ISET Journal of Earthquake Technology, Paper No. 396, Vol. 36, No. 2-4, June-Dec. 1999, pp. 139-167

COMPUTATIONAL PROCEDURE FOR EARTHQUAKE RESPONSE ANALYSIS OF HORIZONTAL TUBE ARRAY IN CALANDRIA

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ABSTRACT

A rigorous computational procedure is presented for dynamic response analysis of horizontal tube array in partially-filled calandria including hydrodynamic interaction effects. The procedure is general enough to consider transfer of energy between the fluid-coupled tubes, effects of moderator sloshing in terms of magnitude and the distribution of hydrodynamic forces on tubes, and wave absorption by the fluid boundaries. The analysis method is based on the substructure approach in which individual tubes, calandria shell, inside moderator domain and the outside vault water domain are treated as different substructures. Numerical evaluation procedures for various substructures and sample response results for typical tubes have been presented to demonstrate the capabilities of the developed procedure.

KEYWORDS: Tube Array, Calandria, Fluid-Structure Interaction, Hydrodynamic Effects, Dynamic Response.

INTRODUCTION

Earthquake response analysis of horizontal tubes in partially-filled calandria of pressurized heavy water reactor must recognize the hydrodynamic interaction forces and modifications in their vibration properties to ensure non-vulnerability during design seismic event. Because of moderator sloshing in partially filled calandria, the generated hydrodynamic forces on tubes and calandria walls may not be accurately accounted for by simple added mass approach based on rigid structures and incompressible water assumptions. The presence of a large number of horizontal tubes further complicates the fluid-structure interaction problem, as their arrangement, spacing, and relative motions among the tubes are additional important considerations in the computation of hydrodynamic forces and added damping. The magnitude and the distribution of hydrodynamic forces also depend on the extent of tuning between the uncoupled frequencies of calandria and tubes, and the sloshing frequencies of moderator contained in calandria (Liu and Ma, 1982). Additionally, because of hydrodynamic coupling, the resultant hydrodynamic forces on horizontal tubes may not be in phase or in the direction of applied ground excitation (Chen, 1987; Chen et al., 1976), thus requiring simultaneous solution for multi-component excitation. Therefore, effects of moderator compressibility, its sloshing and hydrodynamic coupling between different tubes and the calandria walls should be considered in developing computational methods for the analysis of horizontal tube array.

For multiple rigid submerged tubes under seismic excitations, the concept of self-added and coupled-added mass based on potential theory has been applied, although the experimental confirmation is far less extensive for large tube array (Dong, 1978). An analytical procedure is required to determine the design values of earthquake responses for large tube array including the effects of moderator sloshing, and hydrodynamic coupling between the tube array and the calandria shell. At the same time, the developed method should account for added damping due to wave absorption at the structure-fluid interfaces and sloshing surfe of the fluid.

For partially-filled horizontal cylindrical tanks, the theoretical solutions only for the transverse slosh response are available (Budiansky, 1960). The longitudinal slosh response is computed utilizing the available results for rectangular tanks. The cylindrical tank is converted into an equivalent rectangular tank based on its aspect ratio, liquid level parameter and excitation amplitude parameter (Kobayashi et al., 1989). Thus, the current methods of calculating the natural frequencies and slosh forces for longitudinal and transverse directions rely heavily on few available experimental results. The finite element solution to the solid-fluid-surface wave problem has been proposed by several authors (Dubois and Rouvrary, 1978) where the virtual mass and damping matrices for the bounded fluid domains have been computed using finite element solutions of the Navier-Stokes equations.

The objective of this paper is to develop computational procedure to determine the design values of earthquake responses for horizontal tubes. The presented procedure accounts for the effects of fluid compressibility, viscosity, sloshing and surface wave absorption by boundaries of fluid domain, and the flexibility of all the substructures.

SYSTEM IDEALIZATION AND VAULT MOTION

The simplified system consists of partially-filled cylindrical vessel, namely calandria. The ends of calandria are welded to the end shields grouted in vault walls. This horizontal vessel is penetrated by a large number of horizontal tubes (Figure 1). The calandria is filled with moderator (heavy water) up to its normal level of ninetysix percent of the calandria diameter, referred in following discussions as inside moderator domain. The hydrodynamic effects of the water between the calandria shell and the vault walls, referred in following discussions as the outside water domain, are also included in the formulation.

The analysis procedure has been developed based on the substructure method in frequency domain (Goyal and Chopra, 1989). The calandria walls have been idealized as finite element system consisting of 9-node shell elements, and the dynamic equilibrium equations are formulated in modal coordinates corresponding to natural vibration mode shapes of calandria without any hydrodynamic interaction effects. The horizontal tubes have been idealized as one-dimensional flexural beams, and the dynamic equilibrium equations are formulated in modal coordinates corresponding to natural vibration mode shapes of horizontal tubes without any hydrodynamic interaction effects.

The frequency response functions of hydrodynamic pressure because of moderator on the tubes are determined by solving the linearized Navier-Stokes equation for the three-dimensional compressible, viscous fluid for the inside moderator domain subject to continuity boundary conditions at the structure-fluid interface and the gravity wave condition at the free surface. Damping associated with vibration of free surface has been included through modified gravity wave condition at the free surface (Chen et al., 1996). Similarly, the effects of acoustic wave absorption at the fluid boundaries have been accounted for by introducing surface damping factor in acceleration boundary condition (Fenves and Chopra, 1984). The frequency response functions for the equivalent hydrodynamic forces are computed by integrating hydrodynamic pressure on the fluid-structure interface.

The frequency response functions of hydrodynamic pressure because of water outside the calandria are similarly determined by solving the linearized Navier-Stokes equation for the three-dimensional compressible, viscous fluid for the outside water domain subject to continuity boundary conditions at the structure-fluid interface. The hydrodynamic analysis has been performed for both horizontal and vertical component of vault excitations separately. Solutions of the boundary value problems for fluid domain have been obtained by solving three-dimensional problem using fluid elements in Eulerian formulation with hydrodynamic pressures as unknowns at nodal points.

Since the ends of idealized calandria are welded to the end shields grouted in vault walls, it becomes necessary to specify acceleration time histories at various levels of the vault. In this analysis, however, the vault walls have been assumed rigid and the procedure is presented for the same time history applied at all points. The dynamic response is determined for the harmonic ground motions at discrete frequencies covering the entire range of interest. The desired response histories are then synthesized using Fourier synthesis techniques.

FREQUENCY DOMAIN EQUATIONS

The governing equations of motion for the tubes and the calandria including the effects of hydrodynamic interaction are conveniently written in the Fourier-transformed frequency domain because the hydrodynamic pressures depend on the excitation frequency. The idealized system consists of four main substructures: the calandria, horizontal tubes, the inside moderator domain, and the outside vault water domain. The governing equations for these substructures are presented next, for both horizontal and vertical components of vault excitation in the frequency domain followed by a general analytical procedure based on the substructure method.

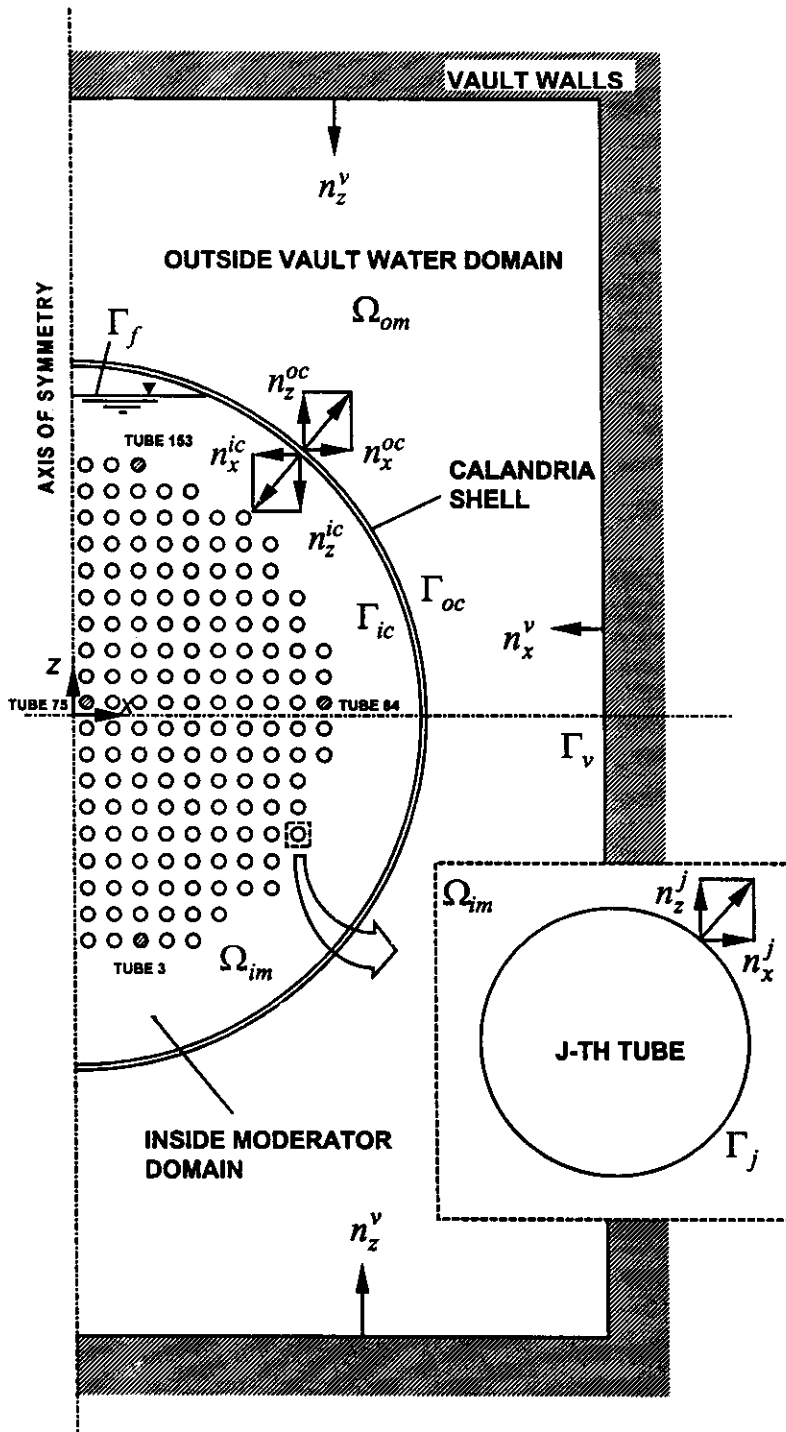

Fig. 1 Horizontal tube array in partially filled calandria; domain definitions and normal directions

1. Calandria Substructure

The equations of motion for calandria shell idealized as a finite element system subject to harmonic excitation $a_x(t) = r_x e^{i\omega t}$ of the calandria vault along the transverse direction of calandria, $a_y(t) = r_y e^{i\omega t}$ along the longitudinal direction of calandria and $a_z(t) = r_z e^{i\omega t}$ along the vertical direction of calandria are written in frequency domain as:

$$\mathbf{M}_c \overline{\ddot{\mathbf{U}}}_c^{tot}(\omega) + (1 + i\eta_c)\mathbf{K}_c \overline{\mathbf{U}}_c(\omega) = -\overline{\mathbf{f}}_c^{i}(\omega) - \overline{\mathbf{f}}_c^{o}(\omega) \tag{1}$$

in which $\mathbf{M}_c$ is the mass matrix of calandria structure without moderator, $\mathbf{K}_c$ is the stiffness matrix of the calandria structure, and η_c is the constant hysteretic damping factor for the calandria; and $\overline{\ddot{\mathbf{U}}}_c^{tot}(\omega)$ is the vector of complex frequency response functions of total accelerations at the nodal points of the shell elements corresponding to five degrees of freedom (three translations and two in-plane rotations). In this equation, $\overline{\mathbf{f}}_c^{i}(\omega)$ is the vector of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressures on the inside surface and $\overline{\mathbf{f}}_c^{o}(\omega)$ is the vector of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressures on the outside surface.

Assuming small displacements and rotations, the frequency response functions for total accelerations can be expressed in the following form:

$$\overline{\ddot{\mathbf{U}}}_c^{tot}(\omega) = \mathbf{r}_x + \mathbf{r}_y + \mathbf{r}_z - \omega^2 \overline{\mathbf{U}}_c(\omega) \tag{2}$$

in which $\overline{\mathbf{U}}_c(\omega)$ is the vector of complex frequency response functions for relative displacements at nodes in the finite element system of calandria. The elements of influence vector $\mathbf{r}_x$ are non-zero and equal to r_x only for translational degrees of freedom along x-axis (transverse direction). Similarly, the elements of influence vector $\mathbf{r}_y$ and $\mathbf{r}_z$ are non-zero and equal to r_y and r_z only for translational degrees of freedom along y-axis (longitudinal direction) and z-axis (vertical direction) respectively.

