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ANALYSIS AND DESIGN OF PILE FOUNDATIONS UNDER DYNAMIC CONDITION

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ABSTRACT

Pile foundations are subjected to vibrations due to earthquakes and under machines. The nature and magnitude of vibrations is different under buildings, machines and off-shore structures.

The piles may lose contact with soil particularly under lateral vibrations

In this paper, methods of analysis for earthquake (passive) loading and machine (active) loading have been described. A comparison of predicted and measured response shows that there are considerable gaps in present day understanding of the dynamic behavior of single piles as well as piles in groups.

INTRODUCTION

A sand mass under vibrations tends to increase in density with a corresponding decrease in voids. In a mass of saturated sand below ground-water level, soils may be subjected to liquifaction resulting in increases in density. The movement of soil grains is associated with the decrease of effective stresses. If the soil is under a certain initial shear stress, the effect of vibrations is felt to a different degree (Prakash, 1981).

A pile introduces additional shear stresses in the soil mass. Excessive settlements are likely to occur under vibrations.

Agarwal (1967) and Prakash and Agarwal (1971) reported tests on vertical model piles embedded in sand at 33 percent relative density. The piles were loaded with a predetermined fraction of upward static pullout resistance. The tank containing piles was subjected to vertical vibrations at 2.3 Hz and 5.2 Hz. It was found that the number of cycles of motion needed to pull out the pile a predetermined distance of 2 cm (0.8 in) decreased with (1) an increase in the static vertical upward load and (2) the acceleration of motion. Ghumman (1985) conducted a comprehensive series of model tests on penetration testing ot piles

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under vertical vibrations. A model pile 6 cm in diameter and 160 cm long was subjected to a predetermined static load. The vertical vibrations were then imparted by a fully counterbalanced mechanical oscillator, which could be excited to different frequencies (Figure 1). A typical penetration record with time at a frequency of oscillations of 10 Hz is shown in Figure 2. A static load of 75 Kg (165 lbs) had been applied on the pile head and the dynamic force level had been varied from 45 Kg (99 lbs in Test no. 1.5) to 60 Kg (132 lbs in Test no. 1.6) and 90 Kg (198 lbs in Test no. 1.8). Both the rate of penetration and total penetration increased with increase in the dynamic force.

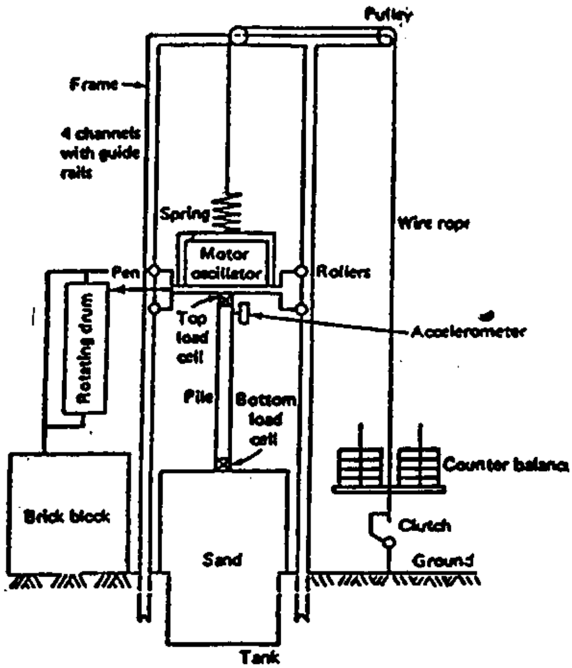


Fig. 1. A Setup for study of penetration of piles under axial vibrations (After Ghumman, 1985).

The above experimental behavior highlights the importance of vibrations in inducing settlement of piles.

Earthquakes introduce lateral force on piles. The energy supplied to a structure may be absorbed in the elastic and plastic deformations of both the substructure. Eccentric and inclined load and moments may be introduced on the pile heads and pile caps.

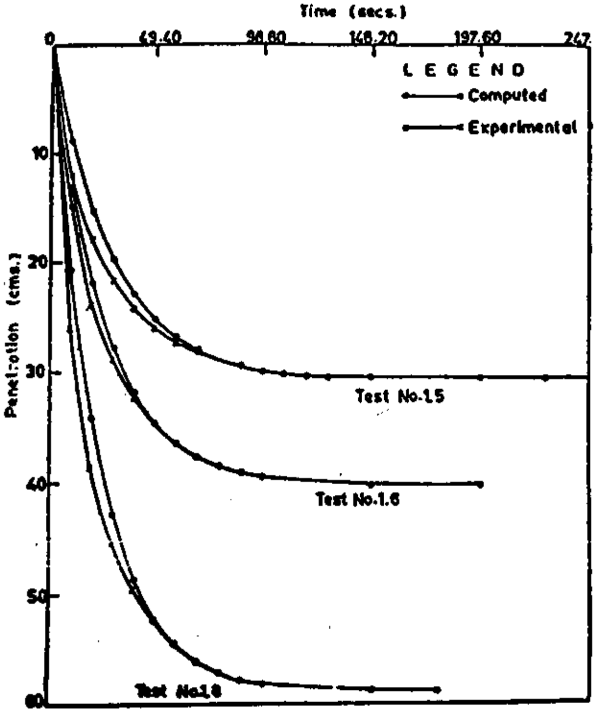


fig. 2. Penetration versus time graph for test no. 1.5, 1.6, and 1.8 (After Ghumman, 1985).

Lateral forces on the superstructure are assumed to be transferred to the ground through the pile cap as lateral loads and moments, and the stabiliiy of the piles is checked against these loads. Vertical loads are always present. These may cause buckling of the piles, particularly, if free-standing lengths are large, or they may increase the deflections. Therefore, buckling of the piles and the beam-column action become important (Prakash 1985, 1987). The pile caps of individual columns are interconnected by grade beams.

Fukuoka (1966) reported classical damage to Showa bridge in the Niigata earthquake of 1964 due to vibrations of piles and pile supported piers. This Bridge had been completed about a month before this earthquake, and had 12 composite girders. Its breadth is about 24 m, (80 ft) and its total length is about 307 m (1023.3 ft). Main span length is about 28 m, (93.3 ft) and side span length is about 15 m (50 ft). A bridge pier is composed of steel pipe, piles of 60 cm (2 ft) diameter and thickness of 16-9 mm (0.64 in—0.36 in) Figure 3. Five main spans out of ten fell down.

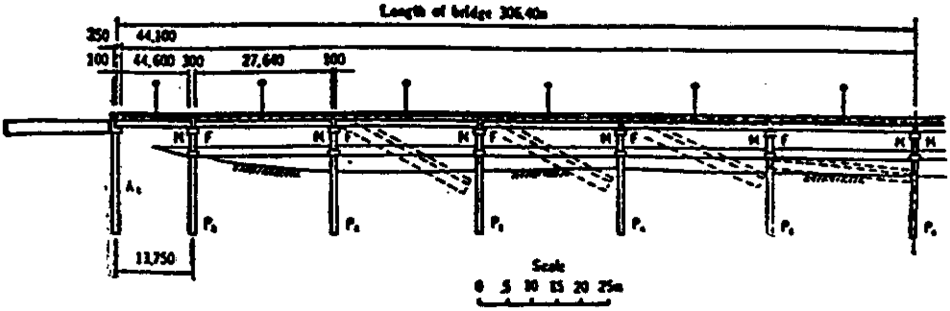


Fig. 3. Profile of Showa-Bridge Showing Damage to Deck Slabs Due to Out-of-Phase Motions of Piers (Fukuoka 1966)

A pile of pier No. 4 (P_4) was taken out after the earthquake, Figure 4 the maximum deflection of the pile at mud line is approximately 1000mm (40 in). Bending and buckling of the pile shows important soil-pile-soil interaction effects.

Piles may be used to support the foundations in (1) buildings, (2) machines and (3) offshore structures.

In buildings, the soils near the ground surface will be of poor quality necessitating the transfer of loads to deeper depths. In machine foundations in addition to the above consideration, it may be necessary to increase the natural frequency of foundation soil system and decrease their amplitude. In off-shore structures, piles may be of very large lengths (1000 ft or so) always with considerable free standing lengths.

The introduction of piles alters the elastic coefficients of the soil-pile system. Both the natural frequency and the amplitudes of motion are affected. In all vibration problems, resonace needs to be avoided. Hence, the natural frequency of the soil-pile system is necessarily evaluated.

In the following sections, methods for determination of the natural frequency of the pile-soil system, dynamic analysis, and the design of piles against earthquakes and under machine foundations have been discussed.

PILES UNDER VERTICAL VIBRATIONS

Barken (1962) proposed determination of soil pile stiffness from a

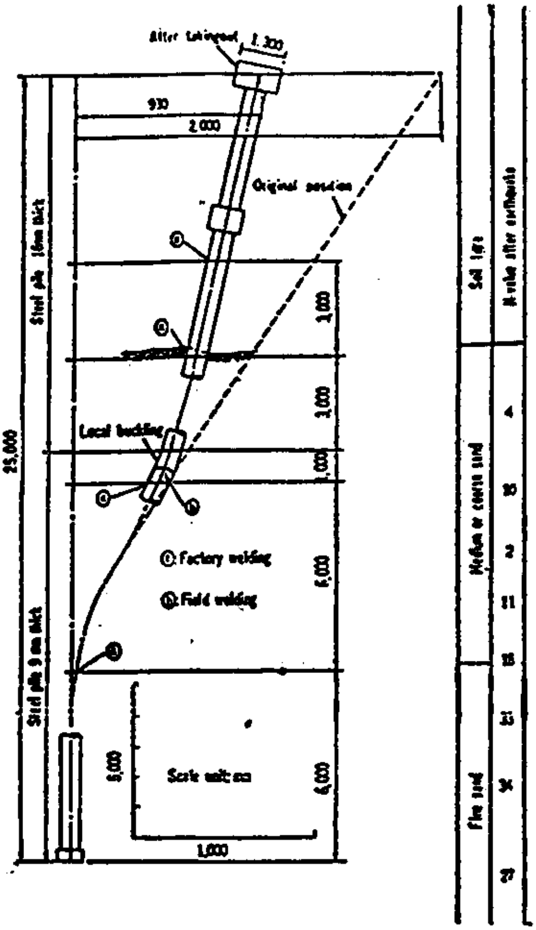


Fig. 4. Pipe Pier No. 4, Taken Out From the Ground after Niigata Earthquake (Fukuoka 1966).

cyclic vertical pile load test similar to a cyclic plast load test (Prakash and Puri 1988). A plot of load 'P' and elastic settlment 'Z_1' may be represented by a straight line up to the working load in many situations. The constant of proportionality (k), **the coefficient of elastic resistance of the pile** is then :

$$P = KZ_1 \tag{1}$$

K is the load required to induce a unit elastic settlement of the pile.

The natural frequency (f_{nz}) of the pile in vertical vibration is then given by;

$$f_{nz}=\frac{1}{2\pi}\sqrt{\frac{K}{m}} \tag{2}$$

where m = static load on the pile.

Based upon the above simple concepts, Barkan (1962) described test data and typical values of elatic constants of piles and pile groups under both vertical and horizontal vibrtations. This analysis does not consider damping in the system and the dynamics of the problem.

The coefficient K in equation 1 depends upon the soil and pile properties and their geometry. Also K will have different values for machine foundation problem and for earthquake loading ie it is strain (or displacement) dependent. No simple and direct relationship between stratins in the soil along a pile, particularly in horizontal vibrations and soil deformations around the pile are available. Since elastic soil constant, E, G and k are strain or displacement dependent, therefore, the values of the elastic constants (k) determined from a lateral deflection of the order of 3.4 mm in Berkan's test are not applicable to machine foundation problems (Prakash and Sharma 1989).

END BEARING PILES

If piles are driven in soft soil and are embedded in sound rock or a hard stratum at their tip, the piles may be considered as end bearing piles. Deformations of pile tip will not occur when dynamic loads are transferred to the pile. The pile may then be considered as an elastic rod fixed at its tip (base) and free at the top, with a mass (m) resting on the top Figure 5. If the weight of the pile is negligible as compared to the supported mass, the natural frequency may be obtained by Equation (Prakash and Puri 1988).

$$\frac{A l r}{W}=\frac{\omega_n l}{V_r}\tan\frac{\omega_n l}{V_r} \tag{3}$$

where Alr = weight of rod and

W weight of added mass

V_r = velocity of longitudinal wave propogation in rod (ft/sec)

l = length of rod (ft)

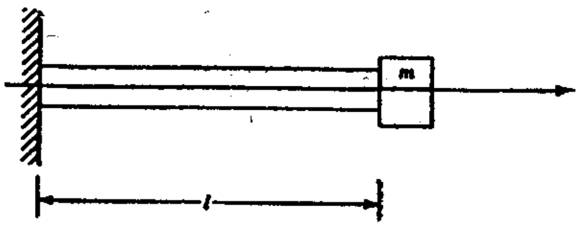


Fig. 5. Fixed-free rod pile with mass attached to free end.

Richart (1932) developed a plot of the natural frequency of end bearing piles, considering the effect of axial load, pile length, and pile material, Figure 6.

