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RESPONSE OF BRIDGES SUBJECTED TO A SYSTEM OF MOVING FORCES

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ABSTRACT

The two dimensional behaviour of the structure subjected to moving load with sprung and unsprung masses has been considered. The moving load is assumed to be of two degrees of freedom. The theory encountered in this paper will find its widest application though the actual conditions are more complicated.

The response study has been made in the form of dynamic coefficient. Its variation has been studied under parametric study taking into account the mass of the moving vehicle, mass of the bridge, weight of the vehicle, weight of the bridge, speed of the moving vehicle, and unsprung and sprung parts of the vehicle. It is observed from the results that speed parameter and the sprung and unsprung parts of the vehicle will give rise to the increase in value of dynamic coefficient.

1. INTRODUCTION

Transport Engineering structures are subjected to loads that vary both in time and space. Such loads are called as moving loads. In recent years all branches of transport have experienced great advances characterised by increasing speeds and weights of vehicles. As a result, structures and media over or in which two vehicles move have been subjected to vibrations and dynamic stresses larger than ever before.

Theoretically the problem of moving load was first tackled for the case in which the beam mass was considered small against the mass of a single constant load. The original approximate solution is due to Willis (1851), being one of the early experiments in the field. Stokes (1849) and Zimmermann (1896) approached the problem under similar assumptions. This case applies to railway bridges and has been analysed by Stokes (1849) and Sundara Raja Iyengar and Jagadish (1968). Stokes (1849) solved the problem with an uniformly distributed mass with constant velocity. Similarly Inglis (1934)

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Krylov (1905) and Timoshenko (1911) solved the problem by considering the load mass small against the beam mass traversed by a constant force with constant velocity on the beam. Timoshenko (1922) integrated the differential equation of the forced vibrations of a simply supported beam by modal superposition. He stated that the maximum dynamic deflection is 1.5 times the static deflection, when the traversing time is 1/2 the first natural period of the beam. It was recognised late by Eichmann (1953) and Warburton (1964) that the situation is less conservative and the maximum deflection is 1.743 times the static deflection. This corresponds to a traversing time of 0.81 times the first natural period. This conclusion was further supported by using finite elements by Vananeio Filho (1978).

A masslessload traversing with constant velocity on a beam uniform mass was studied by Lowan and Bondar (1954), and solved it with the aid of Green's functions and integral equations respectively. Timoshenko (1922) is also credited with the solution to the problem of the effects of harmonic force moving over a beam at a constant speed—an idealization of the effects of counter weights on the locomotive driving wheels. Similarly, Ayre et al. (1950) Nagaraju et al. (1973) and Sundara Raja Iyengar and Jagadish (1970) studied the dynamic response for a two span continuous beams and cantilever beams. The case of moving sprung load has also been treated by the analysis based on the theory of orthotropic plate traversed by a constant force.

In the above paragraphs it can be seen that the response study is made with (i) a massless beam subjected to a load with a mass and (ii) a massless load on a beam with uniform mass moving with constant velocity. In practice it is not so, since both the moving load and the structure over which it moves will have respective masses. The first investigators to tackle this problem were Saller (1921), Jaffcott (1975) whose iterative method fails to converge in some cases and Stending (1935) who treated several specific cases. A satisfactory method (Fourier series with unknown coefficients for the trajectory of the moving mass) was devised by Schallankamp (1937), Muchnikov (1953) and Ryazanova (1958) and solved the problem by the method of integral equations. Whereas Nalerkiewicz (1953) solved the problem by Galerkin method and Bolotin (1961) by a method that leads to approximate solutions in quadrature.

Inglis (1934) used harmonic analysis to solve important cases likely to come up in dynamic calculations of railway bridges traversed by steam locomotives. General system as well as statically complex systems have been studied by the help of normal mode analysis by Kolousek (1967).

In all the references mentioned so far, the vehicle was idealised by a single mass point. It goes without saying that, for modern means of transport with distinctly differentiated unsprung and sprung masses, such a simplification is no longer in order. The first solution of the motion of sprung masses on a beam is due to Hillerborg (1951) who obtained it by means of Fourier method and by the method of numerical differences. Further advances in the solution of the problems were made possible by the arrival of digital computers. Using these techniques Tung et al. (1956) and Biggs et al. (1959) solved the problem originally treated by Hillerborg and applied the solution to vibration of highway bridges. It was assumed that the static and dynamic deflection curves were instantaneously proportional. The same problem has been analysed on the basis of a simplifying assumption that the dynamic deflection is proportional to the first normal mode of vibration of simply supported beam as studied by Biggs et al. (1959).

The finite element method was first used in the 1960's by Fleming and Romualdi (1961), Veletsos and Haung (1970) and Wen and Toridis (1962). A stiffness formulation of the equations of motion was used for a three span continuous beam (Fleming and Remualdi 1961). The moving load with sprung and unsprung masses was considered in the analysis. A numerical step by step integration showed the influence of mass ratio on the time required for the load to cross the beam, and of the stiffness of the suspension on structural response. Cantilever beams under massless moving load were analysed by Veletsos and Haung (1970) using flexibility formulation. The equations of motion are formed and integrated by the modal superposition method. Single span, three span continuous beam and cantilever beams were analysed using flexibility approach (Wen and Toridis 1962). A three axle vehicle and nonlinear spring for the suspension were used. The equations of motion were integrated numerically. For a thorough treatment of the analytical methods used for problem of moving loads with and without masses in both structures and solids was very well treated in the excellent book by Fryba (1972).

In reviewing the above work on the effect of masses due to moving load and bridge structure, the various investigators formulated the problem, based on beam approach. As such the theories developed are not taking into account the transverse modes of vibration. The aim of the present study is to take into consideration the effect of the frequencies and mode shapes resulting due to the secondary beams of transverse beams in addition to the longitudinal beams to study the response of highway bridges (Rajanna et al. 1972). In other words the two dimensional behaviour of the structure subjected to a moving load with sprung and unsprung masses is considered. The moving load is assumed to be of two degrees of freedom system. The theory encountered in this paper will find its widest field of application, though the actual conditions are more complicated. It is now a well known fact that track irregularities, elastic properties on highway bridges, unbalanced components of unsprung masses and other factors are apt to bear considerable effect on dynamic stresses included in respective structures.

THEORETICAL ANALYSIS

In studying this problem the following assumptions are made:

(i) The beam behaviour is described by Bernoulli-Euler's differential equation based on the assumption of the theory of small deformations, Hookes' law, Navier hypothesis and St. Venant's principle. The beam is of constant cross section and constant mass per unit length.

(ii) The load moves at constant speed.

(iii) The damping is proportional to the velocity of vibration.

(iv) The computation will be carried throughout for a simply supported beam, i.e. beam with zero deflection and zero bending moment at both ends. Further, at the instance of force arrival, the beam is at rest, i.e. possesses neither deflection nor velocity.

Formulation of the Problem

Figs. 1 and 2 represent the model of a beam system. The total weight of the vehicle is given by

$$P = P_1 + P_2 \tag{1}$$

where $P_1 = m_1 g$ is the weight of unsprung parts and $P_2 = m_2 g$ is the weight of sprung parts of the vehicle. The two masses are connected by means of springs k and coefficient of viscous damping c. Their vertical displacements are denoted by $y_1(t)$ and $y_2(t)$. Displacement $y_2(t)$ is measured from the position in which spring 'k' is deformed by force P_2. Displacement $y_1(t)$ is measured from the marked dashed lines, Figs. 1 and 2.