The natural frequencies and mode shapes of the calandria without moderator and vault water are given by solutions of the following associated eigenvalue problem for Equation (1):

$$\mathbf{K}_c \, \mathbf{\Phi}_n^c = \left[\omega_n^c\right]^2 \mathbf{M}_c \, \mathbf{\Phi}_n^c \tag{3}$$

The n-th mode shape of calandria is completely defined by vector $\mathbf{\Phi}_n^c$ of the nodal displacements of calandria and associated frequencies ω_n^c. The displacements and rotations of the calandria walls, $\overline{\mathbf{U}}_c(\omega)$, can be expressed as a linear combination of the first few natural modes of calandria without hydrodynamic effects:

$$\overline{\mathbf{U}}_c(\omega) \approx \sum_{n=1}^{N_c} \mathbf{\Phi}_n^c \, \overline{Y}_n^c(\omega) \tag{4}$$

where $\overline{Y}_n^c(\omega)$ is the frequency response function for the generalized (modal) coordinate associated with the n-th mode of vibration of calandria without moderator and vault water. In this summation, only the first N_c modes have been included.

The equations of motion for the calandria are transformed to modal coordinates by substituting Equation (4) into Equation (1), using the principle of virtual work and the orthogonality properties of normal modes. This leads to

$$M_n^c \left[-\omega^2 + (1 + i\eta_c)\left[\omega_n^c\right]^2\right] \overline{Y}_n^c(\omega) = -L_{nx}^c - L_{ny}^c - L_{nz}^c - \overline{l}_n^{ic}(\omega) - \overline{l}_n^{oc}(\omega) \tag{5}$$

in which the generalized mass M_n^c, and generalized excitation terms L_{nx}^c, L_{ny}^c and L_{nz}^c are given by

$$M_n^c = \left[\Phi_n^c\right]^T \mathbf{M}_c \left[\Phi_n^c\right] \quad (6)$$

$$L_{nx}^c = \left[\Phi_n^c\right]^T \mathbf{M}_c \, \mathbf{r}_x \quad (7a)$$

$$L_{ny}^c = \left[\Phi_n^c\right]^T \mathbf{M}_c \, \mathbf{r}_y \quad (7b)$$

$$L_{nz}^c = \left[\Phi_n^c\right]^T \mathbf{M}_c \, \mathbf{r}_z \quad (7c)$$

The hydrodynamic terms in Equation (5) are given by

$$\overline{l}_n^{ic}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_c^i(\omega) \quad (8a)$$

$$\overline{l}_n^{oc}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_c^o(\omega) \quad (8b)$$

The vector of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressure on the inside surface of calandria, $\overline{\mathbf{f}}_c^i(\omega)$, will be expressed in terms of accelerations of modal coordinates of the calandria and the tubes by analysis of the inside moderator domain substructure. Similarly, the vector of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressures on the outside surface of calandria, $\overline{\mathbf{f}}_c^o(\omega)$, will be expressed in terms of accelerations of modal coordinates of the calandria by analysis of the outside water domain substructure.

2. Tube Substructure(s)

The equations of motion for j-th tube idealized as a one-dimensional finite element system and subject to harmonic excitation $a_x(t) = r_x e^{i\omega t}$ along the transverse direction, $a_y(t) = r_y e^{i\omega t}$ along the longitudinal direction and $a_z(t) = r_z e^{i\omega t}$ along the vertical direction are written in frequency domain as:

$$\mathbf{M}_j \ddot{\overline{\mathbf{U}}}_j^{tot}(\omega) + (1 + i\eta_j)\mathbf{K}_j \, \overline{\mathbf{U}}_j(\omega) = -\overline{\mathbf{f}}_j(\omega) \quad (9)$$

in which $\mathbf{M}_j$ is the mass matrix of j-th tube without moderator, $\mathbf{K}_j$ is the stiffness matrix of the j-th tube structure, and η_j is the constant hysteretic damping factor for the j-th tube; and $\ddot{\overline{\mathbf{U}}}_j^{tot}(\omega)$ is the vector of complex frequency response functions of total accelerations at the nodal points of the beam elements used for the j-th tube corresponding to four degrees of freedom (two translations and two rotations). In this equation, $\overline{\mathbf{f}}_j(\omega)$ is the vector of response functions for equivalent forces at the nodes in the finite element system of j-th tube due to hydrodynamic pressures on the outside surface. The motion in x-y plane and y-z plane of the tube are coupled through the hydrodynamic forces.

Assuming small displacements and rotations, the frequency response functions for total accelerations can be expressed in the following form:

$$\ddot{\overline{\mathbf{U}}}_j^{tot}(\omega) = \mathbf{r}_x + \mathbf{r}_y + \mathbf{r}_z - \omega^2 \overline{\mathbf{U}}_j(\omega) \quad (10)$$

where $\overline{\mathbf{U}}_j(\omega)$ is the vector of complex frequency response functions for relative displacements at nodes in the finite element system of j-th tube.

The natural frequencies and mode shapes of the j-th tube without moderator are given by solutions of the following associated eigenvalue problem for Equation (9):

$$\mathbf{K}_j \, \Phi_n^j = \left[\omega_n^j\right]^2 \mathbf{M}_j \, \Phi_n^j \tag{11}$$

The n-th mode shape of j-th tube is completely defined by vector Φ_n^j of the nodal displacements of j-th tube and associated frequencies ω_n^j. The displacements of the j-th tube, $\overline{\mathbf{U}}_j(\omega)$, can be expressed as a linear combination of the first few natural modes of j-th tube without hydrodynamic effects:

$$\overline{\mathbf{U}}_j(\omega) \approx \sum_{n=1}^{N_j} \Phi_n^j \, \overline{Y}_n^j(\omega) \tag{12}$$

where $\overline{Y}_n^j(\omega)$ is the frequency response function for the generalized (modal) coordinate associated with the n-th mode of vibration of j-th tube without moderator. In this summation, only the first N_j modes have been included.

The equations of motion for the j-th tube are transformed to modal coordinates by substituting Equation (12) into Equation (9), using the principle of virtual work and the orthogonality properties of normal modes. This leads to

$$M_n^j\left[-\omega^2 + (1+i\eta_j)[\omega_n^j]^2\right]\overline{Y}_n^j(\omega) = -L_{nx}^j - L_{ny}^j - L_{nz}^j - \overline{l}_n^j(\omega) \tag{13}$$

in which the generalized mass M_n^j, and generalized excitation terms L_{nx}^j, L_{ny}^j and L_{nz}^j are given by

$$M_n^j = \left[\Phi_n^j\right]^T \mathbf{M}_j \left[\Phi_n^j\right] \tag{14}$$

$$L_{nx}^j = \left[\Phi_n^j\right]^T \mathbf{M}_j \, \mathbf{r}_x \tag{15a}$$

$$L_{ny}^j = \left[\Phi_n^j\right]^T \mathbf{M}_j \, \mathbf{r}_y \tag{15b}$$

$$L_{nz}^j = \left[\Phi_n^j\right]^T \mathbf{M}_j \, \mathbf{r}_z \tag{15c}$$

The hydrodynamic term in Equation (13) is given by

$$\overline{l}_n^j(\omega) = \left[\Phi_n^j\right]^T \overline{\mathbf{f}}_j(\omega) \tag{16}$$

The vector of response functions for equivalent forces at the nodes in the finite element system of j-th tube due to hydrodynamic pressures on the outside surface of j-th tube, $\overline{\mathbf{f}}_j(\omega)$, will be expressed in terms of accelerations of modal coordinates of the calandria and the tubes by analysis of the inside moderator domain substructure.

3. Inside Moderator Domain Substructure

Boundary Value Problem

The vector of frequency response functions of unknown hydrodynamic forces $\overline{\mathbf{f}}_c^j(\omega)$ on the inside surface of calandria and the vector of response functions for equivalent forces $\overline{\mathbf{f}}_j(\omega)$ due to hydrodynamic pressures on the j-th tube because of vault excitation along x, y and z axes, can be expressed in terms of accelerations of modal coordinates of the calandria and the tubes by analysis of the inside moderator domain (Figure 1). The small amplitude, irrotational motion of the moderator fluid is governed by the three-dimensional linearized Navier-Stokes equation:

$$\nabla^2 \bar{p} + i\frac{4\mu}{3k}\omega\,\nabla^2 \bar{p} + \frac{\omega^2}{C^2}\bar{p} = 0 \tag{17}$$

where $\bar{p}(\mathbf{x},\omega)$ is the frequency response function for hydrodynamic pressure (in excess of hydrostatic pressure); i.e. the hydrodynamic pressure $p(\mathbf{x},t)$, where $\mathbf{x} = (x,y,z)$ defines the coordinate vector of a point, due to harmonic excitation $a_x(t) = r_x e^{i\omega t}$ along the transverse direction, $a_y(t) = r_y e^{i\omega t}$ along the longitudinal direction, and $a_z(t) = r_z e^{i\omega t}$ along the vertical direction is given by $p(\mathbf{x},t) = \bar{p}(\mathbf{x},\omega)\, e^{i\omega t}$. In this equation, μ is viscosity of moderator, k is the bulk modulus, and C is the velocity of sound in the moderator. The hydrodynamic pressures in the moderator inside the calandria are generated by acceleration of the inside surface of the calandria, and the acceleration of the outside surfaces of the tubes. The motion of these boundaries is related to the hydrodynamic pressure by the following boundary conditions:

On the calandria-moderator interface, Γ_{ic},

$$\frac{\partial}{\partial n^{ic}}\bar{p}(\mathbf{x},\omega) = -\rho\, a_n^{ic}(\mathbf{x},\omega) + i\omega q_{ic}\,\bar{p}(\mathbf{x},\omega) \tag{18}$$

in which ρ is the mass density of the moderator, n^{ic} represents the direction of the normal to the inside surface of calandria; $a_n^{ic}(\mathbf{x},\omega)$ is the spatial distribution of the acceleration of the inside surface of calandria in its normal direction, and q_{ic} is the surface damping coefficient for inside surface of calandria.

On the j-th tube-moderator interface, Γ_j,

$$\frac{\partial}{\partial n^{j}}\bar{p}(\mathbf{x},\omega) = -\rho\, a_n^{j}(\mathbf{x},\omega) + i\omega q_{j}\,\bar{p}(\mathbf{x},\omega) \tag{19}$$

in which n^j represents the direction of the normal to the surface and $a_n^j(\mathbf{x},\omega)$ is the spatial distribution of the acceleration of the outside surface of j-th tube in its normal direction; and q_j is the surface damping coefficient for outside surface of j-th tube.

On the inside surface of the rigid vault walls, Γ_v,

$$\frac{\partial}{\partial n^{v}}\bar{p}(\mathbf{x},\omega) = -\rho\, a_n^{v}(\mathbf{x},\omega) \tag{20}$$

in which n^v represents the direction of the normal to the surface and $a_n^v(\mathbf{x},\omega)$ is the spatial distribution of the acceleration of the inside surface of vault walls in its normal direction; and q_v is the surface damping coefficient for inside surface of vault walls.

On the free surface of the moderator, Γ_f, assuming small free surface waves:

$$(1+i\eta_s)\frac{\partial}{\partial z}\bar{p}(\mathbf{x},\omega) = \frac{\omega^2}{g}\bar{p}(\mathbf{x},\omega) \tag{21}$$

in which η_s is the damping factor associated with sloshing of free surface (Chen et al., 1996).

For harmonic excitation $a_x(t) = r_x e^{i\omega t}$, $a_y(t) = r_y e^{i\omega t}$, and $a_z(t) = r_z e^{i\omega t}$, the surface accelerations $a_n^{ic}(\mathbf{x},\omega)$ on the calandria-moderator interface Γ_{ic} are related to the modal accelerations of the calandria in the following way:

$$a_n^{ic}(\mathbf{x},\omega) = n_x^{ic}(\mathbf{x})\left[r_x - \omega^2 \sum_{n=1}^{N_c} \phi_{nx}^c(\mathbf{x})\overline{Y}_n^c(\omega)\right] + n_y^{ic}(\mathbf{x})\left[r_y - \omega^2 \sum_{n=1}^{N_c} \phi_{ny}^c(\mathbf{x})\overline{Y}_n^c(\omega)\right]$$

$$+ n_z^{ic}(\mathbf{x})\left[r_z - \omega^2 \sum_{n=1}^{N_c} \phi_{nz}^c(\mathbf{x})\overline{Y}_n^c(\omega)\right] \quad (22)$$

in which n_x^{ic}, n_y^{ic} and n_z^{ic} are the direction cosines of the normal at a point $\mathbf{x}$ on the inside surface of the calandria with respect to x, y and z axes, respectively; and $\phi_{nx}^c(\mathbf{x})$, $\phi_{ny}^c(\mathbf{x})$ and $\phi_{nz}^c(\mathbf{x})$ are the displacement functions along x, y and z axes for n-th mode shape of the calandria. These displacement functions can be evaluated using the shape functions of the finite element system and mode shapes of the calandria.