Friction Piles

In floating piles unlike end-bearing piles, load is transferred from the shaft to the soil and their analysis under vertical vibrations is quite different than that for end-bearing piles. Some of the methods employed to determine the response of floating piles to vertical loads are :

(1) A three-dimensional analysis (e. g., using the finite element method) considering the propagation of waves through the the pile and soil.

(2) Solution of the one-dimensional wave equation, for example, in a manner similar to the solution of this equation to analyze the pile-driving process.

(3) An analysis of the resp(nse of a lumped mess-spring-dashpot system representing the pile and soil.

(4) An approximate elastic analysis in which a simple problem of plane strain is considered. It is assumed that the elastic waves propagate only horizontally.

A three dimensional analysis to too expensive and involved for every day use. For pile supported turbo-generator foundations in nuclear power plants where tolerance limits are very critical, such methods are however jn use. Solution of one-dimensional wave equations, involving extension of the numerical method of pile response under vertical vibrations (Poulos and Davis, 1980). A single degree of freedom lumped-mass-spring-dashpot system has been used for solution of vertical vibrations of piles by Barkan (1962) and Maxwell et al. (1969). Madhav and Rao (1971) used a two degree-of-freedom model.

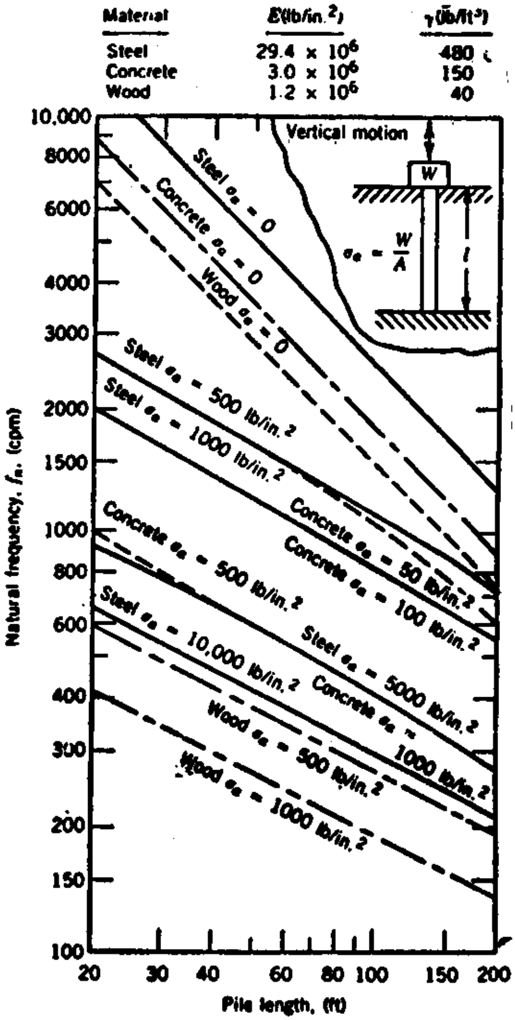


Fig. 6. Resonant frequency of vertical oscillation for a point-bearing pile resting on a rigid stratum and carrying a static load W. (After Richart, 1962)

The fourth approach has been used by Novak (1974; 1977) and Sheta and Novak (1982) to obtain an approximate solution for pile response to vertical loading. The soil has been assumed as composed of a set of independent infinitesimally-thin horizontal layers of infinite extent. This model could be thought of as a generalized Winkler material that possesses inertia and dissipates energy. By applying small harmonic excitations, Novak derived solutions for the equivalent stiffness and damping constants of the pile-soil system. This model predicts response of vertically vibrating piles better than that of Maxwell **et al.** (1969).

Maxwell's Lumped-mass spring-dashpot model :

The vibrating pile is shown in Figure 7a and its single-degree-of-freedom model is shown in Fig. 7b. With appropriate values of the mass, damping, and spring constant selected for the system, the response can be determined from solutions of elementary theory of mechanical vibrations.

The solution for such a system is given by Prakash and Puri (1988).

$$A_z = \frac{Q_o}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \quad (4)$$

Where Q_o = magnitude of exciting force

m = equivalent mass of the system

c = damping coefficient

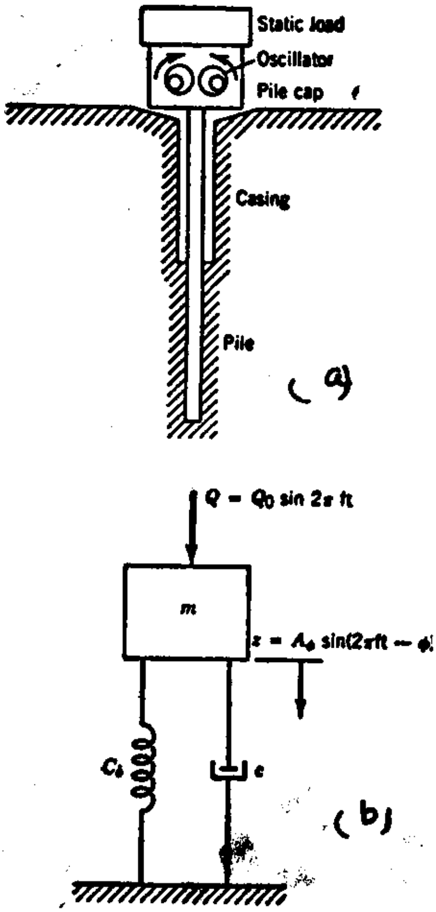
A_z = maximum amplitude

ω = frequency of oscillations.

This solution differs from Barkan's solution since Maxwell **et al.** (1969) considered damping in the system.

In this model, the equivalent mass m has been considered as the mass of the oscillator, the pile cap, and the static load above the ground. Tests were performed on steel H-piles and concrete-filled pipe piles, in silty sand and clay overlying sand. The values of equivalent stiffness K and damping ratio ξ had been back-calculated from the test results. At resonance, the dynamic value of k_n was found to be greater than the static stiffness for comparable piles (Figure 8).

The computed damping ratio ξ for single piles was of the order of 0.00 to 0.04. A singnificant finding was that both the stiffness and the damping ratio varied with frequency. In particular, the response at resonance was not reliably predicted from data on stiffness and damping computed at non-resonant frequencies. The variation of stiffness, expressed in terms of a stiffness ratio k/k_n (where k_n = stiffness at resonant frequency), and damping ratio ξ with frequency ration f/f_n, for pipe pile D-1 are plotted in Figure 8.



c = damping coefficient
C_s = effective spring constant
m = equivalent mass of system
Q = periodic exciting force
Q_0 = magnitude of exciting force
t = time
z = periodic displacement
A_ϕ = amplitude of displacement
ϕ = phase angle between Q_0 and z
f = frequency (Hz)

Fig. 7. Analytical model for floating pile (Maxwell et al., 1969).

The contact of cap is an importact factor which may affect both the natural frequency and amplitude of vibrations of the system in all modes of vibrations. In this case typical test results (1) with the cap contact with the soil, and (2) after excavating beneath the cap, showed that the dynamic displacements of the pile cap were approximately

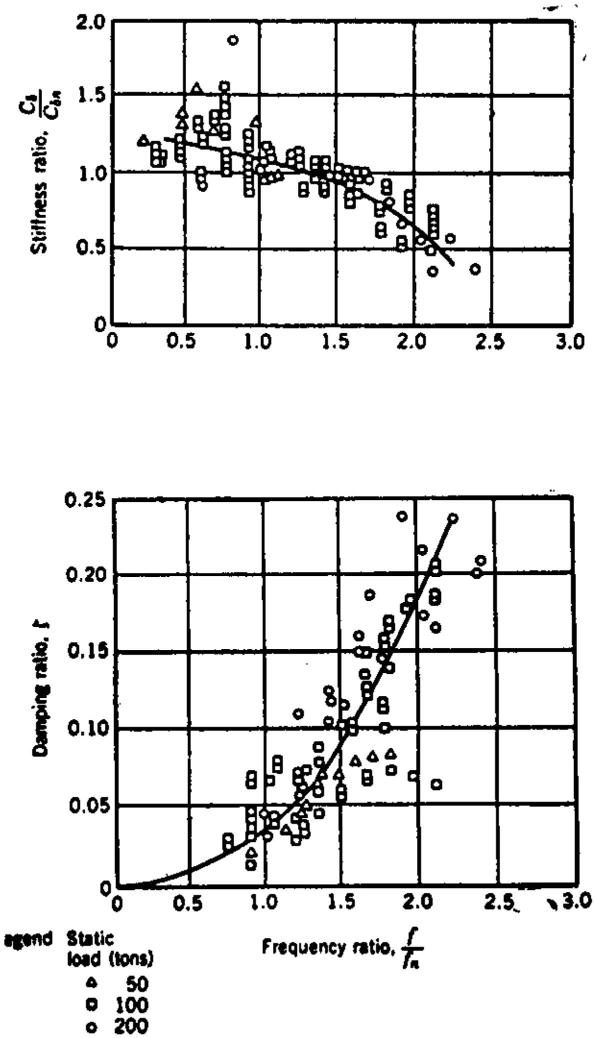


Fig. 8. Stiffness and damping ratio vs. frequency ratio, pipe pile D-1 (Maxwell et al., 1969).

0.0385 in. and 0.145 in. respectively with a constant force excitation of 4T. Since the stiffness of a pile foundation is generally greater than that of a corresponding surface foundaiion, the natural frequency of the foundatian-soil sestem will be increased by the use of piles.

PILES UNDER LATERAL VIBRATIONS

The response of a single pile subjected to a time-dependent horizontal force and moment has been studied by several methods, including the following :

(1) The pile is considered to be an equivalont cantilever and the effect of the soil is neglected.

Hayashi (1973), Prakash and Sharma (1969), and Prakash and Gupta (1970) determined the natural frequencies of the soil pile system in this manner. The soil-pile system is idealized as a massless equivalent cantilever with a single concentrated mass at the top. Its natural frequedcy is determined by using Rayleigh's method. The exciting frequency is used to check the freqency of the system for resonance. This is not a realistic approach and no frequency dependence on the vibration parameter and damping are considered. Also, noin formation can be obtained on the moments, stresses, and displacements along the length of the pile for dynamic loads.

2. The pile is considered as a beam on an elastic foundation subjected to time-dependent loading and analyzed by finite differences. Moments, stresses, and displacements along the length of the pile may be analyzed, and impact loads as well as harmonic loads can be considered (Tucker, 1964).

3. The approximate analytical technique developed by Novak (1974) derives stiffness and damping constants for piles and pile groups, with the help of which lateral response is determined.

4. The fourth approach is in which the soil-pile system has been modeled by a set of discrete (lumed) masses, springs and dashpots. This approch can be used to incorporate the depth and nonlinearity variations of the soil properties which depend upon the definition of the local soil stiffness and geometric damping (Penzien et al., 1964; Penzien, 1970; Prakash and Chandrasekaran 1980). This analysis is based on the following assumptions :

1. The pile is divided into a convenient number of segments and mass of each segment is concentrated at its center.
2. The soil is considered as a linear Winkler's spring. The soil modulus variation is considered both linearly varying with depth and contant with depth.
3. A fraction of the mass of the superstructure is concentrated at the pile top as M_t.
4. The system is one-dimensional.

5. The pile top conditions are either (a) completely free to undergo translation and rotation (f) or (b) completely restrained against rotation but free to undergo translation (ft). Partial fixity at the top can be solved by interpolation. Pile tip is free.

For determination of the free-vibration characteristics, model analysis was performed by using successive approximations of the natural frequencies of the system with an initially assumed value and related end conditions. The assumed end conditions are also utilized to generate the transfer equations and to evaluate the unknown quantities in terms of the known quantities, either at the pile top or the pile tip. These modal quantity values at different station points define the mode shapes. Values at the bottom or top of the piles assist in determining the natural frequencies of vibrations in different modes. For details refer to Chandrasekaran, (1974).

The soil stiffness has been defined by a modulus of horizontal reaction k_h (FL^{-2}), which may be either constant with depth or may vary linearly with depth. In both of these cases, solutions have been obtained for (1) natural frequency, (2) modal displacements (3) slopes (4) bending moments (5) shear forccs, and (6) soil reactions along the lengths of the piles in the f rst three modes of vibrations (Chandrasekaran, 1974; Prakash and Chandrasekaran, 1980). Based on the above analysis, nondimensional frequency factors have been obtained with respect to the basic soil parameters.

