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FUNDAMENTAL PERIOD FOR SHEAR WALLED STRUCTURES

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ABSTRACT

The paper proposes a rigid-ended element for studying fundamental period of reinforced concrete frames having significant joint sizes, shear walls and frame-shear wall structures. The stiffnes and the consistent mass matrices of the proposed element have been developed. It is shown that the well known empirical expression do not carrectly predict the fundamental period as compared to the analytical values. The paper also gives a brief description of the computer program prepared for this purpose.

Introduction :

Use of framed skeletal structures becomes uneconomic beyond a certain height, requiring construction of shear walls which are connected to the frame by linkage beams at each floor level. Shear walls may also be arranged to form bearing wall type structures for lesser heights. Shear walls are primarily designed to resist lateral loads arising from wind, seismic or blast effects. Since these loads are vibratory in nature, and their magnitudes depend on the dynamic characteristics of the structure itself, the present study has been undertaken to develop a procedure to estimate the fundamental time period of vibration for shearwalled structures using an idealisation which makes the procedure computationally efficient for handling large real life structures.

Figs.1 and 2 snow a shear wall and a shear wall-frame structure. As a first step, the shear wall and the shear wallframe structure are idealised by rigid-ended elements havig an over-all length L, rigid portions of lenthgs a and b at the two ends and an intermediate elastic length l (=L—a—b) having the usual cross-sectional properties like the area A, the moment of inertia I, etc. It has been found (1) that for proper simulation of axial characteristics, A, I etc. must be associated with the entire length L. The rigid-ended elements may also be used for idealising framed structures in which joint size effect is considered significant.

The paper presenrs dynamic analysis for fundamental time period for

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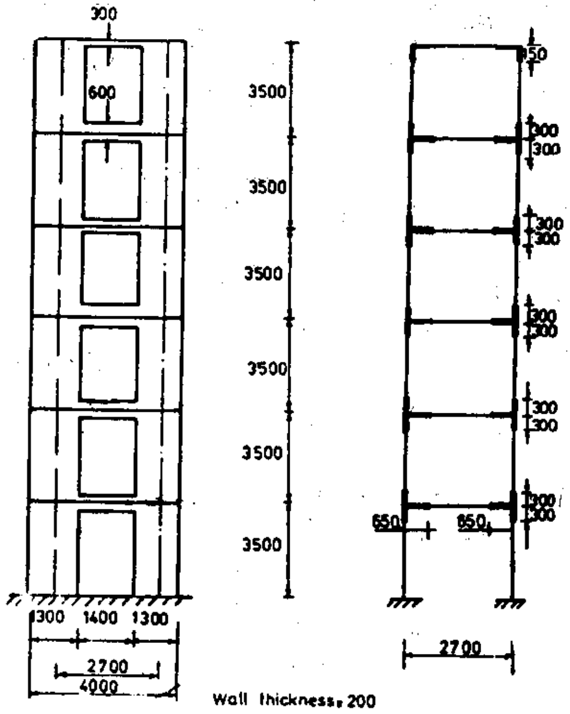

Fig. 1 Idealisation of a shear wall

frames, shear walls and shearwall-frame structures using the proposed rigid-ended elements.

Most codes of practice recommended (2) the use of one of the two expressions given below for estimating fundamental period T in seconds for moment resisting frame without bracings or shear walls

$$T = 0.1\ N \qquad (1)$$

$$T = 0.09\ H\sqrt{D} \qquad (2)$$

where N is the number of storeys, and H and D are the height and the width in metres in the direction of the lateral force in metres. These expressions are thus unsuitable for use for shear walled structures. They also do not take congnisance of either the maierial of construction or the extent of occpancy. The present paper also gives the comparison of the results obtained by the proposed analysis with the estimates by the above expressions.

