z}_0(\omega))-\gamma_r\ddot{\theta}_0(\omega)+\phi_p^{(r)}v(\omega)\right);\quad r=1,2,...,n \tag{6}$$

where

$$H_r(\omega)=\frac{1}{\omega_r^2-\omega^2+2i\zeta_r\omega_r\omega} \tag{7}$$

represents the modal transfer function relating the displacement of the SDOF oscillator in the rth (fixed-base) primary mode to the input ground acceleration. On using Equation (6), Equation (5) may be expressed as

$$X_p(\omega)=-B_p(\omega)(\ddot{z}(\omega)+\ddot{z}_0(\omega))-Y_p(\omega)\ddot{\theta}_0(\omega)+D_{pp}(\omega)v(\omega) \tag{8}$$

where

$$B_p(\omega)=\sum_{i=1}^{n}F_{pi}M_i+\sum_{r=1}^{\hat{n}}\frac{\phi_p^{(r)}}{\omega_r^2}(\omega^2-2i\zeta_r\omega\omega_r)\alpha_rH_r(\omega) \tag{9}$$

and

$$Y_p(\omega)=\sum_{i=1}^{n}F_{pi}M_ih_i+\sum_{r=1}^{\hat{n}}\frac{\phi_p^{(r)}}{\omega_r^2}(\omega^2-2i\zeta_r\omega\omega_r)\gamma_rH_r(\omega) \tag{10}$$

represent the transfer functions of the displacement response along the *p*th DOF for the translational and rocking accelerations, respectively, at the base of the fixed-base primary system, if the interaction forces between the primary and secondary systems are ignored. Further,

$$D_{pk}(\omega)=F_{pk}+\sum_{r=1}^{\hat{n}}\frac{\phi_p^{(r)}}{\omega_r^2}(\omega^2-2i\zeta_r\omega\omega_r)\phi_k^{(r)}H_r(\omega) \tag{11}$$

represents the transfer function of the same response (of the fixed-base primary system) when a force is applied along the *k*th primary DOF, and both ground excitation and secondary system are absent. On taking $\hat{n}=n$, all three transfer functions take the following exact forms,

$$B_p(\omega)=\sum_{r=1}^{n}\alpha_r\phi_p^{(r)}H_r(\omega) \tag{12}$$

$$Y_p(\omega) = \sum_{r=1}^{n} \gamma_r \phi_p^{(r)} H_r(\omega) \tag{13}$$

$$D_{pk}(\omega) = \sum_{r=1}^{n} \phi_p^{(r)} \phi_k^{(r)} H_r(\omega) \tag{14}$$

requiring the determination of all n primary modes. It is, therefore, simpler to use the approximate expressions given by Equations (9)–(11) when $\hat{n} \ll n$.

In order to determine the interaction accelerations, $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$, in Equation (8), we consider the complete primary structure-foundation system in translation and rotation and write the equations of equilibrium in frequency domain involving interaction forces, $V_s(\omega)$ and $M_s(\omega)$, primary system displacements and interaction forces from the secondary system. On expressing the interaction forces in terms of interaction accelerations and on using Equation (8) for system displacements, two simultaneous equations are obtained in $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$ (see Dey and Gupta (1999) for details). Solving those leads to

$$\ddot{z}_0(\omega) = \chi_{zz}^{(1)}(\omega)\ddot{z}(\omega) + v(\omega)\chi_{(zz)}^{(2)}(\omega) \tag{15}$$

$$\ddot{\theta}_0(\omega) = \chi_{\theta z}^{(1)}(\omega)\ddot{z}(\omega) + v(\omega)\chi_{(\theta z)}^{(2)}(\omega) \tag{16}$$

where $\chi_{zz}^{(1)}(\omega)$ and $\chi_{\theta z}^{(1)}(\omega)$, respectively, represent the transfer functions relating the interaction accelerations, $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$, to the input ground acceleration, $\ddot{z}(\omega)$ (in the absence of the oscillator). The terms, $\chi_{(zz)}^{(2)}(\omega)$ and $\chi_{(\theta z)}^{(2)}(\omega)$, denote the transfer functions relating the interaction accelerations to the interaction force, $v(\omega)$, acting at the pth attachment point of the oscillator (in the absence of the ground excitation). Expressions for these transfer functions are given in the Appendix. On substituting $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$, as in Equations (15) and (16), into Equation (8), and since $v(\omega) = (i\omega C_s + K_s)u(\omega)$, the displacement response of the pth primary DOF is obtained as

$$X_p(\omega) = -B'_p(\omega)\ddot{z}(\omega) + (i\omega C_s + K_s)D'_{pp}(\omega)u(\omega) \tag{17}$$

where

$$B'_p(\omega) = (1 + \chi_{zz}^{(1)}(\omega))B_p(\omega) + \chi_{\theta z}^{(1)}(\omega)Y_p(\omega) \tag{18}$$

and

$$D'_{pp}(\omega) = D_{pp}(\omega) - \chi_{(zz)}^{(2)}(\omega)B_p(\omega) - \chi_{(\theta z)}^{(2)}(\omega)Y_p(\omega) \tag{19}$$

are the modified forms of the transfer functions, $B_p(\omega)$ and $D_{pp}(\omega)$, respectively, to account for the effects of translation and rotation of the foundation relative to the soil. As soil becomes stiffer with respect to the primary system, χs tend to zero, and then, these functions approach $B_p(\omega)$ and $D_{pp}(\omega)$, respectively, as defined for a fixed-base primary system. Further, in Equation (17), $u(\omega)$ is the Fourier transform of $u(t)$.

Equation (17) describes how the primary system displacements depend on the input (free-field) ground excitation (see the first term) and on the oscillator displacement (see the second term). For the latter, we write the equation of motion for the decoupled oscillator (Gupta, 1997) as

$$\ddot{u}(t) + 2\xi_s\omega_s\dot{u}(t) + \omega_s^2 u(t) = -\ddot{X}_p(t) - \ddot{z}(t) - \ddot{z}_0(t) - h_p\ddot{\theta}_0(t) \tag{20}$$

where ω_s and ξ_s denote the natural frequency and damping ratio of the oscillator, respectively. Height of the oscillator mass above the base of the primary system has been assumed here to be same as h_p. On Fourier-transforming this equation, using Equations (15) and (16) for $\ddot{z}_0(\omega)$ and $\ddot{\theta}_0(\omega)$ respectively, and on taking $v(\omega) = (i\omega C_s + K_s)u(\omega)$, the oscillator displacement may be expressed as

$$u(\omega) = A_p(\omega)X_p(\omega) + E_p(\omega)\ddot{z}(\omega) \tag{21}$$

where $A_p(\omega)$ represents the transfer function relating oscillator displacement to the displacement of the pth primary DOF in the absence of the ground excitation, and $E_p(\omega)$ represents the transfer function relating oscillator displacement to the ground acceleration when the oscillator is supported on a rigid primary system. These transfer functions are given by

$$A_p(\omega) = \frac{\omega^2 h_s(\omega)}{1 + (\chi^{(2)}_{(zz)}(\omega) + h_p\chi^{(2)}_{(\theta z)}(\omega))(i\omega C_s + K_s)h_s(\omega)} \tag{22}$$

$$E_p(\omega) = -\frac{[1 + \chi^{(1)}_{(zz)}(\omega) + h_p\chi^{(1)}_{(\theta z)}(\omega)]h_s(\omega)}{1 + (\chi^{(2)}_{(zz)}(\omega) + h_p\chi^{(2)}_{(\theta z)}(\omega))(i\omega C_s + K_s)h_s(\omega)} \tag{23}$$

where $h_s(\omega) = (\omega_s^2 - \omega^2 + 2i\xi_s\omega_s\omega)^{-1}$ is the transfer function relating the secondary system displacement to the input base excitation. On substituting the expression of $u(\omega)$, as in Equation (21), in Equation (17), $X_p(\omega)$ becomes

$$X_p(\omega) = H'_p(\omega)\ddot{z}(\omega) \tag{24}$$

with

$$H'_p(\omega) = \frac{-B'_p(\omega) + (i\omega C_s + K_s)D'_{pp}(\omega)E_p(\omega)}{1 - (i\omega C_s + K_s)D'_{pp}(\omega)A_p(\omega)} \tag{25}$$