Since the system is assumed to be subjected to a moving vehicle which moves from left to right, with constant velocity 'v', the coordinate of the moving vehicle w.r.t. a constant point is:

$$x_1 = vt. \tag{1}$$

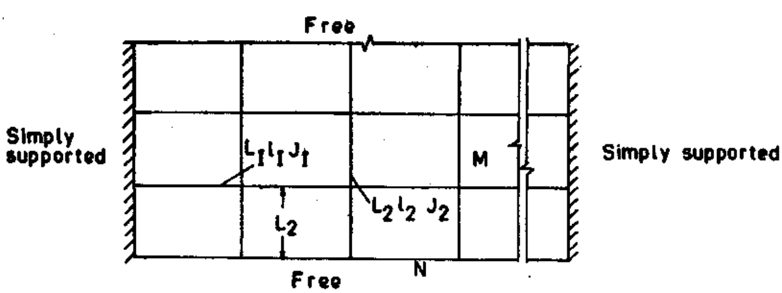

Fig. 1. Rectangular Grid with M Cross Beams and N Longitudinal Beams

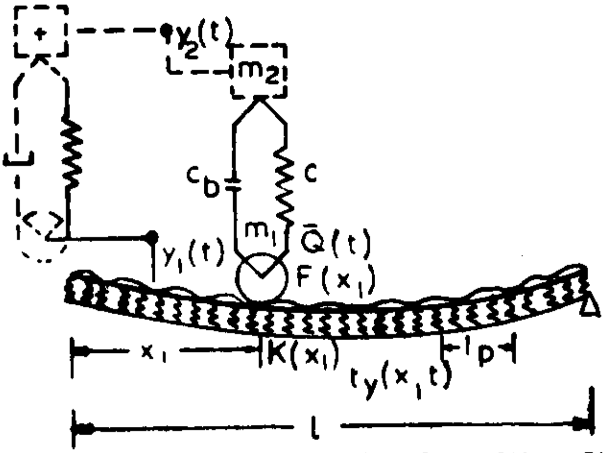

Fig. 2. Moving Sys𝔤m with Two Degrees of freedom and Force $Q(t)$

The unsprung mass m_1g is assumed to be acted upon by a harmonic force given by:

$$\bar{Q}(t)=Q \sin \Omega t \tag{3}$$

where Q is the amplitude of the force, $\Omega=2\pi v/\beta$ is the circular frequency of the force and β is the circumference of the driving wheels.

The top surface of the beam is covered with an elastic layer of variable stiffness and it is given by

$$k(x)=k_1+k_2 \cos 2\pi x/l_p \tag{4}$$

The meaning of k_1 and k_2 is defined in Fig. 3(a) where l_p is the length equal to the span of the longitudinal beams, spacing of sleepers or cross beams etc. Equation (4) expresses the idealised elastic properties of a roadway which may be expected to vary harmonically along the length of the bridge. This is a generalised equation which takes into account the cross beam effect, if $k_2=0$. The effect of the spring k_2 represents the spring constant of tyres.

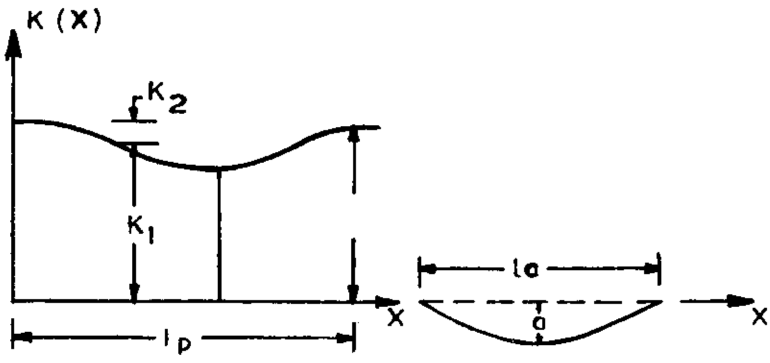

Fig. 3 Properties of the Elastic Layer Below the Vehicles.

Referring to Figs. 1 and 2 the track irregularity is assumed to vary harmonically along the bridge span in the form:

$$\bar{r}(x)=\tfrac{1}{2}la(1-\cos 2\pi x/l_a) \quad (5)$$

where

$\bar{r}$=maximum depth of the track unevenness

l_a=length of the track irregularity (Fig. 3b).

Making use of the above assumptions the equations are formulated with respect to figures 1 and 2 resulting in a set of three simultaneous differential equations with variable coefficients describing respectively the vertical displacement of sprung and unsprung masses and the beam vibration within the interval $0 \leqq t \leqq l/V$, $0 \leqq x \leqq l$. The equations related to sprung mass, unsprung mass, and beam vibrations are of the following form (Fryba, 1972).

Sprung mass:

$$m_2\ddot{y}_2+c(\dot{y}_2-\dot{y}_1)+k(y_2-y_1)=0 \quad (6)$$

Unsprung mass:

$$P+\bar{Q}(t)-m_1\ddot{y}_1+k(y_2-y_1)+c(\dot{y}_2-\dot{y}_1)-\bar{R}(t)=0 \quad (7)$$

$$m_1\ddot{y}_1-c(\dot{y}_2-\dot{y}_1)-k(y_2-y_1)-P-\bar{Q}(t)+\bar{R}(t)=0 \quad (8)$$

Beam vibration:

$$EI\frac{\partial^4 y}{\partial x^4}(x, t)+\mu\frac{\partial^2 y}{\partial t^2}(x, t)+2\mu\omega\frac{\partial y}{b\partial t}(x, t)=\delta(x-x_1)\bar{R}(t) \quad (9)$$

where

$$\bar{R}(t)=k(x)[y_1(t)-y(x_1, t)+\bar{r}(x_1)]\geqq 0 \quad (10)$$

is the force by which a moving system acts on a beam at the point of contact x_1 and the over-dots correspond to the differentiation with respect to time, t.

The set of equations (6) to (9) should satisfy the boundary conditions of simply supported beam, i.e.,

$$y=\frac{\partial^2 y}{\partial x^2}=0 \quad \text{at } x=0 \text{ and } l \quad (11)$$

and the initial conditions become:

$$y(x, 0);\ y_1(0)=y_{10};\ y_2(0)=y_{20}$$

$$\left.\frac{\partial y(x, t)}{\partial t}\right|_{t=0};\ \left.\frac{dy_1(t)}{dt}\right|_{t=0}=\dot{y}_{10};\ \left.\frac{dy_2(t)}{dt}\right|_{t=0}=\dot{y}_{20} \quad (12)$$

The equations 6 to 9 are very general statement of the problem of vibration excited by a system of masses moving along a beam.