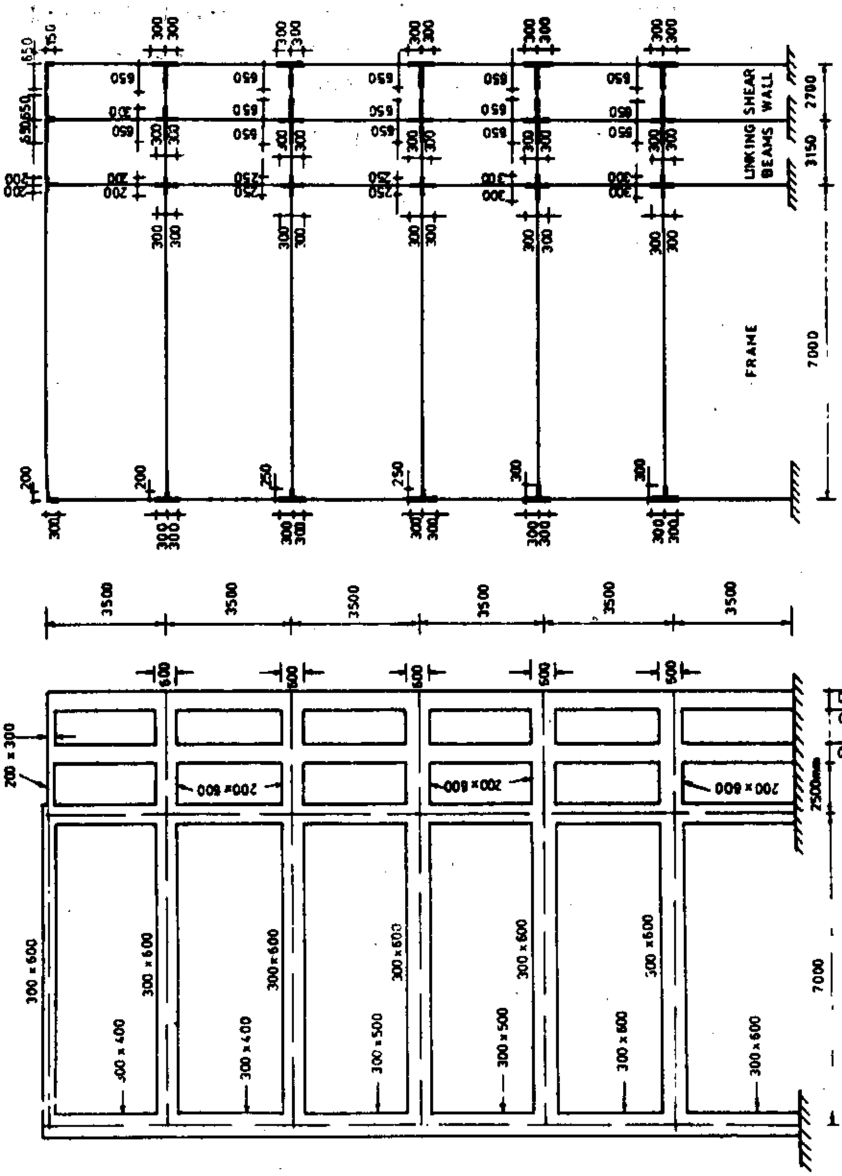

Fig. 2, Idealisation of a Frame-shear wall Structure

2. Method Of Analysis

The given structure, whether a frame, a shear wall or a shear wall-frame combination, is idealised by rigid-ended elementns having lengths of rigid portions equal to half the depth of the intersecting members. The centre-line distances remain unchanged. Since the influance of different degrees of freedom (DOF) on the fundamental period of the structure idealised in this manner is not obvious, three DOF consisting of two translations and one rotation are assigned to each node. The dynamic equation of equilibrium for free undamped vibrations take the following form :

$$[M] <\ddot{Y}> + [k] <Y> = <0> \qquad (3)$$

Where [M[and [K] are symmetric structure mass and stiffness matrices respectively of order nxn, and $<Y>$ and $<\ddot{Y}>$ are the displacements and acceleration vectors respectivcly of order nx1; where n is the total DOF of the struciure equal to 3 times the number of nodes.

To develop equations (3), stiffness and consistent mass metrices are evaluated for each element in local axes and then transformed to a common set of axes. The structure stiffness and mass materices are than assembled in the usual manner. For introducing boundary constraints, terms in the corresponding rows and columns of both [M] and [K] matrices are made zero except the diagonal terms which are chosen judiciously so that the corresponding frequencies are abnormally high and easily recognisable. Equation (3) then leads to the following eigen problem :

$$([K] - w^2 [M]) <\phi> = <0> \qquad (4)$$

There are several methods for solving the above eigen-problem for natural frequencies wi, i = 1,2,..n and the corresponding eigen vectors. In the present study, the sub-space iteration procedure (3) has been used to determine the first few lowest frequencies and the corresponding normalised model vectors $<\phi>_i$ i, = 1,2,...n.

A combuter program, DYNEL, has been written in Fortan based on the above formulation. The program has options to use ordinary prismatic rigid-ended elements. Fig. 3 shows the flow chart given major steps in the algorithm of the program.