Similarly, the surface accelerations $a_n^j(\mathbf{x},\omega)$ on the j-th tube-moderator interface, Γ_j, are related to the modal accelerations of the tubes in the following way:

$$a_n^j(\mathbf{x},\omega) = n_x^j(\mathbf{x})\left[r_x - \omega^2 \sum_{n=1}^{N_j} \phi_{nx}^j(y)\overline{Y}_n^j(\omega)\right] + n_z^j(\mathbf{x})\left[r_z - \omega^2 \sum_{n=1}^{N_j} \phi_{nz}^j(y)\overline{Y}_n^j(\omega)\right] \quad (23)$$

in which n_x^j, and n_z^j are the direction cosines of the normal at a point $\mathbf{x}$ on the outside surface of the j-th tube with respect to x and z axes, respectively (Figure 1); and $\phi_{nx}^j(y)$ and $\phi_{nz}^j(y)$ are the displacement functions along x and z axes for n-th mode shape of the j-th tube. These displacement functions can be evaluated using the shape functions of the finite element system and mode shapes of the j-th tube.

The surface accelerations $a_n^v(\mathbf{x},\omega)$ on vault walls-inside moderator interface, Γ_v, are given as:

$$a_n^v(\mathbf{x},\omega) = r_x\, n_x^v(\mathbf{x}) + r_y\, n_y^v(\mathbf{x}) + r_z\, n_z^v(\mathbf{x}) \quad (24)$$

Solution for Hydrodynamic Pressure

The linear form of the governing equation and boundary conditions allow $\overline{p}(\mathbf{x},\omega)$ to be expressed as

$$\overline{p}(\mathbf{x},\omega) = \overline{p}_{0x}^i(\mathbf{x},\omega) + \overline{p}_{0y}^i(\mathbf{x},\omega) + \overline{p}_{0z}^i(\mathbf{x},\omega) - \omega^2 \sum_{n=1}^{N_c} \overline{p}_n^{ic}(\mathbf{x},\omega)\, \overline{Y}_n^c(\omega)$$

$$-\omega^2 \sum_{j=1}^{N_t} \sum_{n=1}^{N_j} \overline{p}_n^j(\mathbf{x},\omega)\, \overline{Y}_n^j(\omega) \quad (25)$$

In Equation (25), the hydrodynamic pressure $\overline{p}_{0x}^i(\mathbf{x},\omega)$ due to vault acceleration along x axis is the solution of Equation (17) for a rigid calandria and rigid tubes subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{ic}} \overline{p}_{0x}^i(\mathbf{x},\omega) = -\rho r_x n_x^{ic}(\mathbf{x}) + i\omega q_{ic} \overline{p}_{0x}^i(\mathbf{x},\omega) \text{ on } \Gamma_{ic} \quad (26a)$$

$$\frac{\partial}{\partial n^j} \overline{p}_{0x}^i(\mathbf{x},\omega) = -\rho r_x n_x^j(\mathbf{x}) + i\omega q_j \overline{p}_{0x}^i(\mathbf{x},\omega) \text{ on } \Gamma_j \quad (26b)$$

$$\frac{\partial}{\partial n^{v}}\overline{p}_{0x}^{i}(\mathbf{x},\omega)=-\rho r_{x}n_{x}^{v}(\mathbf{x})+i\omega q_{v}\overline{p}_{0x}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{v} \tag{26c}$$

$$(1+i\eta_{s})\frac{\partial}{\partial z}\overline{p}_{0x}^{i}(\mathbf{x},\omega)=\frac{\omega^{2}}{g}\overline{p}_{0x}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{f} \tag{26d}$$

In Equation (25), the hydrodynamic pressure $\overline{p}_{0y}^{i}(\mathbf{x},\omega)$ due to vault acceleration along *y-axis* is the solution of Equation (17) for a rigid calandria and rigid tubes subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{ic}}\overline{p}_{0y}^{i}(\mathbf{x},\omega)=-\rho r_{y}n_{y}^{ic}(\mathbf{x})+i\omega q_{ic}\overline{p}_{0y}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{ic} \tag{27a}$$

$$\frac{\partial}{\partial n^{j}}\overline{p}_{0y}^{i}(\mathbf{x},\omega)=i\omega q_{j}\overline{p}_{0y}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{j} \tag{27b}$$

$$\frac{\partial}{\partial n^{v}}\overline{p}_{0y}^{i}(\mathbf{x},\omega)=-\rho r_{y}n_{y}^{v}(\mathbf{x})+i\omega q_{v}\overline{p}_{0y}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{v} \tag{27c}$$

$$(1+i\eta_{s})\frac{\partial}{\partial z}\overline{p}_{0y}^{i}(\mathbf{x},\omega)=\frac{\omega^{2}}{g}\overline{p}_{0y}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{f} \tag{27d}$$

In Equation (25), the hydrodynamic pressure $\overline{p}_{0z}^{i}(\mathbf{x},\omega)$ due to vault acceleration along z-axis is the solution of Equation (17) for a rigid calandria and rigid tubes subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{ic}}\overline{p}_{0z}^{i}(\mathbf{x},\omega)=-\rho r_{z}n_{z}^{ic}(\mathbf{x})+i\omega q_{ic}\overline{p}_{0z}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{ic} \tag{28a}$$

$$\frac{\partial}{\partial n^{j}}\overline{p}_{0z}^{i}(\mathbf{x},\omega)=-\rho r_{z}n_{z}^{j}(\mathbf{x})+i\omega q_{j}\overline{p}_{0z}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{j} \tag{28b}$$

$$\frac{\partial}{\partial n^{v}}\overline{p}_{0z}^{i}(\mathbf{x},\omega)=-\rho r_{z}n_{z}^{v}(\mathbf{x})+i\omega q_{v}\overline{p}_{0z}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{v} \tag{28c}$$

$$(1+i\eta_{s})\frac{\partial}{\partial z}\overline{p}_{0z}^{i}(\mathbf{x},\omega)=\frac{\omega^{2}}{g}\overline{p}_{0z}^{i}(\mathbf{x},\omega)\text{ on }\Gamma_{f} \tag{28d}$$

In Equation (25), the hydrodynamic pressure $\overline{p}_{n}^{ic}(\mathbf{x},\omega)$ is the solution of the governing equation, due to excitation of calandria in its n-th mode with no excitation of the vault or tubes, subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{ic}}\overline{p}_{n}^{ic}(\mathbf{x},\omega)=-\rho\left[n_{x}^{ic}(\mathbf{x})\,\phi_{nx}^{c}(\mathbf{x})+n_{z}^{ic}(\mathbf{x})\,\phi_{nz}^{c}(\mathbf{x})\right]+i\omega q_{ic}\overline{p}_{n}^{ic}(\mathbf{x},\omega)\text{ on }\Gamma_{ic} \tag{29a}$$

$$\frac{\partial}{\partial n^{j}}\overline{p}_{n}^{ic}(\mathbf{x},\omega)=i\omega q_{j}\overline{p}_{n}^{ic}(\mathbf{x},\omega)\text{ on }\Gamma_{j} \tag{29b}$$

$$\frac{\partial}{\partial n^{v}}\overline{p}_{n}^{ic}(\mathbf{x},\omega)=i\omega q_{v}\overline{p}_{n}^{ic}(\mathbf{x},\omega)\text{ on }\Gamma_{v} \tag{29c}$$

$$(1+i\eta_s)\frac{\partial}{\partial z}\overline{p}_n^{ic}(\mathbf{x},\omega)=\frac{\omega^2}{g}\overline{p}_n^{ic}(\mathbf{x},\omega) \text{ on } \Gamma_f \tag{29d}$$

These boundary value problems are solved for each mode of calandria. Similarly, in Equation (25), the hydrodynamic pressure $\overline{p}_n^j(\mathbf{x},\omega)$ is the solution of the governing equation, due to excitation of j-th tube in its n-th mode with no excitation of the vault or calandria walls, subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{ic}}\overline{p}_n^j(\mathbf{x},\omega)=i\omega q_{ic}\overline{p}_n^j(\mathbf{x},\omega) \text{ on } \Gamma_{ic} \tag{30a}$$

$$\frac{\partial}{\partial n^j}\overline{p}_n^j(\mathbf{x},\omega)=-\rho\left[n_x^j(\mathbf{x})\phi_{nx}^j(y)+n_z^j(\mathbf{x})\phi_{nz}^j(y)\right]+i\omega q_j\overline{p}_n^j(\mathbf{x},\omega) \text{ on } \Gamma_j \tag{30b}$$

$$\frac{\partial}{\partial n^v}\overline{p}_n^j(\mathbf{x},\omega)=i\omega q_v\overline{p}_n^j(\mathbf{x},\omega) \text{ on } \Gamma_v \tag{30c}$$

$$(1+i\eta_s)\frac{\partial}{\partial z}\overline{p}_n^j(\mathbf{x},\omega)=\frac{\omega^2}{g}\overline{p}_n^j(\mathbf{x},\omega) \text{ on } \Gamma_f \tag{30d}$$

These boundary value problems are solved for each mode of and each tube in the calandria.

Hydrodynamic Forces on Calandria

The linear form of the governing equation and boundary conditions allow the vector of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressures on the inside surface, $\overline{\mathbf{f}}_c^i(\omega)$, to be expressed as:

$$\overline{\mathbf{f}}_c^i(\omega)=\overline{\mathbf{f}}_{0x}^{ic}(\omega)+\overline{\mathbf{f}}_{0y}^{ic}(\omega)+\overline{\mathbf{f}}_{0z}^{ic}(\omega)-\omega^2\sum_{n=1}^{N_c}\overline{\mathbf{f}}_n^{ic}(\omega)\,\overline{Y}_n^c(\omega)$$

$$-\omega^2\sum_{j=1}^{N_t}\sum_{n=1}^{N_j}\overline{\mathbf{f}}_n^{icj}(\omega)\,\overline{Y}_n^j(\omega) \tag{31}$$

In Equation (31), $\overline{\mathbf{f}}_{0x}^{ic}(\omega)$, $\overline{\mathbf{f}}_{0y}^{ic}(\omega)$, $\overline{\mathbf{f}}_{0z}^{ic}(\omega)$, $\overline{\mathbf{f}}_n^{ic}(\omega)$, and $\overline{\mathbf{f}}_n^{icj}(\omega)$ are the vectors of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressures $\overline{p}_{0x}(\mathbf{x},\omega)$, $\overline{p}_{0y}(\mathbf{x},\omega)$, $\overline{p}_{0z}(\mathbf{x},\omega)$, $\overline{p}_n^{ic}(\mathbf{x},\omega)$, and $\overline{p}_n^j(\mathbf{x},\omega)$ respectively, i.e.

$$\overline{\mathbf{f}}_{0x}^{ic}(\omega)=\int_{\Gamma_{ic}}\mathbf{N}_{ic}(\mathbf{x})\;\overline{p}_{0x}(\mathbf{x},\omega)\,d\Gamma \tag{32a}$$

$$\overline{\mathbf{f}}_{0y}^{ic}(\omega)=\int_{\Gamma_{ic}}\mathbf{N}_{ic}(\mathbf{x})\;\overline{p}_{0y}(\mathbf{x},\omega)\,d\Gamma \tag{32b}$$

$$\overline{\mathbf{f}}_{0z}^{ic}(\omega)=\int_{\Gamma_{ic}}\mathbf{N}_{ic}(\mathbf{x})\;\overline{p}_{0z}(\mathbf{x},\omega)\,d\Gamma \tag{32c}$$

$$\overline{\mathbf{f}}_n^{ic}(\omega)=\int_{\Gamma_{ic}}\mathbf{N}_{ic}(\mathbf{x})\;\overline{p}_n^{ic}(\mathbf{x},\omega)\,d\Gamma \tag{32d}$$

$$\overline{\mathbf{f}}_n^{icj}(\omega) = \int_{\Gamma_{ic}} \mathbf{N}_{ic}(\mathbf{X})\ \overline{p}_n^{j}(\mathbf{X},\omega)\, d\Gamma \tag{32e}$$

In these equations, $\mathbf{N}_{ic}(\mathbf{X})$ is the vector of shape functions defined on the inside surface of calandria and corresponds to its finite element discretization.