It may be observed that neglecting dynamic interaction between the primary system and the oscillator leads to $H'_p(\omega) = -B'_p(\omega)$. In view of Equation (24), $u(\omega)$ finally becomes

$$u(\omega) = h'(\omega)\ddot{z}(\omega) \tag{26}$$

with $h'(\omega) = A_p(\omega)H'_p(\omega) + E_p(\omega)$. Now, since $(-\omega^2 u(\omega) - \omega^2 X_p(\omega) + \ddot{z}(\omega) + \ddot{z}_0(\omega) + h_p\ddot{\theta}_0(\omega))$ represents the absolute acceleration response of the oscillator, the transfer function, $\psi(\omega)$, relating this response to the ground acceleration may be expressed as

$$\begin{aligned}\psi(\omega) = 1 &+ (\chi^{(1)}_{(zz)}(\omega) + h_p\chi^{(1)}_{(\theta z)}(\omega)) + (\chi^{(2)}_{(zz)}(\omega) + h_p\chi^{(2)}_{(\theta z)}(\omega))(i\omega C_s + K_s)h'(\omega) \\ &- \omega^2(h'(\omega) + H'_p(\omega))\end{aligned} \tag{27}$$

It may be observed here that the terms, $(\chi^{(1)}_{(zz)}(\omega) + h_p\chi^{(1)}_{(\theta z)}(\omega))$ and $(\chi^{(2)}_{(zz)}(\omega) + h_p\chi^{(2)}_{(\theta z)}(\omega))(i\omega C_s + K_s)h'(\omega)$, respectively, represent the contributions of the primary system and the oscillator due to foundation translation and rocking; and the term, $\omega^2(h'(\omega) + H'_p(\omega))$, is due to the flexibility of primary system plus oscillator.

An alternative formulation for the transfer function may be obtained by expressing $v(\omega)$, instead of $u(\omega)$ (see Equation (21)), in terms of $X_p(\omega)$ and $\ddot{z}(\omega)$, and by following the same steps as before:

$$\psi(\omega) = \frac{\psi_1(\omega)}{\psi_2(\omega)} \tag{28}$$

with

$$\begin{aligned}\psi_1(\omega) = &\left(1 + \omega^2 h_s(\omega)\right)\Big[\omega^2 B'_p(\omega) + (1 + \chi^{(1)}_{(zz)}(\omega) + h_p\chi^{(1)}_{(\theta z)}(\omega)) \\ &\times\left\{1 + M_s(1 + \omega^2 h_s(\omega))(\chi^{(2)}_{(zz)}(\omega) + h_p\chi^{(2)}_{(\theta z)}(\omega))\right\}\Big]\end{aligned} \tag{29}$$

and

$$\psi_2(\omega) = 1 + M_s\left(1 + \omega^2 h_s(\omega)\right)\left(\chi^{(2)}_{(zz)}(\omega) + h_p\chi^{(2)}_{(\theta z)}(\omega) - \omega^2 D'_{pp}(\omega)\right) \tag{30}$$

This formulation explicitly involves the oscillator mass, M_s, and leads to the following transfer function in case of the fixed-base primary system (Rao, 1998),

$$\psi(\omega)=\frac{\left(1+\omega^2 h_s(\omega)\right)\left(1+\omega^2 B_p(\omega)\right)}{1-\omega^2 M_s D_{pp}(\omega)\left(1+\omega^2 h_s(\omega)\right)} \tag{31}$$

Once the transfer function relating the absolute acceleration response of the oscillator to the ground acceleration is determined, the PSDF of the oscillator response process may be obtained, by using the stationary random vibration theory, as squared modulus of the transfer function multiplied by the PSDF of the ground acceleration process (Crandall and Mark, 1963). The input PSDF may be taken as an idealized PSDF in a functional form (e.g., band-limited white noise, or Clough-Penzien PSDF (Clough and Penzien, 1993)) for studying the system behaviour, or as design spectrum-compatible PSDF (e.g., see Unruh and Kana (1981)) in order to account for the non-stationary nature of the output process and thus to obtain results for the practical situations. The area under the response PSDF gives the mean-square value of the response process, the square root of which may be multiplied with a suitable peak factor to estimate the largest peak in the response process (see Gupta (2002) for further details). This peak value forms one ordinate of the floor response spectrum for damping ratio ξ_s. By varying ω_s, the complete curve for ξ_s may be obtained, and by considering different values of ξ_s, a set of floor response spectra may be generated.

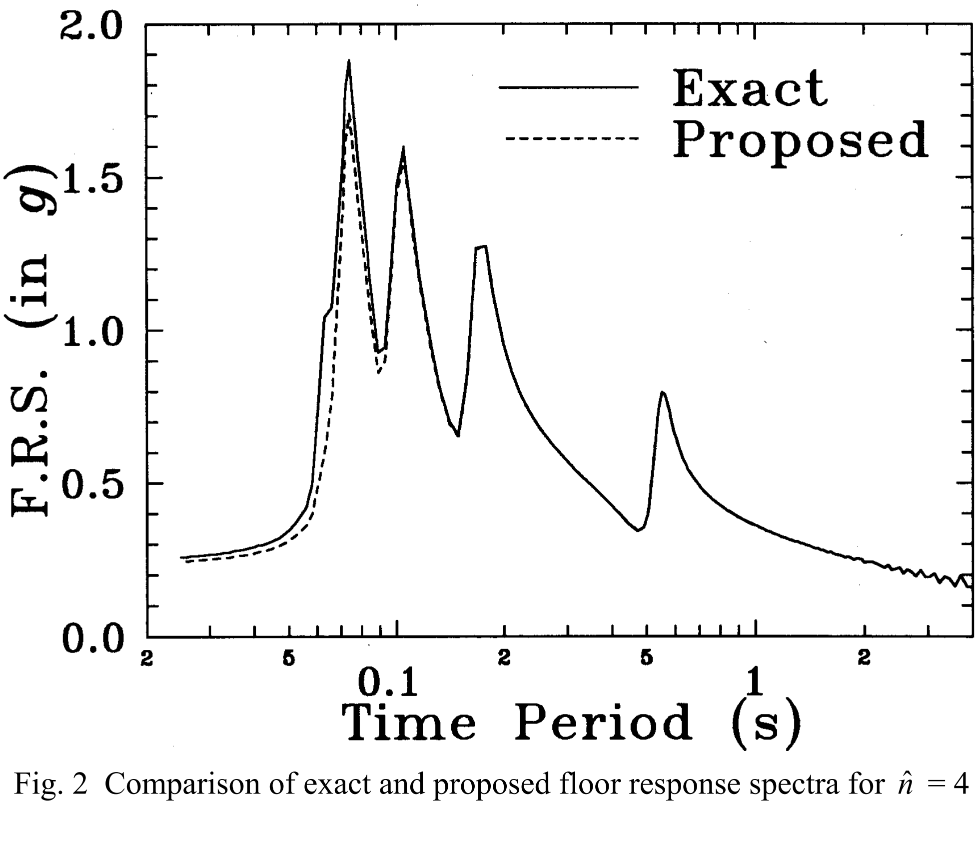

Fig. 2 Comparison of exact and proposed floor response spectra for $\hat{n}$ = 4

RESULTS AND DISCUSSION

The proposed formulation has been illustrated by considering a 15-story shear building. Floor mass is taken as 2.8×10^5 kg for the first floor and 2.0×10^5 kg each for all other floors. The inter-story stiffness is taken as 3.5×10^6 kN for the first story and 3.15×10^6 kN each for all other stories. The corresponding fixed-base natural frequencies are obtained as 12.79, 38.17, 62.95, 86.75, 109.30, 130.54, 150.58, 169.51, 187.15, 203.18, 217.27, 229.14, 238.59, 245.45 and 249.60 rad/s. The modal damping ratio is assumed to be 0.05 for all the modes of the building. The story height is considered to be 3.5 m for the first story and 3.0 m each for all other stories. The mass moments of inertia of the floor masses, i.e. I_js, are assumed to be negligible. The foundation is assumed to be a square-shaped slab with length of reference, L = 9 m, and of negligible mass as compared to the masses of the floors. Impedance functions are taken for the uniform visco-elastic half-space with the mass density, ρ, hysteretic damping ratio, ζ, and Poisson's ratio, ν, equal to 1800 kg/m^3, 0.02 and 0.3, respectively. The oscillator is assumed to be of 200 kg mass and supported at the first floor. The oscillator damping ratio is assumed to be 0.02.

