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DYNAMIC BEHAVIOUR OF HOLLOW CYLINDERS SURROUNDED BY WATER*

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INTRODUCTION

Dynamic analysis of structures when partly or fully submerged in water has additional dynamic forces caused due to surrounding water. The motion of the fluid in contact with dynamic system gives rise to an effect which is equivalent to an increase in the apparent mass of the system. The determination of this added mass due to submergence in a fluid has been the subject of many analytical and experimental investigations (Stelson and Mavis (1957), Liaw and Chopra (1973)). Theoretical and experimental studies have shown that a system has a longer period of vibration and a higher damping when vibrating in water as compared to that in air (Clough(1960), Chandrasekaran et al. (1972)). This increase in the natural period of vibration of the system is due to an increase in the apparent mass of the system as a result of interaction between the structure and surrounding water. Experiments, though adequate enough for period estimation, cannot give the hydrodynamic pressure distribution. If the hydrodynamic pressure distribution can be known with accuracy, the resulting additional mass which will be moving with the structure can be evaluated.

Theoretical studies have now established that compressibility of water can be ignored in water-structure interaction analysis (Chandrasekaran et al. (1970), Liaw and Chopra (1973)). Closed form solutions are now available for simple geometric shapes and considering flexibility of structures subjected to harmonic ground excitations. However, for more complicated geometry, finite element techniques are ideally suited for numerical evaluation of hydrodynamic pressure and interaction forces.

The concept of finite elements in fluids has been proposed to solve the Laplace's equation with appropriate boundary conditions (Zienkiewicz et al. (1965)). Techniques are also now available to evaluate coupled vibration in which the structure is also idealized by finite elements (Zienkiewicz and Newton (1969), Liaw and Chopra (1973)).

In this study, axisymmetric finite elements have been used to discretise both the structure and the surrounding water (Figs. 1 & 2). The

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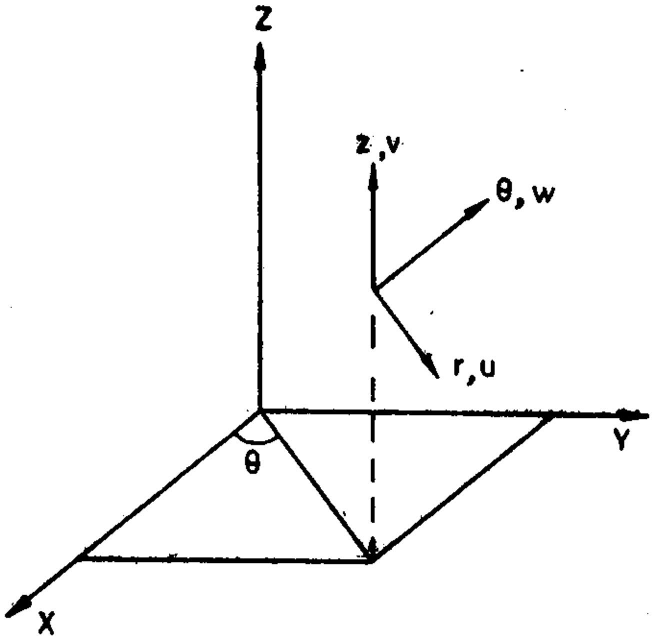