The above equation are reduced to dimensionless form by introducing the following parameters:

$$x=l\xi;\ t=\frac{\tau l}{\pi v};\ y^*(\xi,\tau)=y(x,t)/yo,$$

$$y_i^*(\tau)=y_i(t)/yo; \tag{13}$$

where $i=1, 2, \ldots$ etc. and $y_0=2Pg/G\omega^2$ (1) represents the deflection at mid span of the beam loaded with force, P, at $x=l/2$. Substituting the above non-dimensional parameters, equations (6) and (7) reduce to:

$$\ddot{y}_2=\frac{1}{\alpha^2}\gamma_2^2(y_1-y_2)+\frac{\psi_2\gamma_2}{\pi\alpha}(\dot{y}_1-\dot{y}_2) \tag{14}$$

$$\ddot{y}_1=\frac{1}{\alpha^2}\frac{1+\chi_0}{2\chi\chi_0}(1+Q-R)-\frac{1}{\alpha^2}\frac{\gamma_2^2}{\chi_0^{1/2}}(y_1-y_2)-\frac{\psi_2\gamma_2}{\pi\alpha\chi_0^{1/2}}(\dot{y}_1-\dot{y}_2) \tag{15}$$

where the asterisks on y_1 and y_2 are omitted for simplicity and the over-dots stand for the differentiation with respect to t. Comparing equation (14) with the corresponding equation of (Fryba, 1972; Eqn: 8.43) there is a mistake in the second term of the RHS of Fryba's equation. In the case of Fryba in the denominater we have χ_0. Actually it should be $\chi_0^{1/2}$ which is taken care of in equation (14) of the present paper. The general expression for the dynamic displacement 'y' can be written in terms of Fourier sine series in the form:

$$y(\xi,\tau)=\sum_{J=1}^{\infty} q_J(\tau)\sin(J\pi\xi) \tag{16}$$

Non-dimensionalising the equation (9) and using equation (16) we find that each Fourier coefficients $q_J(\tau)$ satisfy the equations:

$$\ddot{q}_J=R/\alpha^2\sin(J\tau)-4\pi^2 f(1)^2\frac{l^2}{\pi^2 q^2}q_J-\frac{\psi}{\pi\alpha}\dot{q}_J \tag{17}$$

Fryba (1972) has shown that in the case of a constant moving force and in the case of harmonic force, series (16) converges at a rapid rate. Thus in dealing with the large-span bridges it will be sufficient to consider only the first term of the series (16). Then taking $q_1(\tau)=q$, equation (17) can be replaced by a single equation:

$$q=R/\alpha^2\sin(\tau)-\frac{1}{\alpha^2}q(\tau)-\frac{\psi}{\pi\alpha}\dot{q}(\tau) \tag{18}$$

where:

$$\bar{R}(\tau)=\frac{\bar{R}(t)}{P}=\gamma_1'(1+a\cos b\tau)$$

$$[y_1(\tau)-q(\tau)\sin\tau-\gamma(\tau)]=\geqslant 0 \tag{19}$$

is the dimensionless force according to equation (19) by which a moving system acts on a beam at the point of contact.

$$Q(\tau)=\frac{\bar{Q}(t)}{P}=a_1\sin(b_1t)\text{ is the dimensionless force}$$

$$\text{and } T(t)=\frac{\bar{\gamma}(x_1)}{y_0}=\tfrac{1}{2}a_2[1-\cos b_2\tau] \tag{20}$$

is the dimensionless ordinate of track irregularities. The initial conditions are:

$$q=q_0;\ \frac{dq(\tau)}{d\tau}=\dot{q}_0 \qquad \text{at } t=0 \tag{21}$$

Though the equations and the boundary conditions represented in equations (21) are the same as those of Fryba (1972) we take different values of the parameter associated with the free vibration data of the bridge analysed as a 2-D structure in evaluating the dynamic coefficient, δ. Because of the change in values of the frequencies and modal values of parameters α, χ, ψ will be effected.

Non-Dimensional Parameters:

The various non-dimensional parameters used in the analysis are as follows:

(i) Speed parameter $\alpha=c/2f_{(1)}l$.

(ii) The ratio between the weights of the vehicle and beem $\chi=P/G$, where $G=\mu gl$ is the self weight of the beam.

(iii) The ratio between the weights of unsprung and sprung parts of vehicle

$$\chi_0=P_1/P_2.$$

(iv) The frequency parameter of unsprung mass:

$$\gamma_1=\frac{f_1}{f_{(1)}}$$

where f_1 is the natural frequency of the unsprung mass and

$$f_{(1)}=\frac{1}{2\pi}(k_1/m_1)^{1/2}.$$

(v) The frequency parameter of sprung mass:

$$\gamma_2=f_2/f_{(2)}$$

where f_2 is the natural frequency of the sprung mass and

$$f_{(2)}=\frac{1}{2\pi}(k_2/m_2)^{1/2}.$$

(vi) Parameter a expressing the variable stiffness of the elastic layer of the roadway

$$a=k_2/k_1.$$

(vii) Parameter, b, a function of beam span and length

$$b=2l/l_p.$$

(viii) The logarithmic decrement of vehicle sprung damping

$$\psi_2=C/2m_2f_2.$$

(ix) The initial parameters

$$\tau_0,\ y_{20},\ \dot{y}_{20},\ y_{10},\ \dot{y}_{10},\ q_0,\ \dot{q}_0.$$

For the bridge grid under consideration free vibration analysis is carried out and the results used with respect to the dynamic response of the bridge grid are reduced to a beam subjected to a loading system as shown in Fig. 2 and the solution to the equations (14), (15) and (18) are obtained by numerical methods.

NUMERICAL SOLUTION

For the numerical solution, the method developed by Gill (1951) is used. He developed a step-by-step integration procedure based on the Runge Kutta method (Ralston and Wilf 1965). This procedure has the following advantages over other available methods: (i) This needs less storage register (ii) it controls the growth of rounding errors and is usually stable (iii) it is computationally economical. The procedure was programmed for an IBM 360 computer and later on for DEC 1090 computer.

The solution starts with the printing of input data and the calculation and printing of α_1 and α_2. Following this, Gill's method is applied to the computation of various functional values in the specified time steps. The computed values of τ, $y_2(\tau)$, $y_1(\tau)$, $q(\tau)$ and $R(\tau)$ are printed at every Nth step, the number N having chosen in advance. At the end of the computation, the machine also prints the largest value of q and R i.e. maximum q and maximum R and the value of τ at which the maximum occurs.

The choice of the integration step length, h, was made by Collatz (1942) method. The accurate results are obtained for steps of $h = \pi/2000$.

Using the input data as shown in Table (1) the effect of some of the dimensionless parameters on the maximum value of $q(\tau)$, i.e. on the maximum value of the beam deflections are considered. These initial input data are valid for large span bridges of the span length equal to 50 m, wherein bridge is assumed to be traversed by electric or diesel locomotives.

In the figures referred to in our discussion the maximum value of $q(\tau)$ is represented by δ, such that:

$$\delta = \text{Maxm } q(\tau)$$

where δ is the dynamic coefficient.

TABLE 1. Input Parameters

$\tau_0 = 0.0$	$Y_{20} = \frac{1}{13}$	$\dot{Y}_{20} = 0.0$	$y_{10} = 1/13$	$\dot{y}_{10} = 0.0$	$q_0 = 0$
$q_0 = 0.0$	$a = 0.3$	$\gamma_1' = 1.0$	$b = 20$	$\gamma_2' = 0.2$	$a_1 = 0$
$a_2 = 0.0$	$b_1 = 0$	$b_2 = 0.00$	$\chi = 0.5$	$\chi_0 = 0.25$	$\psi = 0.08$
$\psi_2 = 0.5$	$S_1 = 0$	$S_2 = 0.00$	$\alpha = 0.12$	$h = \frac{\pi}{2000}$	$L = 0.0$

DISCUSSION AND CONCLUSIONS

The Effect of Speed

The effect of speed as defined by parameter, α is varied from 0 to 0.14 in the steps of 0.005. The variation of δ with respect of α is represented in Fig. 4. The graph displays generally ascending tendency with a number of local peaks. One of the first well defined peak occurs at $\alpha = 0.03$, while at for $\alpha > 0.04$, there is hardly any peak at all and remains constant from $\alpha = 0.08$ onwards. The reasons for the local peaks may be due to the fact that the system under study is very complex.