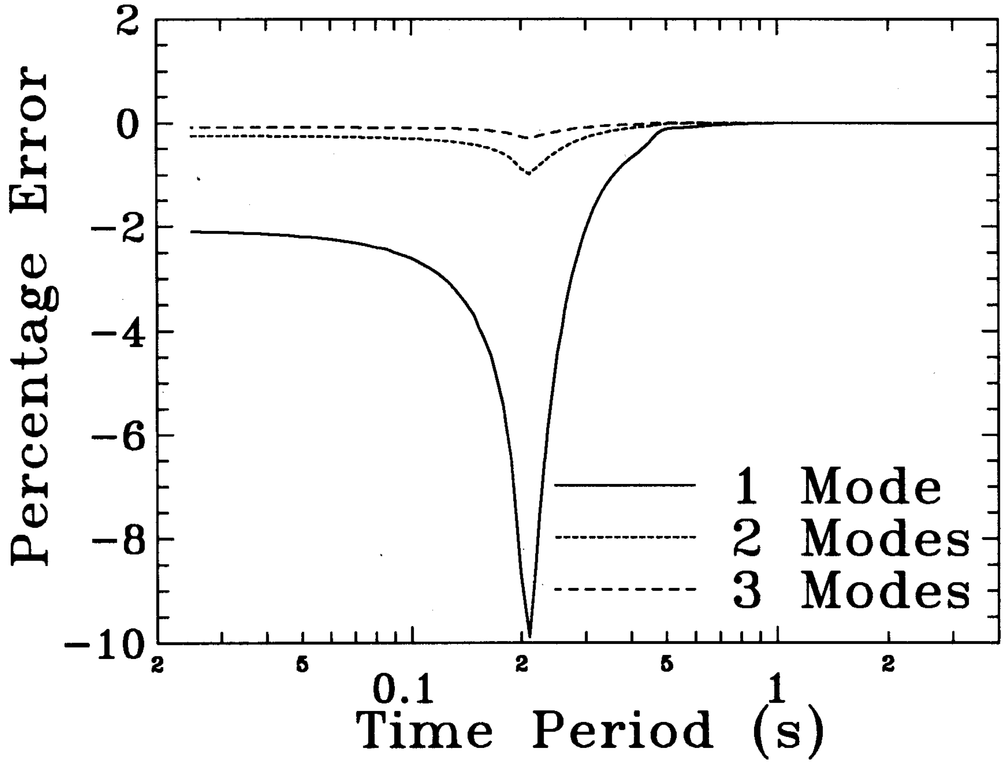

Fig. 3 Percentage error spectra in case of proposed floor response spectra for $\hat{n}$ = 1, 2 and 3

To generate floor response spectra, the building is assumed to be excited by a band-limited white noise process with 0.005 m^2/s^3 intensity within a band of 0.5-100.5 rad/s. Peak factor is assumed to be 3.0, and, thus, the example excitation corresponds to a peak ground acceleration of 0.2162*g*. Figure 2 shows a comparison of the proposed floor response spectrum (F.R.S.), obtained by 'truncating' last 11 primary modes (i.e., $\hat{n}$ = 4), with the exact spectrum (i.e., $\hat{n}$ = 15), for the shear wave velocity equal to 600 m/s. It may be observed that all 11 primary modes which are stiffer than the excitation process have been assumed to respond pseudo-statically. Figure 2 shows that the proposed formulation gives excellent approximation of floor response spectra for oscillator frequencies less than 70 rad/s. The proposed spectrum however deviates from the exact spectrum for stiffer oscillators. In order to appreciate the resulting errors for such oscillators, we consider the band-limited excitation in the range of 0.5–30.5 rad/s, and obtain the variation of percentage error associated with 'proposed' F.R.S. with time period for the fixed-base condition. Figure 3 shows such variations, being called as percentage error spectra, for $\hat{n}$ = 1, 2 and 3. Negative errors here indicate that the F.R.S. ordinates estimated by the proposed formulation are less than the exact ordinates. The errors are observed to be maximum for $\hat{n}$ = 1 and when the oscillator is at the upper cut-off frequency of the excitation. The errors remain negligible when the oscillator is more flexible compared to the first primary frequency. However, as the oscillator becomes stiffer and its frequency approaches the second primary frequency, the oscillator gets more tuned with the second mode, thus leading to an increased participation of this mode. Ignoring the dynamic response in the second mode therefore leads to greater error in the transfer function for the frequencies between the oscillator frequency and the second primary frequency. Greater is the tuning, more is this error. The effect of this error on F.R.S. ordinates however does not increase further as soon as the oscillator frequency exceeds the upper cut-off frequency of the excitation. This happens because the frequencies falling between the oscillator frequency and the second primary frequency are no longer excited by the excitation. In fact, a sudden fall is observed in the percentage error as the oscillator frequency increases and the frequencies significantly affected by the oscillator tuning with the second and higher (pseudo-static) modes move away from the frequency band of excitation. The percentage error however stabilizes beyond certain oscillator frequency (i.e., at lower time periods), due to ignoring the dynamic response of those primary modes (second, third, ...) which are not sufficiently stiff to the excitation. It may be noted that these modes have increased participation due to the oscillator becoming stiffer compared to those modes. In the cases of $\hat{n} = 2$ and 3, the stabilized error becomes lesser as the dynamic response of more higher modes is accounted for. Also, the two dashed curves for these cases show much lesser maximum errors (at the upper cut-off frequency of the excitation) since, at this frequency, the oscillator is not tuned to the third primary mode (in case of $\hat{n} = 2$) or to the fourth primary mode (in case of $\hat{n} = 3$) as strongly as it was tuned to the second primary mode in the case of $\hat{n} = 1$.

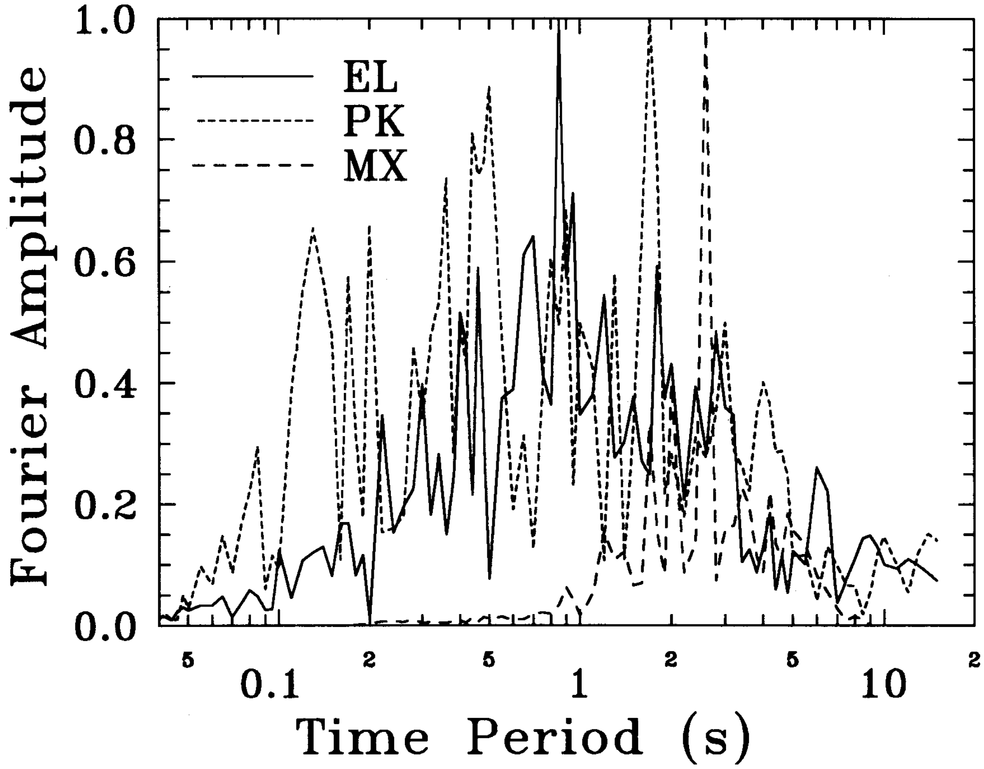

Fig. 4 Comparison of the normalized Fourier spectra of the El Centro (EL), Parkfield (PK), and Mexico City (MX) ground motions