3. Characteristics Of Rigid-Ended Elements

The stiffness and the mass matrices constitute the element charcteristics required for setting up the dynamic Eqn. 3 stated above. The stiffness matrix of rigid ended elements having unequal rigid portions a and b has earlier been used extensively by the authors in studying elastic-plastic behaviour of framed stbructures (1). The same is reproduced in the Appendix for ready reference.

The use of consistent mass materix [M] for rigid ended elements will be more appropriate than the use of lumped mass matrix, for greater accuracy and especially, in view of the three DOF assigned to each node.

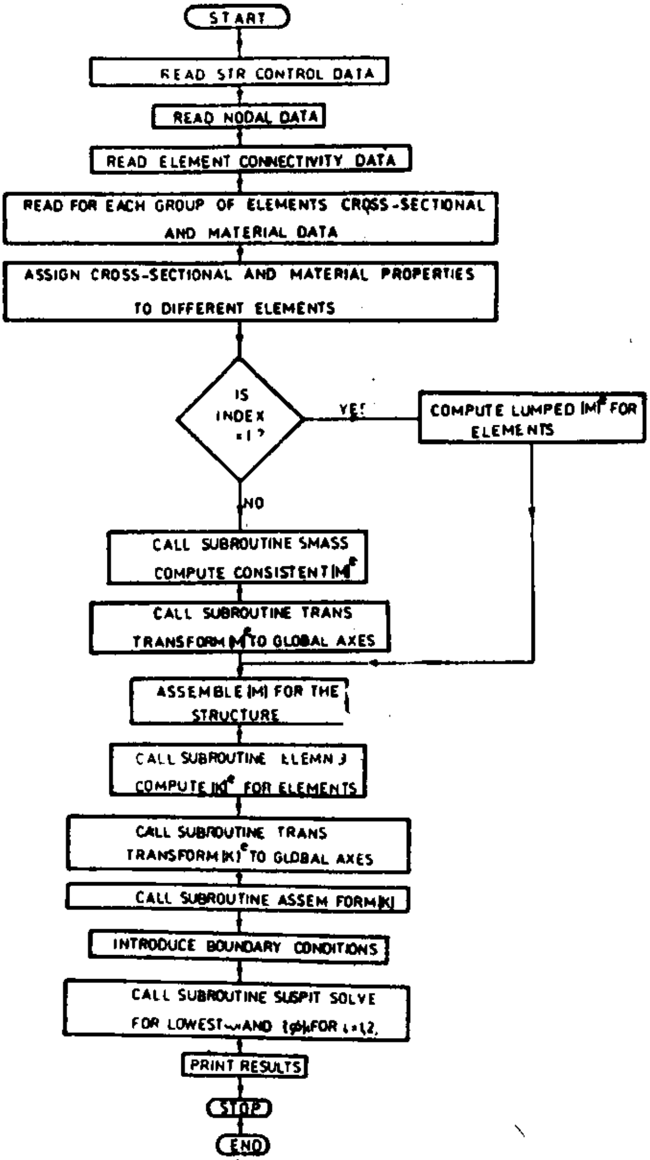

Fig. 3 Flow Chart for the Program 'DYENL'

The terms of the matrix [M] are easily shown (4) to be given by the following expression.

$$m_{ij} = \int_0^L \overline{m}(x)\,\Psi_i(x)\,\Psi_j(x)\,dx \qquad (5)$$

where m(x) is the distributed mass per unit length of the member, and Ψ' r (x) is the function giving dynamic displacement along the member resulting from unit dinamic displacement along the rth DOF. It is further assumed that dynamic displacement funtions are similar to those oblained from static considerations.

Fig. 4 shows a rigid ended element with the six DOF at its two ends, Fig. 5 shows the deformed shapes of the element under unit nodel displacements at the ends. The displacement functions may be defined as follows in Table 1. These functions are written with the origin being different for diffent portions of the element for convenience.