Hydrodynamic Forces on j-th Tube:

The linear form of the governing equation and boundary conditions allow the vector of response functions for equivalent forces at the nodes in the finite element system of j-th tube due to hydrodynamic pressures on its surface, $\overline{\mathbf{f}}_j(\omega)$, to be expressed as:

$$\overline{\mathbf{f}}_j = \overline{\mathbf{f}}_{0x}^{j} + \overline{\mathbf{f}}_{0y}^{j} + \overline{\mathbf{f}}_{0z}^{j} - \omega^2 \sum_{n=1}^{N_c} \overline{\mathbf{f}}_n^{jc}\, \overline{Y}_n^{c}(\omega) - \omega^2 \sum_{k=1}^{N_t} \sum_{n=1}^{N_j} \overline{\mathbf{f}}_n^{jk}\ \overline{Y}_n^{k}(\omega) \tag{33}$$

In Equation (33), $\overline{\mathbf{f}}_{0x}^{j}(\omega)$, $\overline{\mathbf{f}}_{0y}^{j}(\omega)$, $\overline{\mathbf{f}}_{0z}^{j}(\omega)$, $\overline{\mathbf{f}}_n^{jc}(\omega)$, and $\overline{\mathbf{f}}_n^{jk}(\omega)$ are the vectors of response functions for equivalent forces at the nodes in the finite element system of j-th tube due to hydrodynamic pressures $\overline{p}_{0x}^{i}(\mathbf{X},\omega)$, $\overline{p}_{0y}^{i}(\mathbf{X},\omega)$, $\overline{p}_{0z}^{i}(\mathbf{X},\omega)$, $\overline{p}_n^{ic}(\mathbf{X},\omega)$, and $\overline{p}_n^{k}(\mathbf{X},\omega)$ respectively, i.e.

$$\overline{\mathbf{f}}_{0x}^{j}(\omega) = \int_{\Gamma_j} \mathbf{N}_{sj}(\mathbf{X})\ \overline{p}_{0x}^{i}(\mathbf{X},\omega)\, d\Gamma \tag{34a}$$

$$\overline{\mathbf{f}}_{0y}^{j}(\omega) = \int_{\Gamma_j} \mathbf{N}_{sj}(\mathbf{X})\ \overline{p}_{0x}^{i}(\mathbf{X},\omega)\, d\Gamma \tag{34b}$$

$$\overline{\mathbf{f}}_{0z}^{j}(\omega) = \int_{\Gamma_j} \mathbf{N}_{sj}(\mathbf{X})\ \overline{p}_{0z}^{i}(\mathbf{X},\omega)\, d\Gamma \tag{34c}$$

$$\overline{\mathbf{f}}_n^{jc}(\omega) = \int_{\Gamma_j} \mathbf{N}_{sj}(\mathbf{X})\ \overline{p}_n^{ic}(\mathbf{X},\omega)\, d\Gamma \tag{34d}$$

$$\overline{\mathbf{f}}_n^{jk}(\omega) = \int_{\Gamma_j} \mathbf{N}_{sj}(\mathbf{X})\ \overline{p}_n^{k}(\mathbf{X},\omega)\, d\Gamma \tag{34e}$$

in which $\overline{p}_n^{k}$ is the hydrodynamic pressure due to motion of the k-th tube in its n-th mode. In Equation (34), $\mathbf{N}_{sj}(\mathbf{X})$ is the vector of shape functions defined on the outside surface of the j-th tube corresponding to six degrees of freedom per node of finite element discretization of outside tubes.

4. Outside Vault Water Domain Substructure

Boundary Value Problem

The vector of frequency response functions of unknown hydrodynamic forces $\overline{\mathbf{f}}_c^{o}(\omega)$ on the outside surface of calandria because of vault excitation along x, y and z axes, can be expressed in terms of accelerations of modal coordinates of the calandria by analysis of the outside water domain (Figure 1). The small amplitude, irrotational motion of the moderator fluid is also governed by the three-dimensional linearized Navier-Stokes equation (Equation (17)) subject to the following boundary conditions:

On the calandria-moderator interface, Γ_{oc},

$$\frac{\partial}{\partial n^{oc}} \overline{p}(\mathbf{X},\omega) = -\rho\, a_n^{oc}(\mathbf{X},\omega) \tag{35}$$

in which ρ is the mass density of the moderator, n^{oc} represents the direction of the normal to the outside surface of calandria; and $a_n^{oc}(\mathbf{x},\omega)$ is the spatial distribution of the acceleration of the outside surface of calandria in its normal direction.

On the inside surface of the rigid vault walls, Γ_v,

$$\frac{\partial}{\partial n^v}\overline{p}(\mathbf{x},\omega) = -\rho\, a_n^v(\mathbf{x},\omega) \tag{36}$$

For harmonic excitation $a_x(t) = r_x e^{i\omega t}$, $a_y(t) = r_y e^{i\omega t}$, and $a_z(t) = r_z e^{i\omega t}$, of the calandria vault along the transverse, longitudinal and vertical directions of calandria, the surface accelerations $a_n^{oc}(\mathbf{x},\omega)$ on the calandria-moderator interface Γ_{oc}, and surface accelerations $a_n^{ov}(\mathbf{x},\omega)$ on the vault-moderator interface, Γ_v, are related to the modal accelerations of the calandria in the following way:

$$a_n^{oc}(\mathbf{x},\omega) = n_x^{oc}(\mathbf{x})\left[r_x - \omega^2 \sum_{n=1}^{N_c} \phi_{nx}^c(\mathbf{x})\overline{Y}_n^c(\omega)\right] + n_y^{oc}(\mathbf{x})\left[r_y - \omega^2 \sum_{n=1}^{N_c} \phi_{ny}^c(\mathbf{x})\overline{Y}_n^c(\omega)\right]$$

$$+ n_z^{oc}(\mathbf{x})\left[r_z - \omega^2 \sum_{n=1}^{N_c} \phi_{nz}^c(\mathbf{x})\overline{Y}_n^c(\omega)\right] \tag{37}$$

in which n_x^{oc}, n_y^{oc} and n_z^{oc} are the direction cosines of the normal at a point $\mathbf{x}$ on the outside surface of the calandria with respect to x, y and z axes, respectively (Figure 1); and $\phi_{nx}^c(\mathbf{x})$, $\phi_{ny}^c(\mathbf{x})$ and $\phi_{nz}^c(\mathbf{x})$ are the displacement functions along x, y and z axes for n-th mode shape of the calandria. These displacement functions can be evaluated using the shape functions of the finite element system and mode shapes of the calandria. Similarly, on the moderator-vault wall interface:

$$a_n^v(\mathbf{x},\omega) = r_x\, n_x^v(\mathbf{x}) + r_y\, n_y^v(\mathbf{x}) + r_z\, n_z^v(\mathbf{x}) \tag{38}$$

in which n_x^v, n_y^v and n_z^v are the direction cosines of the normal at a point $\mathbf{x}$ on the inside surface of the vault walls with respect to x, y and z axes, respectively (Figure 1).

Solution for Hydrodynamic Pressure

The linear form of the governing equation and boundary conditions allow $\overline{p}(\mathbf{x},\omega)$ to be expressed as

$$\overline{p}(\mathbf{x},\omega) = \overline{p}_{0x}^{o}(\mathbf{x},\omega) + \overline{p}_{0y}^{o}(\mathbf{x},\omega) + \overline{p}_{0z}^{o}(\mathbf{x},\omega) - \omega^2 \sum_{n=1}^{N_c} \overline{p}_n^{oc}(\mathbf{x},\omega)\, \overline{Y}_n^c(\omega) \tag{39}$$

In Equation (39), the hydrodynamic pressure $\overline{p}_{0x}^{o}(\mathbf{x},\omega)$ due to vault acceleration along x-axis is the solution of Equation (17) for a rigid calandria subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{oc}}\overline{p}_{0x}^{o}(\mathbf{x},\omega) = -\rho r_x n_x^{oc}(\mathbf{x}) + i\omega q_{oc}\overline{p}_{0x}^{o}(\mathbf{x},\omega) \text{ on } \Gamma_{oc} \tag{40a}$$

$$\frac{\partial}{\partial n^{v}}\overline{p}_{0x}^{o}(\mathbf{x},\omega) = -\rho r_x n_x^{v}(\mathbf{x}) + i\omega q_v\overline{p}_{0x}^{o}(\mathbf{x},\omega) \text{ on } \Gamma_v \tag{40b}$$

In Equation (39), the hydrodynamic pressure $\overline{p}_{0y}^{o}(\mathbf{x},\omega)$ due to vault acceleration along y-axis is the solution of Equation (17) for a rigid calandria subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{oc}} \bar{p}^{o}_{0y}(\mathbf{x},\omega) = -\rho r_y n^{oc}_y(\mathbf{x}) + i\omega q_{oc} \bar{p}^{o}_{0y}(\mathbf{x},\omega) \text{ on } \Gamma_{oc} \tag{41a}$$

$$\frac{\partial}{\partial n^{v}} \bar{p}^{o}_{0y}(\mathbf{x},\omega) = -\rho r_y n^{v}_y(\mathbf{x}) + i\omega q_{v} \bar{p}^{o}_{0y}(\mathbf{x},\omega) \text{ on } \Gamma_{v} \tag{41b}$$

In Equation (39), the hydrodynamic pressure $\bar{p}^{o}_{0z}(\mathbf{x},\omega)$ due to vault acceleration along z-axis is the solution of Equation (17) for a rigid calandria subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{oc}} \bar{p}^{o}_{0z}(\mathbf{x},\omega) = -\rho r_z n^{oc}_z(\mathbf{x}) + i\omega q_{oc} \bar{p}^{o}_{0z}(\mathbf{x},\omega) \text{ on } \Gamma_{oc} \tag{42a}$$

$$\frac{\partial}{\partial n^{v}} \bar{p}^{o}_{0z}(\mathbf{x},\omega) = -\rho r_z n^{v}_z(\mathbf{x}) + i\omega q_{v} \bar{p}^{o}_{0z}(\mathbf{x},\omega) \text{ on } \Gamma_{v} \tag{42b}$$

In Equation (39), the hydrodynamic pressure $\bar{p}^{oc}_{n}(\mathbf{x},\omega)$ is the solution of the governing equation, due to excitation of calandria in its n-th mode with no excitation of the vault, subject to the following boundary conditions:

$$\frac{\partial}{\partial n^{oc}} \bar{p}^{oc}_{n}(\mathbf{x},\omega) = -\rho\left[n^{oc}_x(\mathbf{x})\phi^{c}_{nx}(\mathbf{x}) + n^{oc}_z(\mathbf{x})\phi^{c}_{nz}(\mathbf{x})\right] + i\omega q_{oc} \bar{p}^{oc}_{n}(\mathbf{x},\omega) \text{ on } \Gamma_{oc} \tag{43a}$$

$$\frac{\partial}{\partial n^{v}} \bar{p}^{oc}_{n}(\mathbf{x},\omega) = i\omega q_{v} \bar{p}^{oc}_{n}(\mathbf{x},\omega) \text{ on } \Gamma_{v} \tag{43b}$$

Hydrodynamic Forces on Calandria

The linear form of the governing equation and boundary conditions allow the vector of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressures on the outside surface, $\bar{\mathbf{f}}^{o}_{c}(\omega)$, to be expressed as:

$$\bar{\mathbf{f}}^{o}_{c}(\omega) = \bar{\mathbf{f}}^{oc}_{0x}(\omega) + \bar{\mathbf{f}}^{oc}_{0y}(\omega) + \bar{\mathbf{f}}^{oc}_{0z}(\omega) - \omega^2 \sum_{n=1}^{N_c} \bar{\mathbf{f}}^{oc}_{n}(\omega)\, \bar{Y}^{c}_{n}(\omega) \tag{44}$$

In Equation (44), $\bar{\mathbf{f}}^{oc}_{0x}(\omega)$, $\bar{\mathbf{f}}^{oc}_{0y}(\omega)$, $\bar{\mathbf{f}}^{oc}_{0z}(\omega)$, and $\bar{\mathbf{f}}^{oc}_{n}(\omega)$ are the vectors of response functions for equivalent forces at the nodes in the finite element system of calandria due to hydrodynamic pressures $\bar{p}^{o}_{0x}(\mathbf{x},\omega)$, $\bar{p}^{o}_{0y}(\mathbf{x},\omega)$, $\bar{p}^{o}_{0z}(\mathbf{x},\omega)$ and $\bar{p}^{oc}_{n}(\mathbf{x},\omega)$, respectively:

$$\bar{\mathbf{f}}^{oc}_{0x}(\omega) = \int_{\Gamma_{oc}} \mathbf{N}_{oc}(\mathbf{x})\; \bar{p}^{o}_{0x}(\mathbf{x},\omega)\, d\Gamma \tag{45a}$$

$$\bar{\mathbf{f}}^{oc}_{0y}(\omega) = \int_{\Gamma_{oc}} \mathbf{N}_{oc}(\mathbf{x})\; \bar{p}^{o}_{0y}(\mathbf{x},\omega)\, d\Gamma \tag{45b}$$

$$\bar{\mathbf{f}}^{oc}_{0z}(\omega) = \int_{\Gamma_{oc}} \mathbf{N}_{oc}(\mathbf{x})\; \bar{p}^{o}_{0z}(\mathbf{x},\omega)\, d\Gamma \tag{45c}$$

$$\bar{\mathbf{f}}^{oc}_{n}(\omega) = \int_{\Gamma_{oc}} \mathbf{N}_{oc}(\mathbf{x})\; \bar{p}^{oc}_{n}(\mathbf{x},\omega)\, d\Gamma \tag{45d}$$

In these equations, $\mathbf{N}_{oc}(\mathbf{x})$ is the vector of shape functions defined on the outside surface of the calandria and corresponds to finite element discretization of calandria.