It may be mentioned that the above observations do not change when the upper cut-off frequency of the excitation is raised to 38.0 rad/s (from 30.5 rad/s) which is very close to the second natural frequency (i.e., 38.17 rad/s) of the primary system. However, a different picture emerges when, instead of the band-limited white noise process, we consider realistic ground motion processes corresponding to (i) recorded S00E component of Imperial Valley earthquake of 18 May 1940 at El Centro site, (ii) recorded vertical component of Parkfield earthquake of June 1966 at Cholame, Shandon site, and (iii) synthetic accelerogram for the horizontal component of Michoacan earthquake, 1985 at Mexico City site (Gupta and Trifunac, 1990). The Fourier spectra of these ground motions, as normalized to the respective maximum amplitudes, are shown in Figure 4. It may be observed from this figure that the Parkfield motion is rich in high-frequency waves compared to El Centro motion, and Mexico City ground motion is dominated by low-frequency waves. The three example processes correspond to the percentage error spectra for $\hat{n} = 1$, as shown in Figure 5 by EL, PK and MX respectively, when the three processes are characterized by the spectrum-compatible PSDFs, as obtained in Ray Chaudhuri and Gupta (2002). For comparison, Figure 5 also shows the '1 Mode' curve of Figure 3, describing the error variation for the band-limited white noise excitation (see the WN curve). It is seen that in the absence of an upper cut-off frequency, as the oscillator frequency approaches the second primary frequency from below, the percentage error continues to rise till the second primary frequency for all three realistic excitations. In case of $\hat{n} = 2$, however, the error would rise to a local peak at the third primary frequency from a negligible value at the second primary frequency. Figure 5 also shows that in contrast with the WN curve, the error spectra curves for the realistic excitations do not fall off to stabilized values for very stiff oscillators. This again happens due to the absence of an upper cut-off frequency and a wider range of frequencies getting excited, and, therefore, we get local error peaks at the primary system frequencies. We also get a local peak in between two adjoining primary system frequencies. This happens due to a local minimum in the exact F.R.S. ordinate at the corresponding frequency (see Figure 2) while the F.R.S. based on the use of pseudo-static modes for those primary system frequencies fails to capture this trend. The local peaks in between the primary system frequencies appear to increase in magnitude for stiffer oscillators, due to the increased participation of those 'pseudo-static' modes (second, third, ...) which are more flexible compared to the oscillator. However, a sudden fall-off is observed for the oscillators having periods less than 0.04 s, just as it was observed in case of the band-limited white noise (for the oscillators having natural frequencies greater than the upper cut-off frequency of the excitation). This is due to the fact that the PSDFs of the realistic excitations are assumed to have no energy beyond 157 rad/s, and, thus, 157 rad/s effectively becomes the upper cut-off frequency for the realistic excitation processes as considered in this study.

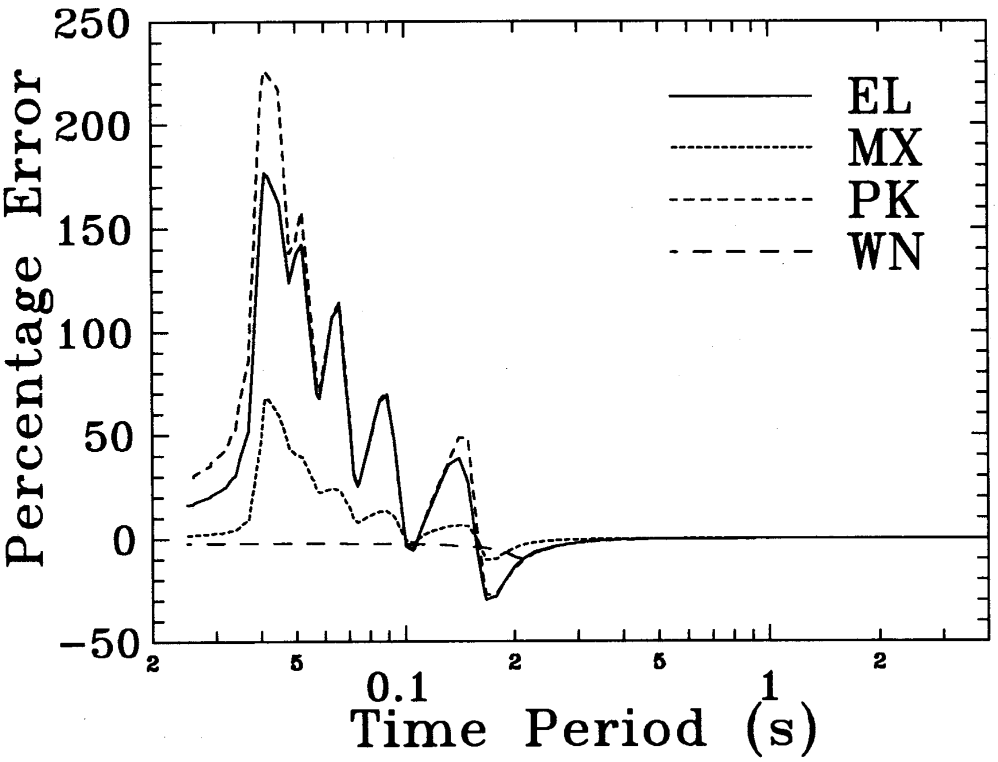

Fig. 5 Percentage error spectra in case of proposed floor response spectra for the El Centro (EL), Mexico City (MX), Parkfield (PK), and white noise (WN) excitation processes

Returning to Figure 3, the maximum percentage error at the upper cut-off frequency as observed here is going to be more when the oscillator is placed at those floors where there is a greater interaction of oscillator with the second mode. Figure 6 shows a comparison of the errors for the 1st floor location with those for the 4th and 7th floor locations in case of $\hat{n} = 1$. It is seen that the maximum error increases from 10% to more than 30% for the 4th floor location and that the stabilized percentage error in case of the 7th floor location is zero. The latter observation implies that the second mode has negligible participation in the oscillator response, when it is placed at the 7th floor. In order to see how far the flexibility of the base affects the observations of Figure 3, we compare the percentage error spectra for shear wave velocity, β = 100, 200, and 1000 m/s in case of $\hat{n} = 1$ (see Figure 7). Though the value of β = 100 m/s is inconsistent with the type of foundation we have assumed in the paper, this has been chosen to emphasize the effects of shear wave velocity on the errors associated with the 'mode truncation'. Further, it may be mentioned that the error spectrum for β = 1000 m/s is almost identical to that for the fixed-base condition, as at this value of β, there is a negligible soil-structure interaction. It is seen in Figure 7 that while the error spectra for β = 200 and 1000 m/s show the same trends as for the fixed-base, the error spectrum for β = 100 m/s shows a shift to the right in the peak for the maximum error. This peak corresponds to the decreased second primary frequency (due to base-flexibility) which now falls within the frequency band of the excitation. For stiffer oscillators, the error spectrum shows the same trends as seen earlier (see Figure 5), with a transition to positive error peak at the upper cut-off frequency, as the oscillator frequency is increased, and then the fall-off to a stabilized value for frequencies greater than the upper cut-off frequency.