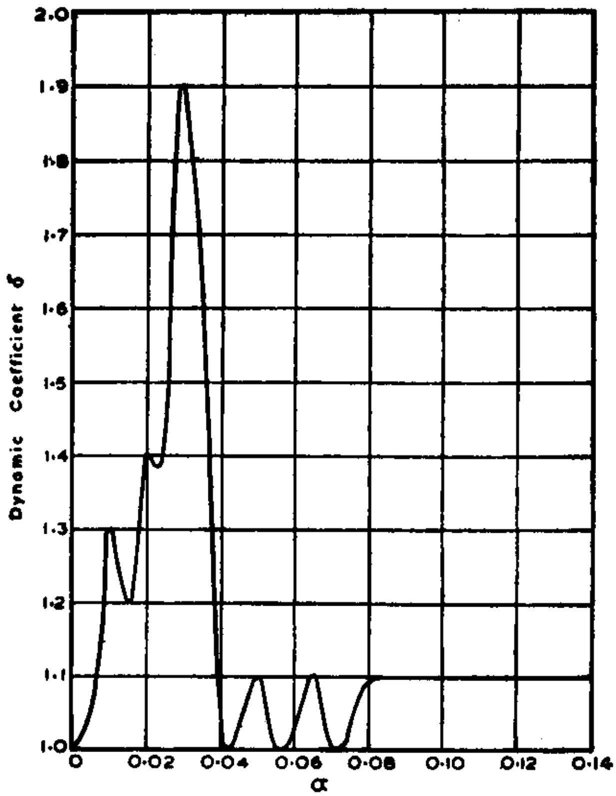

Fig. 4. Effect of the Speed Parameter, α

The Effect of Frequency Parameter of Unsprung Mass, γ_1

Fig. 5 represents the response curve of dynamic coefficient, δ, against speed parameter for different values of unsprung mass. Figure 6 relates to the variation of dynamic coefficient as against unsprung mass for different values of α. From Fig. 5 it can be seen as γ_1 falls off the dynamic coefficient δ grows very large and very quickly. The greatest effect takes place in the range of $0 \leqslant \alpha \leqslant 0.06$. From Fig. 6 it can be seen that for $\gamma_1 > 5$ and for given α, the dynamic effects are nearly constant. This means that a very hard elastic layer on the beam surface has virtually the same effect as an infinitely rigid layer (i.e. the unsprung mass is in direct contact with the beam).

The Effect of the Frequency Parameter of Sprung Mass, γ_2

The dependence of δ on (i) parameter α for various values of γ_2 and (ii) parameter γ_2 for various values of α is illustrated in Figs. 7 and 8 respectively. Figures indicate that the dynamic effects have a tendency to grow with growing γ_2. The maximum value of δ occurs when the value of γ_2 lies

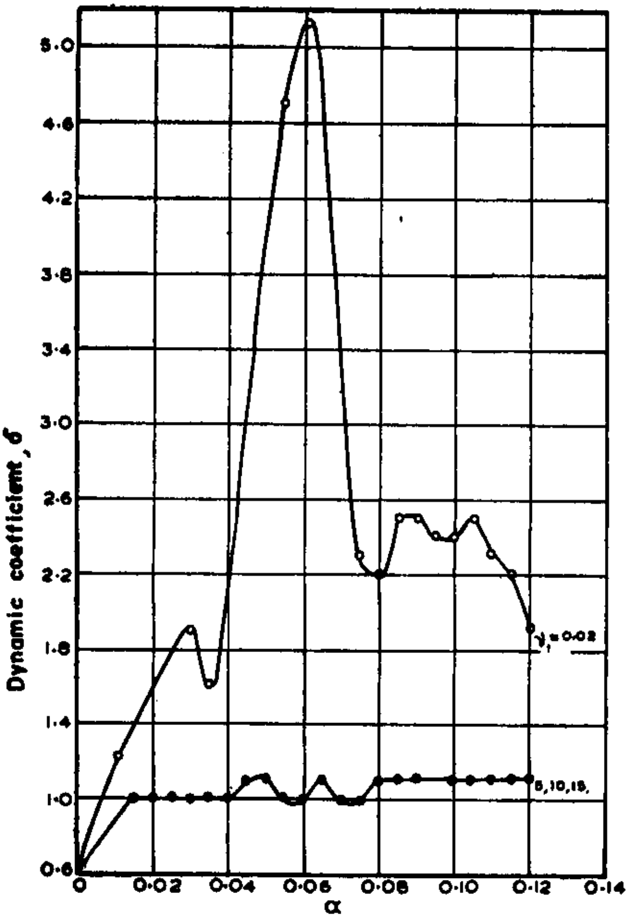

Fig. 5. Effect of the Frequency Parameter of the Unsprung Mass, γ_1

between 0.2 and 0.4 and in the range of α varying between 0.040 to 0.065. The diagram herein also represents number of local peaks.

Effect of Variable Stiffness of the Elastic Layer, *a*

Figure 9 depicts the relation of the dynamic coefficient, δ, with reference to the speed parameter, α, for different values of, a. It can be seen from the Fig. 9 that δ increases rapidly in the range of a varying between 0.100 to 0.300 and α lying between 0.040 and 0.065. The maximum value of δ occurs at $\alpha = 0.06$ and $a = 0.300$. It is observed from the numerical values, that δ is virtually independent of parameter a at $\alpha = 0.0225$ and varies continuously with respect to a. The same results can also be seen from Fig. 10.