5. Coupled Equations of Motion

Modal Equations of Motion for Calandria

The modal equations of motion (Equation (5)) for the calandria in N_c unknowns, $\overline{Y}_n^c(\omega)$ (the frequency response function for the generalized (modal) coordinate associated with the first N_c modes of vibration of calandria without moderator) can be written in the following form by substituting Equations (31) and (44) into Equation (8):

$$-\omega^2 M_n^c \overline{Y}_n^c(\omega) - \omega^2 \sum_l^{N_c}\left[\overline{M}_{nl}^{ica}(\omega) + \overline{M}_{nl}^{oca}(\omega)\right]\overline{Y}_l^c(\omega) - \omega^2 \sum_j^{N_t}\sum_l^{N_j} \overline{M}_{nl}^{cja}(\omega)\overline{Y}_l^j(\omega)$$

$$+M_n^c\left[(1+i\eta_c)[\omega_n^c]^2\right]\overline{Y}_n^c(\omega) = -r_x\left[L_{nx}^c + \overline{L}_{nx}^{ica}(\omega) + \overline{L}_{nx}^{oca}(\omega)\right]$$

$$-r_y\left[L_{ny}^c + \overline{L}_{ny}^{ica}(\omega) + \overline{L}_{ny}^{oca}(\omega)\right] - r_z\left[L_{nz}^c + \overline{L}_{nz}^{ica}(\omega) + \overline{L}_{nz}^{oca}(\omega)\right] \qquad n = 1,\cdots N_c \tag{46}$$

in which

$$\overline{M}_{nl}^{ica}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_n^{ic}(\omega) \tag{47a}$$

$$\overline{M}_{nl}^{oca}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_n^{oc}(\omega) \tag{47b}$$

$$\overline{M}_{nl}^{cja}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_n^{icj}(\omega) \tag{47c}$$

$$\overline{L}_{nx}^{ica}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_{0x}^{ic}(\omega) \tag{48a}$$

$$\overline{L}_{nx}^{oca}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_{0x}^{oc}(\omega) \tag{48b}$$

$$\overline{L}_{ny}^{ica}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_{0y}^{ic}(\omega) \tag{48c}$$

$$\overline{L}_{ny}^{oca}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_{0y}^{oc}(\omega) \tag{48d}$$

$$\overline{L}_{nz}^{ica}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_{0z}^{ic}(\omega) \tag{48e}$$

$$\overline{L}_{nz}^{oca}(\omega) = \left[\Phi_n^c\right]^T \overline{\mathbf{f}}_{0z}^{oc}(\omega) \tag{48f}$$

In Equation (46), the real and imaginary parts of the functions $\overline{M}_{nl}^{ica}(\omega)$ and $\overline{M}_{nl}^{oca}(\omega)$ represent the self-added hydrodynamic mass and self-added hydrodynamic damping respectively on the calandria in its n-th mode due to its own motion in l-th mode. The real and imaginary parts of function $\overline{M}_{nl}^{cja}(\omega)$ represents the coupled-added hydrodynamic mass and coupled-added hydrodynamic damping respectively on the calandria in its n-th mode due to the motion of j-th tube in its l-th mode. In Equation (46), the functions $\overline{L}_{nx}^{ica}(\omega)$ and $\overline{L}_{nx}^{oca}(\omega)$ are the added hydrodynamic excitations in the n-th mode of vibration of calandria due to moderator inside the calandria, and outside the calandria, respectively due to vault excitation along

x-axis. Similarly, the functions $\overline{L}_{ny}^{ica}(\omega)$ and $\overline{L}_{ny}^{oca}(\omega)$ are the added hydrodynamic excitations due to vault excitation along y-axis and the functions $\overline{L}_{nz}^{ica}(\omega)$ and $\overline{L}_{nz}^{oca}(\omega)$ are the added hydrodynamic excitations due to vault excitation along z-axis.

Modal Equations of Motion for j-th Tube

The modal equations of motion (Equation (13)) for tubes in $\sum_{j}^{N_t} N_j$ unknowns, $\overline{Y}_n^j(\omega)$ (the frequency response function for the generalized (modal) coordinate associated with the first N_j modes of vibration of j-th tube without moderator) can be written in the following form by substituting Equation (33) into Equation (16):

$$-\omega^2 M_n^j \overline{Y}_n^j(\omega) - \omega^2 \sum_{l}^{N_c} \overline{M}_{nl}^{jca}(\omega)\overline{Y}_l^c(\omega) - \omega^2 \sum_{k}^{N_t} \sum_{l}^{N_k} \overline{M}_{nkl}^{ja}(\omega)\overline{Y}_l^k(\omega)$$

$$+ M_n^j\left[(1+i\eta_j)[\omega_n^j]^2\right]\overline{Y}_n^j(\omega) = -r_x\left[L_{nx}^j + \overline{L}_{nx}^{ja}(\omega)\right] - r_y\left[L_{ny}^j + \overline{L}_{ny}^{ja}(\omega)\right]$$

$$-r_z\left[L_{nz}^j + \overline{L}_{nz}^{ja}(\omega)\right] \quad n = 1,\cdots N_j \tag{49}$$

in which N_k is the number of modes considered for k-th tube and

$$\overline{M}_{nl}^{jca}(\omega) = \left[\Phi_n^j\right]^T \overline{\mathbf{f}}_n^{jc}(\omega) \tag{50a}$$

$$\overline{M}_{nkl}^{ja}(\omega) = \left[\Phi_n^j\right]^T \overline{\mathbf{f}}_l^k(\omega) \tag{50b}$$

$$\overline{L}_{nx}^{ja}(\omega) = \left[\Phi_n^j\right]^T \overline{\mathbf{f}}_{0x}^j(\omega) \tag{51a}$$

$$\overline{L}_{ny}^{ja}(\omega) = \left[\Phi_n^j\right]^T \overline{\mathbf{f}}_{0y}^j(\omega) \tag{51b}$$

$$\overline{L}_{nz}^{ja}(\omega) = \left[\Phi_n^j\right]^T \overline{\mathbf{f}}_{0z}^j(\omega) \tag{51c}$$

In Equation (49), for j=k, the real and imaginary parts of function $\overline{M}_{nkl}^{ja}(\omega)$ represent the self-added hydrodynamic mass and self-added hydrodynamic damping on the j-th tube in its n-th mode due to its own motion in l-th mode. For j≠k, the real and imaginary parts of function $\overline{M}_{nkl}^{ja}(\omega)$ represent the coupled-added hydrodynamic mass and coupled-added hydrodynamic damping on the j-th tube in its n-th mode due to motion of k-th tube in l-th mode. The real and imaginary parts of function $\overline{M}_{nl}^{jca}(\omega)$ represent the coupled-added hydrodynamic mass on the j-th tube in its n-th mode due to the motion of calandria in its l-th mode. In Equation (49), the term $\overline{L}_{nx}^{ja}(\omega)$, $\overline{L}_{ny}^{ja}(\omega)$ and $\overline{L}_{nx}^{ja}(\omega)$ are the added hydrodynamic excitation in the n-th mode of vibration of j-th tube due to vault excitation along x, y and z axes respectively.

Thus, there are a total of $N_c + \sum_{j}^{N_t} N_j$ frequency-dependent equations that are coupled through the hydrodynamic forces. These equations must be solved simultaneously for each excitation frequency of interest to get the frequency response functions $\overline{Y}_n^c(\omega)$ (the generalized (modal) coordinate associated with the first N_c modes of vibration of calandria) and $\overline{Y}_n^j(\omega)$ (the generalized (modal) coordinate associated with the first N_j modes of vibration of j-th tube). Repeated solution for the excitation frequencies covering the range over which the earthquake ground motion and structural response have significant components leads to the complete frequency response functions for the modal coordinates.

The frequency response function of any response quantity for calandria, $R_c(\omega)$ can be expressed in terms of the frequency response functions for modal coordinates $\overline{Y}_n^c(\omega)$ of calandria:

$$R_c(\omega) = \sum_{n=1}^{N_c} R_n^c \overline{Y}_n^c(\omega) \tag{52}$$

in which the coefficients R_n^c are known for the calandria system under consideration. Similarly, the frequency response function of any response quantity for a tube can be expressed in terms of the frequency response functions for modal coordinates as follows:

$$R_j(\omega) = \sum_{n=1}^{N_j} R_n^j \overline{Y}_n^j(\omega) \tag{53}$$

in which the coefficients R_n^j are known for the j-th tube under consideration.

RESPONSE TO SPECIFIED VAULT EXCITATION

The equations of motion in frequency domain have been derived for harmonic excitation $a_x(t) = r_x e^{i\omega t}$ of the calandria vault along the transverse direction of calandria, $a_y(t) = r_y e^{i\omega t}$ of the calandria vault along the longitudinal direction of calandria and $a_z(t) = r_z e^{i\omega t}$ along the vertical direction of calandria. The frequency response function of any response quantity $R(\omega)$, for calandria, or a tube, is computed for three sets of values of r_x, r_y and r_z, thus yielding $R_x(\omega)$ for $r_x = 1$, $r_y = 0$, $r_z = 0$; $R_y(\omega)$ for $r_x = 0$, $r_y = 1$, $r_z = 0$; and $R_z(\omega)$ for $r_x = 0$, $r_y = 0$, $r_z = 1$.

1. Response for Deterministic Vault Excitations

The time-history of response quantity $R(t)$, for calandria or j-th tube, is given by the Fourier integral as a superposition of responses to individual harmonic components of the vault excitation (Clough and Penzien, 1993):

$$R(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[A_x(\omega) R_x(\omega) + A_y(\omega) R_y(\omega) + A_z(\omega) R_z(\omega) \right] e^{i\omega t} d\omega \tag{54}$$

where $A_x(\omega)$, $A_y(\omega)$ and $A_z(\omega)$ are the Fourier transforms of the specified vault accelerations $a_x(t)$, $a_y(t)$ and $a_z(t)$:

$$A_x(\omega) = \int_0^{T_d} a_x(t) e^{-i\omega t} dt \tag{55a}$$

$$A_y(\omega) = \int_0^{T_d} a_y(t) e^{-i\omega t} dt \tag{55b}$$

$$A_z(\omega) = \int_0^{T_d} a_z(t) e^{-i\omega t} dt \tag{55c}$$

where T_d is the duration of vault excitation. The Fourier integrals in Equations (55) are computed in their discrete forms using the fast Fourier transform (FFT) algorithm.

2. Response for Stochastic Vault Excitations

The power spectral density of response $S_{RR}(\omega)$ of response quantity $R(t)$, for calandria or j-th tube, is given by (Clough and Penzien, 1993):

$$S_{RR}(\omega) = \begin{Bmatrix} R_x(\omega) \\ R_y(\omega) \\ R_y(\omega) \end{Bmatrix} \begin{bmatrix} S^v_{xx}(\omega) & S^v_{xy}(\omega) & S^v_{xz}(\omega) \\ S^v_{yx}(\omega) & S^v_{yy}(\omega) & S^v_{yz}(\omega) \\ S^v_{zx}(\omega) & S^v_{zy}(\omega) & S^v_{zz}(\omega) \end{bmatrix} \begin{Bmatrix} R_x(\omega) \\ R_y(\omega) \\ R_y(\omega) \end{Bmatrix} \tag{56}$$

where $S^v_{xx}(\omega) \cdots S^v_{zz}(\omega)$ are the terms of power spectral density function matrix for vault wall accelerations.

NUMERICAL EVALUATION PROCEDURES

Evaluation of added mass and damping terms in the coupled equations of motion requires natural modes and frequencies of calandria and tubes, and numerical solution of boundary value problems for inside moderator domain and outside water domain. For this purpose, efficient numerical procedures are presented next for four substructures: the calandria shell, the horizontal tube(s), the inside moderator domain and the outside water domain.

1. Calandria Substructure

The calandria substructure is idealized as finite element system using standard nine-node shell element. The element mass matrix and elastic stiffness matrix are obtained using standard procedures and assembled to yield the global mass matrix, $\mathbf{M}_c$, and stiffness matrix, $\mathbf{K}_c$ for the calandria without moderator (Zienkiewicz and Taylor, 1991).

The natural frequencies and mode shapes for the calandria substructure are given by solutions of the associated eigenvalue problem (Equation (3)). The n-th mode shape of calandria is completely defined by vector $\mathbf{\Phi}^c_n$ of the nodal displacements of calandria and associated frequencies ω^c_n. For n-th mode of vibration of calandria, the sub-vectors $\mathbf{\Phi}^c_{nx}$, $\mathbf{\Phi}^c_{ny}$ and $\mathbf{\Phi}^c_{nz}$ containing displacements along x, y and z-axes, respectively, at nodes of finite element discretization of Γ_c are evaluated. For a point $\mathbf{x}$ on the calandria surface, the values of displacement functions $\phi^c_{nx}(\mathbf{x})$, $\phi^c_{ny}(\mathbf{x})$, and $\phi^c_{nz}(\mathbf{x})$ representing the motion of the calandria surface along x, y and z-axes respectively are computed as:

$$\phi^c_{nx}(\mathbf{x}) = \{\mathbf{N}_c(\mathbf{x})\}^T \mathbf{\Phi}^c_{nx} \tag{57a}$$

$$\phi^c_{ny}(\mathbf{x}) = \{\mathbf{N}_c(\mathbf{x})\}^T \mathbf{\Phi}^c_{ny} \tag{57b}$$

$$\phi^c_{nz}(\mathbf{x}) = \{\mathbf{N}_c(\mathbf{x})\}^T \mathbf{\Phi}^c_{nz} \tag{57c}$$

where $\mathbf{N}_c(\mathbf{x})$ is the vector of interpolation functions defined over the calandria moderator interface and corresponds to finite element discretization of calandria.