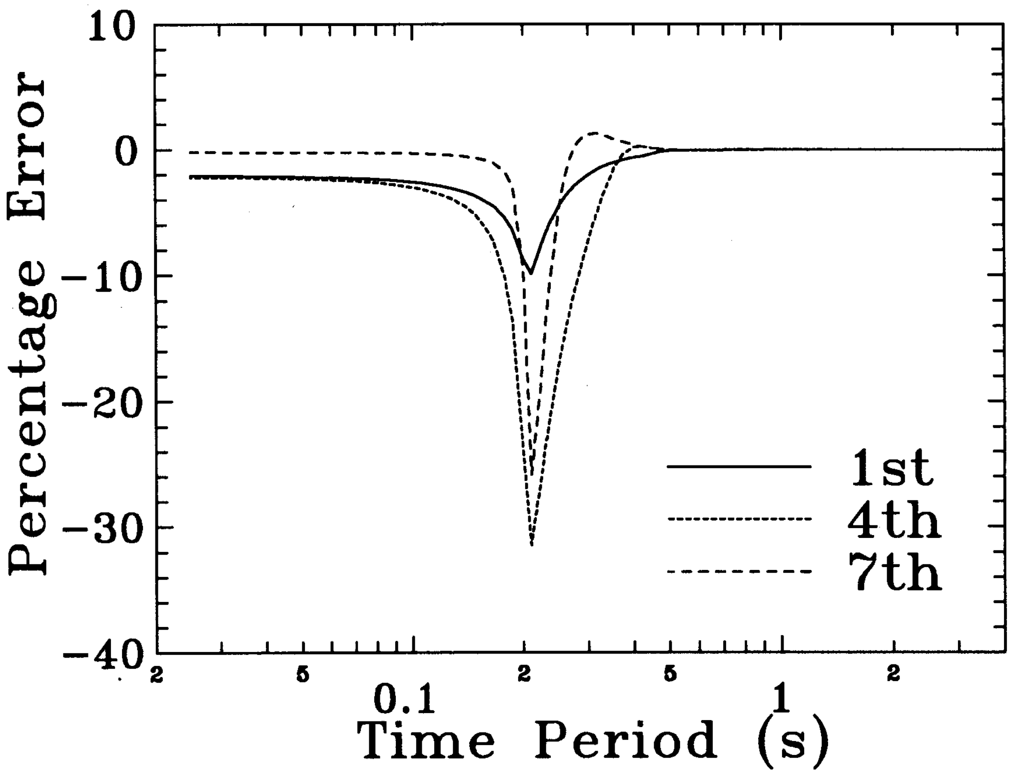

Fig. 6 Percentage error spectra in case of proposed floor response spectra for the oscillator on 1st, 4th and 7th floors

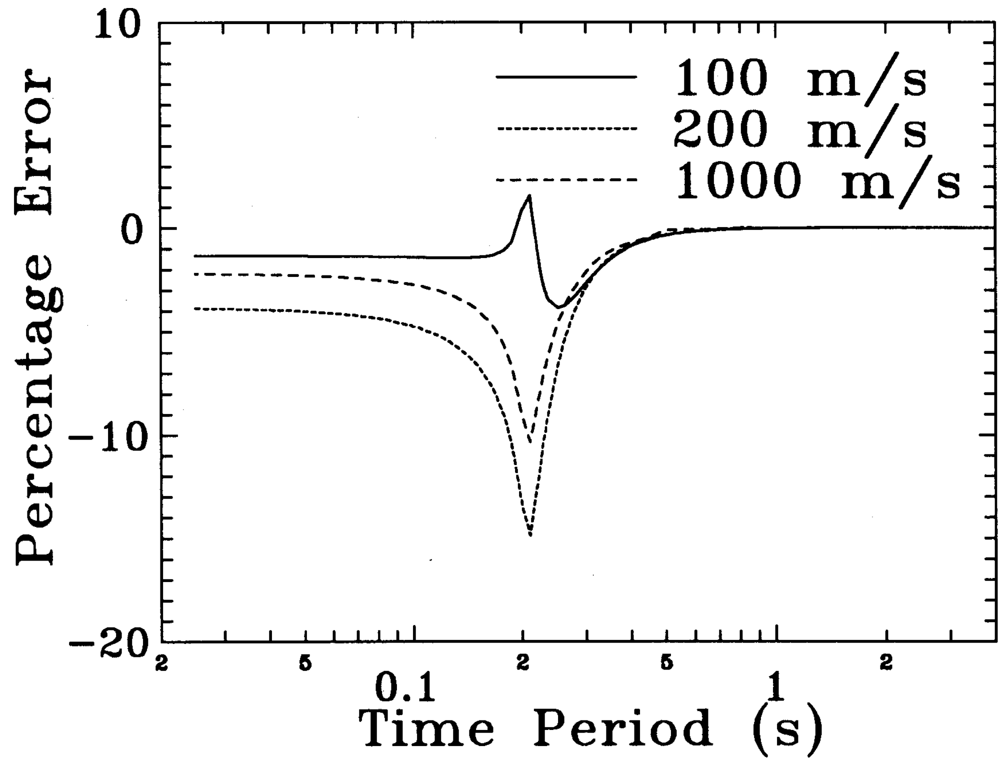

Fig. 7 Percentage error spectra in case of proposed floor response spectra for β = 100, 200 and 1000 m/s

