Bull. Ind. Soc. Earth. Tech. Paper No. 262, Vol. 25, No. 1, March 1988, 11-22

AN INVESTIGATION FOR THE PROBABILITIES OF MAXIMUM EARTHQUAKE MAGNITUDES IN NORTH WEST HIMALAYAN REGION

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ABSTRACT

A Triple-Exponential extreme value distribution based on Poisson assumption and Adler-Lomnitz and Lomnitz's (1978, 1979) magnitude-frequency relation has been applied to the annual maximum earthquake magnitudes in the northwest Himalayan region (Lat. 71°—82° E and Long. 28°—38° N) and the results have been compared with those from commonly used Gumbel's distribution and Extreme Value type-III distributian. Maximum magnitudes for different return periods evaluated from Triple-Exponential distribution show lesser dispersion compared to the results from other extreme value distribution, and hence this distribution may be surmised to be better. The results from the Log—Pearson Type-III distribution, which is very commonly used for the flood data, has been also found to be in excellent agreement with those from Triple Exponential Distribution. From the Triple - exponential distribution for northwest Indian region, probability of occurrence of different magditudes has been plotted as a function of return period. This may be useful to find the design earthquake magnitude during a given life period with any desired confidence level.

INTRODUCTION

Theory of extreme values has been applied to evaluate the expectancy of large mognitude earthquakes in many seismic risk studies. This has the advantage that a detailed information on all the data is not required, and one needs a knowledge of only the largest earthquake events, which are generally more accurate and complete for longer time span. Nordquist (1945) was the first to apply the extreme value theory to seismic data. Subsequently, it has been used by several investigators (e.g., Epstein and Lomnitz, 1966; Karnic and Schenkova. 1974; Shrivastava. et al., 1976; Knopoff and Kagan, 1977; Mc Cue and Papastamatiou. 1978; Su 1978; Burton, 1979; Slemmons, 1982, Kim and Kim, 1982; Gan and Tung, 1983; Goswami and Sarmah, 1982, 1983; Al-Abbasi and Fahmi, 1985; etc.) for the purpose of earthquake risk analysis in different parts of the world. Most

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of these studies have applied the Extreme Value Type-I and Type III (Gumbel, 1958) distributions, which are more general type of extreme value distributions, and they are not based on the statistics of earthquake events.

In the present study, using the fact that the occurrence of large magnitude earthquakes in a region can be approximated by the Poisson distribution (Kiremidjian, 1982; Adeli et al., 1978; Lomnitz, 1966, etc.), we have derived the probability function for the maximum earthquake magnitude in T years. To evaluate this probability function one needs to know the distribution function of earthquake magnitudes. Two different magnitude distributions, based on the Gutenberg and Richter (1944), and Adler-Lomnitz and Lomnitz (1978,1979) frequeucy relations; respectively have been considered for this purpose. The more conventional Extreme Value Type I and Type III districutions have been also applied to compute the average maximum magnitudes expected during different time intervals. Incidentally, the maximum magnitude probability function derived on the basis of Gutenberg-Richter's frequency relation is found to be similar to Gumbel's Type-I distributions.

A comparison of the goodness of fit of different probability functions to the observed data on annual maximum magnitudes in the northwest Indian region shows that the overall fit for the probability function derived from Adler-Lomnitz and Lomnitz's frequency distribution is better compared to Extreme value Type I and Type-III distributions. Using this distribution function. probability of occurrence of different maximum magnitudes have been presented as the plots of probability versus return period. Such results may be useful to predict the design earthquake magnitude expected during a given life-period with a desired confidence level.

DISTRIBUTION OF MAXIMUM MAGNITUDE IN T YEARS

According to Poisson distribution, the probability that exactly n events will occur in T years is given by

$$\text{Probe}\,\{n|T\} = \frac{(\lambda T)^n}{n!}\, e^{-\lambda T} \qquad \ldots (1)$$

where λ is the annual rate of events.