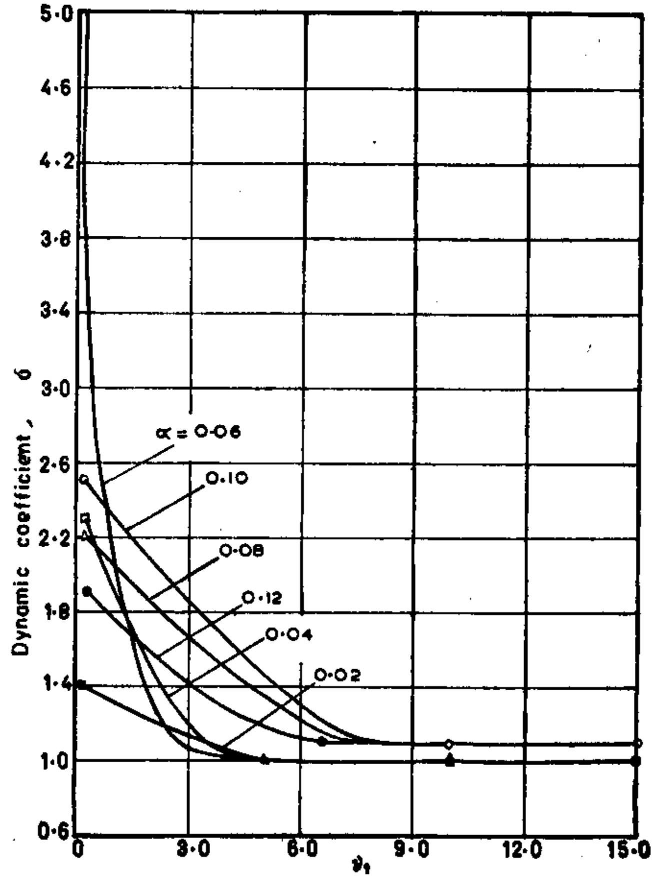

Fig. 6. Effect of the Frequency Parameter of the Unsprung Mass, γ_1

The Effect of χ, the Ratio between the Weights of Vehicle and Beam

The variation of δ on parameter, χ, at various values of α is shown in Fig. 11 and variation of δ on parameter α at various values of χ in Fig. 12. At constant α, δ and χ relation is fairly complicated, but generally it is seen that δ varies with increase in values of χ. The values of δ is maximum for $\chi = 0.50$ and 0.750 in the range of $\alpha = 0.05$ to 0.06. The diagram displays number of local peaks.

The Effect of the Ratio between the Weights of Unsprung and Sprung Parts of the Vehicle, χ_0

The variation of δ with respect to α for different values of χ_0 are plotted and are shown in Fig. 13. Judging by the general pattern of the diagram, at constant speed parameter, χ_0, bears but a small effect on the dynamic coefficient. From the graph it can be observed that the local peaks are much

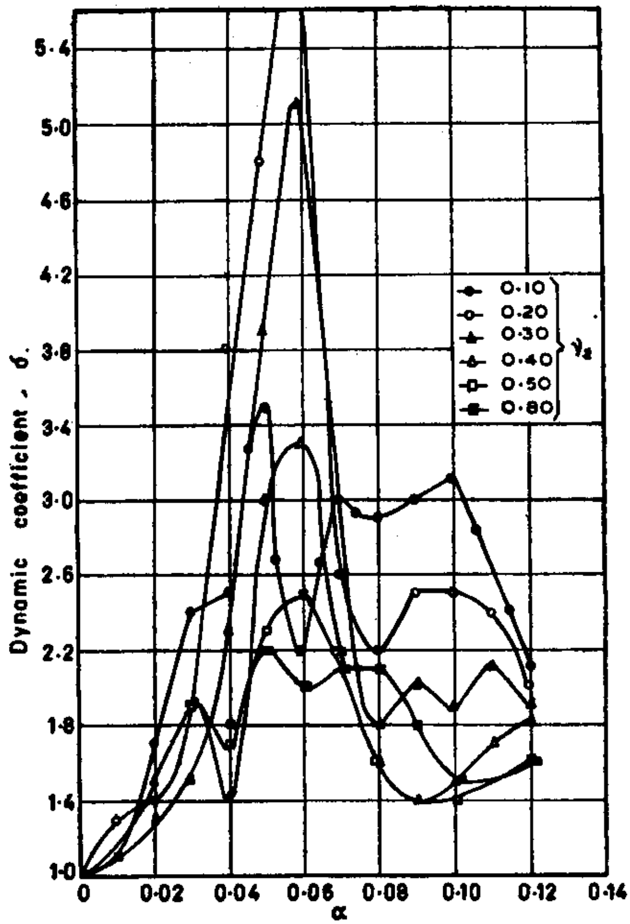

Fig. 7. Effect of the Frequency Parameter of the Sprung Mass, γ_1

minimised as it can be seen from the effect of the ratio between the weights of the vehicle and beam (Fig. 11). Maximum value of δ of 6.00 can be seen at $\alpha = 0.055$. It can also be seen from the Fig. 14 that the values of δ decreases as α increases.

The Effect of Vehicle Sprung Damping, ψ_s

The effect of the logarithmic decrement of vehicle sprung damping, ζ_s, is indicated in Fig. 15 and 16. Even though the dynamic coefficient, δ, falls off with growing ψ_s in most of the cases, they have an increasing tendency at some speeds with decrease in value of the ν_s (Fig. 15). From Fig. 15 it can be seen that δ increases to a very large value of 8.7 against a speed parameter $\alpha = 0.06$ and for $\psi_s = 0$.

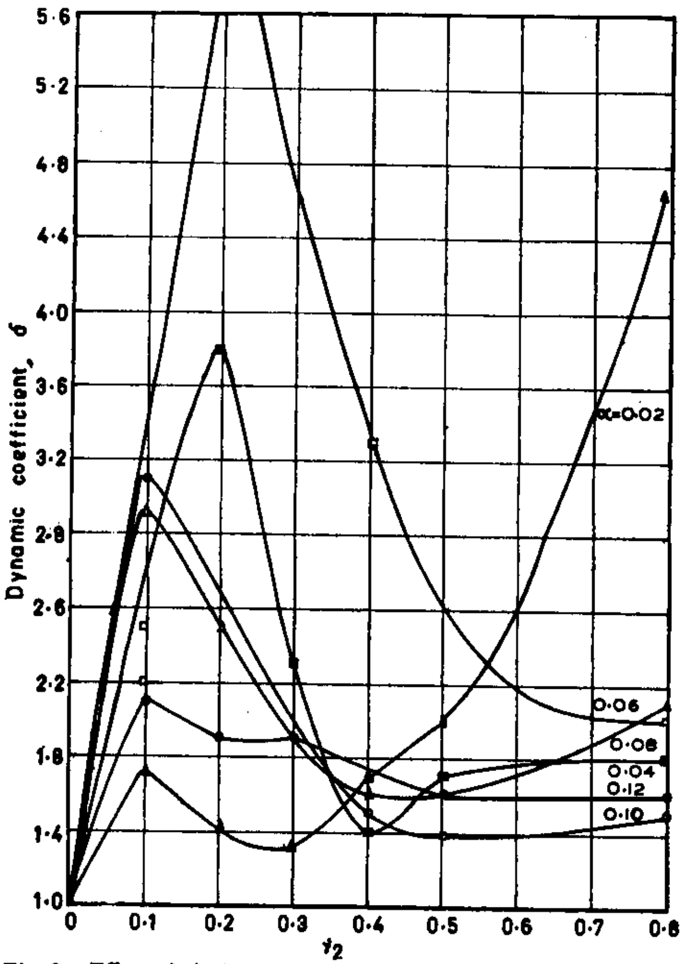

Fig. 8. Effect of the Frequency Parameter of the Sprung Mass, γ_2

The Effect of Span Rotio *b*

The dependence of δ on b reveals that at constant speeds the ratio of the beam span to longitudinal beam length has a slight effect on δ (Figs. 17 and 18). The variation of δ in b for speed parameter α is represented in Fig. 17. Here δ possesses the maximum value for different values of $b = 16, 20, 24$ and 28 which almost remains the same and represents local peaks at increasing speed parameter. The maximum value of 'δ' occurs in each case, 'α' lying at 0.04, 0.055, 0.06 and 0.0755.