The variables constituting F_{cl}, the nondimensional frequency factor for piles embedded in soils in which the soil modulus remains constant with depth, is given by :

$$\omega_{n1} = F_{CLI} \text{ (or } F_{CLI}'\text{)} \div \sqrt{\frac{W}{gKR}} \tag{5}$$

where ω_{n1} = the first natural angular frequency in radians per second

$\frac{W}{g}$ = lumped mass at the top of the pile

K = soil modulus and

R = relative stiffness factor, defined as follows :

$$R = \sqrt[4]{\frac{EI}{K}} \tag{6}$$

In Figure 9 the variation of frequency factor F'_{CL} or F'_{CL} with Z_{max} has been plotted, in which $Z_{max} = L/R$. F_{CL} and F'_{CL} refer to cases with pile top

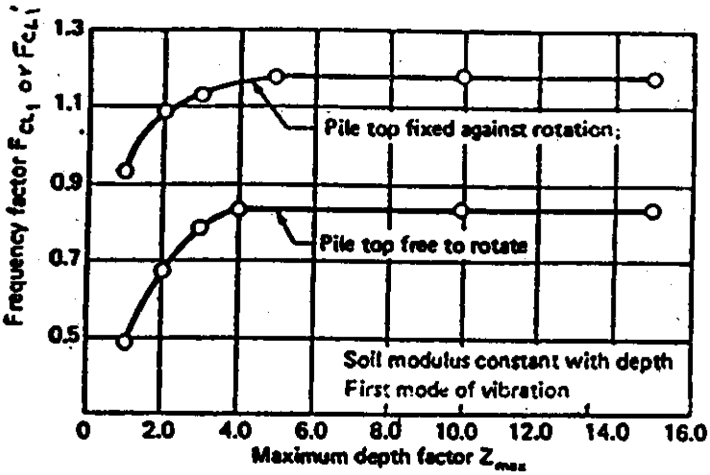


Fig. 9. Nondimensional frequency factors in first mode of vibrations. Soil modulus constant with depth. (After Prakash and Chandrasekaran, 1977).

free to rotate and pile top restrained against rotation, respectively. It may be seen that for a given soil pile characteristics, such a flexural stiffness EI, soil stiffness in terms of k, sustained vertical load, W and Z_{max} unique frequency factor values exist. Similarly in Figure 10 and b, frequency factors F_{SL1} and F'_{SL1} for soils whose moduli vary linearly with depth have also been plotted for the pile top free to rotate and the pile top restrained against rotation, respectively.

The definition of F_{SL1} and F'_{SL1} for the pile tops free to rotate and the pile top restrained against rotation are identical and given by :

$$\omega_{n1} = F_{SL1} \text{ (or } F'_{SL1} \text{)} \div \sqrt{\frac{\omega}{g} \frac{I}{n_h T^3}} \quad (7)$$

in which n_h is the constant of horizontal subgrade reaction of $7_x = n_h x$ and

$$T = \sqrt[5]{\frac{EI}{n_h}} \quad (8)$$

It will be seen from Figures 9 and 10 that the natural frequency attains a constant value for $Z_{max} \geq 4.5$ in all cases. Therefore, piles with embedded depths $(Ls) \geq 5T$ behave as "long" piles as under static loading. Similar frequency factors and mode shapes parameters for determining natural frequencies and mode shapes in the second and third modes of vibrations have been p'otted by Chandrasekaran (1974).

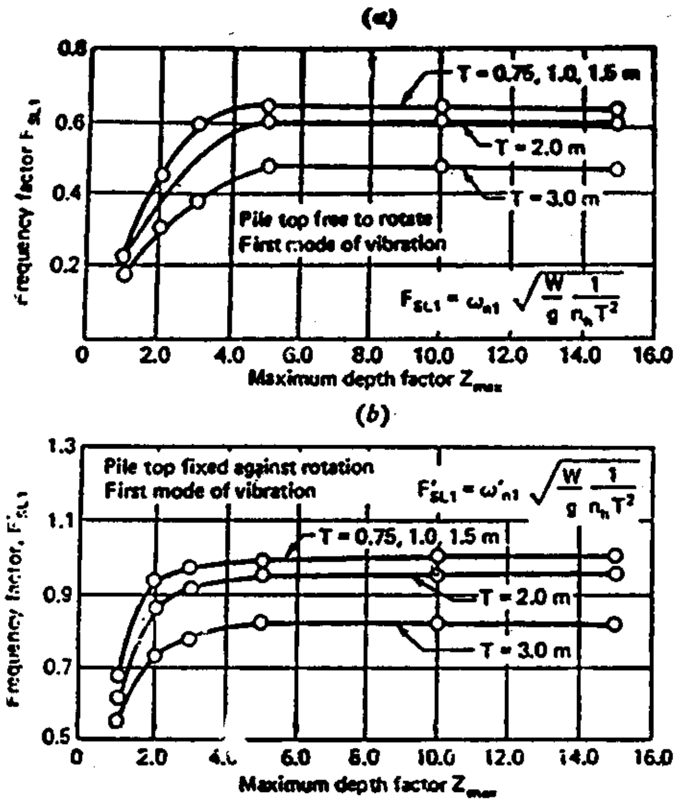


Fig 10. Nondimensional frequency factors in first mode of vibrations. (a) Soil modulus linearly varying with depth and pile top free. (b) Soil modulus linearly varying with depth and pile top restrained against rotation. (After Prakash and Chandrasekaran, 1977).

From the mode shapes and frequencies of the system, the overall response can be computed by principle of modes superposition.

ASEISMIC DESIGN OF PILES

Based upon the above analysis and the concept of the spectral response technique, the following method of analysis and design of piles against earthquakes may be used (Prakash 1981).

For this analysis, the following data must be obtained first :

1. Soil characteristics, and bore log of soils
2. Pile characteristics, size, EI, length, type of pile

3. Lateral load-deflection of the pile under static conditions for estimation of 'k' or n_h.

   The steps in design are :

DESIGN STEPS

1. Estimate the dynamic soil modulus k or n_h. In the absence of realistic data, the values from a static lateral load test may be modified based upon engineering judgement.
2. Compute the relative stiffness factor R or T.
3. Calulate the maximum depth factor Z_{max} for a pile; Z_{max} in most practical cases will be greater than 5.
4. For the computed value of the maximum depth factor and the pile end condition, read the frequency factor (Figs. 9 or 10).
5. Estimate the dead load on the pile. The mass at the pile top is only a fraction of this load.
6. Determine the natural frequency ω_{n1} in first mode of vibrations by Equations 5 or 7 as the case may be. Then

$$T_{n1} = 2\pi/\omega_{n1} \tag{9}$$

7. For the above time period, determine the spectral displacement S_d for assumed damping from a spectral displacement curve. If an accelogram for a site has been selected, spectral response is determined for this ground motion. For the soil-pile system, 5-10 percent damping may be assumed This is the maximum displacement of the pile head.
8. Estimate the maximum bending moment in the pile section (Prakash 1981, Prakash and Sharma 1989).

   (a) Soil modulus constant with depth.

$$\text{Bending moment} = A_{me} \times KR^2 \times S_d \tag{10}$$

   maximum values of A_{me} are given in Table 1

   (b) Soil modulus increasing linearly with depth.

$$\text{Bending moment} = B_{me} \times n_h T^3 + S_d$$

   The maximum values of B_{me} are given in Table 2. The pile section should be able to stand the above moments.
9. For the maximum ground displacement computed above, the the displacement all along the length of the pile may be determined by assuming that the deflected shape in vibrations is

TABLE 1 MAXIMUM VALUES OF COEFFICIENT A_{me}*

Maximum depth factor Z_{max}	COEFFICIENT A_{me}		
	pile top free to rotate	Pile fixed at top against rotation	
		— ive	+ ive
2	0.13	0.9	0
3	0.24	0.9	0.04
5-15	0.32	0.9	0.18

*After Chandrasekaran (1974)

TABLE 2 MAXIMUM VALUES OF COEFFICIENT B_{me}*

Maximum depth factor Z_{max}	COEFFICIENT B_{me}		
	pile top free to rotate	Pile fixed at top against rotation	
		— ive	+ ive
2	0.100	0.93	0
3	0.255	0.93	0.10
5-15	0.315	0.90	0.28

*After Chandrasekaran [1974]

similar to one under static load. For (1) soil modulus constant with depth and (2) or soil modulus linearly varying with depth, the solutions of Davisson and Gill (1963) and Reese and Matlock (1956) respectively may be used for the two cases of soil modulus. The soil reaction is then computed all along the pile lengths as :

(a) For soil modulus constant with depth.

$$P_x = k\, y_x \qquad (12a)$$

(b) For soil modulus linearly varying with depth :

$$P_x = n_k . x . y_x \tag{12b}$$

The allowable soil reaction may be taken as that corresponding to the Rankine passive pressure at all depths (Terzaghi and Peck, 1967 Prakash et al.. 1979).

The deflections, bending moments, and soil reactions under static loading are added to the corresponding values under dynamic loading to arrive at the final values. For this analysis, the soil modulus values recommended in Table 3 and modified for appropriate dynamic conditions may be used.

TABLE 3a
ESTIMATED VALUES FOR k_h*

Soil Type	Value
Granular soils	n_h ranges from 1.5 to 200 lb/in³, is generally in the range from 10 to 100 lb/in³, and is approximately proportional to relative density
Normally loaded	n_h ranges from 1.4 to 3.0 lb/in³
organic silt	n_h is approximately 0.2 lb/in³
Peat	k_h is approximately 67 S_u, where
Cohesive soils	S_u is the undrained shear strength of the soil

Note: The effects of group action and repeated loading are not included in these estimates.

* After Davisson 1970.

TABLE 3b

RECOMMENDED VALUES OF n_h FOR SUBMERGED SAND

1. TERZAGHI (1955)

Relative Density	Loose	Medium	Dense
Range of Values of n_h (lbs/in³)	2.6—7.7	7.7—26	26—51

2. REESE et al (1974)

(Static and Cyclic Loading)

Relative Density	Losse	Medium	Dense
Recommended n_h (1b/in³)	20	60	125

GROUP ACTION

The value of k needs to be corrected for group action. The following guidelines are recommended (Praksh 1981, Prakash and Sharma 1989).

1. In case, the center-to-center spacing of piles is 8 d in the direction of loading where d is the diameter of the pile, and the center-to-center spacing is at least 2.5 d in the direction perpendicular to the load, group action is neglected. The piles may be arranged to behave as **individual** piles. If the spacing in the direction of the load is 3 d, the effective value of k (k_{eff}) is 0.25 k. For other spacings , a linear interpolation may be made. This recommendation is based on model tests on piles in sands under static loads (Prakash 1962)
2. If a cyclic load is applied, the deflections increase and k_{eff} decreases. It has been observed that the deflections after 50 cycles of load application are double the deflections under the first cycle (Prakash, 1962). The soil modulus decreases to 0.30 times and 0.4 times for soils with linearly increasing and constant modulus with depth respectively.
3. If the load is applied in an oscillatory manner, the deflections increase about seven times that under the first cycle of loading (Prakash and Sharma, 1969). The soil modulus decreases to a larger extent in this case.

If group action and oscillatory loads are considered, the soil modulus is decreased on two counts, and the final value may be less than 10 percent of k for a single pile.

These recommeddations may be regarded as tentative. When more data become available, these recommendations may need to be revised.

NOVAK'S DYNAMIC ANALYSIS OF PILES

Novak (1974) developed soil-pile analyses for (1) vertical (2) lateral and rocking and (3) torsional vibrations. In these procedures, the soil pile stiffness and damping have been evaluatad for the system. A complete dynamic analysis can then be performed.

Analysis of Piles Under Vertical Vibrations

The main assumption in Novak's analysis are (Novak, 1974, 1977)

1. The pile is vertical, and of circular cross section;
2. The pile material is linearly elastic.
3. The pile is perfectly connected to the soil ie there is no separation between soil and pile under vibrations.
4. The pile is a floating pile.
5. The soil above the tip is modelled as a linearly elastic layer composed of infinitesimally thin independent layers, which means that the elstic waves propogate only horizontally. The soil reaction acting on the tip is assumed to be equal to that of an elastic halfspace.
6. The motion is small and the excitation is harmonic, which yields the impedance functions and the equivalent stiffness and damping constants of the soil-pile system that can be used in structural analysis.

The stiffness constant K_w of one pile has been expressed as

$$K_w^{l} = \frac{E_p A}{r_o} f_{w_1} \tag{13}$$

where

$$f_{w_1} = \frac{F_w(\Lambda)_1}{\frac{l}{r_o}} \tag{13a}$$

The constant of equivalent viscous damping may be written as :

$$C_w^{l} = \frac{E_p A}{V_s} f_{w_2} \tag{14}$$

where

$$f_{w_2} = \frac{F_w(\Lambda)_2}{a_o \frac{l}{r_o}} \tag{14b}$$

where E_p = Young's modulus of pile material
- A = area of pile
- r_o = radius of pile
- V_s = shear wave velocity in soil around the piles

The stiffness and damping of piles vary with frequency, as shown in Figure 11. In this figure, parameter f_{w1} characterizes stiffness, and parameter f_{w2} characterizes damping. These parameters have been plotted for a few typical cases (Novak 1977). This figure shows that :

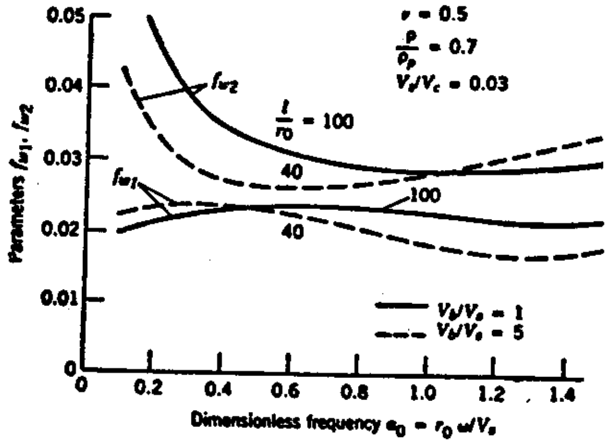


Fig. 11. Variations of stiffness and damping parameters of pile with frequency. (After Novak, 1977).