Fig. 1. **Displacements in cylindrical coordinates (r, θ, z)**

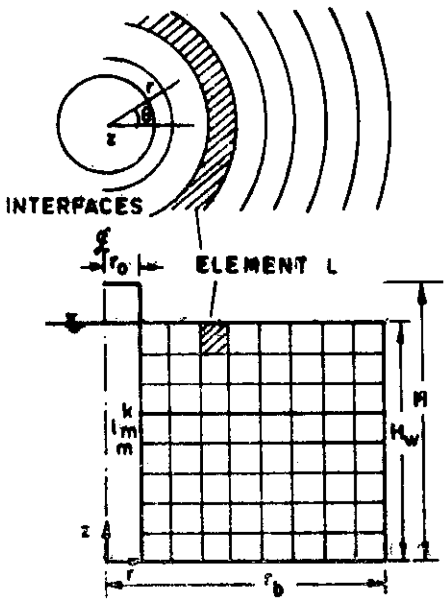

Fig. 2. Finite element idealizaiton of water surrounding the structure

resulting hydrodynamic pressure is interpreted to give the added mass or virtual mass. In one case, the virtual mass has been taken equal to hydrodynamic pressure corresponding to a uniform unit acceleration along the height and equivalent nodal masses obtained for a lumped mass matrix. This has been termed as uncoupled vibration here. In the second case, the flexibility of the structure has been taken into account while obtaining virtual mass matrix. This has been termed as coupled vibration. The structure analysed had a hollow right cylindrical shape and free vibration characteristics have been evaluated for different slenderness ratios of the cylinder and different levels of its submergence.

PARAMETERS CONSIDERED FOR STUDY

The structure considered had a right circular hollow cylindrical shape. The following parameters were considered:

(i) Outside radius to height of cylinder (r_0/H):
1.00, 0.5, 0.33, 0.25, 0.125, 0.083, 0.0625, and 0.05

(ii) Wall thickness (t):
0.20 r_0 and 0.033 r_0

(iii) Depth of submergence (H_w):
1.00H, 0.83H, 0.67H, and 0.50H

The thicknesses have been chosen such that they cover both thick and thin axisymmetric type of structures. The values of modulus of elasticity, Poisson's ratio and unit weight of concrete were assumed to be $3 \times 10^6 T/m^2$, 0.20 and 2.40 T/m^3 respectively.

Analysis was done for both uncoupled and coupled vibrations. The fluid finite element was an eight noded parabolic element with 2×2 integration scheme. The structure was idealized by paralinear elements with 3×2 integration scheme (Chandrasekaran and Singhal (1979,1981)).

RESULTS

(a) Hydrodynamic Pressure

The hydrodynamic pressure distribution assuming uniform unit acceleration along the height of the cylinder for different values of outside radius to height of the cylinder (r_0/H) is shown in Fig. 3 with firm lines. The dotted lines show the curves plotted as per Indian Standard Code IS: 1893–1975. There are only four such curves as the IS Code gives the values only for four r_0/H ratios. Also it assumes the same shape of hydrodynamic pressure distribution irrespective of r_0/H ratio. It can be inferred from Fig. 3 that the hydrodynamic pressure diagram

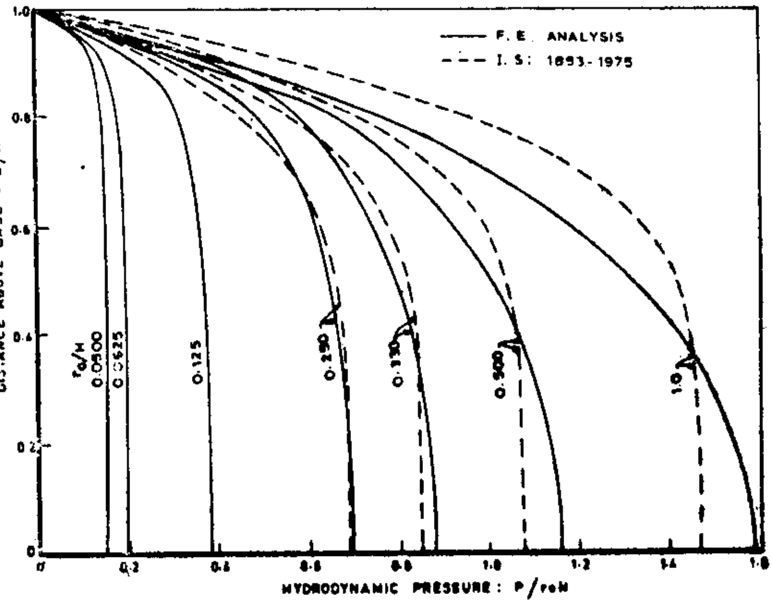

Fig. 3. Hydrodynamic pressure distribution

changes with r_0/H and is almost constant for very slender cylinders. Also the pressure distribution is not uniform along the height.