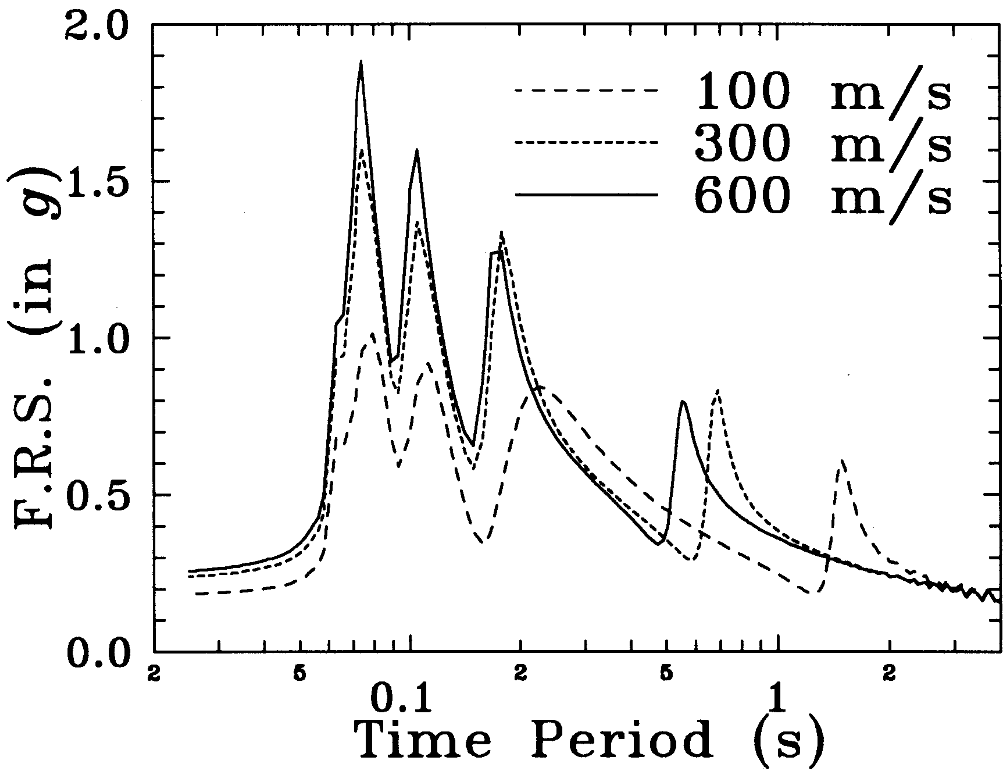

Fig. 8 Comparison of exact floor response spectra for β = 100, 300 and 600 m/s

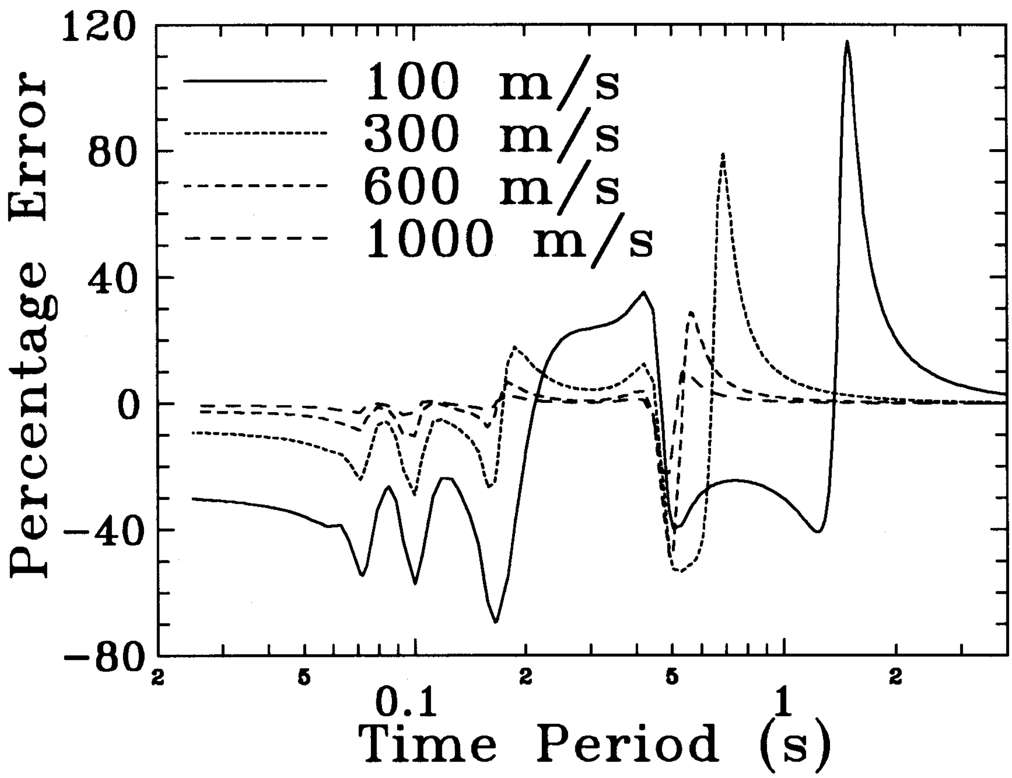

Fig. 9 Percentage error spectra due to fixed-base assumption in case of β = 100, 300, 600 and 1000 m/s

The above observations imply that the proposed formulation is capable of giving quite accurate F.R.S. ordinates as long as $\hat{n}$ is carefully chosen. It appears that this should be so large as to include all modes falling within the frequency range of excitation plus 1–2 next higher modes, provided the oscillator is stiff to the excitation. However, if the oscillator frequency falls within the frequency range of excitation, it may be enough to take as many modes as are necessary to contain the oscillator frequency. Thus, in the present case, 5 modes may be sufficient (i.e., $\hat{n} = 5$), if the oscillator frequency is less than 109.3 rad/s and the excitation range extends beyond 109.3 rad/s. In other words, only those modes should be assumed to respond pseudo-statically which are sufficiently stiff (after accounting for the base flexibility) to the more flexible of the oscillator and the excitation.

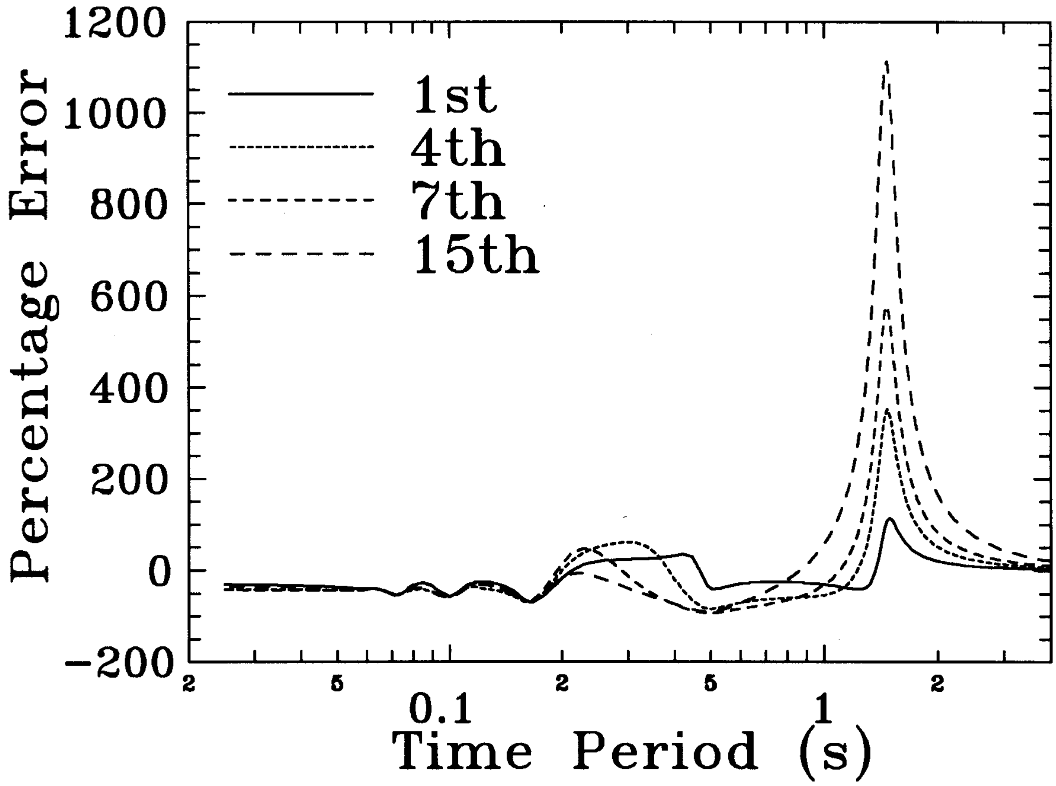

Fig. 10 Percentage error spectra due to fixed-base assumption in case of oscillator on 1st, 4th, 7th and 15th floors

In order to see how the shear wave velocity (representing the level of soil interaction) affects the F.R.S., a comparison of the proposed spectra for $\beta = 100$, 300 and 600 m/s in case of $\hat{n} = 15$ is shown in Figure 8 for 0.5–100.5 rad/s band-limited white noise excitation. It is obvious that the peaks corresponding to the first two primary natural frequencies shift towards right as a result of soil interaction, with this shift being more in case of the fundamental frequency. Increased interaction may also lower various peaks significantly. As a result, there may be considerable underestimation or overestimation (depending upon the frequency of the oscillator) if the effects of soil interaction are ignored. To illustrate this, we compare the percentage error spectra for $\beta = 100$, 300, 600 and 1000 m/s in Figure 9, where the percentage error refers to the error associated with the fixed-base assumption of a flexible-base primary structure. It is seen that while the errors are small for $\beta = 1000$ m/s case, those may be as large as 120% in case of $\beta = 100$ m/s. This implies that unless the soil is reasonably stiff relative to the primary system, the soil flexibility must be accounted for in estimating the F.R.S. The maximum error may increase to a very high value for a different location of the oscillator as shown by Figure 10 for 1st, 4th, 7th and 15th floor locations in case of $\beta = 100$ m/s. This may however reduce in case of higher oscillator damping as shown by Figure 11 for the damping values of 2%, 5%, 7% and 10%. This is also dependent on the excitation process as shown by the comparison of the percentage error spectrum for the band-limited white noise (WN) process (with $\beta = 100$ m/s, 2% oscillator damping, first floor location of the oscillator, and $\hat{n} = 15$) with the error spectra for the El Centro (EL), Mexico City (MX), and Parkfield (PK) processes (see Figure 12). It may be observed here that the fixed-base assumption is not as critical in case of the Mexico City process as it is in the cases of the El Centro and Parkfield processes, particularly for the oscillator periods less than 1 s. This may be due to negligible energy in the Mexico City ground motion for the periods less than 1 s.