1. The dynamic stiffness of the soil-pile system varies only moderately with frequency both for slender as well as rigid piles.

2. The damping decrases rapidly with incressing frequency but levels off in the range of moderate frequencies.

Since stiffness and damping do not depend much on frequency, Novak (1977) has recommended parameters f_{w1} and f_{w2} for design purposes which are independent of frequency.

Figure 12 shows the variation of the stiffness and damping parameters of the pile with the shear wave velocity ratio, V_b/V_s, of the soil below and above the pile tip. The slenderness ratios (l/r_o) used in this plot vary from 10-100, and $V_s/V_c=0.03$. It is seen from this figure that :

(1) With increasing stiffness of the soil below the tip, the stiffness of the pile increases while the damping decreases,

(2) with increasing length, the stiffness of end bearing piles decreases while the stiffness of floating piles increases, and

(3) damping increases with pile length in most cases.

In Figure 13, stiffness and damping parameters have been plotted against slenderness ratio (l/r_o) for floating as well as end bearing piles.

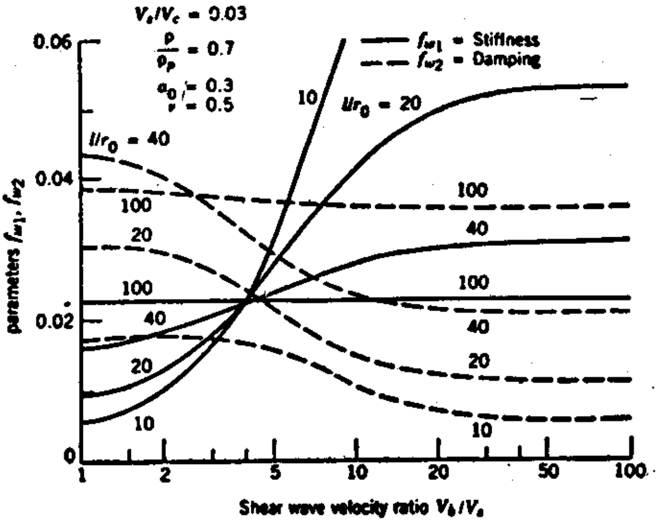


Fig. 12. Variation of stiffness and damping parameters of pile with ratio of shear wave velocities of soil below and above tip. (After Novak, 1977).

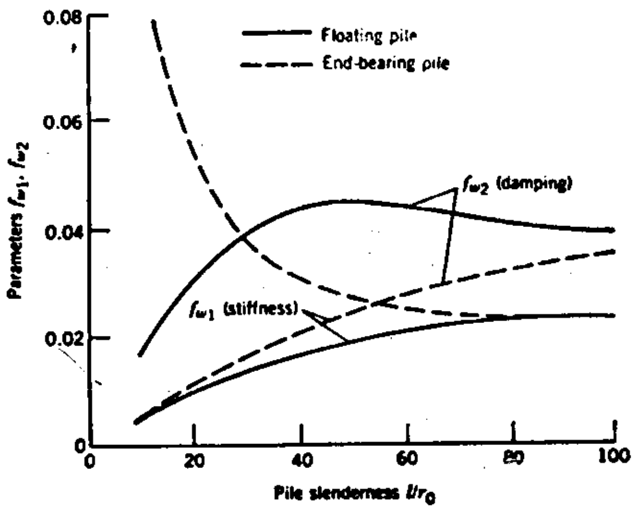


Fig. 13 Comparison of floating piles with end bearing piles ($\rho/\rho_s=0.7, \nu=0.5$., $a_0=0.3$, $v_s/v_c=0.03$) After Novak, 1977).

For design of both end bearing and floating piles, the constants f_{w1} and f_{w2} in Equations 13a and 14a had been solved by Novak (1974, 1977), for soil modulus constant with depth. Novak and EL-Sharnouby

(1983) included solutions for shear modulus decreasing upwards in a quadratic manner. In Figure 14 stiffness and damping parameters have been plotted for fixed tip piles for two cases. Similar solutions for floating piles are presented in Figure 15.

LATERAL VIBRATIONS

Novak (1974) had derived lateral stiffness and damping constants for single piles with soil modulus constant with depth. He considered

(1) translation alone,

(2) rotation alone, and

(3) coupled rotation and translation. Novak and El-Sharnouby (1983) extended these solutions to include parabolic variation of soil-shear modulus also. Equations (7.51)-(7.56) summarize the stiffness and damping coefficients and Table 7.3 lists values of constants used.

Translation stiffness constant,

$$k_x^l = \frac{E_p I_p}{r_0^3}(f_{x1}) \tag{15}$$

Translation damping constant,

$$c_x^l = \frac{E_p I_p}{r_0^2 V_s}(f_{x2}) \tag{15a}$$

Rotation stiffness constant.

$$k_\phi^l = \frac{E_p I_p}{r_0}(f_{\phi 1}) \tag{16a}$$

Rotation damping constant,

$$c_\phi^l = \frac{E_p I_p}{V_s}(f_{\varphi 2}) \tag{16b}$$

Cross-stiffness constant,

$$k_{x\phi}^l = \frac{E_p I_p}{r_0^2} f(x\phi_1) \tag{17a}$$

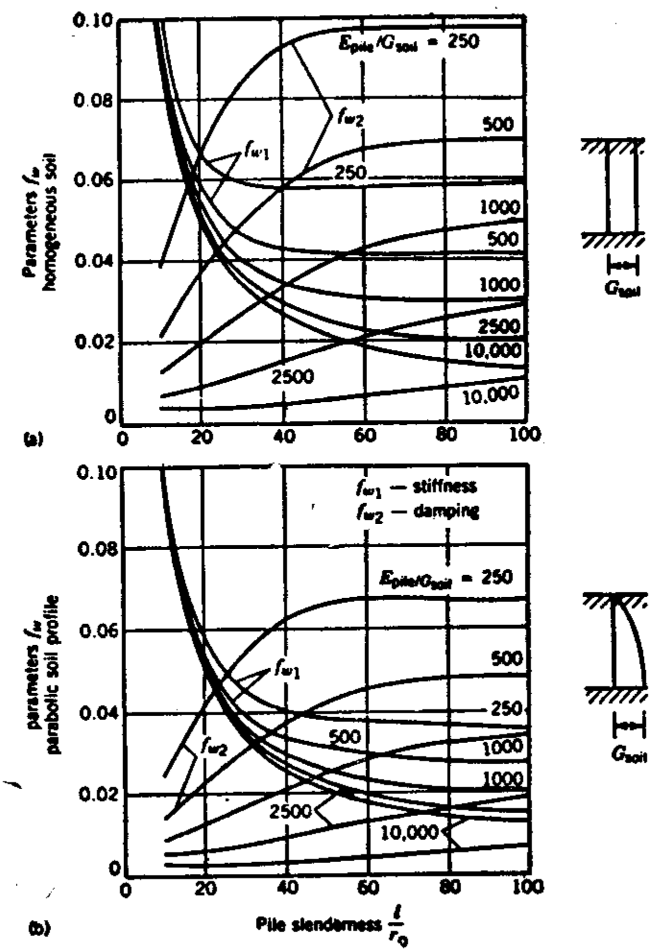


Fig. 14 Stiffness and Damping Factors for Fixed-Tip Vertically Vibrating piles (After Novak and E1 Sharnouby, 1983).

Cross-damping constant,

$$c^{l}_{x\phi} = \frac{E_p I_p}{r_0 V_s} f(x\phi_2) \tag{17b}$$

in which

I_p = moment of inertia of pile cross-section

E_p = Young's moduls of pile

V_s = shear wave velocity in soil

V_c = longitudinal wave velocity in pile

r_o = pile diameter

f = constants in Table 3.

It was found, as in case of vertical vibrations, that the frequency dependence of stiffness and damping can generally be ignored, and that the important parameters are 1) the ratio of shear wave velocities in the pile and soil, and 2) the slenderness ration l/r_o.

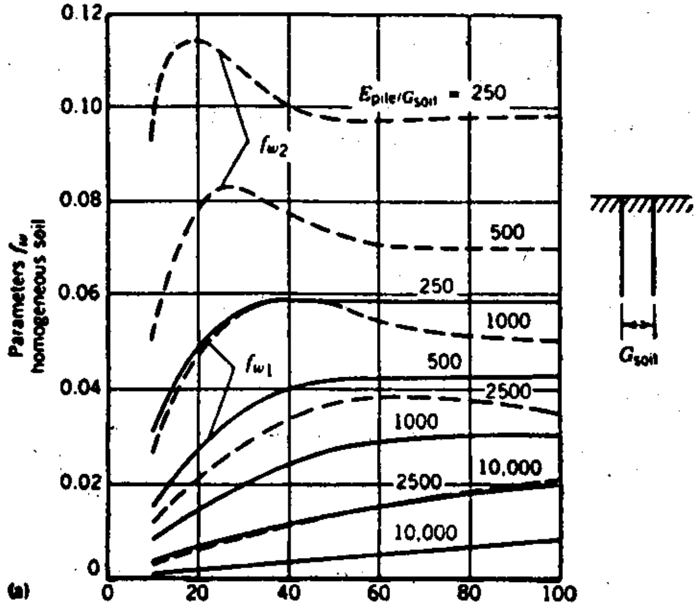
Also in Table 3, coefficients have been included for both pin-headed and fixed-translating headed piles. For pin-headed pile f_{x1} gives translation stiffness and $f\phi_1 = 0$ (i. e. $k'\phi_1 = 0$). The stiffness and damping of pin-headed piles are much leas than for fixed (translating) head piles.

TABLE 3 —Stiffness and Damping Parameters of Horizontal Response for Piles with $\ell/R > 25$ for Homogeneous Soil Profile and $\ell/R > 30$ for Parabolic Soil Profile

ν	E_{pile}/G_{soil}	Stiffness Parameters				Damping Parameters			
		$(f_{\phi 1})$	$f(x\phi_1)$	(f_{x1})	(f_{x1}^P)	$(f_{\phi 2})$	$f(x\phi_2)$	(f_{x2})	(f_{x2}^P)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				(a) Homogeneous Soil Profile					
0.25	10,000	0.2135	−0.0217	0.0042	0.0021	0.1577	−0.0333	0.0107	0.0054
	2,500	0.2998	−0.0429	0.0119	0.0061	0.2152	−0.0646	0.0297	0.0154
	1,000	0.3741	−0.0668	0.0236	0.0123	0.2598	−0.0985	0.0579	0.0306
	500	0.4411	−0.0929	0.0395	0.0210	0.2953	−0.1337	0.0953	0.0514
	250	0.5186	−0.1281	0.0659	0.0358	0.3299	−0.1786	0.1556	0.0864
0.40	10,000	0.2207	−0.0232	0.0047	0.0024	0.1634	−0.0358	0.0119	0.0060
	2,500	0.3097	−0.0459	0.0132	0.0068	0.2224	−0.0692	0.0329	0.0171
	1,000	0.3860	−0.0714	0.0261	0.0136	0.2677	−0.1052	0.0641	0.0339
	500	0.4547	−0.0991	0.0436	0.0231	0.3034	−0.1425	0.1054	0.0570
	250	0.5336	−0.1365	0.0726	0.0394	0.3377	−0.1896	0.1717	0.0957
				(b) Parabolic Soil Profile					
0.25	10,000	0.1800	−0.0144	0.0019	0.0008	0.1450	−0.0252	0.0060	0.0023
	2,500	0.2452	−0.0267	0.0047	0.0020	0.2025	−0.0484	0.0159	0.0076
	1,000	0.3000	−0.0400	0.0086	0.0037	0.2499	−0.0737	0.0303	0.0147
	500	0.3489	−0.0543	0.0136	0.0059	0.2910	−0.1008	0.0491	0.0241
	250	0.4049	−0.0734	0.0215	0.0094	0.3361	−0.1370	0.0793	0.0398
0.40	10,000	0.1857	−0.0153	0.0020	0.0009	0.1508	−0.0271	0.0067	0.0031
	2,500	0.2529	−0.0284	0.0051	0.0022	0.2101	−0.0519	0.0177	0.0084
	1,000	0.3094	−0.0426	0.0094	0.0041	0.2589	−0.0790	0.0336	0.0163
	500	0.3596	−0.0577	0.0149	0.0065	0.3009	−0.1079	0.0544	0.0269
	250	0.4170	−0.0780	0.0236	0.0103	0.3468	−0.1461	0.0880	0.0443

Source - Novak and El-Sharnouby (1983)

The soils very near the surface control the load deformation properties of the pile. In addition, a gap may be formed behind a pile under lateral vibrations (Figure 16 (Prakash and Sharma 1989). There-the value of G or V_m to be used for such a case is smaller than the value used for vertical analysis. This holds both for static as well as dynamic analysis.



