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DYNAMIC RESPONSE OF MAT FOUNDATIONS

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Introduction

Mat foundations are commonly adopted for industrial and Power Structures, which are generally subjected to both static and dynamic loads. He therto, the analysis available for mat foundations is by taking (a) the effects of dynamic load, as an equivalent static load and (b) by taking the effect of Soil on foundation as a constant known as modulus of subgrade reaction. In all the methods of analysis available, the actual mat foundation consisting of beams running two directions or one direction with a slab resting over it is replaced by an equivalent isotropic or orthotropic plate having the same regrdites as that of the original mat.

The common methops of analysis used in Static analysis are (a) finite difference method (5, 4, 3) (b) finite element method (6) and (c) classical thin or thick plate theory (1, 2, 8) (d) as a ract grid with slab resting on it (9, 13). The mat under dynamic condition is analysed by method of finite differences (10,) and by using finite element techniques (1, 11).

The present paper uses an approach based on the calculus of finite differences (7) for analysis of mat foundation. This method is used in Engineering Structures which are uniform in nature (beams on equidistant supports, grid works with equally spaced and geometrically similar stiffening components). In this method the technique of finite difference is used to derive the governing difference equations as governing differential equation derived for actual structure. The method has following advantages over the usual matrix methods; (a) the method is exact with in the bounds of established simplifying assumptions (b) the determinative equations result in operational matrix orders of magnitude less than those necessary in the force or displacement methods.

The soil reaction on the foundation is taken into account by modulus of subgrade reaction (K), based on Winkler's hypothesis. The solution obtained for mat foundation is compared with the available methods of finite differences and finite element methods. The Solution requires less computer time and fairly accurate for practical purposes.

Method of Analysis

The governing differential equation for vibration of an orthotropic plate resting on elastic subgrade acted by a transmit force is given by

$$D_x \frac{\delta^4 \omega}{\delta x^4} + 2H \frac{\delta^4 \omega}{\delta x^2 . \delta y^2} + D_y \frac{\delta^4 \omega}{\delta y^4} + K\omega = p \sin \omega t \quad (1)$$

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Equation (1) can be converted into difference equation by using finite difference shift operators which are defined as

$$E^{x}_{r}\, Y_{r,s} = Y_{r+x,s} \tag{2}$$

$$E^{x}_{s}\, Y_{r,s} = Y_{r,s+x} \tag{3}$$

The bouudary conditions to be satisfied by a mat foundation resting on elastic foundation are given below.

$$\frac{\delta^2\omega}{\delta x^2} + \mu \cdot \frac{\delta^2\omega}{\delta y^2} = 0 \qquad M_x = 0 \text{ along simply supported edges}$$

$$\frac{\delta^3\omega}{\delta x^3} + (2-\mu)\frac{\delta^3\omega}{\delta x \cdot \delta y^2} = 0 \qquad M_y = 0 \text{ along free edges}$$

The above boundary conditions when expressed in the form.

Shift operators, the above B—C are given by

$$\left.\begin{aligned} &Y_{r,s} = 0 \\ &M_{r,s} + M^{L}_{\wedge,s} = 0 \end{aligned}\right\} \text{ along simply supported edges.}$$

$$\left.\begin{aligned} &\theta_{r,s} = 0 \\ &\overline{M}_{r,s} = 0 \\ &\overline{V}^{L}_{r,s} + V^{R}_{r,s} + V^{-L}_{r,s} = P_{r,s} \sin \omega t \end{aligned}\right\} \text{ along free edges}$$

Making use of the Shift operators and the boundary conditions described above and on simplification, the governing difference equation of an ortho-tropic plate resting on elastic foundation can be reduced to

$$L_{r,s}\, Y_{\wedge,s} = \frac{P_{r,s} a^3}{6EI} \tag{4}$$

$$\text{where } L_{r,s} = \left[\left\{\frac{3(E_\wedge - E_\wedge^{-1})^2}{(E_r - E_\wedge^{-1}) - \sim(E_s + E_s^{-1}) + 4 + 2\sim} - 2(E_r + E_\wedge^{-1} - 2)\right\}\right.$$
$$\left. + \lambda\left\{\frac{3(E_s - E_s^{-1})^2}{(E_s - E_s^{-1}) - \approx(E_\wedge + E_\wedge^{-1}) + 4 + 2\approx} - 2(E_s + E_s^{-1} - 2) + K\right\}\right] \tag{5}$$

Where
$$\lambda = \frac{Ia^3}{\bar{I}\,\bar{a}^3};\ \sim = \frac{G\bar{J}a}{2EI\bar{a}};\ \approx = \frac{GJ\bar{a}}{2E\bar{I}a}$$

E_r, E_s are shift operators in r and S directions of a grid-work

EI=flexural rigidity of beams

GJ=Torsional regidity of beams

Let the Solution to the above equation 4 using half beam concepts be of the form $=Y_1$ Where $Y^{(1)}_{r,s}$ is complementary function of the Solution for free vibration case and $Y^{(2)}_{r,s}$ is the particular intigral part of Solution representing the forced vibration of mat foundation.