COMPARISON OF RESULTS WITH THAT OF FRYBA

The results as obtained by Fryba (1972) has been illustrated in his Figs. 8.4 to 8.11 and 8.13. In the cases of the parameters considered δ vary

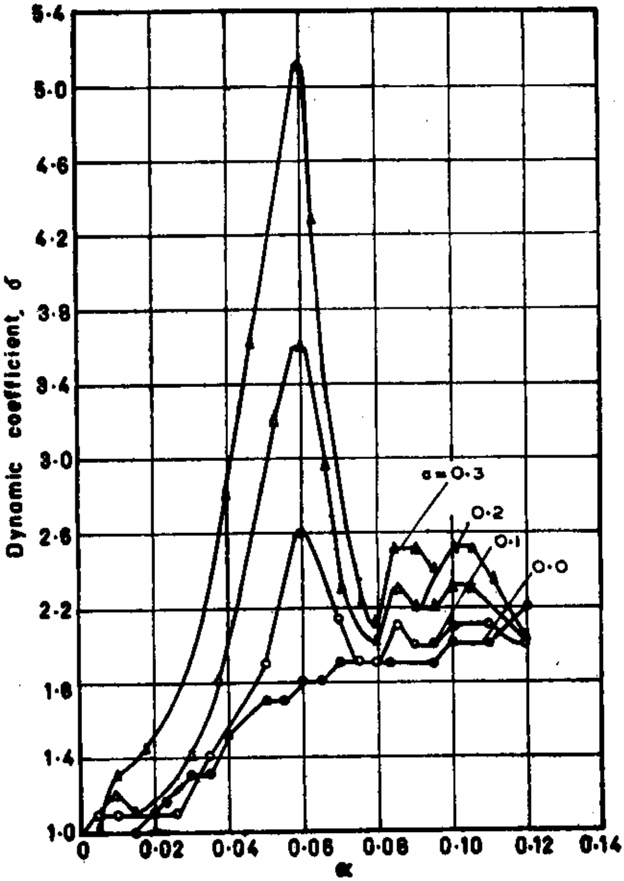

Fig. 9. Effect of Variable Stiffness of the Elastic Layer.

with increasing in value of α, whereas in the results reported in the paper, the maximum value of δ is obtained at the speed parameter lying in the range of 0.045 to 0.065. At higher speeds the value of the dynamic coefficients gets reduced and it is appreciable. Other than these the other effects like local peaks at $\alpha = 0.02$ onwards the δ variation remains constant with respect to speed parameter α, with almost having the same values of δ. The effect of frequency parameter of unsprung mass, ν_1, on δ variation, in case of Fryba and the present work remains the same with change in the magnitude of the δ values. Similar variation can be observed with respect to effect of the variable stiffness of the layer a, χ, χ_0, ψ_2 and b.

CONCLUSIONS

From the results obtained, the variable quantity that mostly influence the dynamic stresses in bridges is the vehicle speed. Speaking generally, the

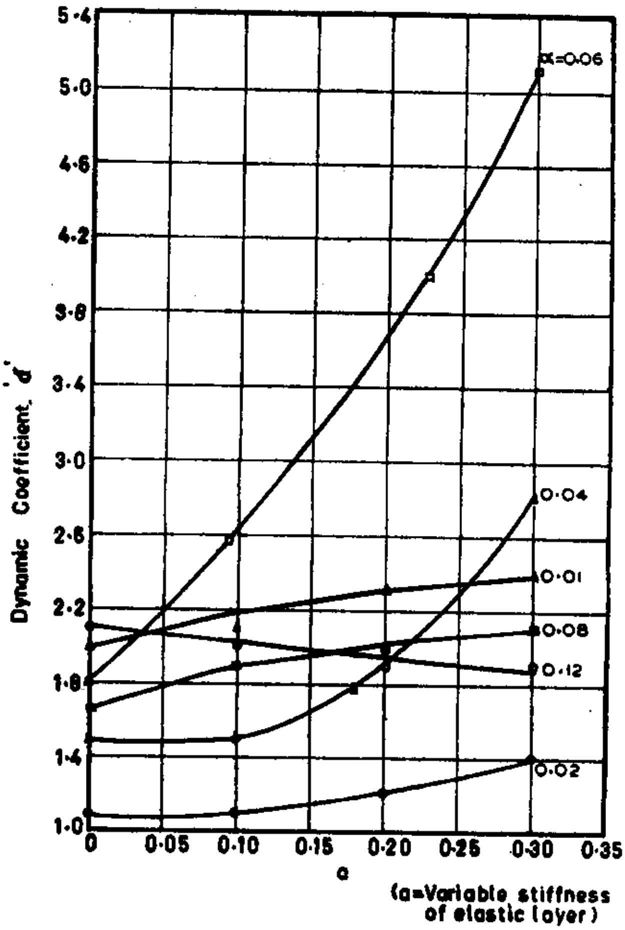

Fig. 10. Effect of the Speed Parameter α

high vehicle speed always produces an increase in the deflection and stresses in bridges.

Second is the cross beam effect, that is, uniform spacing of the sleepers (in case of railway bridges) and other regular unevenness enlarge the local peaks in the dynamic coefficient. Also it is seen from the results that sprung and unsprung masses of the vehicles contribute very much in increasing the value of the dynamic coefficient. This gives rise to an increase in the dynamic stresses e.g. in the case of a steam locomotive the predominant cause of dynamic stresses in the bridges are produced by the unbalanced weights of the counter wheels. More often they are responsible for the single well defined peak that appears in the resonance curves at critical speed. Now-a-days, the dynamic effects of railway vehicles grow larger approximately proportional to the frequency of sprung masses and vehicle weight. In large span bridges a predominant role is played by the initial conditions of motion as the vehicle

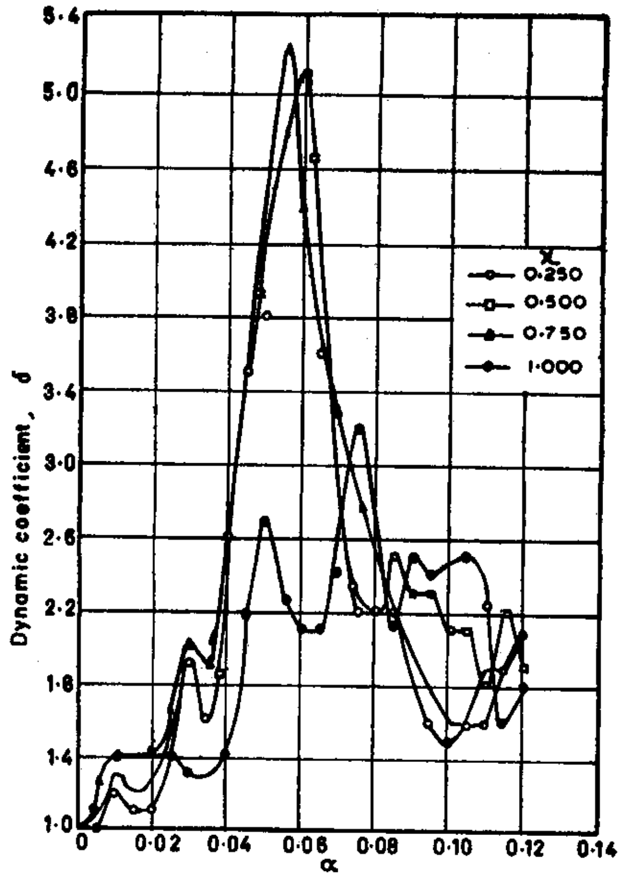

Fig. 11. Effect of the Ratio between the Weights of the Vehicle and the Beam, χ

starts to traverse the bridge, hence the increase in dynamic stresses by the state of track ahead of the entrance to the bridge. The theory reported herein is very well suited for highway bridges made of steel, reinforced and prestressed concrete of large spans.