(b) Increase in period of vibration

The following table gives the increase in period of vibration of the structure due to different submergence levels. The range corresponds to different r_0/H ratios considered.

TABLE–1 PERCENTAGE INCREASE IN PERIOD OF VIBRATION

	Uncoupled Vibration		Coupled Vibration	
	Ist mode	Higher modes	Ist mode	Higher modes
Thickness $t = 0.20\ r_0$				
$H_w/H = 1.00$	26–32	30–40	25–35	22–32
$H_w/H = 0.83$	14–15	24–32	13–16	21–28
$H_w/H = 0.67$	5–6	20–25	5–6	18–25
$H_w/H = 0.50$	1–2	13–19	1–1.5	12–17
Thickness $t = 0.033\ r_0$				
$H_w/H = 1.00$	108–126	126–137	105–136	98–133
$H_w/H = 0.83$	66–69	95–150	64–76	77–100
$H_w/H = 0.67$	28–33	80–140	30–33	70–90
$H_w/H = 0.50$	7–11	60–90	9–12	50–70

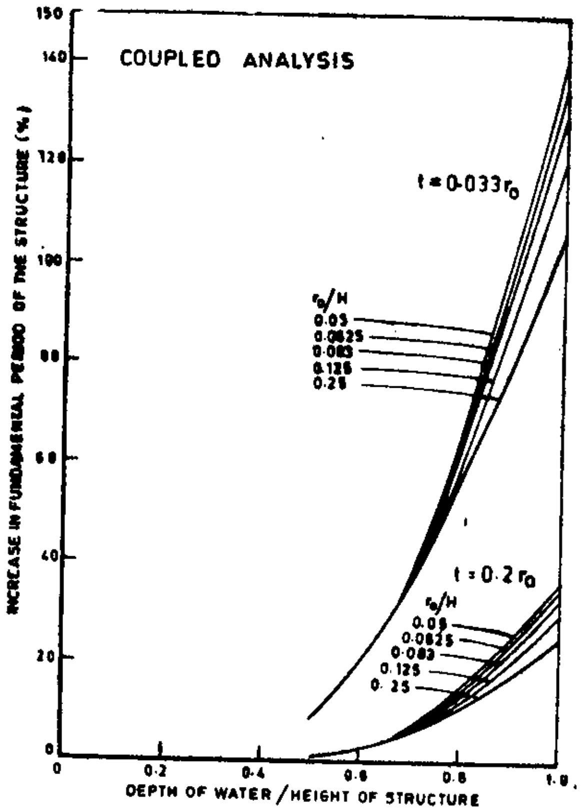

Fig. 4. Effect of surrounding water on fundamental period of vibration of the structure

The increase in the fundamental period of vibration due to surrounding water is plotted in Fig. 4 as a function of the ratio of depth of surrounding water to height of the structure for different values of the ratio r_0/H.

The results show that increase in period of vibration reduces with the decrease in submergence level and is negligible when the structure is half submerged. Therefore, for $H_w/H \leqslant 0.50$ the effect of water-structure interaction need not be considered. Also the period increases substantially in case of thin cylinders. This is not unexpected because the virtual mass remains the same for both thick and thin cylinders as it depends on outside radius of the cylinder. Higher modes exhibit greater increase in period of vibration than fundamental mode. As can be seen from the above table the uncoupled and coupled analyses give reasonably identical results. Therefore, virtual mass obtained from uncoupled analysis can be used for period estimation.

(c) Mode Shapes and Participation Factors

First three modes of vibration obtained from (i) uncoupled, (ii) coupled and (iii) without water-structure interaction analysis for a typical case with $t=0.033\ r_o$ and $r_o/H=0.25$ are drawn in Fig. 5. Mode shapes are almost coincident in fundamental mode of vibration but in higher modes they differ.

Table 2 gives the mode participation factors evaluated with and without water effect. These factors increase by about 10% in the fundamental mode and by about 20-40% in higher modes for $t=0.20\ r_o$ case.