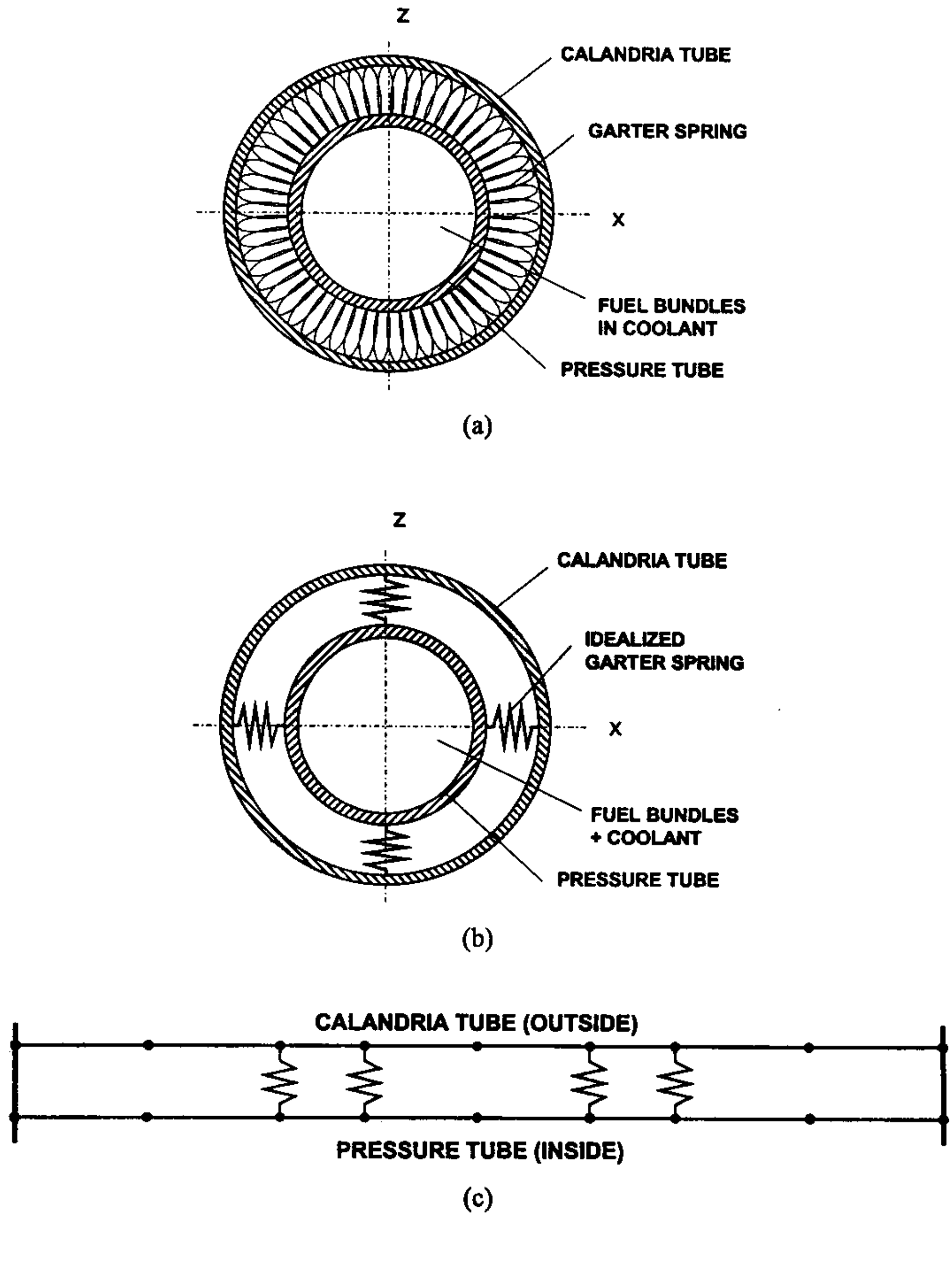

Fig. 2 Horizontal tube assembly: idealization of garter springs and tubes

The natural modes of vibration of calandria can be classified into three groups:

1. Modes that are excited due to vault excitation along x-axis. These modes are symmetric about $y = H/2$ plane and anti-symmetric about y-z plane.
2. Modes that are excited due to vault excitation along y-axis. These modes are anti-symmetric about $y = H/2$ plane and symmetric about y-z plane.
3. Similarly, the modes that are excited due to vault excitation along z-axis. These modes are symmetric about $y = H/2$ and y-z planes.

To avoid numerical difficulties due to repeated eigenvalues, the calandria shell is analyzed with different boundary conditions at the planes of symmetry to evaluate modes that are excited by excitation along x, y and z-axes separately.

2. Horizontal Tube(s) Substructures

The cross-section of a horizontal tube is shown in Figure 2. The substructure consists of two concentric tubes separated by spacers (garter springs) at four locations along the length of the tube. The outside tube is referred as calandria tube and the inside tube is referred as pressure tube. The fuel bundles are kept inside the pressure tube and coolant flows at high pressure inside the pressure tube.

The finite element model of a typical horizontal tube substructure is shown in Figure 2. The calandria (outside) tube and pressure (inside) tube are modeled as beam elements. The garter springs are idealized as equivalent springs having stiffness k_g. The hydrodynamic forces due to motion of the tubes and calandria are applied on the outside tube whereas the mass of the fuel bundles and coolant inside the pressure tube is added to the mass of the inside tube.

To compute equivalent spring stiffness k_g, the garter spring is replaced by a series of radially placed annular rings of the same cross-sectional properties as those of garter spring. The separation between these rings is taken equal to the mean pitch of the garter spring. Stiffness of such a ring with mean radius equal to r_{mg} and subject to diametrically opposite forces can be given as:

$$k_{rg} = \frac{EI}{r_{mg}^3}\left(\frac{4\pi}{\pi^2 - 8}\right) \tag{58}$$

To further simplify the model, the discrete rings are replaced by a continuous radial spring bed of Winkler-type with distributed stiffness equal to $k_{rg}s_r^{-1}$ where k_{rg} is the stiffness of individual ring and s_r is the separation between the rings. The distributed stiffness is then integrated over contact angle π to obtain equivalent spring stiffness k_g. While calculating k_g, the lack of fit, if any, has been ignored. The spring stiffness value thus calculated is very high compared to the stiffness of individual tubes. The natural frequencies of the twin tube system are not sensitive to the variation of value of k_g near its calculated value (Joshi, 1998) and therefore the garter springs have been idealized as rigid links.

The stiffness and mass properties of the tubes are not coupled for vibrations in x-y plane and z-y plane. Therefore, natural frequencies and mode shapes are evaluated independently by planar analysis considering only two-degrees-of-freedom at a node. Element stiffness and mass matrices are evaluated using Hermitian interpolation functions. Global stiffness and mass matrices are assembled from these element matrices taking into account connecting rigid links. The natural frequencies and mode shapes of the horizontal tube substructure are evaluated by solution of associated eigenvalue problem (Equation (11)).

The displacement functions $\phi_{nx}^j(y)$ and $\phi_{nz}^j(y)$ are evaluated from the vectors Φ_n^j, using interpolation functions with nodal displacements equal to the displacements in a mode shape of horizontal tube obtained by solution of Equation (11), i.e.

$$\phi_{nx}^j(y) = \phi_{nz}^j(y) = [\mathbf{N}_{cj}(y)]^T \Phi_n^j \tag{59}$$

in which $\mathbf{N}_{cj}$ is matrix of interpolation functions defined on centroidal axis of the j-th horizontal tube. Since the displacement functions are required only for outer tubes, the above interpolation is carried out only for outer tube.

The numerical results are presented for tube assembly with the following properties. The outside and inside diameters of calandria tube are 110.2 mm and 107.77 mm respectively. Similarly, outside and inside diameters of pressure tube are 90.61 mm and 82.55 mm respectively. The elastic modulus of the tube material is assumed to be 96,500 N/mm^2 whereas its mass density is taken equal to 6,660 kg/m^3. The first three symmetric and anti-symmetric mode shapes of the horizontal tube without hydrodynamic effects are shown in Figures 3.

3. Inside Moderator Domain

The boundary value problems for inside moderator domain to evaluate hydrodynamic pressures $\overline{p}_{0x}^{i}$, $\overline{p}_{0y}^{i}$, $\overline{p}_{0z}^{i}$, $\overline{p}_{n}^{ic}$, and $\overline{p}_{n}^{j}$ (Equations (26)-(30)) are solved by minimization of the following functional (Mikhlin, 1964):

$$\Pi = \frac{1}{2}\left[1+i\omega\frac{4\mu}{3k}\right]\int_{\Omega_{im}} \nabla p \cdot \nabla p \, d\Omega - \frac{\omega^2}{2C^2}\int_{\Omega_{im}} (\overline{p})^2 \, d\Omega + \left[1+i\omega\frac{4\mu}{3k}\right] i\omega q_c \int_{\Gamma_{ic}} (\overline{p})^2 d\Gamma$$

$$-\rho\left[1+i\omega\frac{4\mu}{3k}\right]\int_{\Gamma_{ic}} b^{ic}\,\overline{p}\, d\Gamma - \frac{\omega^2}{2g(1+i\eta_s)}\left[1+i\omega\frac{4\mu}{3k}\right]\int_{\Gamma_f} (\overline{p})^2 d\Gamma$$

$$+\left[1+i\omega\frac{4\mu}{3k}\right]\sum_j i\omega q_j \int_{\Gamma_j} (\overline{p})^2 d\Gamma - \rho\left[1+i\omega\frac{4\mu}{3k}\right]\sum_j \int_{\Gamma_j} b^j\,\overline{p}\, d\Gamma$$

$$+\left[1+i\omega\frac{4\mu}{3k}\right] i\omega q_v \int_{\Gamma_v} (\overline{p})^2 d\Gamma - \rho\left[1+i\omega\frac{4\mu}{3k}\right]\int_{\Gamma_v} b^v\,\overline{p}\, d\Gamma \qquad (60)$$

using standard finite-element procedures (Zienkiewicz and Taylor, 1991). The inside moderator domain Ω_{im} is idealized as an assemblage of three-dimensional finite elements and consequently, the calandria-inside moderator domain interface Γ_{ic}, the j-the tube-inside moderator interface Γ_j, the vault walls-moderator interface Γ_v and the free surface of inside moderator Γ_f get discretized into a number of subdivisions (Figure 4).

In Equation (60), b^{ic}, b^j and b^v are the known functions defining distribution of surface accelerations at calandria-inside moderator interface, j-th horizontal tube-inside moderator interface and inside surface of vault walls respectively.

For

$$b^{ic} = r_x n_x^{ic}(\mathbf{x}) \text{ on } \Gamma_{ic} \qquad (61a)$$

$$b^{j} = r_x n_x^{j}(\mathbf{x}) \text{ on } \Gamma_{j} \qquad (61b)$$

$$b^{v} = r_x n_x^{v}(\mathbf{x}) \text{ on } \Gamma_{v} \qquad (61c)$$

the pressure function which renders functional of Equation (60) minimum is equal to $\overline{p}_{0x}^{i}(\mathbf{x},\omega)$.