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SYMBOLS

a	—	Variable stiffness of the elastic layer of the road way
b	—	function of the beam span and length

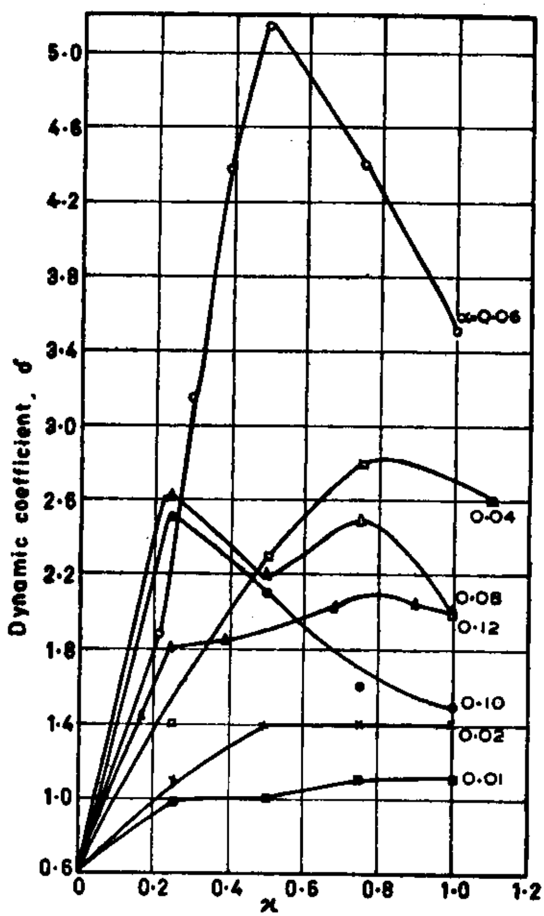

Fig. 12. Effect of the Ratio between the Weights of the Vehicle and the Beam, χ

C	—	viscous damping coefficient
E	—	Young's modulus
f_1	—	natural frequency of unsprung mass
f_2	—	natural frequency of sprung mass
g	—	acceleration due to gravity
G	—	self weight of the beam
h	—	integration step length
I	—	moment of inertia
k	—	elastic spring constant
k_1	—	spring constant of elastic layer below the tyre
k_2	—	spring constant of tyres

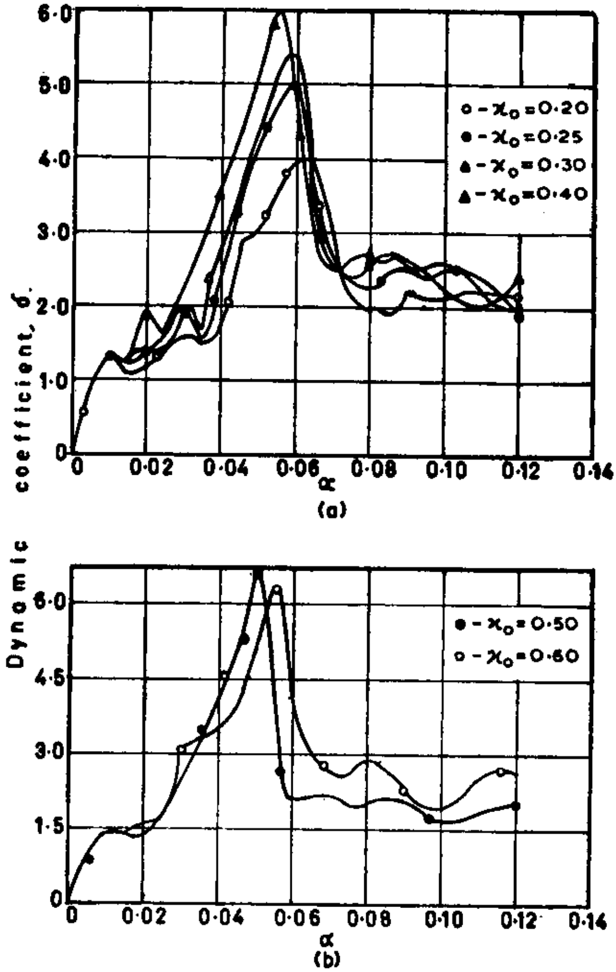

Fig. 13. Effect of the Ratio between Weights of the Unsprung and Sprung Vehicle Parts

l_a	—	length of the track irregularity
l_p	—	span of the longitudinal beams
m_1	—	mass of the unsprung mass
m_2	—	mass of the sprung mass
P	—	total weight of the vehicle
P_1	—	weight of the unsprung mass of the vehicle
P_2	—	weight of the sprung mass of the vehicle
q	—	Fourier coefficients
Q	—	amplitude of the force $\bar{Q}(t)$