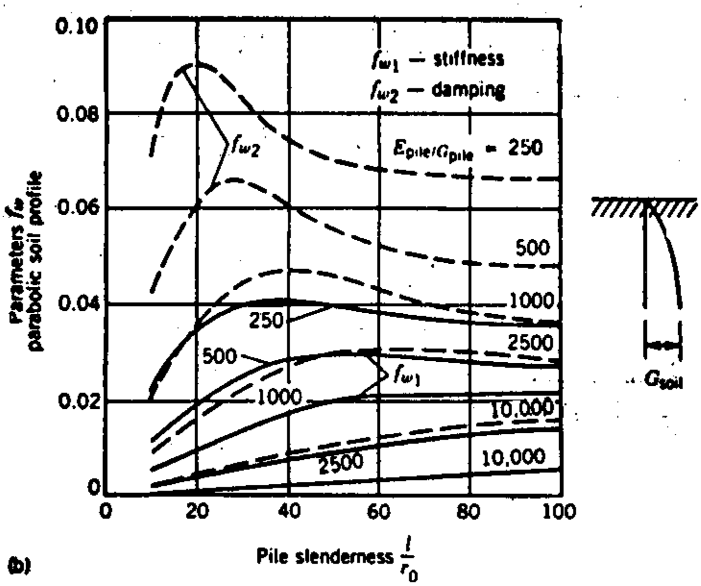


Fig. 15 Stiffness and damping parameters of vertical response of floating piles (After Novak and E1 Sharnouby, 1983).

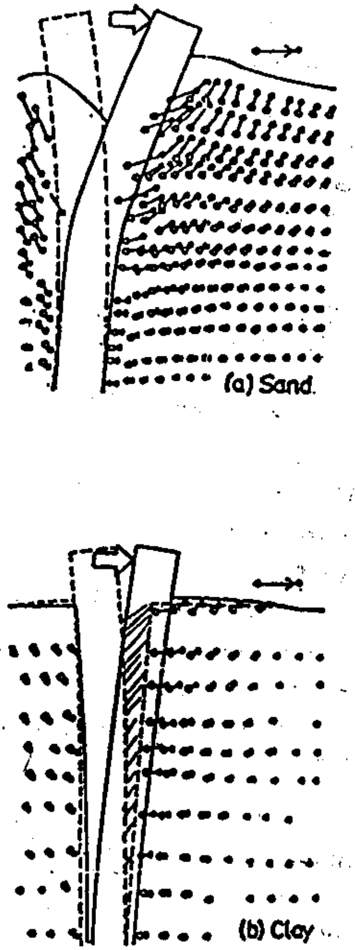


Fig. 16 Movements of Soils (a) Sand (b) Clay (After Kishida et al 1985)

TORSIONAL VIBRATIONS

Novak and Howell (1977) developed an analysis for torsional vibrations of piles. The main assumptions in this analysis are :

(1) The pile has a circular cross-section. It is vertical and elastic It is perfectly connected to the soil ;

(2) The pile is end bearing.

(3) The soil is modeled as a linear viscoelastic medium with frequency independent material damping of the hysteretic type ;

(4) The soil reaction per unit length of the pile is assumed to be equal to that derived for plane strain conditions, i.e. for uniform rotation of an infinitely long pile; and

(5) The excitation is harmonic and the motion of the pile is small. Stiffness and damping constants K^1_Ψ and C^1_Ψ for fixed tip single piles are given by, as for vertical vibrations

$$K^1_\Psi = \frac{G_p J}{r_0} f_{T,1} \qquad (18\ a)$$

and

$$C^1_\Psi = \frac{G_p J}{V_s} f_{T,2} \qquad (18b)$$

In which G_p=Shear modulus of pile material

J =polar moment of inertia of the cross section with circular cross section

r_0=effective radius of one pile and

V_s=shear wave velocity of soil

$f_{T,1}$, $f_{T,2}$ = parameters in Figures 17 and 18 which have been plotted for dimensionless input parameters for reinforced concrete and timber piles respectively.

These figures show that damping parameter $f_T.2$ varies with frequency much more than the stiffness parameter $f_T.1$.

The marked effect of material damping may be seen from the broken lines in Figures 17 and 18, which were calculated with tan $\delta=0.1$, a representative value for soils. The material damping of the soil increases significantly the total torsional damping of the pile, particularly at low frequencies, and makes the equivalent viscous damping constant somewhat less frequency dependent than it is with tan $\delta=0$ (for higher frequencies). The effect of material damping on the torsional stiffness of the pile is negligible.

Stiffness and Damping Constants of Footing. The torsional stiffness and damping constants of a pile have been obtained in the above analysis as moments that correspond to unit rotational displacement and velocity respectively. For a pile located beyond the reference point, these moments are composed of two parts :

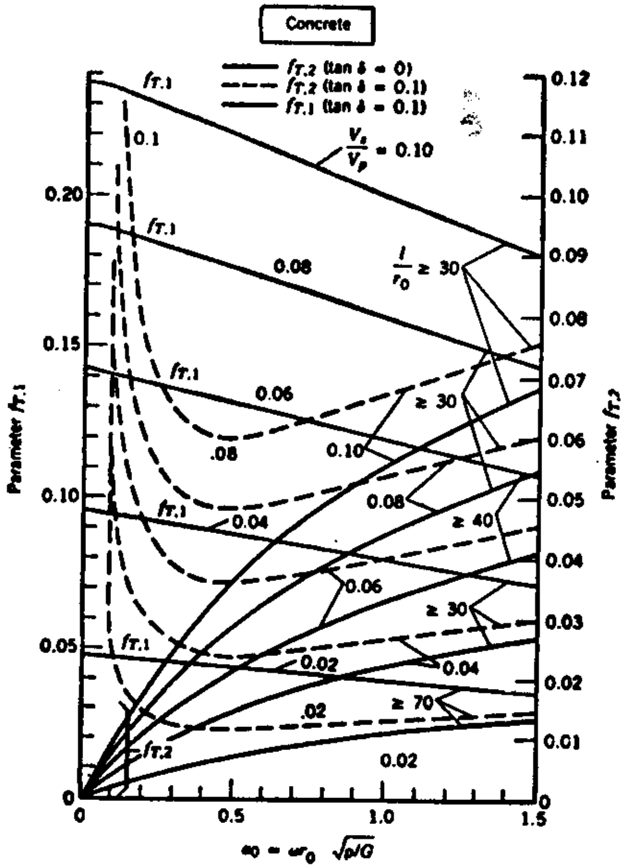


Fig. 17 Torsional stiffness and damping parameters of reinforced concrete piles ($p'p_p=0.7$) After Novak and Howell, 1977).

(1) That twists the pile and

(2) That translates it. With reference to Figure 19, the torsional stiffness constant of a pile-footing is

$$K_{\Psi} = \Sigma \left[K^{1}_{\Psi} + K^{1}_{X}(x_r^{2} + y_r^{2}) \right] \tag{19}$$

and the torsional damping constant is

$$C_{\Psi} = \Sigma \left[C^{1}_{\Psi} + C^{1}_{X}(x_r^{2} + y_r^{2}) \right] \tag{20}$$

The summation is extended over all the Piles.

Equations 19 and 20 show clearly that the contribution of the translation component increases with the square of the distance from the

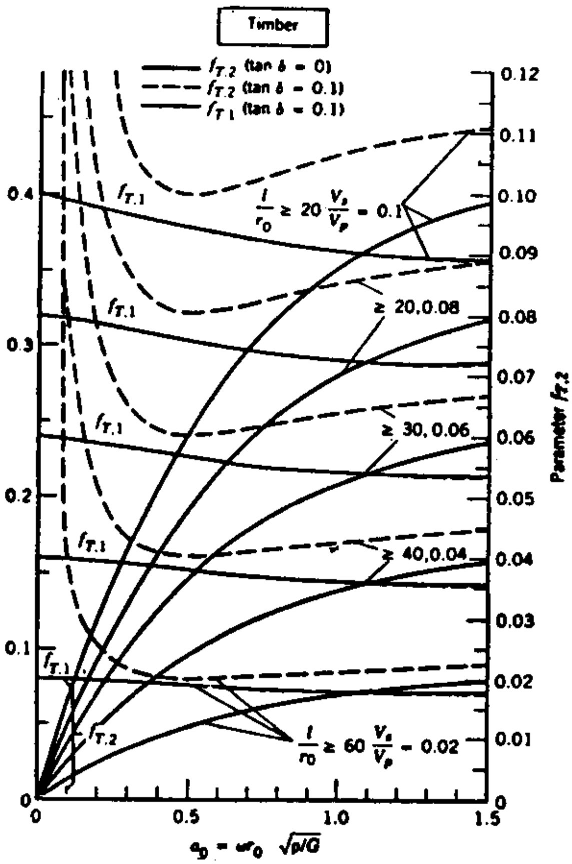


Fig. 18 Torsional stiffness and damping parameters of timber piles (ρ/ρ_p= 2) After Novak and Howell, 1977)

reference point, $R=\sqrt{X_r^2+y_r^2}$. If the centroid of the footing coincides with the elastic center of the piles in plan, the excitation moment, $M_0 \cos \omega t$, produces pure torsional response of the footing, Ψ_0, given by

$$\Psi_0=\frac{M_0}{\sqrt{(K_\Psi - I_p\ \omega^2)^2+(C_\Psi\omega^2)^2}} \tag{21}$$

where I_p=polar mass moment of intertia of the footing about the vertical axis.

GROUP ACTION UNDER DYNAMIC LOADING

Piles are always used in groups. The stiffness and damping of pile groups need be evaluated from considerations of group action.

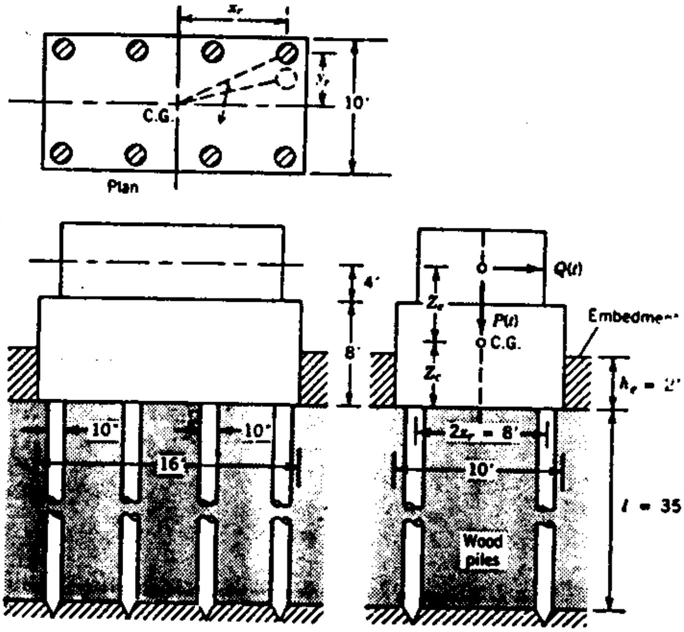


Fig. 19 Rotational constants of footing in torsion.

VERTICAL VIBRATIONS

Novak and Grigg (1976) proposed that the deflection factors of Poulos for groud action of statically loaded piles based on elastic analysis may also be applied to a pile group undergoing steady-state vibration. Therefore, stiffness of pile group K_w may be obtained from Eq. (22)

$$\bar{K}^{y}_{w} = \frac{\Sigma^{n}_{1} K^{1}_{w}}{\Sigma^{n}_{1} \alpha_{a}}$$

where n = number of piles and

α_a = axial displacement interaction factor for a typical reference pile in the group relative to itself and to all other piles in the group, assuming the reference pile and all other piles carry the same load.

The factor α_a is obtained from Figure 20.

The equivalent geometric damping ratio for the group is given by

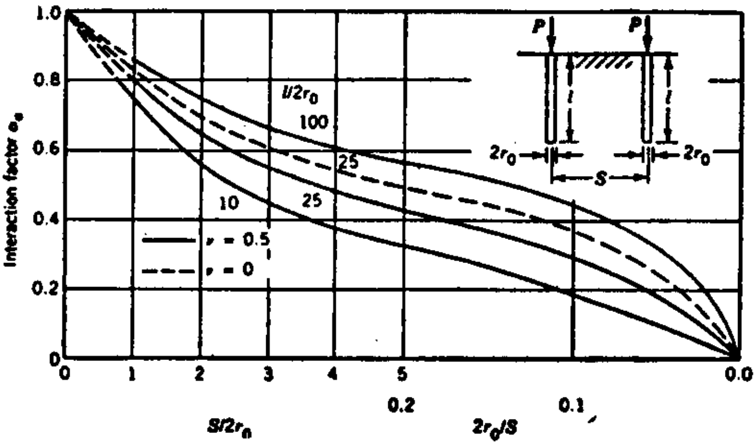


Fig. 20. L$_A$ *as a function of pile length and spacing for vertical loading (After Poulos, 1968).*

$$C_z^g = \frac{\Sigma_1^n C_w^1}{\Sigma_1^n \alpha_a} \tag{23}$$

Embedment of the pile cap results in increase of the stiffness and damping values of the pile group. For details, refer to Prakash and Puri (1988). It may however be assumed that in practice embedment is provided only in the development of side friction between the cap and soil and only when dense granular backfill is used.