Now, let $M_1, M_2, \ldots\ldots\ldots\ldots, M_n$ be the magnitudes of the n earthquakes in T years, and let M_{max} be their maximum. Assuming these n earthquakes to be statistically independent and assuming their magnitudes to be identically distributed with probability distribution F (M), the probability that M_{max} will not exceed a magnitude M due to on earthquakes in T years can be written as

$$\text{Prob}\{M_{max} \leqslant M/n, T\} = \text{Prob}\{M_1 \leqslant M, M_2 \leqslant M, \ldots\ldots\ldots \text{and } M_n \leqslant M\}$$
$$= \prod_{i=1}^{n} \text{Prob}\{M_i \leqslant M\}$$
$$= [F(M)]^n \qquad \ldots\ldots (2)$$

Thus the probability that M_{max} will not exceed magnitude M in T years is given by

$$\text{Prob}\{M_{max} \leqslant M/T\} = \sum_{n=0}^{\infty} \text{Prob}\{M_{max} \leq M/n, T\}.\ \text{Prob}, \{n/T\}$$
$$= \sum_{n=0}^{\infty} [F(M)]^n \frac{(\lambda T)^n}{n!} e^{-\lambda T}$$
$$= e^{-\lambda T} \sum_{n=0}^{\infty} \frac{[\lambda T\, F(M)]^n}{n!}$$
$$= \exp\{-\lambda T [1 - F(M)]\} \qquad \ldots\ldots (3)$$

From Gutenberg — Richter's frequency distribution it follows that

$$F(M) = 1 - e^{-\beta M}, \qquad \ldots\ldots (4)$$

where β is the parameter of the distribution. The relation of equation (4), generally, shows poor fit to the observed data on very low and very high magnitude earthquakes. Therefore, for the distribution of maximum magnitude, where one deals with the large earthquakes, it does not provide a good description of magnitude distribution. Adler-Lomnitz and Lomnitz (1978, 1979) have suggested a double exponential magnitude distribution which greatly improved the estimate of the frequency of large earthquakes

$$F(M) = 1 - e^{-e^{(\alpha + \beta M)}} \qquad \ldots\ (5)$$

In this relation, α and β are the constant parameters of the distribution. Using the distribution of equations (4) and (5) into equation (3) gives; respectively, the following probability functions for the maximum magnitude in T years.

$$P(M/T) = \exp\{-\lambda T \exp(-\beta M)\} \qquad \ldots\ldots (6)$$

$$P(M/T) = \exp\{-\lambda T \exp\{-\exp(\alpha + \beta M)\}\} \qquad \ldots\ldots (7)$$

It should be noted here that the relation of eqn. (6) can be put into the form of Gumbel's Type-1 distribution (Goswami and Sarmah, 1982) with slight rearrangements.

Many workers also recommend the use of Extreme Value Type-III distribution

$$P(M_{max} \leqslant M) = \exp[-\{(M_u - M)/(M_u - \mu)\}^k] \qquad \text{...... (8)}$$

In this relation M_u is the physical upper bound of earthquake magnitude, and K and μ are constant parameters. For the northwest Indian region, M_u has been assigned a value of 9.1, which is the largest earthquake magnitude

DATA USED

In order to investigate the relative goodness of fit of the extreme value distribution of equations (6) to (8), they have been applied to the earthquakes In northwest Indian region between latitudes 28° and 38° N; and longitudes 71° and 82° E. For this purpose we have used the annual maximum earthquake magnitudes for the period 1925 to 1980. These data have been extracted from the catalogue of Bapat et al (1983), and they are listed in Table 1. For some of the years, marked by asterisks, the magnitudes were not available in the catalogue, and the authors have assigned those by personal judgement using an idea about the threshold of detectability during different past periods. The magnitude for the year 1951 has been assigned from the intensity value.

METHOD OF ANALYSIS AND THE RESULTS

To find the observed probabilities of not exceedance in one year for different magnitudes, the annual maximum magnitudes from Table-1 were arranged rn ascending order. Then the observed probability that the magnitude, M_j, at j-th serial number will not be exceeded is given by

$$P(M_j) \; \frac{j}{N+1} \qquad \text{...... (9)}$$

Here N is the total number of years for which the data are used. From the observed values $P(M_j)$ and M_j of P and M in equations (6) through (8), least square estimates have been obtained for the parameters of the distributions. This gives $\lambda = 741.45 \times 10^3$ and $\beta = 2.375$ for Type-I distribution (6); $\lambda = 8.0814$, $\alpha = --4.7231$ and $\beta = 0.9451$ for the Triple Exponetial distribution of equation (7); and $k = 7.589$ and $\mu = 5.7218$ for Extreme Value Type-III distribution of equation (8). Observed data and the least square fit of all the three distributions are shown in Figure-1.