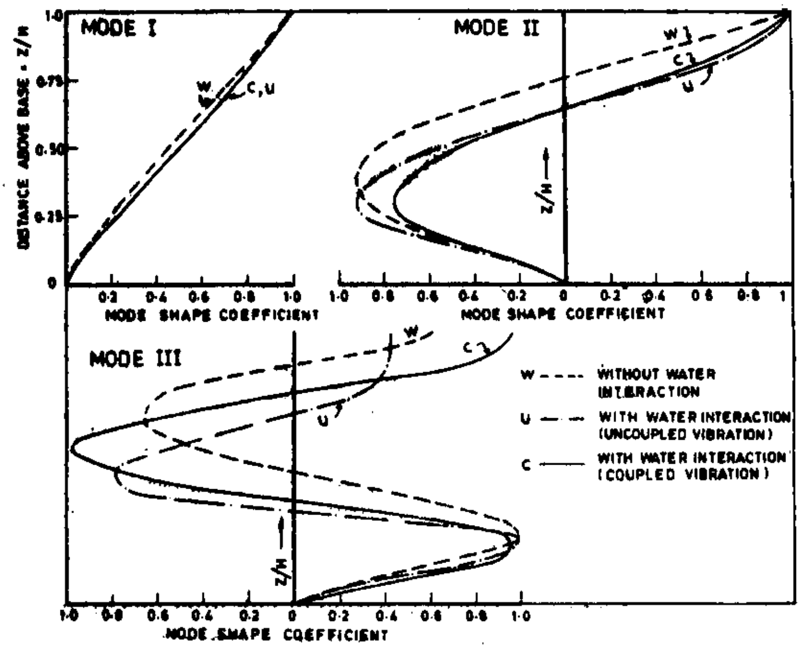

Fig. 5. Mode shapes for a typical case ($t=0.033\ r_0$, $r_0/H=0.25$)

APPROXIMATE DESIGN PROCEDURE

For the purpose of preliminary design only the effect due to fundamental mode can be considered as the contribution due to higher modes will be much less. This fundamental frequency of vibration of the cylinder can be approximately determined using Rayleigh's method. The fundamental frequency of the cylinder submerged in water is determined by the same procedure but now considering the virtual mass. Let the cylinder be submerged in water upto a height H_w as shown in Fig. 6(a). The condition of Fig. 6(a) can be represented as in Fig. 6(b) where the cylinder is idealized as a beam and the virtual mass due to water as a uniformly distributed load along the height of submergence

TABLE-2 MODE PARTICIPATION FACTORS

r_o/H	Mode	In Air	In water							
			Coupled Vibration				Uncoupled Vibration			
			H_w/H				H_w/H			
			1.00	0.83	0.67	0.50	1.00	0.83	0.67	0.50
(a) Thickness $t = 0.2\ r_o$										
0.25	I	1.45	1.59	1.63	1.63	1.55	1.54	1.59	1.59	1.54
	II	0.67	0.85	0.91	0.86	0.78	0.77	0.86	0.83	0.79
	III	0.32	0.36	0.36	0.38	0.38	0.31	0.27	0.35	0.44
0.125	I	1.52	1.64	1.73	1.73	1.65	1.63	1.71	1.71	1.63
	II	0.77	0.96	1.09	1.04	0.96	0.91	1.04	1.01	0.94
	III	0.40	0.49	0.52	0.42	0.47	0.46	0.50	0.47	0.48
0.0625	I	1.54	1.64	1.76	1.77	1.68	1.64	1.74	1.74	1.66
	II	0.81	0.99	1.15	1.12	1.04	0.97	1.11	1.08	1.01
	III	0.43	0.56	0.62	0.57	0.55	0.53	0.59	0.56	0.57
0.05	I	1.56	1.64	1.97	1.78	1.70	1.64	1.75	1.75	1.66
	II	0.83	1.00	1.18	1.15	1.08	0.99	1.14	1.12	1.04
	III	0.46	0.60	0.67	0.61	0.63	0.57	0.63	0.60	0.61
(b) Thickness $t = 0.033\ r_o$										
0.25	I	1.44	1.67	1.83	1.95	1.93	1.59	1.73	1.86	1.88
	II	0.65	0.86	1.28	1.55	1.21	0.66	0.91	1.25	1.11
	III	0.25	0.30	0.47	0.39	0.60	0.26	0.20	0.17	0.28
0.125	I	1.51	1.74	2.02	2.22	2.12	1.72	1.97	2.16	2.07
	II	0.75	1.07	1.58	1.91	1.48	1.09	1.45	1.81	1.42
	III	0.39	0.50	0.76	0.66	0.73	0.58	0.64	0.59	0.56
0.0625	I	1.54	1.72	2.04	2.29	2.18	1.72	2.02	2.23	2.11
	II	0.81	1.11	1.72	2.11	1.64	1.12	1.65	2.01	1.57
	III	0.43	0.62	1.01	0.83	0.91	0.66	0.90	0.81	0.81
0.05	I	1.55	1.71	2.03	2.30	2.22	1.73	2.04	2.26	2.12
	II	0.83	1.11	1.79	2.21	1.72	1.15	1.75	2.09	1.63
	III	0.45	0.68	1.13	0.92	1.03	0.71	1.02	0.90	0.96