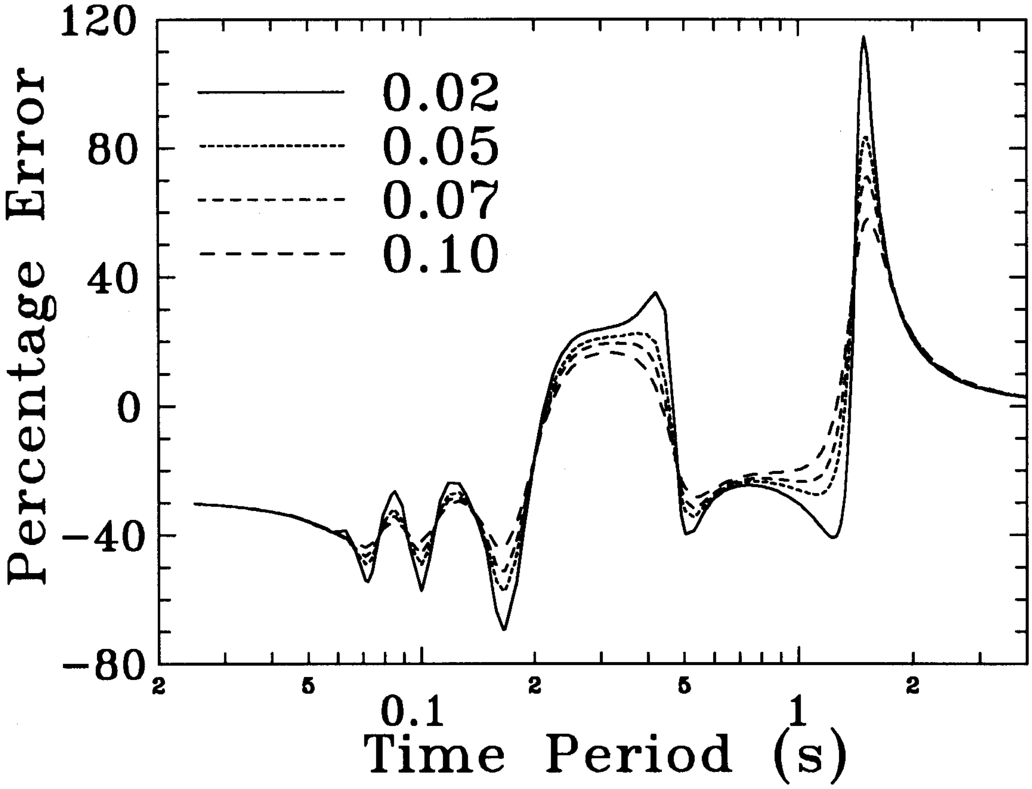

Fig. 11 Percentage error spectra due to fixed-base assumption in case of ξ_s = 0.02, 0.05, 0.07 and 0.10

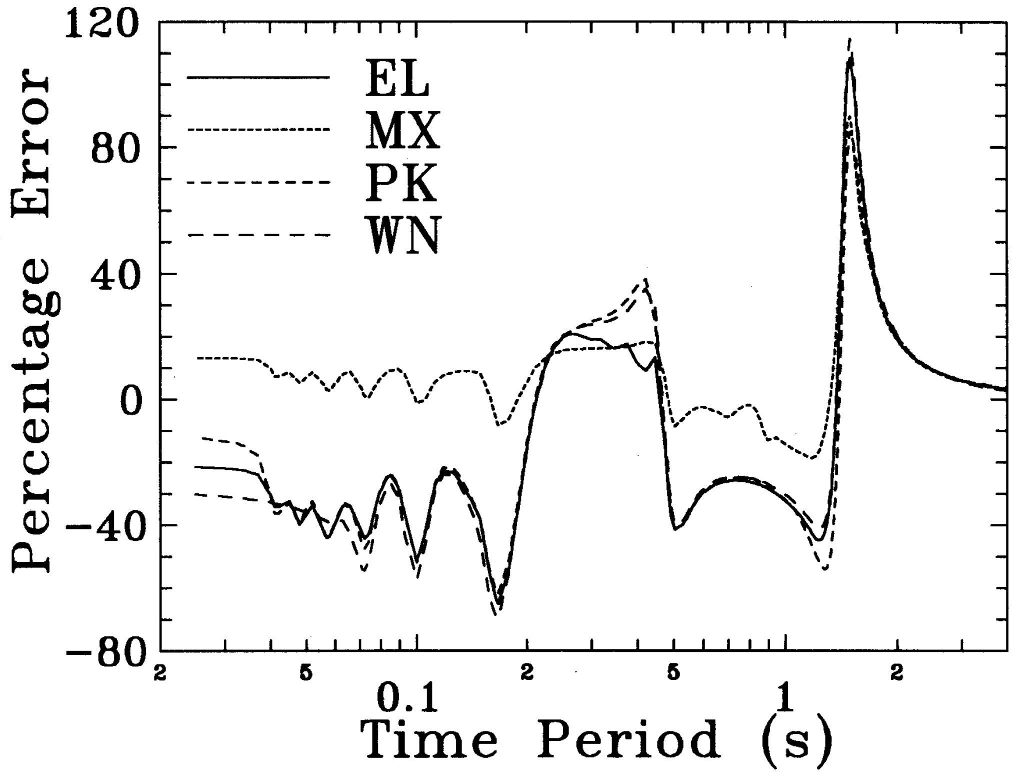

Fig. 12 Percentage error spectra due to fixed-base assumption in case of El Centro (EL), Mexico City (MX), Parkfield (PK), and white noise (WN) excitation processes

It may be noteworthy that the extent to which the fixed-base assumption affects the F.R.S. ordinates does not depend on the oscillator mass unless the oscillator is heavy, say more than 10% of the mass of the supporting floor. This happens because, with the fixed-base frequencies of the two sub-systems kept unchanged, the level of soil interaction remains insensitive to the oscillator mass. For heavy oscillators, however, damping of the combined system may change with the oscillator mass, particularly when the oscillator frequency is close to one of the natural frequencies of the system, and, then, the oscillator mass (relative to the mass of the supporting floor) may become an important parameter.

CONCLUSIONS

A mode acceleration formulation has been proposed for the absolute acceleration response transfer function of a SDOF oscillator which is attached to the "stick" model of a base-excited, classically damped primary system supported on compliant soil. The proposed formulation uses fixed-base primary mode shapes. Effects of soil-structure interaction have been taken into account by using the sub-structure approach. As the primary system response has been approximated by considering pseudo-static response in a chosen number of higher modes, those modes need not be evaluated which are well outside the frequency range of the excitation. The proposed transfer function has been then used together with a band-limited white noise ground PSDF to generate floor response spectra in case of a 15-story shear building. The numerical results show that in case of the proposed formulation, it is important to consider the dynamic response of all those modes of the flexible-base building which fall within the frequency range of excitation, and that soil-structure interaction should not be neglected in generating floor response spectra unless the soil is quite stiff relative to the superstructure. The extent of soil-structure interaction effects is found to depend significantly on the damping ratio, natural period and location of the oscillator, and on the energy distribution in the excitation process, besides the shear wave velocity of the soil medium.

APPENDIX: TRANSFER FUNCTIONS, χs

The transfer functions for the interaction acceleration, as in Equations (15) and (16), may be expressed as (Dey and Gupta, 1999)

$$\chi_{zz}^{(1)}(\omega) = \frac{\omega^2}{\Delta}\Big(L\big(Lm_T(\omega)K_{MM} - m_{HT2}(\omega)K_{VM}\big) + \omega^2\big(m_{HT1}(\omega)m_{HT2}(\omega) - m_T(\omega)I_T(\omega)\big)\Big) \quad \text{(A.1)}$$

$$\chi_{(zz)}^{(2)}(\omega) = \frac{\omega^2}{\Delta}\Big(G_p(\omega)\big(\omega^2 I_T(\omega) - L^2 K_{MM}\big) + P_p(\omega)\big(LK_{VM} - \omega^2 m_{HT1}(\omega)\big)\Big) \quad \text{(A.2)}$$

$$\chi_{\theta z}^{(1)}(\omega) = \frac{\omega^2}{\Delta}\big(m_{HT2}(\omega)K_{VV} - Lm_T(\omega)K_{VM}\big) \quad \text{(A.3)}$$

$$\chi_{(\theta z)}^{(2)}(\omega) = \frac{\omega^2}{\Delta}\Big(G_p(\omega)\big(LK_{VM} - \omega^2 m_{HT2}(\omega)\big) + P_p(\omega)\big(\omega^2 m_T(\omega) - K_{VV}\big)\Big) \quad \text{(A.4)}$$

with

$$\Delta = L^2(K_{VV}K_{MM} - K_{VM}^2) + \omega^2\Big(L\big(m_{HT1}(\omega) + m_{HT2}(\omega)\big)K_{VM} - I_T(\omega)K_{VV} - L^2 m_T(\omega)K_{MM}\Big)$$
$$+ \omega^4\big(m_T(\omega)I_T(\omega) - m_{HT1}(\omega)m_{HT2}(\omega)\big) \quad \text{(A.5)}$$