Novak and Beredugo (1972) have developed expressions for calculating stiffness and geometric damping constants for the embedded footings which can be applied to pile caps. The stiffness (K_w) and damping (C_w) values due to side friction of the pile is expressed as (Prakash and Puri, 1988)

$$K_\omega^f = G_s \; h \; \bar{S}_1 \tag{24}$$

$$C_w^f = h r_0 \; \bar{S}_2 \sqrt{G_s \rho_s} \tag{25}$$

where h = depth of embedment of the cap

r_0 = equivalent radius of the cap

G_s and ρ_s are the shear modulus and total mass density of the backfill and S_1 and S_2 = constants and are 2.70 and 6.70 respectively, in which ν_s is the Poisson's ratio of the backfill soil.

LATERAL VIBRATIONS

In lateral vibrations, the stiffness and damping for groups of piles is given by

$$K_x^g = \Sigma_1^n K^1_{xx} / \Sigma_1^n \alpha_L \qquad (26)$$

$$C_x^g = \frac{\Sigma_1^n C_x^1}{(K^n_1 \alpha_L} \qquad (27)$$

where α_L = displacement interaction factor for lateral translation and adopted from Figure 21

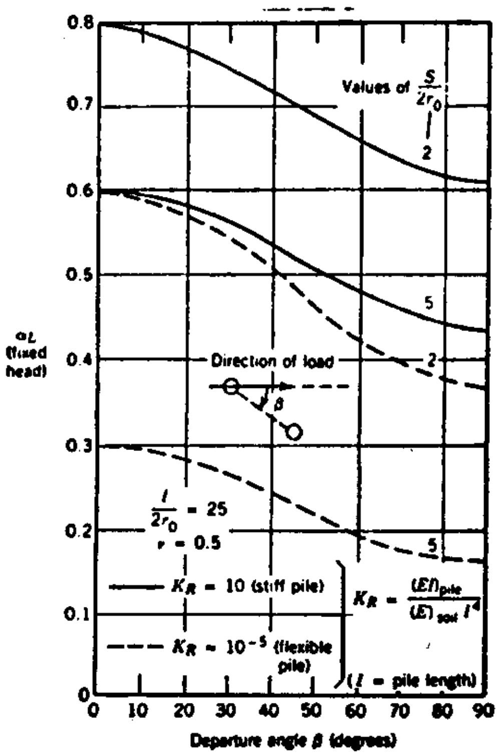


Fig. 21 Graphical solution for α_L (After Poulss, 1972).

Again, as for vertical vibrations, the spring constant and damping due to pile cap translation are respectively.

$$k^{f}_{x} = G_s h \bar{S}_{x1} \tag{28}$$

and

$$C^{f}_{x} = h r_0 \sqrt{G_s \gamma_s / g}\ \bar{S}_{x2} \tag{29}$$

where h = depth of embedment

r_0 = equivalent radius of the cap

G_s and P_s = the shear modulus and total mass density of the back fill and S_{x1} and S_{x2} = constants in Table 4

Table 4

Stiffness and Damping for Half Space and

Side Layers for Sliding Vibrations

(After Beredngo and Novak, 1972)

Poissons Ratio ν	Validity Range	Constant Parameter
0.0	$0 < a_o < 1.5$	$\bar{S}_{x1} = 3.6$
	$0 < a_0 < 01.5$	$\bar{S}_{x2} = 8.2$
0.25	$0 < a_0 < 2$	$\bar{S}_{x1} = 4.0$
	$0 < a_0 < 1.5$	$\bar{S}_{x2} = 9.1$
0.4	$0 < a_0 < 2.0$	$\bar{S}_{x1} = 4.1$
	$0 < a_0 < 1.5$	$\bar{S}_{x2} = 10.6$

The total stiffness and total damping values are sums of Eqs. 26 and 28 and Eqs. 27 and 29 respectively as below :

$$\text{Total } K^{g}_{x} = K^{g}_{x} + K^{f}_{x} \tag{30}$$

and

$$\text{Total } C^{g}_{x} = C^{g}_{x} + C^{f}_{x} \tag{31}$$

For rocking vibrations, the effect of pile groups and the pile cap is accounted for as for sliding as :

Compute stiffness and damping of pile only in the pile group (Novak 1974)

$$K^{g}_{\phi} = \Sigma^{h}_{1} [K^{1}_{\phi} + K^{1}_{\omega} X^{2}_{r} + K^{1}_{x} Z^{2}_{c} - 2 Z_c K^{1}_{x\phi}] \tag{32}$$

$$C^{g}_{\phi} = \Sigma [C^{l}_{\phi} + C^{l}_{w} X^{2}_{r} + C^{l}_{x} Z^{2}_{c} - 2 Z_c C^{l}_{x\phi}] \tag{33}$$

in which x_r=spacing of piles,

z_c = height of C.G. of the pile cap above its base and

$$\delta = \frac{h}{r_0}$$

Compute stiffness and damping of pile cap as :

$$K^f_\phi = G_s\, r_0^2\, h\, \overline{S}_{\phi 1} + G_s\, r_0^2 h[(\delta^2/3) + (Z_{c/r_0})^2 - \delta\,(z_{c/r_0})\; \overline{S}_{x1} \quad (34)$$

$$C^f_\phi = \delta r_0^4 \sqrt{G_s \gamma_s / g}\, [\overline{S}\phi_2 + [\frac{\delta^2}{3} + (\frac{Z_c}{r_0})^2 - \delta\,(\frac{Z_c}{r})[\overline{S}_{x2}] \quad (35)$$

Once the stiffness and damping of the system are computed, its response can be determined from principles of elementary mechanical vibrations (Prakash and Puri 1988)

DESIGN PROCEDURE OF PILES UNDER DYNAMIC LOADS

The design procedure essentially consists of computation of

(1) Stiffness of the pile group considering group action and

(2) damping of the pile group considering group action.

This procedure has been developed based on the analytical formulation of stiffness and damping in different modes of vibrations in the preceding sections.

The response of the foundations may then be computed either by the spectral response technique described for earthquake loading (Prakash 1981) or by response equations for machine foundations (Prakash and Puri 1988, Prakash and Sharma 1989).

CENTRIFUGE MODEL TESTS

In order to check the various methods of analysis of piles under dynamic conditions, it is desirable to carry out field dynamic testing of full-scale piles. Only a few such tests have been conducted. Novak and Grigg (1976) carried out vibration tests on large model (or small prototype) piles and normal sized pile dynamic tests have been performed by Scott (1982). Alpan (1973) studied 0.3 m square prestessed reinforced concrete piles 4.7 m long in ringdown tests. These tests were not performed at large enough amplitudes in the soils to cause substantial changes in the soils properties (Scott, Ting & Lee 1882). In particular. the question of soil liquefaction was not addressed.

In many fields of engineering, scaled models of large structures are used to study physical phenomena. Scaled models of geotechnical structures under earths gravity, however, do not satisfy similitude conditions because the stress levels in the model do not match those in the prototype. By placing the model in an appropriately increased gravitational field, the model material is made heavier, and prototype stress levels in the model are achieved. Such a gravitational field is created by spinnining the centrifuge arm at an appropriate angular speed such that the centrifugal acceleration at the location of the model on the arm is ng where g is the acceleration due to gravity and n is the model scale. The scaling relationships used in centrifugal modelling are summarizee in Table 5.

Table 5

Table (after Scott, 1979)
Scaling Relations

Quantity	Full Scale (Prototype)	Model at n g's
Linear Dimension, Displacement	1	1/n
Area	1	$1/n^2$
Volume	1	$1/n^3$
Stress	1	1
Strain	1	1
Force	1	$1/n^2$
Acceleration	1	n
Velocity	1	1
Time - In Dynamic Terms	1	1/n
- In Diffusion Cases	1	$1/n^2$
Frequency in Dynamic Problems	1	n

Since each model is of finite size, different parts of the model are at different radii from the rotational axis of the centrifuge. Therefore, different parts of the model will be subjected to different gravitational intensities. The greater is the radial distance of the model compared with the dimension of the model in the direction of the centrifuge the more uniform the acceleration field across the model will result.

A comprehensive series of centrituge model tests have been reported by Finn and Gohl (1987) which will now be described.

Studies an Models

The tests by Finn and Gohl (1987) represent very carefully conducted test on piles in the centrifuge and provide a data base against which currently available analytical models used to predict the lateral response of piles to earthquake loading could be checked. Several tests on single pile and pile groups were performed but data on a single pile and 2-pile groups embedded in dry sand under lateral loading were presented. The single pile was subjected to both sinusoidal and random earthquake motion while the pile groups were subjected to both sinusoidal and random earthquake motion while the pile group were subjected to sinusoidal wave motion only. The excitation levels for the pile groups were kept low enough to ensure approximatly linear elastic response so that the accuracy of elastic solutions could be checked. The distribution of shear moduli in the foundation layer were measured while the centrifuge was in flight using piezoceramic binder elements.

This has been achieved in centrifuge tests for the first time and should make predictions and checking of data against analytical methods more reasonable and accurate (Finn and Gohl 1987).

In these tests, the acceleration varied from 55 g at the surface of the model to 61; g at the base an average centrifuge scale factor, n, equal to 60, was used in converting model test quantities to prototype scale.

Pile tests were carried out in both "loose" and "dense" sands at void ratios of 0.83 and 0.57 respectively. Instrumented piles were pushed into the soil by hand in loose sand. In dense sands a low level vibration of the sand foundation was used to assist penetration.

Tests on 2-pile groups were conducted at various spacings to evaluate interaction effects. Both piles were instrumented to measure bending strains. In addition, one pile was instrumented to record axial strains caused by rocking of the pile foundation during shaking. The piles in the group were rigidly attached to a pile cap and an additional mass was bolted to the pile cap to simulate the effects of a supertructure as in the case of a single pile. The center of gravity of the pile cap assembly was 17.0 mm above the base of the pile cap. The pile cap mass assembly was instrumented with an accelerometer and displacement L.E.D. as for the single pile.

After model pile installation, four lightweight settlement plates were placed at a minimum of eight pile diameters from the center of any pile to measure surface settlement due to : 1) the increase in self-weight of the soil during spin-up of the centrifuge and 2) the cyclic shear strains generated by the base motion.

All data were presented at prototype scale. In test 12, the pile was subjected to a moderate level of shaking (peak base acceleration 0.15 g), while in test 41 twenty cycles of a sine wave base motion with a peak steady state acceleration of 0.04 g were applied (Table 6).

Table 6

Table 6

Single Pile Test Characteristics (After Finn and Gohl 1987)

Test	Soil Type	Z_{cg} (1) (m)	Base Motion Type	Peak Base Accel. (g)	Peak Pile Head Accel. (g)
12	Loose (avg. Void ratio after consolidation = 0.78)	1.95	Earthquake (30 sec. duration)	0.15	0.18
41	Very dense (avg. void ratio = 0.57)	1.89	Sine wave (20 cycles)	0.04(2)	0.041

The acceleration input at the base of the model accelerations recorded in the free field at the surface of the foundation layer and at the pile head are shown in Figures 22a, b and c respectively. The time histories of pile bending moment at various points along the pile are shown in Figures 22a, b, c for strain gauge stations No. 1, No 4 and No 7 respectively (See Fig. 24). The bending moment distribution along the pile at a time when maximum pile head deflection occurs (t=12.0sec) was also observed.

(1) Distance of center of gravity of pile head mass above ground surface

(2) Averaged over steady state portion of base input motion.