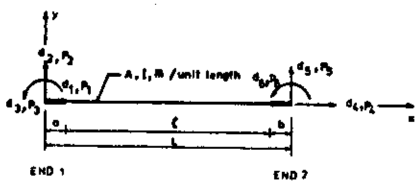

Fig. 4. Rigid - Ended Element

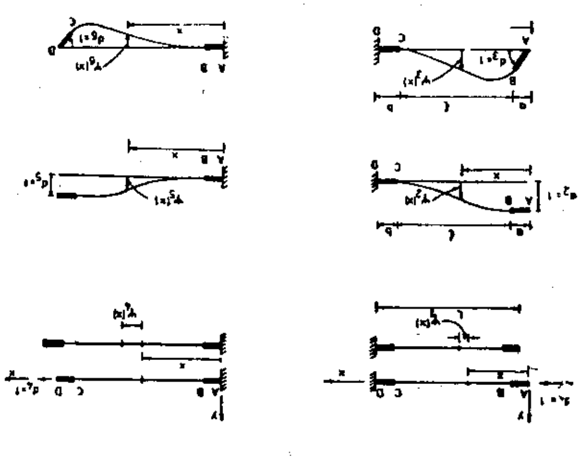

Fig. 5. Deformed Shapes under unit Nodal Displacements

Table 1 : Displacement Functions For Unit Nodal Displacemens

Imposed Nodal Displacement	Portion	Origin	Displacement Functions
d1 = 1	AD	A	$\Psi 1 (x) = 1 - x/1$
d2 = 1	AB	A	$\Psi 2 (x) = 1$
	BC	B	$\Psi 2 (x) = 1 - 3 (x/1)^2 + 2 (x/1)^3$
	CD	C	$\Psi 2 (x) = 0$
d3 = 1	AB	A	$\Psi 3 (x) = x$
	BC	B	$\Psi 3 (x) = a [1 - 3 (x/1)^2 + 2 (x/1)^3] + x (1 - x/1)^2$
	CD	C	$\Psi 3 (x) = 0$
d4 = 1	AD	A	$\Psi 4 (x) = x/L$
d5 = 1	AB	A	$\Psi 5 (x) = 0$
	BC	B	$\Psi 5 (x) = 3 (x/1)^2 - 2 (x/1)^3$
	CD	C	$\Psi 5 (x) = 1$
d6 = 1	AB	A	$\Psi 6 (x) = 0$
	BC	B	$\Psi 6 (x) = -b [3 (x/1)^2 - 2 (x/1)^3] + x^2 / 1 (x/1 - 1)$
	CD	C	$\Psi 6 (x) = (x - b)$

The evaluation of the integral in eqn. 5 has thus been carried out as follows; except for the terms corresponding to axial deformations.

$$m_{ij} = \bar{m} [\int_0^a \psi_i (x) \psi_j (x) dx + \int_0^l \psi_i (x) \psi_j (x) dx$$

$$+ \int_0^b \psi_i (x) \psi_j (x) dx \qquad (6)$$

where $\bar{m}$ is the mass per unit length assumed uniform over the entire length L.

The consistent mass matrix for the rigid-ended element has been aveluated explicitly and given below.

$$[M] = \frac{ml}{420}\begin{bmatrix} 140\frac{L}{l} & & & & & \\ 0 & 420\frac{a}{l}+156 & & & \text{SYMMETRICAL} & \\ 0 & 210\frac{a^2}{l}+221 +156a) & 140\frac{a^3}{l}+156a^2 +4l^2+44al) & & & \\ 70\frac{L}{l} & 0 & 0 & (140\frac{L}{l}) & & \\ 0 & 54 & (54a+13l) & 0 & (\frac{4206}{l}+156) & \\ 0 & (-54b-13l) & -(54ab+13al +13bl+3l^2) & 0 & -(156b+221 +210\frac{b}{l}) & 156b^2+4l^2 44bl+140\frac{b}{l}) \end{bmatrix}$$

The rotational matrix for transforming the element mass matrix from the local to the global axes is the same as that for the stiffness matrix.

4. Example of a Frame

As a first example, a reinforced concrete six storeyed frame consisting of floor beams of cross-section 300×600 mm, and columns of dimensions 300×400 mm (5th and 6th storyes), 300×500 mm (3rd and 4th storeys) and 300×600 mm (1st and 2nd storeys) as shown in Fig. 6, has been chosen for analysis. The bay width is 7 0 m and each storey height equal to 3.5 m. The frame has been idealised by rigid-ended elements as shown in Fig. 6 (b). It is seen that the frame has a total of 42 DOF, with six DOF restrained. Thus the frame has 36 freqencies and an equal number of modal vectors.

The above problem has been analysed for the six lowest freqencies on HP 150 microprocessor, using the program 'DYNEL'. Consistent mass matrices have been used for each rigid-ended element as given by Eqn. 7.

The lowest natural frequency w is found to be 6.203 rad./sec. with fundamental time period T equal to 1.01 seconds.