Response of mat foundation:—Let

$$Y^{(1)}_{r,s}=[(A_1\cosh\alpha s+A_2\sinh\alpha s+A_3\cos\beta s+A_4\sin\beta s+k)]\sin\frac{m\pi\gamma}{R}\sin\omega t \tag{6}$$

be the Solution of mat foundation satisfying the symmetrical mode of vibration. It may be noted that the above equation satisfied the Boundary conditions of mat foundation along simply supported edges. For evaluating the constants A_1, A_2, A_3, A_4, the solution has to satisfy the B.C. along the free edges i.e. by using the condition

$$M_s=0$$

$$M_s=0$$

The free vibration solution satisfying the above B.C. is given by

$$Y^{(1)}_{r,s}=\Bigg[P_{r,s}\Bigg\{\frac{(K_1\cosh\alpha s+\approx K_2\sinh\alpha s)\cos\beta s}{P_1\cosh\alpha s\cos\beta s+P_2\sinh\alpha s\sin\beta s+P_3\cosh\alpha s\sin\beta s+p_4\sinh\alpha s\cos\beta s+K}$$

$$+P_{r,s}\frac{(-K_3\cos\beta s+\sim\bar{K}_4\sin\beta s)\cosh\alpha s.}{p_1\cosh\alpha s\cos\beta s+p_2\sinh\alpha s\sin\beta s+p_3\cosh\alpha s\sin\beta s+p_4\sinh\alpha s\cos\beta s+K}\Bigg\}\Bigg]$$

$$\sin\frac{m\pi\gamma}{R}\sin\omega t \tag{7}$$

Anti Symmetrical Case :—

Let $Y^{(2)}_{r,s}$ be the Solution satisfying the antisymmetric mode of vibration. Proceeding in the same way as described above, we have.

$$Y^{(2)}_{r,s}=P_{r,s}\Bigg\{\frac{(-k_3\sin\beta s+\approx k_4\cos\beta s)+\sin\beta s\,(k_2\sinh\alpha s+\approx k_1\cosh\alpha s)}{p_1\cosh\alpha s\cos\beta s+p_2\sinh\alpha s\sin\beta s+p_3\cosh\alpha s\sin\beta s+p_4\sinh\alpha s\sin\beta s+k}\Bigg\}$$

$$\times\sin\frac{m\pi\gamma}{R}\sin\omega t \tag{8}$$

Hence, the final response of the mat foundation under elastic condition is given by

$$Y_{r,s}=Y^{(1)}_{r,s}+Y^{(2)}_{r,s},$$

which reduces to

$$Y_{r,s}=Ax\,P\sin\frac{m\pi\gamma}{R}\sin\omega t \tag{9}$$

where A=dynamic Coeficient and this is given by

$$A=\Bigg[\frac{(k_1\cosh\alpha s+\approx k_2\sinh\alpha s)\cos\beta s+\cosh\alpha s\,(k_3\cos\beta s+\approx k_4\sin\beta s)}{p_1\cosh\alpha s\cos\beta s+p_2\sinh\alpha s\sin\beta s+p_3\cosh\alpha s\sin\beta s+p_4\sinh\alpha s\cos\beta s+k}$$

$$+\frac{(-k_3 \sin \beta s+\alpha k_4 \cos \beta s) \sinh \alpha s+(k_5 \sinh \alpha s+\alpha k_6 \cosh \alpha s) \sin \beta s}{p_1 \cosh \alpha s \cos \beta s+p_3 \sinh \alpha s \sin \beta s+p_4 \cosh \alpha s \sin \beta s+p_6 \sinh \alpha s \cos \beta s+k}\Bigg] \quad (10)$$

Discussion

In order to check the validity of the above solution for mat foundation resting on elastic subgrade, the values of response obtained by this method are compared with the results obtained from using finite difference (56) and finite element methods (14). It can be seen from table (1) that the Solution obtained by this methods gives values which are very near to the solution obtained by finite element method.

Table I Rate of Dynamic to Static diflection of Square Raft foundation for various points by different theories

S.No.	Case	Rate of Dynamic to Static deflections at x=y=a/4, x=y=a/2							
		Classical theory	finite diff	finite element	finite diff cal	Classi-theory	finite diff-erence	finite element	finite diff calculus
		$x=y=\frac{a}{4}$	$x=y=\frac{a}{4}$	$x=y=\frac{a}{4}$	$x=y=\frac{a}{4}$	$x=y=\frac{a}{2}$	$x=y=\frac{a}{2}$	$x=y=\frac{a}{2}$	$x=y=\frac{a}{2}$
1.	Isotropic case	0.8000	0.9500	1.000	0.9800	0.8200	0.9000	1.000	0.9500
2.	Orthotropic case	0.7500	0.9000	1.000	0.9200	0.8000	0.8900	1.000	0.9400

In the Solution obtained above if K=0, the solution reduces to a plate supported on two opposite sides simply supported and other two sides free. The accuracy of the method for the problem described with K=0 has already been demonstrated by Ist author elsewhere (9).

It may be noted that the subgrade reaction is taken as constant while solving the present problem. In the field, the subgrade reaction is dependent on soil properties and on properties of foundation structure itself (12). Hence the value of subgrade reaction is to be determined and used in the solution.

Conclusions

The following conclusions are arrived at based on the Study.

(1) A classical method of analysis for mat foundation under dynamic loads has been developed and it is found to require less computer time as compared to finite difference and finite element methods.

(2) It was observed that the solutions obtained is nearer to the solution obtained by finite element method.

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