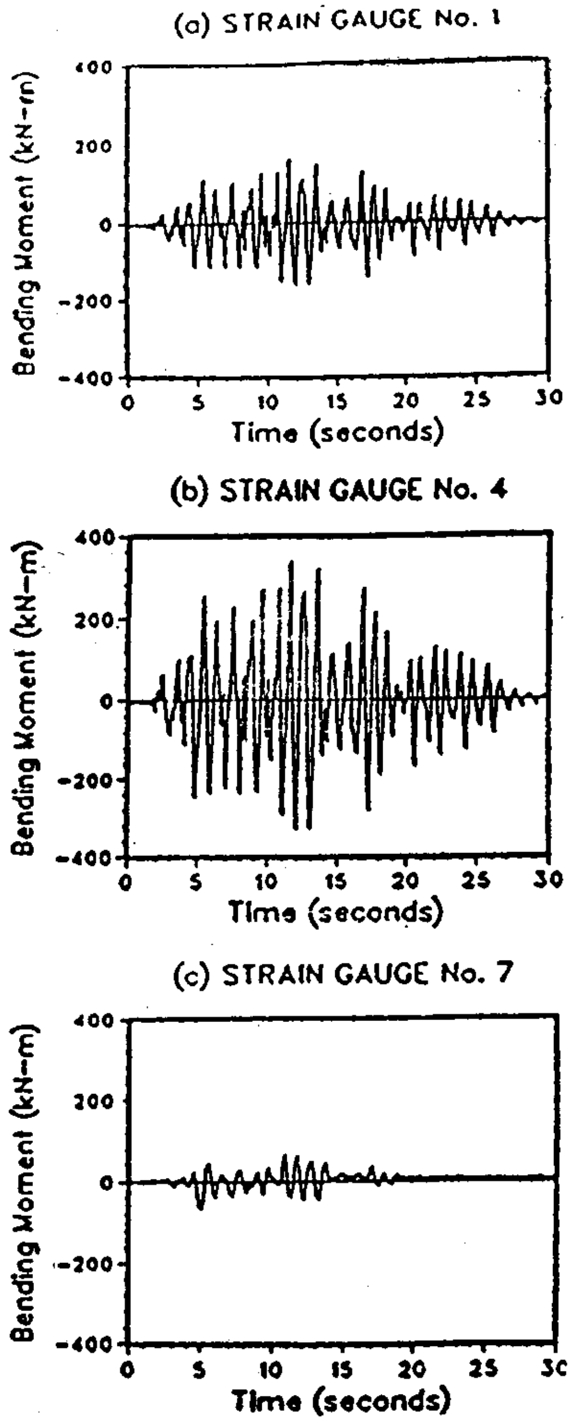


Fig. 23 Bending Moment Time Histories—Single Pile Test Number 12 (Finn and Gohl, 1987); (a) Strain Gauge No. 1; (b) Strain Gauge No. 4; (c) Strain Gauge No. 7.

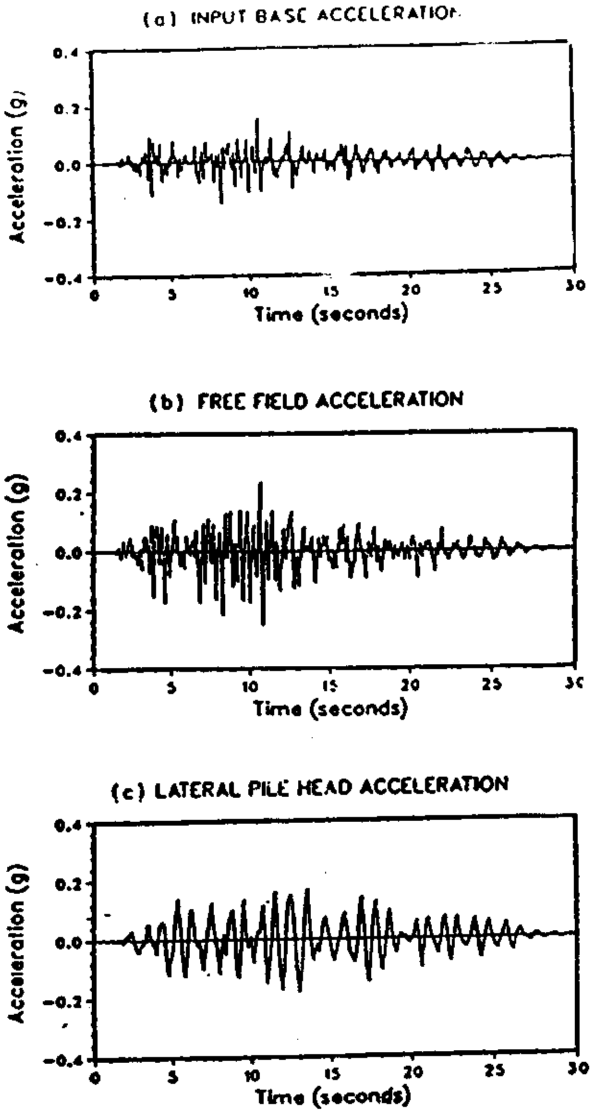


Fig. 23 Bending Moment Time Histories—Single Pile Test Number 12 (Finn and Gohl, 1987); (a) Strain Gauge No. 1; (b) Strain Gauge No. 4; (c) Strain Gauge No. 7.

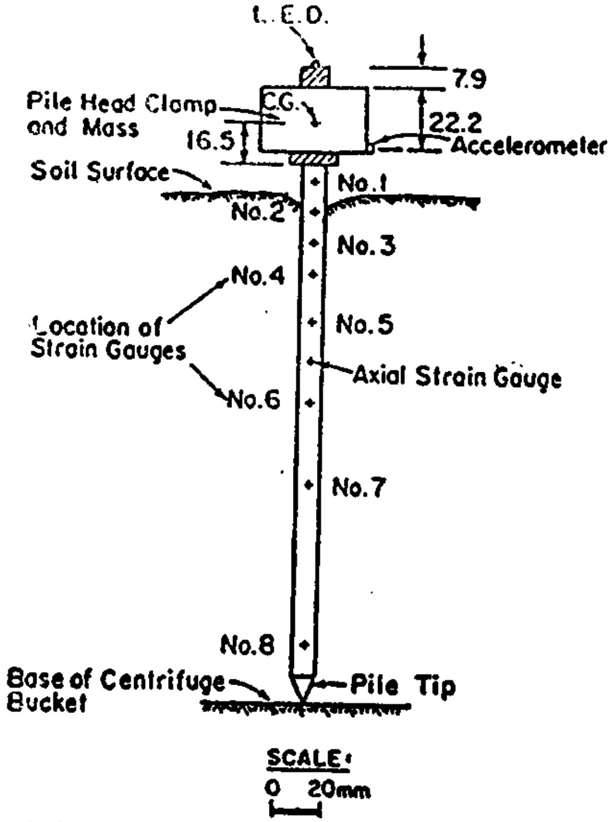


Fig. 24. Single Pile Showing Instrumentation Layout (Finn and Ghol, 1987)

The following observations are significant on a single pile :

1. In Figures 22, the maximum input acceleration was 0.15 g. The peak free-field acceleration was 0.26 g and the peak pile head acceleration was 0.18 g. Thus both, the pile head and free-field peak accelerations were magnified relative to the input base acceleration.

2. The predominant period of the pile head response was longer than that of the free field ground surface response. Therefore strong interaction takes place between them.

Pile Group Response

Two-pile groups were tested at various spacings at low levels of excitation using an approximately harmonic base motion.

Bending moment distributions in piles with a center-to-center spacing equal to two pile diameters for the tenth load cycle at a time when pile cap deflection is a maximum during the cycle is shown in Figure 25.

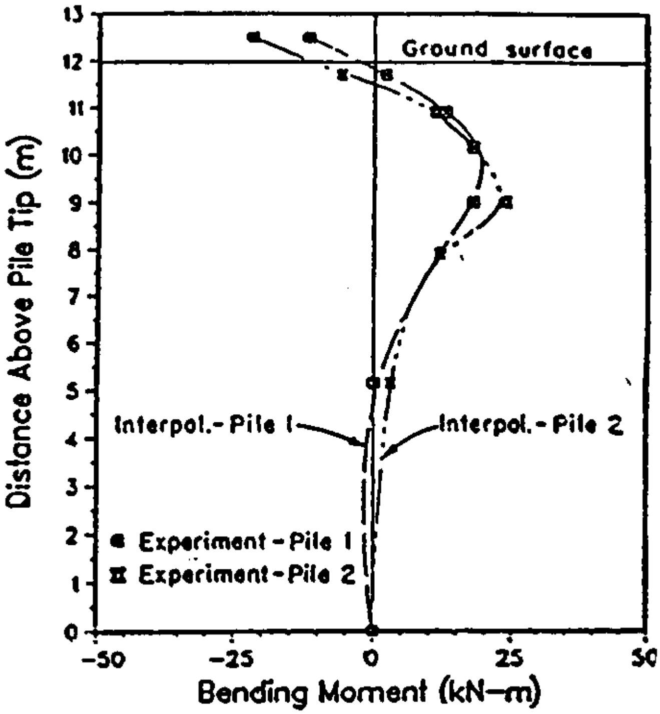


Fig. 25 Bending Moment Versus Depth; Loading in the Direction of Pile Spacing (s/b=2) Dense Sand Group Test Number 21 (Finn and Gohl, 1987).

In this figure, at peak displacement, the bending moment changes sign indicating the restraint of the pile cap against rotation. The moment distributions in the two piles are sufficiently different to suggest significant interaction (Finn and Gohl 1987).

The steady peak pile cap displacement is plotted against the pile spacing ratio, s/b, for ratios between 2 and 6 in Figure 26.

This figure suggests that the pile cap displacements at the same level of excitation depend very strongly on pile spacing. This indicates

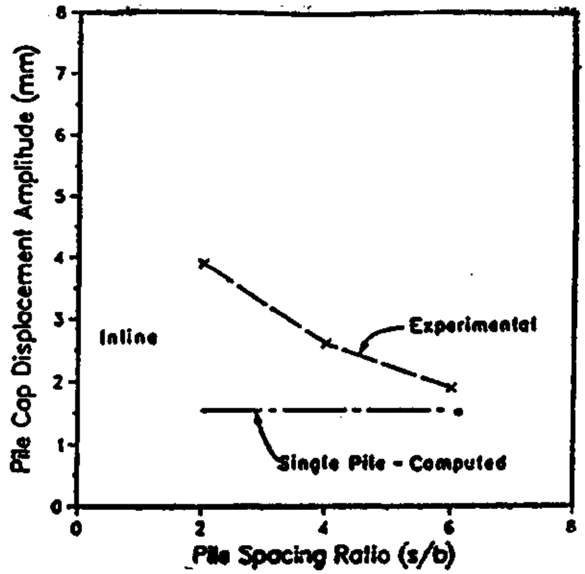


Fig. 26 Influence of Pile Interaction on Pile Cap Displacement in Inline Loading (Finn and Gohl, 1987).

strong interaction between piles in the group. Computed value of displacements for single pile are also shown in Figure 26. The results suggest that interaction effects at spacings beyond about 6 pile diameters are insignificant (Finn and Gohl 1987).

The tests of Finn and Gohl (1987) had been conducted with maximum spacing (S) of 6 times the diameter of the pile. Their data in Figure 26 however, suggests that probably the experimental curve will become asymptotic to the value of the single pile displacement if the s/b ratio approaches 8. This had been determined by Prakash (1962) in his model tests in piles in sand.

The tests data reported above had a unique feature. In these centrifuge tests, the in.situ distribution of shear modulii in the soil was measured during flight using piezo-ceramice bender elements. This gives data to check the measurements of the pile behavior with the predicted response.

Sufficiently more data is needed to check the validity of various analytical formulations with the measured response both in case of 1) single pile and 2) pile groups. It appears that data is being collected by various investigators and in not too distant a future better comparisons of the predicted and measured response be available.

COMPARISON OF PREDICTED RESPONSE WITH OBSERVED RESPONSE OF SINGLE PILE AND PILE GROUPS

Several lateral dynamic tests on full-sized single piles were performed to check if the predicted response tallied with the measured response, (Gle, 1984 and Woods, 1984). No tests had been performed on pile groups. Also, Novak and El-Sharnouby (1984) performed tests on a group of model piles to compare predictions with performance. No single pile tests were performed. The predicted response did not tally with the measured response.

Tests of Full Size Single Piles

Fifty-five steady state lateral vibration tests were performed on eleven pipe piles 14 inches in outside diameter with wall thickness of 0.188 in or 0.375 inch (0 47 cm—0.94 cm) at 3 sites in southeast Michigan (Woods, 1984). The end bearing piles were 50 ft to 160 feet (15m—48m) in length.

Figure 27 shows response curves for the pile GP 13-7, 157 (47. 1m) feet long in soft clay. It was observed that the frequency of maxium

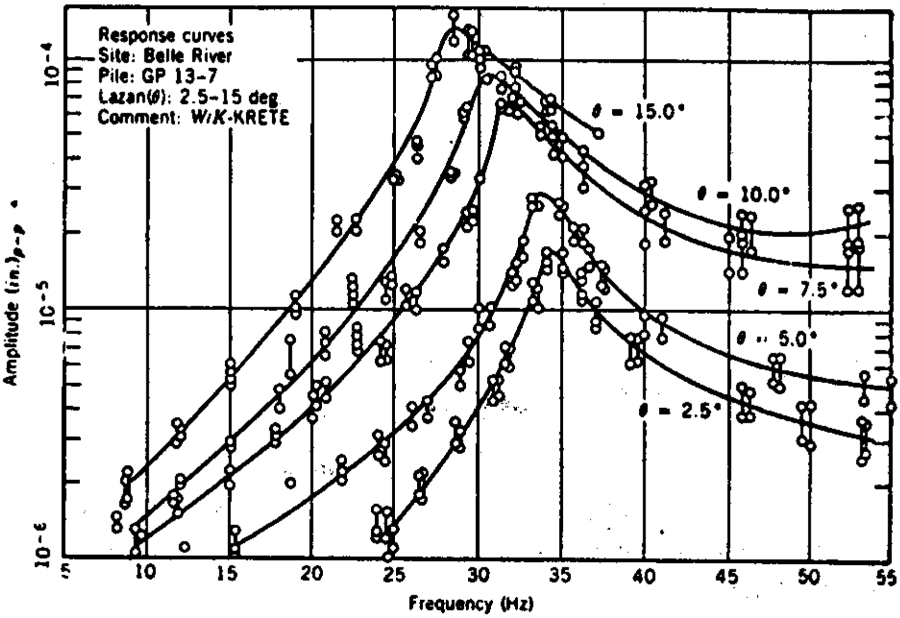


Fig. 27 Response curves. A decrease in resonant freguency with increasing amplitudes (from Gle, 1981).

response decreased as the force level increased indicating non-linear response. PILAY computer program was used by Woods (1994) to determine stiffness and damping elements (Novak and Aboul-Ella, 1977). However, PILAY assumed that the soil surrounding the pile in a given layer is the same at all distances from the pile.