of the cylinder. In order that this equivalence may hold good, it remains to determine this uniformly distributed load such that the period of vibration in water from beam analysis should match that obtained in this study from finite element analysis. This can be done by expressing the total uniformly distributed load on the beam equal to a coefficient times the mass of the enveloping cylinder. That is,

$$\text{Total uniformly distributed load on the beam} = C\pi r_o^2 H_w \rho_w$$

Here ρ_ω is the mass density of water and other notations are as defined earlier. This coefficient is a function of height of submerged portion of the cylinder to the radius of the enveloping cylinder and will vary with the height of the cylinder as shown in Fig. 6.

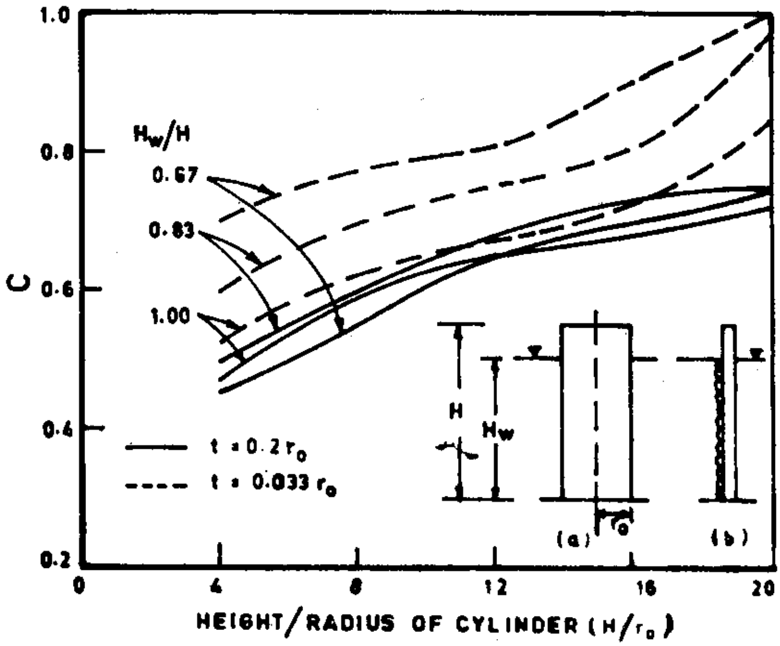

Fig. 6. Variation of C with H/r_0

CONCLUSIONS

Hydrodynamic pressures for various slenderness ratios of hollow cylinders have been evaluated and based on this the virtual mass associated with the cylinder due to water is estimated. Having got the virtual mass the period of the structure in water is determined. The period in water is also determined taking the flexibility of the structure into account.

The period of vibration of hollow cylinders elongates substantially because of the interaction due to water. This elongation in period is dependent on the thickness of the cylinder, being more in case of thin slenders. The increase in period of vibration is negligible when the cylinder is half submerged. There is a reasonably close agreement between the results of uncoupled and coupled analyses and, therefore, the virtual mass obtained from uncoupled analysis can be used for evaluating the period of vibration due to water.

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