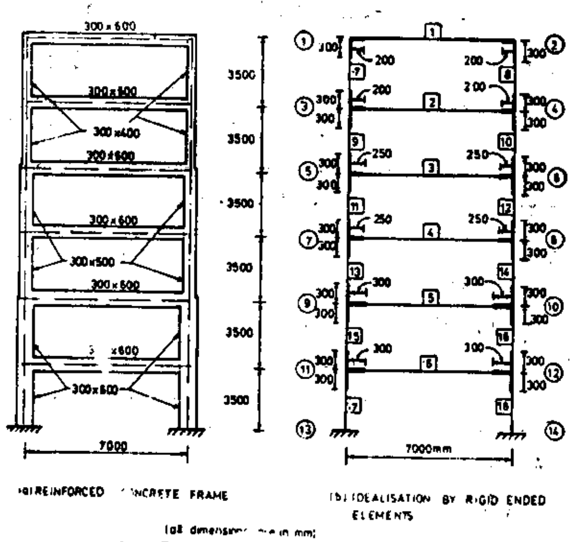

Fig. 6. A Six Storeyed Reinforced Concrete Frame

The same frame has then been analysed by assuming elements to be prismatic without end rigid protions. The lowest natural frequency is found to be 7 693 rad/sec with the fundamental time period T equal to 0.817 seconds. It is seen that even in case of not too-wide flanged column frames, idealisation of the frame by rigid-ended elements, thereby taking into account the joint size effect, makes significant difference in the value of the fundamental period

5. Example of a Shear wall

The six storeyed 21.0 m high reinforced concrete shear wall in Fig. 1 has next been analysed for the fundamental period using rigid-ended elements The shear wall has left and right piers 1300 mm wide with 1400 mm wide central openings. All floors beams are 600 mm deep, the roof beam being 300 mm deep. The wall and the beams are 200 mm thick.

Fig. 1 (b) shows idealisation of the shear wall by 18 rigid-ended elements. The beams have 650 mm long rigid portions are 0, 150 mm and 300 mm.

The natural frequency of the shear wall, as idealised above, is found to be 29 628 rad./second with the fundamental period equal to 0.212 second. These values are examined later in the light of Eqns. 1 and 2.

6. Example of a Frame-Shear Wall Structure

The frame shown in Fig. 6 and the shear wall shown in Fig. 1 have been combined to give a frame-shear wall structure of Fig. 2 to be analysed next for the natural period. The linking beams are 200×600mm at all floor levels, the roof level linking beam being 200×300mm. The total height of the six storeyed structure is 21.0m.

Fig. 2 shows the idealisation of the frame-shear wall structure by 42 rigid-ended elements with 72 unrestrained DOF. The lenghts of the rigid portions of different members are varied depending upon the dimensions of the intersecting members. The structure has been analysed by the pragram 'DYNEL' for the six lowest frequencies out of the total 72 values. The lowest frequency is 20.191 rad./second giving the fundamental period equal to 0.311 second.

7. Discussion of Results

It is worthwhile to examine the results of analysis obtainned above in relation to one another and in the light of Eqns. 1 and 2. Table gives the comparison in a summarised form.

Table 2 : Comparison of Results

Structure	Fundamental Period in Seconds					
	Analysis	Eqn. 1 (T=0.1 N)	Eqn. 2 (T=0.9H/√D) D=7m	D=2.7m	D=4.0m	D=12.85m
Frame with out rigid ends	0.817	0.6	0.714			
Frame with rigid ends	0.013	0.6	0.714			
Shear wall	0.212	0.6		1.150	0.945	
Frame shear wall	0.311	0.6				0.527

Note that Eqns. 1 and 2 are not strictly applicable to the shear wall and the frame-shear wall structures. Also not that whereas the lateral dimension D=7.0 m for the frame for use in Eqn. 2 D has taken = 2.7 (distance between centre lines of the piers), and = 4.0 m (total width) for the shear wall.

It is seen that for frame with rigid ends represnting more closely the actual behaviour, the fundamental pariod obtained by analysis is much higher than the values obtained by using Eqns. 1 and 2. For the shear wall, the analysis gives T= 0.212 sec, which is incorrectly predicted by Eqn. 1 as well as by Eqn. 2 in spite of using different values of D.

8. CONCLUSIONS

The use of rigid-ended elements for idealising frames, shear walls or combinations therof is proposed and the necessary characteristics for such elements have been developed for studying the fundamental periods. It is shown that the analytical results cannot be accurately predicted by the empirical expressions currently in use.

It is to be noted that any computer program available for dynamic analysis of frames can be suitably modified to include rigid-ended elements.

9. REFERENCES.

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