A dynamic response curve with the solutions is shown in. Fig. 28 along with the field data. The correlation between predicted curve and

COMPARISON OF MEASURED AND PREDICTED PILE RESPONSE 543

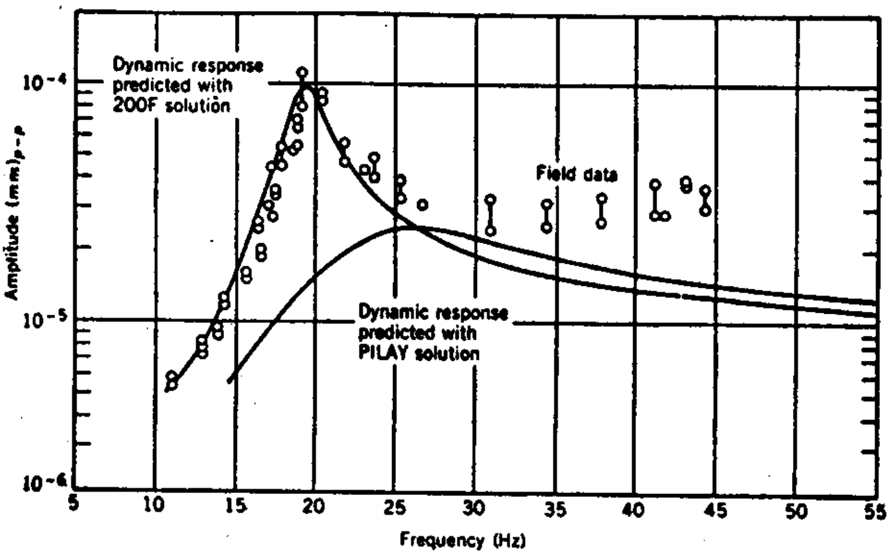


Fig. 28 Typical response curves predicted by Pilay superimposed on measured pile response (After Gle and Woods, 1984b).

measured response is very poor because of the reasons already described.

An attempt was made to match the measured response with the the computed response by altering the stiffness arbitrarily.

A good match of the measured and predicted response could be obtained by a considerably greatly reduced soil modulus in the softened zone (one tenth to two tenths of the original value) and the extent of the softened zone (one half to one times the pile radius). A loss of contact of the soil with pile for a short length close to the ground surface also improved the predicted response.

No tests on pile groups were performed at any of these sites.

Tests on Groups of Model Piles

El-Sharnouby and Novak (1984) performed dynamic tests on a 102 steel pipe piles group. The piles were 106cm long with outside and inside diameters of 26.7mm and 20.93mm respecttvely. The slenderness ratio (l/r_o) of piles was greater than 40 and the pile spacing about 3 diameters. The pile group was placed in a hole in the ground, which, was backfilled with a specially prepared soil mixture. The pile cap was 6cm above the ground level. The pile group was excited by a Lazan oscillator at frequencies of 6-60 Hz. in the vertical and horizontal directions and in torsional mode. Free vibration tests and static tests had also been performed. The measured response curves were very linear for small amplitudes and indicated relatively small non-linearity at amplitudes and indicated relatively small non-linearity at amplitudes of 0.2mm. The test results of Gle (1984) and Woods (1984) show definitely nonlinear behavior of insitu piles.

Novak and El-Sharnouby (1984) analyzed the data as above by the following methods :

1. Using static interaction factors by Poulos (1971, 1975, 1979) and Poulos and Davis (1980).

2. Concept of equivalent piers

3. Using dynamic interaction factors by Kaynia and Kausel (1982).

4. Direct dynamic analysis of Waas and Hartmann (1981).

A critique of their vertical tests has already been presented elsewhere (Prakash 1986, Prakash and Puri 1988).

Horizontal Response

Horizonal, rocking, and cross stiffness and damping constants, K_x, $K_{x\alpha}$, K_α, C_x, C_α, and $C_{x\alpha}$ were calculated for a single pile using the computer program PILAY2. A group interaction factor, $\Sigma\alpha a$, of the group of 102 piles based on Poulos' charts (1975, 1979) was estimated approximately as 13. This interaction factor was applied only to the horizontal stiffness, K. The theoretical horizontal component of coupled response to horizontal excitation, based on the static interaction factor, is shown together with the experimental one in Fig. 29. Four theoretical response curves have been plotted against the experimental one. Curve A represents the group response without any interaction effect, while curve B was calulated using the static interaction factor for stiffness only. It can be seen that a much lower value of the interaction factor

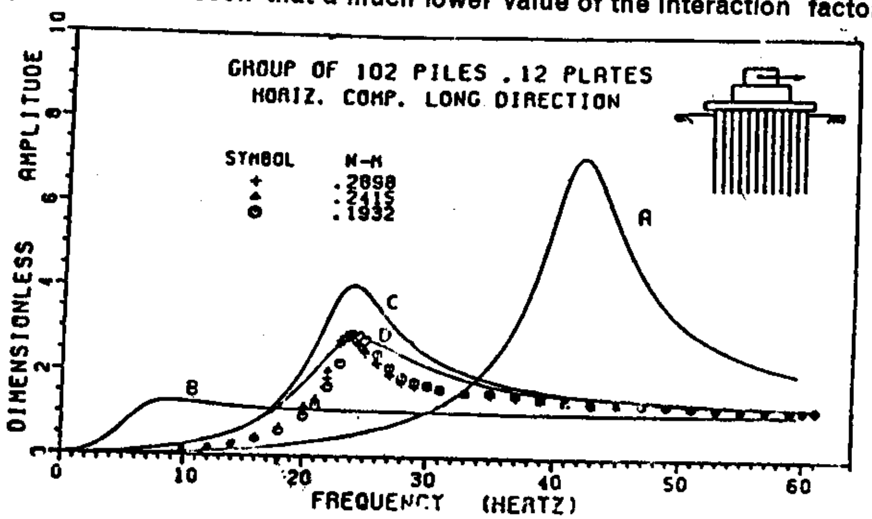


Fig. 29. Experimental Horizontal Response Curve and Theoretical Curves: A, No Interaction; B, with Poulos Static Interaction Factor Applied to Stiffness only; C, with Interaction Factor of 2.85 Applied to Stiffness only; and D, with 2.85 and 1.40 Interaction Factors for Stiffness and Damping, Respectively (After Novak and E1-Sharnouby, 1984).

is needed for the stiffness if the resonant frequency is to be matched. Therefore, an interaction factor of 2.85 was arbitrarily introduced for stiffness to yield curve C. The best agreement between the theoretical and experimental curves was achieved by increasing the damping constent by 45% (curve D). Yet, some discrepancy between the theoret.cal and expcrimental response curves occurs at frequencies other than than the resonant frequency. This indicates the limits of the limits of the applicability of static interaction factors (Novak and E-l Sharnouby, 1984).

Novak and El-Sharnouby (1984) also describe the concept of an equivalent pier which may at best have a limited practical application. Also, comparisons of the theoretical and measured respoose both in vertical as well as in torsional vibrations by several methods have been presented by the authors.

The above discussion points to the fact that dynamic interaction is very complicated and further theoretical and experimental research is needed in dynamic behavior of piles and pile groups.

OVERVIEW

Piles are used extensively for supporting building foundations, in seismic zones, for machine foundations and for off-shore structures.

The natur of pile response and pile interactions are quite different in all the three cases. Earthquake loading for piles under buildings may cause large deformations and soil non-linearity. On the contrary machines may cause only small amplitudes of vibrations and soils may behave as elastic materials. In off-shore structures, the piles are especially long and slender with considerable free standing lengths.

In earthquake loading only lateral vibrations may be important while in machine foundations, the pile-foundation may be subjected to 1) vertical oscillations, 2) horizontal translation and rocking, and 3) torsion.

Solutions based on beam on elastic foundation for static loads has been extended for dynamic loading by Chandra Śekaran (1973) Penzien (1960) and a design procedure has been proposed based on spectral response technique (Prakash 1981). For pile supported machine foundations, simple solutions for single piles in all the modes of vibrations have been included in this paper. Also group action on the behavior of the total system as compared to that of the single pile has been evaluated and a complete analysis has been included. However, there are certain definite gaps in the present (1988) understanding of "single pile" and "pile group" action under dynamic loads.

Initial analyses by Barkan (1962) and Maxwell et al (1969) have been shown to have only limited appplication.

For vertically vibrating piles, Novak's (1974)analysis for single piles is reasonable and uses rational soil and pile properties. However, in case of groups, static interaction factor have been used, Novak and Grigg (1976). Sheta and Novak (1982) developed an approximate theory for vertical vibrations of pile groups.

On the basis of comparison of predicted and measured response of 102 closely spaced pile groups in vertical vibrations, Novak and El-Sharnouby (1985) have shown that :

1. Correction for the apparent mass in vertical vibrations may be necessary, particularly for rigid floating closely spaced piles.

2. The static interaction factor provided quite a good estimate of the group stiffness but the group damping could not be predicted.

3. For closely spaced piles, the equivalent pier concept provided a reasonable agreement with the experimental data if the theoretical damping constant was reduced by about 50 percent constant was recuced by about 50 percent.

The solutions are by no means simple in their present form. Therefore, more research is needed to solve the problem completely and put it in a form which can be easily used by the practicing engineer.

Horizontal vibrations of piles have been investigated by considering the piles as

1. an equivaleet cantilever

2. a beam of elastic foundation (Tucker 1964 and Prakash 1981) and also by

The equivalent cantilever method.

(1977) does not consider realistic behavior of soil-pile. Solutions for beams on elastic foundations need to be developed further to put them in readily, usable forms.

3. Novak's solution for a single pile and for pile groups for horizontal vibrations is subject to the same limitation as that for vertical vibrations. (Prakash 1986, Prakash aud Puri 1988).

The tests of Novak and El-Sharnouby (1985) showed that the static group interaction effects differed considerably from dynamic group effects in horizontal vibrations. However, the equivalent pier concept predicted the stiffness well, but not the damping for that particular case.

For single piles also, Woods (1984) found that softened zone around the pile in clay alters the behavior and needs to be considered in a realistic analysis.

For a group of piles, the contribution from torsion to the total stiffness and damping decreases with the relative distance of the pile from the centroid of the footing. Pile foundations can have smaller natural frequencies in torsion than shallow footings, but the increased damping generated yields lower resonant amplitudes. This contrasts with other modes of vibration. Since pile slippage, and other effects such as method of installing the piles are not accounted for in this theory, comparison with experiments is desirable.

The interaction of pile cap with soil, affectes the dynamic response of the system, which can be accounted for in all modes of vibrations on the basis of principles of embeded foundations (Prakash and Puri 1988).

Based on the approximate solutions in the preceeding sections, a step by step dasign procedure has been developed (Prakash and Puri 1988).

The soil properties used in defining the stiffness and damping parameters are 1) shear wave velocity and shear modulus V_s and G respectively and 2) Poissons ratio, which may be determined as described in chapter 4.

Aubry and Postel (1985) considered the soil-pile syatem as a fiber reinforced composite meterial and the technique of homogenization of composite materials was used to compute equivalent modulus which were used to compute the seismic response of the equivalent foundation at the soil surface. This method has been shown to be useful particularly for very large number of piles beneath a foundation. This method may be regarded as a complimentary solution to Novak's equivalent pier concept for closely spaced piles.

Gazetas and Dobry (1984) proposed a method to compute response of single fixed head pile under horizontal excitation at its head. In the method, the solution is based on realistic estimation of

(1) deflections of the pile under static lateral load,

(2) dashpots attached to the pile at every elevation,

(3) dashpot at its head, and

(4) variation of spring coefficient and damping ratio with frequency.

The applicability of the proposed method has been illustrated in three linearly hysteretic soil deposits (i) homogeneous deposit with modulus constant with depth (ii) in-homogeneous deposit with modulus increasing linearly with depth and (iii) layered deposit.

Centrifuge studies on models of pile foundations have been performed. More recently (1987), the tests of Finn and Gohl have shown quantitatively the extent of group action in a 4-pile group under lateral vibrations. The soil shear velocity was measured with piezocramic under elements in the soil with depth. This data has shown that the carefully conducted centrifuge tests short of full scale tests hold promise for validation of theoretical formulations.

CONCLUSIONS

Based upon a critical review of the available literature on the subject, it is concluded ;

1. The profession has come a long way in tackling the analysis design of pile foundations under dynamic loads.

2. Considerable more research effort is needed to generate carefully controlled test data to match/predictions with performance and thereby revise analytical formulations.

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