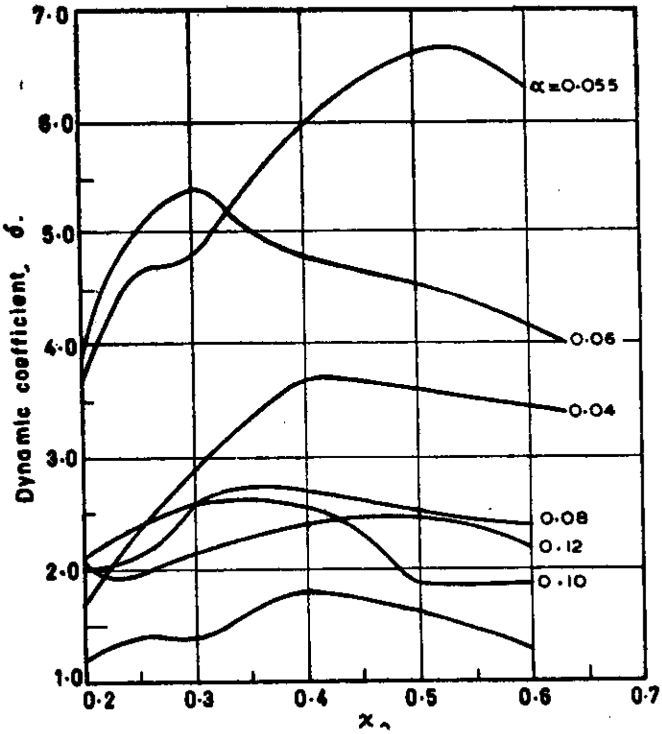

Fig. 14. Effect of the Speed Parameter, α

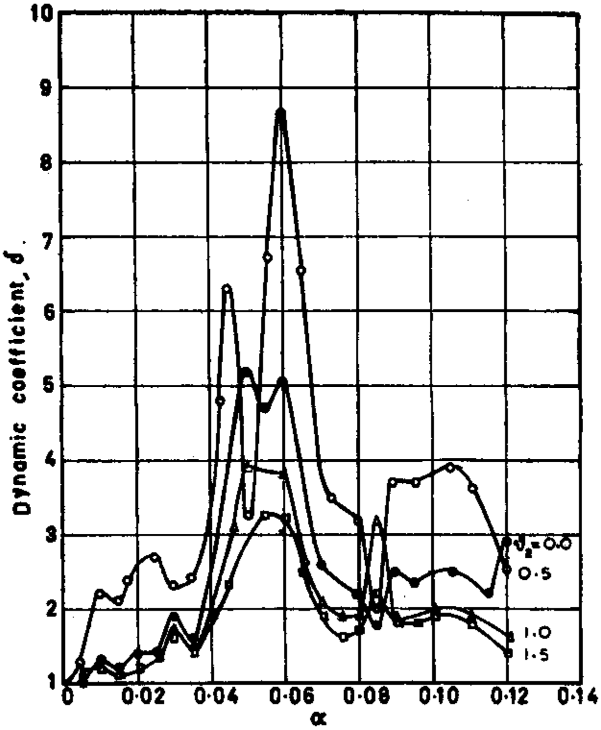

Fig. 15. Effect of the Vehicle Spring Damping, ν_2

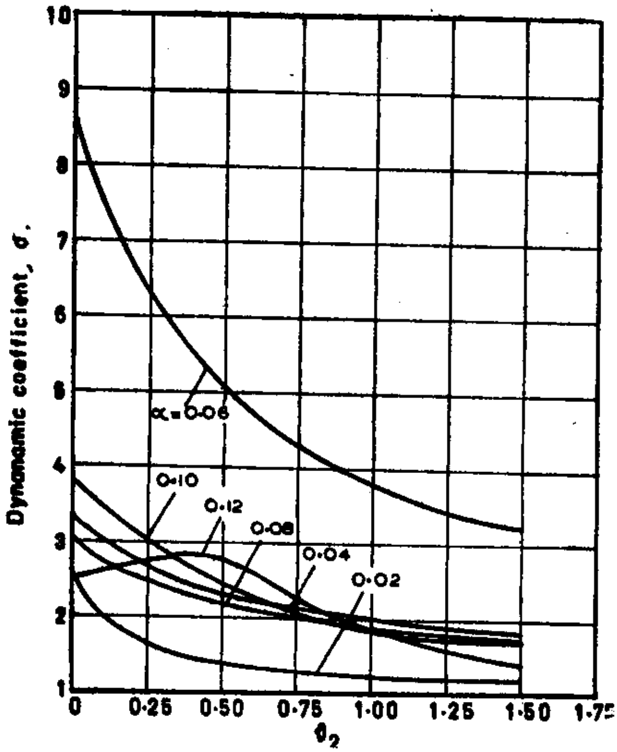

Fig. 16. Effect of the Speed Parameter, α

$\bar{Q}(t)$	—	harmonic force acting on the unsprung mass (m_1)
r	—	maximum depth of the track unevenness
$\bar{r}(x)$	—	assumed track irregularity
T	—	dimensionless parameter of track irregularity
t	—	time
v	—	velocity
x	—	horizontal coordinate
y	—	vertical displacement
α	—	speed parameter
β	—	circumference of the driving wheel
γ_1	—	frequency parameter of the unsprung mass
γ_2	—	frequency parameter of the sprung mass
δ	—	dynamic coefficient
μ	—	mass density
ψ, ψ_1, ψ_2	—	logarithmic decrements of vehicle spring damping

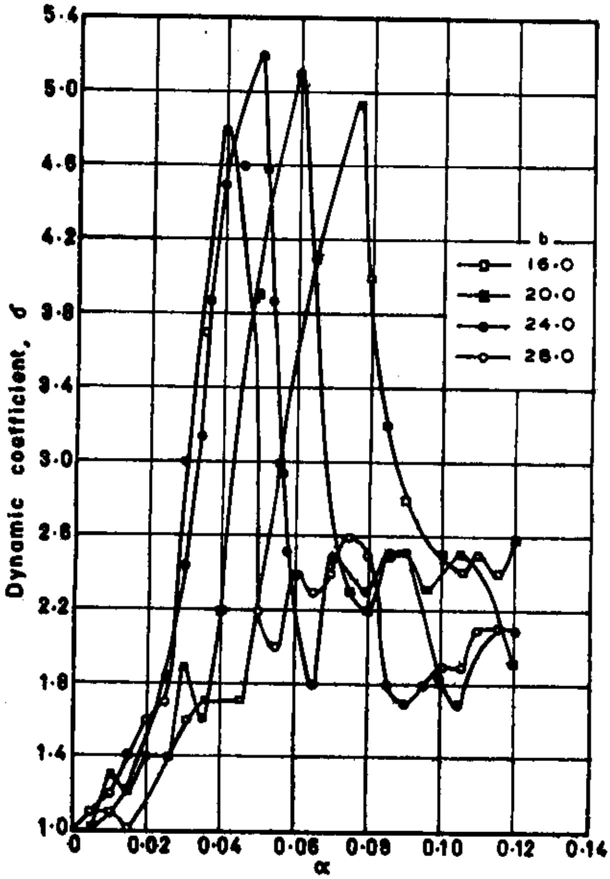

Fig. 17. Effect of Parameter, $b(l/l_p)$

χ — ratio of the weight of the vehicle to that of the beam

χ_0 — ratio of the weight of the unsprung mass to that of the sprung mass

Ω — circular frequency of the driving wheel.

Superscripts

$\cdot$ — first differentiation with respect to time

$\cdot\cdot$ — second differentiation with respect to time.

Subscripts

0 — initial condition

1 — unsprung mass

2 — sprung mass.

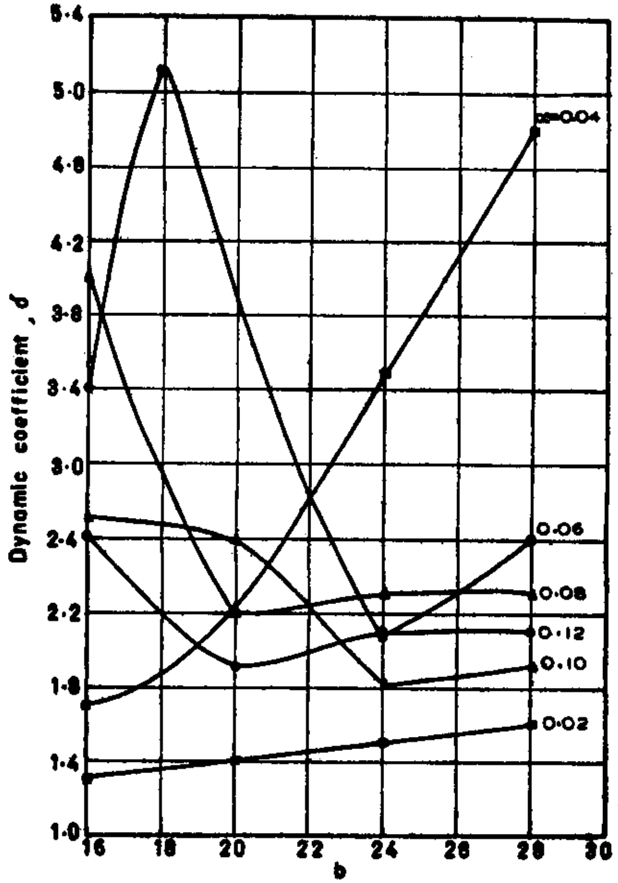

Fig. 18. Effect of Parameter $b(l/l_r)$

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