Here K_{VV}, K_{VM} and K_{MM} are the complex-valued, frequency-dependent impedance functions (with the units of force per unit length). These functions are directly proportional to G and L, where G (= $\rho\beta^2$) is the shear modulus of rigidity, ρ is the mass density, and β is the shear wave velocity of the soil medium. L denotes a suitable length of reference of the foundation. It is taken as the radius of a circular foundation of equal area in case of a rectangular slab foundation. The impedance functions also depend on hysteretic damping ratio, ζ, Poisson's ratio, ν, and aspect ratio of the rectangular foundation slab. Those are assumed to be obtained independently for the given foundation-soil system, e.g. for a surface or embedded foundation resting on visco-elastic uniform or layered half-space, or on layered soil deposit over a rigid half-space. Further,

$$G_p(\omega) = 1 + \omega^2 \sum_{j=1}^{n} M_j D_{jp}(\omega) \tag{A.6}$$

represents the transfer function for the total horizontal force acting on all masses of the fixed-base primary system (including inertia forces) when a force is applied along the *p*th primary DOF and both ground excitation and secondary system are absent, and

$$P_p(\omega) = h_p + \omega^2 \sum_{j=1}^{n} M_j h_j D_{jp}(\omega) \tag{A.7}$$

represents the transfer function for the moment of this horizontal force about the base of the primary system. The functions,

$$m_T(\omega) = m_T + \omega^2 \sum_{j=1}^{n} M_j B_j(\omega) \tag{A.8}$$

and

$$m_{HT2}(\omega) = m_{HT} + \omega^2 \sum_{j=1}^{n} M_j h_j B_j(\omega) \tag{A.9}$$

represent the transfer functions respectively for (i) the total inertia force acting on all primary system masses of the fixed-base primary system, and (ii) moment of this force about the base of the primary system, when the primary system is subjected to translational acceleration at the fixed-base and secondary system is absent. The transfer functions,

$$m_{HT1}(\omega) = m_{HT} + \omega^2 \sum_{j=1}^{n} M_j Y_j(\omega) \tag{A.10}$$

and

$$I_T(\omega) = I_T + \omega^2 \sum_{j=1}^{n} M_j h_j Y_j(\omega) \tag{A.11}$$

are $m_T(\omega)$ and $m_{HT2}(\omega)$, respectively, for the case when the primary system base undergoes rocking acceleration instead of translational acceleration. In case of mode-displacement method, it can be shown that $m_{HT1}(\omega) = m_{HT2}(\omega)$. In Equations (A.8)–(A.11), $m_T = \sum_{j=0}^{n} M_j$ represents the total mass of the primary structure-foundation system, $m_{HT} = \sum_{j=1}^{n} M_j h_j$ represents the moment of the entire primary structure-foundation system about the ground level, and $I_T = I_0 + \sum_{j=1}^{n} (I_j + M_j h_j^2)$ represents the moment of inertia of the primary structure-foundation system about a horizontal axis at the ground level. Here M_0 and I_0, respectively, represent the mass and mass moment of inertia of the foundation, and I_j represents the mass moment of inertia of the *j*th floor mass, M_j, about a horizontal axis through its mass centre.

REFERENCES

1. Borino, G. and Muscolino, G. (1986). "Mode-Superposition Methods in Dynamic Analysis of Classically and Non-classically Damped Linear Systems", Earthq. Eng. Struct. Dyn., Vol. 14, pp. 705-717.
2. Chen, G. and Soong, T.T. (1994). "Energy-Based Dynamic Analysis of Secondary Systems", J. Eng. Mech. (ASCE), Vol. 120, No. 3, pp. 514-534.
3. Clough, R.W. and Penzien, J. (1993). "Dynamics of Structures", McGraw-Hill, Inc., New York, U.S.A.

4. Crandall, S.H. and Mark, W.D. (1963). "Random Vibration in Mechanical Systems", Academic Press, Inc., New York, U.S.A.
5. Dey, A. and Gupta, V.K. (1999). "Stochastic Seismic Response of Multiply-Supported Secondary Systems in Flexible-Base Structures", Earthq. Eng. Struct. Dyn., Vol. 28, pp. 351-369.
6. Foss, K.A. (1958). "Coordinates which Uncouple the Equations of Motion of Damped Linear Dynamic Systems", J. Appl. Mech., Trans. ASME, Vol. 25(E), pp. 361-364.
7. Gupta, V.K. (1997). "Acceleration Transfer Function of Secondary Systems", J. Eng. Mech. (ASCE), Vol. 123, No. 7, pp. 678-685.
8. Gupta, V.K. (2002). "Developments in Response Spectrum-Based Stochastic Response of Structural Systems", ISET J. Earthq. Tech., Vol. 39, No. 4, pp. 347-365.
9. Gupta, V.K. and Trifunac, M.D. (1990). "Response of Multistoried Buildings to Ground Translation and Rocking during Earthquakes", J. Probabilistic Eng. Mech., Vol. 5, No. 3, pp. 138-145.
10. Igusa, T. and Der Kiureghian, A. (1985). "Generation of Floor Response Spectra Including Oscillator-Structure Interaction", Earthq. Eng. Struct. Dyn., Vol. 13, pp. 661-676.
11. Igusa, T., Der Kiureghian, A. and Sackman, J.L. (1984). "Modal Decomposition Method for Stationary Response of Non-classically Damped Systems", Earthq. Eng. Struct. Dyn., Vol. 12, pp. 121-136.
12. Itoh, T. (1973). "Damped Vibration Mode Superposition Method for Dynamic Response Analysis", Earthq. Eng. Struct. Dyn., Vol. 2, pp. 47-57.
13. Perotti, F. (1994). "Analytical and Numerical Techniques for the Dynamic Analysis of Non-classically Damped Linear Systems", Soil Dyn. Earthq. Eng., Vol. 13, pp. 197-212.
14. Rao, V.S.C. (1998). "Mode Acceleration Approach to Seismic Response of Secondary Systems", M.Tech. Thesis, Dept. Civil Eng., I.I.T. Kanpur, Kanpur.
15. Ray Chaudhuri, S. and Gupta, V.K. (2002). "Variability in Seismic Response of Secondary Systems due to Uncertain Soil Properties", Eng. Structures, Vol. 24, pp. 1601-1613.
16. Sackman, J.L. and Kelly, J.M. (1979). "Seismic Analysis of Internal Equipment and Components in Structures", Eng. Structures, Vol. 1, No. 4, pp. 179-190.
17. Sackman, J.L., Der Kiureghian, A. and Nour-Omid, B. (1983). "Dynamic Analysis of Light Equipment in Structures: Modal Properties of the Combined System", J. Eng. Mech. (ASCE), Vol. 109, No. 1, pp. 73-110.
18. Singh, M.P. (1975). "Generation of Seismic Floor Spectra", J. Eng. Mech. Div., Proc. ASCE, Vol. 101, No. EM5, pp. 593-607.
19. Singh, M.P. (1980a). "Seismic Design Input for Secondary Systems", J. Eng. Mech. Div., Proc. ASCE, Vol. 106, No. ST2, pp. 505-517.
20. Singh, M.P. (1980b). "Seismic Response by SRSS for Nonproportional Damping", J. Eng. Mech. Div., Proc. ASCE, Vol. 106, No. EM6, pp. 1405-1419.
21. Suarez, L.E. and Singh, M.P. (1987a). "Seismic Response of SDF Equipment-Structure System", J. Eng. Mech. (ASCE), Vol. 113, No. 1, pp. 16-30.
22. Suarez, L.E. and Singh, M.P. (1987b). "Floor Response Spectra with Structure-Equipment Interaction Effects by a Mode Synthesis Approach", Earthq. Eng. Struct. Dyn., Vol. 15, pp. 141-158.
23. Traill-Nash, R.W. (1981). "Modal Methods in the Dynamics of Systems with Non-classical Damping", Earthq. Eng. Struct. Dyn., Vol. 9, pp. 153-169.
24. Unruh, J.F. and Kana, D.D. (1981). "An Iterative Procedure for the Generation of Consistent Power/Response Spectrum", Nuclear Eng. Design, Vol. 66, pp. 427-435.
25. Veletsos, A.S. and Ventura, C.E. (1986). "Modal Analysis of Non-classically Damped Linear Systems", Earthq. Eng. Struct. Dyn., Vol. 14, pp. 217-243.
26. Wong, H.L. and Luco, J.E. (1978). "Tables of Impedance Functions and Input Motions for Rectangular Foundations", Report CE 78-15, Univ. Southern Calif., Los Angeles, U.S.A.