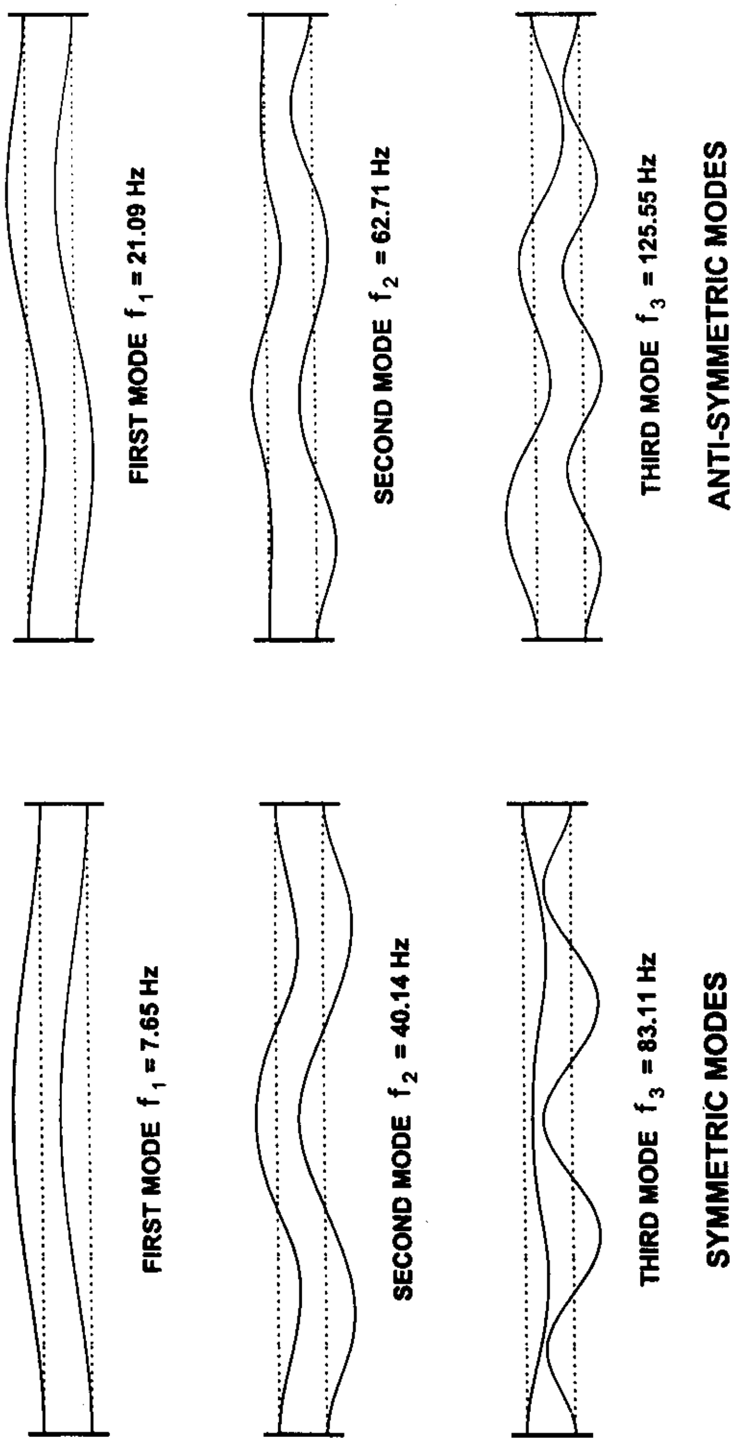

Fig. 3 Vibration mode shapes of horizontal tube

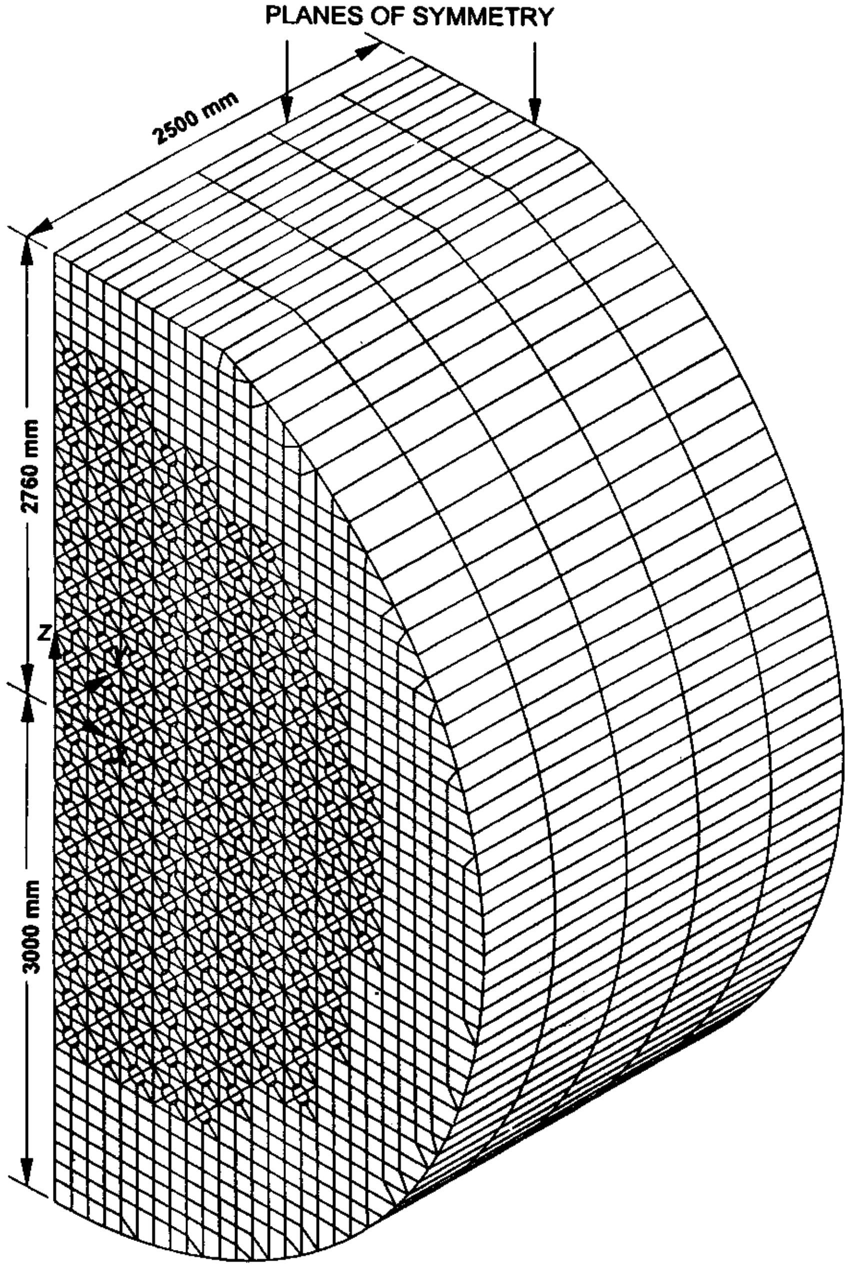

Fig.4 Finite element dicretization of inside moderator domain

For

$$b^{ic} = -\rho r_y n_y^{ic}(\mathbf{x}) \text{ on } \Gamma_{ic} \tag{62a}$$

$$b^{j} = 0 \text{ on } \Gamma_j \tag{62b}$$

$$b^{v} = -\rho r_y n_y^{v}(\mathbf{x}) \text{ on } \Gamma_v \tag{62c}$$

the pressure function which renders functional of Equation (60) minimum is equal to $\overline{p}_{0y}^{i}(\mathbf{x},\omega)$.

For

$$b^{ic} = r_z n_z^{ic}(\mathbf{x}) \text{ on } \Gamma_{ic} \tag{63a}$$

$$b^{j} = r_z n_z^{j}(\mathbf{x}) \text{ on } \Gamma_j \tag{63b}$$

$$b^{v} = r_z n_z^{v}(\mathbf{x}) \text{ on } \Gamma_v \tag{63c}$$

the pressure function which renders functional of Equation (60) minimum is equal to $\overline{p}_{0z}^{i}(\mathbf{x},\omega)$.

For

$$b^{ic} = n_x^{ic}(\mathbf{x})\,\phi_{nx}^{c}(\mathbf{x}) + n_y^{ic}(\mathbf{x})\,\phi_{ny}^{c}(\mathbf{x}) + n_z^{ic}(\mathbf{x})\,\phi_{nz}^{c}(\mathbf{x}) \text{ on } \Gamma_{ic} \tag{64a}$$

$$b^{j} = 0 \text{ on } \Gamma_j \tag{64b}$$

$$b^{v} = 0 \text{ on } \Gamma_v \tag{64c}$$

the pressure function which renders functional of Equation (60) minimum is equal to $\overline{p}_{n}^{ic}(\mathbf{x},\omega)$.

For

$$b^{ic} = 0 \text{ on } \Gamma_{ic} \tag{65a}$$

$$b^{j} = n_x^{j}(\mathbf{x})\phi_{nx}^{j}(y) + n_z^{j}(\mathbf{x})\phi_{nz}^{j}(y) \text{ on } \Gamma_j \tag{65b}$$

$$b^{v} = 0 \text{ on } \Gamma_v \tag{65c}$$

the pressure function which renders functional of Equation (60) minimum is equal to $\overline{p}_{n}^{j}(\mathbf{x},\omega)$.

At each frequency interval, $3 + N_c + \sum_{j}^{N_t} N_j$ solutions of complex equation derived from Equation (60) are required. The pressure values thus obtained are used in Equations (32) and (34) for evaluation of complex valued forces acting on calandria and tubes. The required integration over three-dimensional surface is carried out using Lagrange interpolation of the nodal pressure values and Gaussian numerical integration.

4. Outside Vault Water Domain

The boundary value problems for outside water domain to evaluate hydrodynamic pressures $\overline{p}_{0x}^{o}$, $\overline{p}_{0y}^{o}$, $\overline{p}_{0z}^{o}$, and $\overline{p}_{n}^{oc}$ (Equations (40)-(43)) are solved by minimization of the following functional (Mikhlin, 1964):

$$\Pi = \frac{1}{2}\left[1 + i\omega\frac{4\mu}{3k}\right]\int_{\Omega_{om}} \nabla p \cdot \nabla p d\Omega - \frac{\omega^2}{2C^2}\int_{\Omega_{om}} \left(\overline{p}\right)^2 d\Omega + \left[1 + i\omega\frac{4\mu}{3k}\right] i\omega q_c \int_{\Gamma_{oc}} \left(\overline{p}\right)^2 d\Gamma$$

$$+\left[1+i\omega\frac{4\mu}{3k}\right]i\omega q_v\int_{\Gamma_v}\left(\bar{p}\right)^2 d\Gamma-\rho\left[1+i\omega\frac{4\mu}{3k}\right]\int_{\Gamma_{oc}} b^c\,\bar{p}\,d\Gamma-\rho\left[1+i\omega\frac{4\mu}{3k}\right]\int_{\Gamma_v} b^v\,\bar{p}\,d\Gamma \quad (66)$$

using standard finite-element procedures (Zienkiewicz and Taylor, 1991). The outside water domain Ω_{om} is idealized as an assemblage of three-dimensional finite elements and consequently, the calandria-outside water interface Γ_{oc} and vault walls-outside water interface Γ_v get discretized into a number of subdivisions (Figure 5).

In Equation (66), b^{oc} and b^v are the known functions defining distribution of surface accelerations at calandria-outside water interface and inside surface of vault walls respectively.

For

$$b^{oc}=r_x n_x^{oc}(\mathbf{X}) \text{ on } \Gamma_{oc} \quad (67a)$$

$$b^{v}=r_x n_x^{v}(\mathbf{X}) \text{ on } \Gamma_{v} \quad (67b)$$

the pressure function which renders functional of Equation (66) minimum is equal to $\bar{p}_{0x}^{o}(\mathbf{X},\omega)$.

For

$$b^{oc}=r_y n_y^{oc}(\mathbf{X}) \text{ on } \Gamma_{oc} \quad (68a)$$

$$b^{v}=r_y n_y^{v}(\mathbf{X}) \text{ on } \Gamma_{v} \quad (68b)$$

the pressure function which renders functional of Equation (66) minimum is equal to $\bar{p}_{0y}^{o}(\mathbf{X},\omega)$.

For

$$b^{oc}=r_z n_z^{oc}(\mathbf{X}) \text{ on } \Gamma_{oc} \quad (69a)$$

$$b^{v}=r_z n_z^{v}(\mathbf{X}) \text{ on } \Gamma_{v} \quad (69b)$$

the pressure function which renders functional of Equation (66) minimum is equal to $\bar{p}_{0z}^{o}(\mathbf{X},\omega)$.

For

$$b^{oc}=n_x^{oc}(\mathbf{X})\phi_{nx}^{c}(\mathbf{X})+n_y^{oc}(\mathbf{X})\phi_{ny}^{c}(\mathbf{X})+n_z^{oc}(\mathbf{X})\phi_{nz}^{c}(\mathbf{X}) \text{ on } \Gamma_{oc} \quad (70a)$$

$$b^{v}=0 \text{ on } \Gamma_{v} \quad (70b)$$

the pressure function which renders functional of Equation (66) minimum is equal to $\bar{p}_n^{oc}(\mathbf{X},\omega)$.

At each frequency interval, $3+N_c$ solutions of complex equation derived from Equation (66) are required. The pressure values thus obtained are used in Equations (46) for evaluation of complex valued forces acting on calandria.

EXAMPLE RESULTS

The analytical procedure and numerical evaluation techniques presented earlier for the evaluation of the added hydrodynamic mass, damping and excitation terms in the coupled equations of motion have been implemented in a set of computer programs. These computer programs were validated by comparing the numerically obtained results with explicit mathematical solutions of rectangular fluid domains with free surface (Liu and Ma, 1982), vertical cylindrical domains with free surface (Chen et al., 1996), and for fluid domains between two eccentric cylinders (Chen, 1987). The results of hydrodynamic analysis were further verified by using semi-analytical process for the same problem (Joshi and Goyal, 1999). Details of the validation are documented in Joshi (1998).

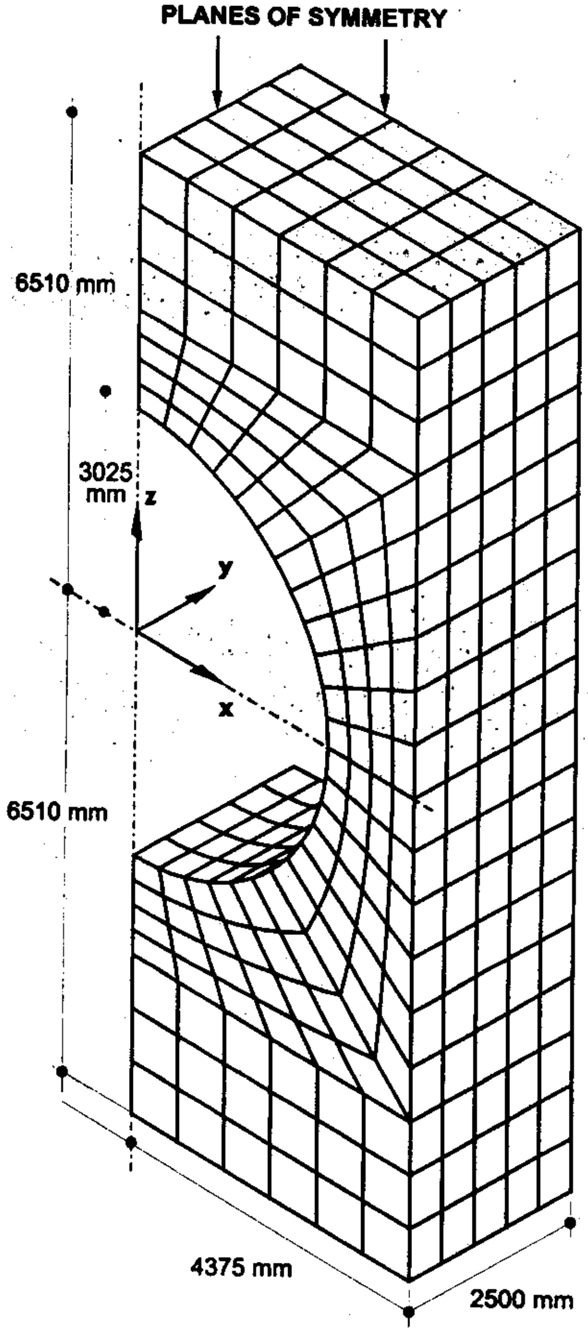

Fig. 5 Finite element discretization of outside vault water domain

The capabilities of the presented procedure are demonstrated by evaluating the response of tube array to harmonic vault excitation varied over a wide range of excitation frequencies. The numerical results are presented for a typical calandria used in 220 MW nuclear power plants of India. The results are presented for a 5000 mm long calandria with inside radius 3000 mm, thickness 25 mm, and with material properties that of stainless steel. The calandria cross-section has been assumed to be uniform along the *y*-axis (longitudinal direction) without steps at the end for simplified analysis. Finite element model of the actual horizontal tube assembly has been used. The inside moderator and outside water domains are of compressible, viscous fluids with mass density, viscosity and bulk modulus equal to that of water. The depth of inside moderator at the vertical center-line of calandria is 96 percent of its inside diameter. Vault is assumed rigid and its translational motions, along *x*, *y* and *z*-axes, have been considered.

Different boundary conditions have been imposed on the line of symmetry for excitation along *x*, *y* and *z* directions. For each direction of vault excitation, six modes of each tube (three in each direction) and three modes of calandria have been considered simultaneously. Analysis has been performed separately for vault excitations along *x*, *y* and *z*-axes. The surface damping for vault walls, calandria and tubes have been neglected. The response results have been evaluated for the frequency range of 0-25 Hz. The element size in the discretization of fluid domain however limits the number of sloshing modes (first twenty modes only) that contribute to tube response.

Response results for two tubes (tube 153 and tube 3) located in the top and bottom rows respectively are presented in Figures 6 and 7. The tubes also vibrate in the direction perpendicular to the direction of excitation. The significant cross response makes it imperative to consider flexibility of tubes along both *x* and *z* directions even for a single component of excitation. This phenomenon is known as hydrodynamic coupling and the developed procedure considers coupling between all flexible substructures. As seen in Figure 6, the sloshing is confined only to a narrow frequency range because, for the system analyzed, the ratio of tube frequency in air to sloshing frequency of moderator in calandria (frequency ratio) is high. Hence, the three fundamental frequencies are far apart, and consequently, tuning is absent. The sloshing forces acting in *x* and *z* directions for the excitation along *x*-axis are almost of the same order. In Figure 6, the peak corresponding to first transverse sloshing frequency representing the contribution of sloshing to response, is somewhat significant only for Tube 153, the tube closest to the free surface, and only for *x*-axis excitation. These results are as expected, that is the sloshing effects diminish fast with depth and they are negligible for *z*-axis excitation. Figure 7 shows the response of the two tubes for vault excitation along *y*-axis in which the sloshing of moderator excites antisymmetric transverse modes of the tubes along *x* and *z* directions. The contribution of *y*-axis excitation to the response, even for a tube close to the free surface, is negligible compared with what is due to excitation along *x* and *z*-axes.

CONCLUSIONS

A rational approach to account for the effects of structure-fluid interaction in analyzing the dynamic response of horizontal tube array in partially-filled calandria has been developed. The computational procedure has the capability of including the effects of fluid compressibility, viscosity, sloshing, surface wave absorption by boundaries of fluid domain, and the flexibility of all the substructures. Numerical procedures are presented for evaluation of natural vibration properties of calandria and horizontal tubes and for the analysis of fluid domains inside and outside calandria shell. The procedure uses substructure formulation in frequency domain and therefore has the flexibility of accomodating realistic idealizations of calandria with stiffeners and end steps. The frequency response functions, obtained for the desired quantities contain complete information and can be used to estimate the design values in both deterministic and stochastic approaches of analysis.

The tubes are strongly coupled through fluid and therefore, coupled analysis considering flexibility of tubes along the two transverse directions is necessary to obtain correct response. No attempt has been made to simplify the analysis for such an important structure. The non-linearities in the free-surface fluid-flow equations and vortex-induced oscillations cannot be simulated in the presented procedure.

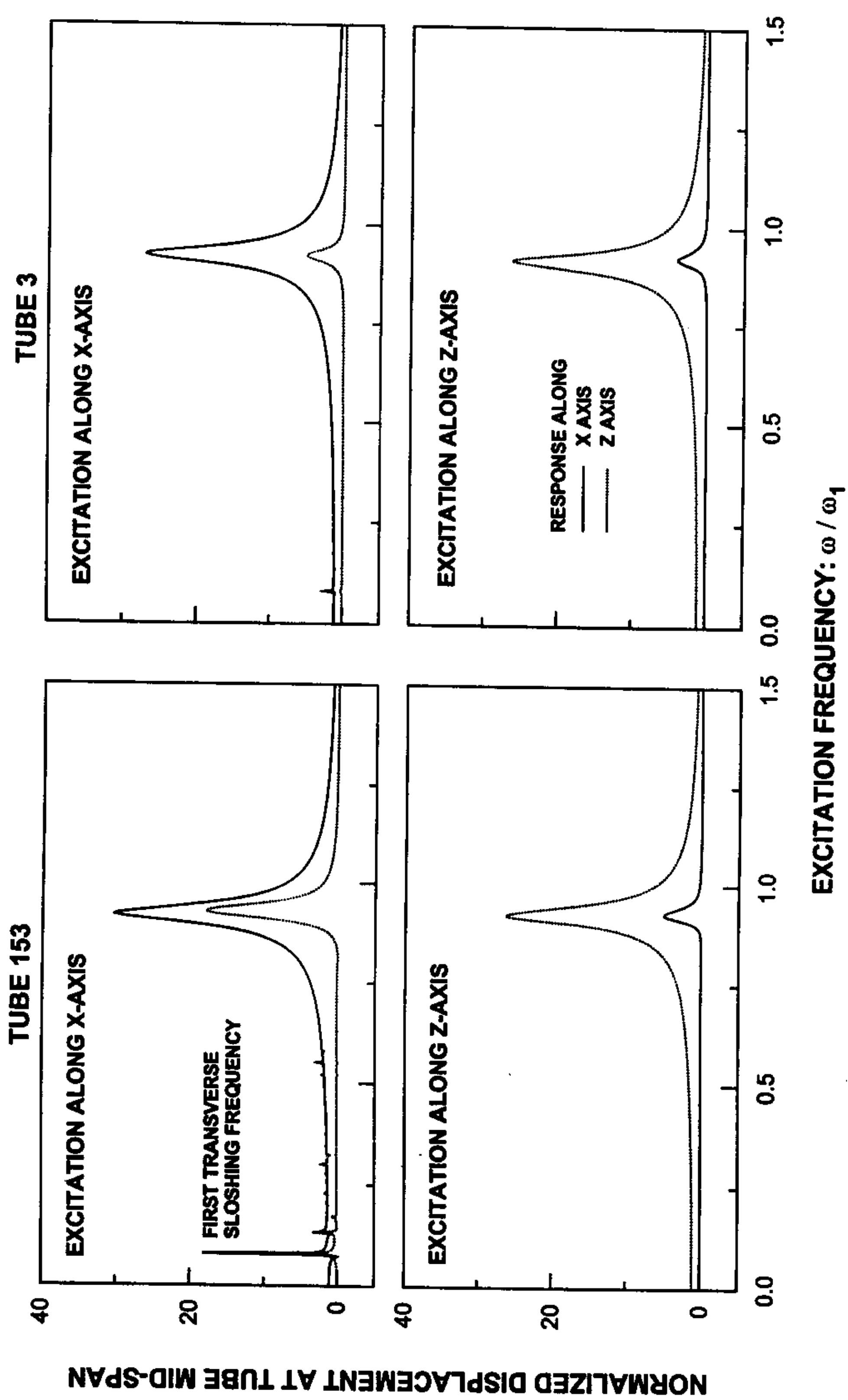

Fig. 6 Response of tubes to vault excitation along x and z axes

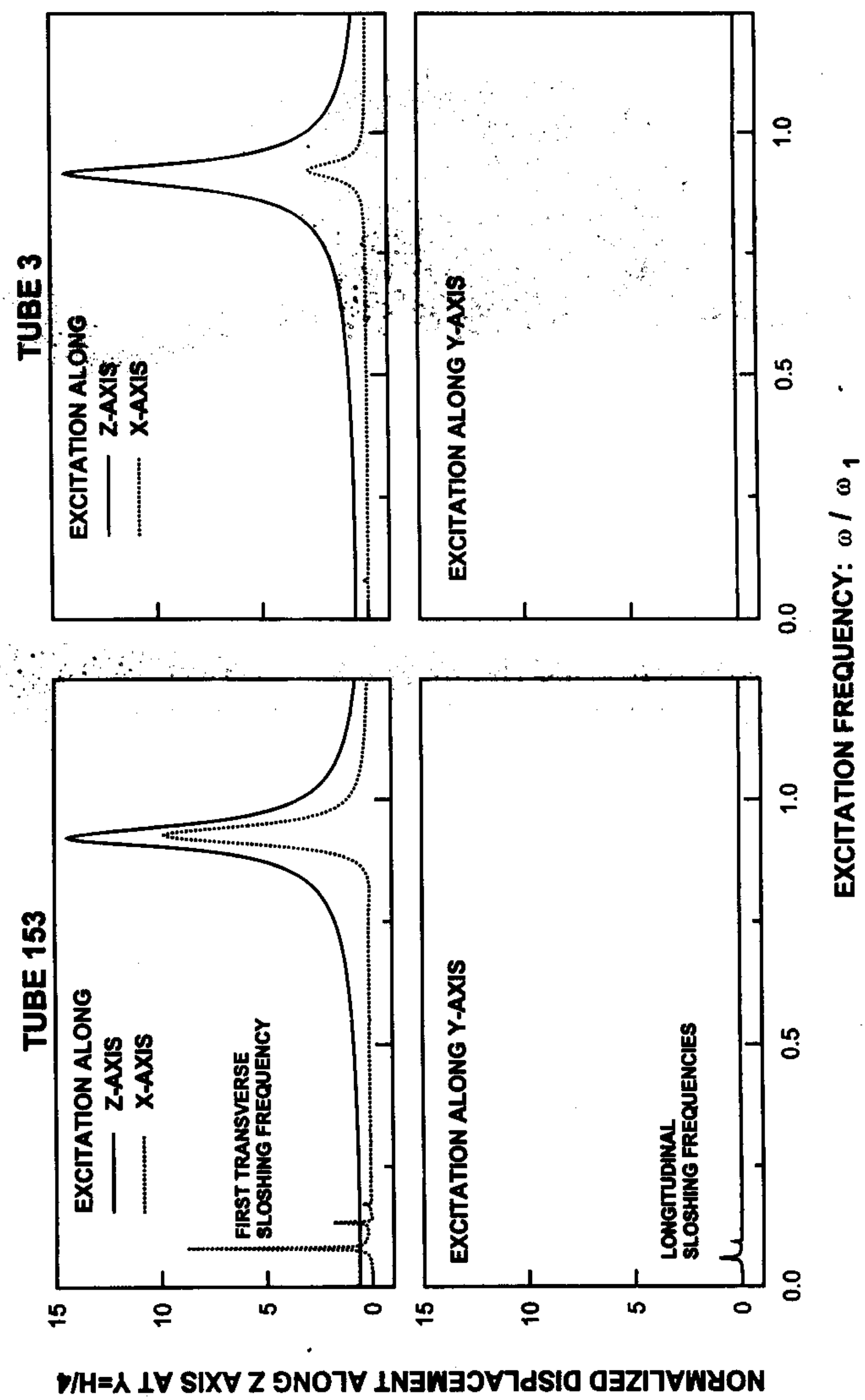

Fig. 7 Response of tubes to vault excitation along *x*, *y* and *z* axes

ACKNOWLEDGEMENTS

This research investigation was supported by the Board of Research in Nuclear Sciences, Department of Atomic Energy, India, under the grant 36/2/93-G/421. The authors are grateful to Mr. H. S. Kushwaha, Head, Reactor Structures Safety Division, BARC, Bombay, for his support.

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