From the results in Figure-1, it is obserxed that Type I distribution does not fit very well to the observed data for the low and the high magnitudes. Type-III distribution is seen to lie above the mean trend of observed data for low and high magnitudes. and below the observed mean trend

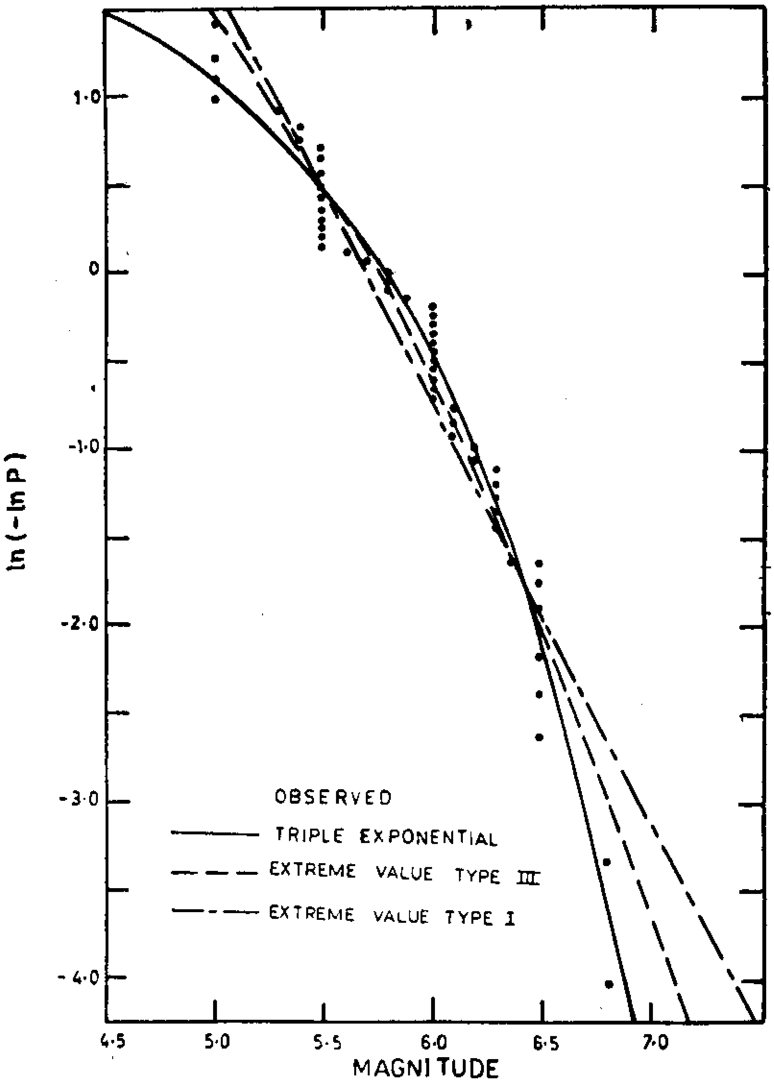

Fig. 1 Comparison of the fitnesses of diffrent extreme value distribution to the observed data.

for intermediate magnitudes. However, the Triple Exponential distribution of equations (7) is found to have very good fit to the observed mean behaviour for all the magnitudes. The standared deviations of the magnitude "σM" for Type-I, Type-III and the Triple Exponential distributions are found to be 0.148, 0.107 and 0.099; respectively.

Table-II gives the estimates of the mean maximum magnitudes ±95% confidence intervals of t-distribution from the distributions of equations (6) to (8). The results on maximum magnitude from Log-Pearson Type-III distribution as evaluated using standard table (Linsley, Jr. et al., 1975, p. 344) are also listed in this table. These values are seen to be in good agreement with the mean maximum magnitude from triple exponential distribution. Triple Exponential distribution has been found to have the smallest 95% confidence intervals; and hence, the results from it will be more reliable. From this maximum magnitude distribution, it is possible to evaluate the time T during which the magnitude will not exceed a given value with any desired confidence level. These results for M = 4.5, 5.0, 5 5 6.0 6.5, 7.0 and 7.5 are presented in Figure-2 as the plots of confidence level versus return period.

DISCUSSION

The frequencies of large magnitude earthquakes in a region are usually found to lie below the straight line relation of Gutenberg and Richter (1944). Therefore, to develop a probability distribution of the maximum earthquake magnitude in a given time, one should use a frequency law which decays faster than the linear decay for large magnitudes. Though there is a tendency for some of the research workers to ascribe the fast deacy of the frequencies of large earthquakes to short period of observations, it may actually be related to the physics of the phenomenon of earthquake occurrence. Assuming the mean stress drop as stationary and the total faulting as homogeneous, Lomnitz (1964) has shown that the earthquake magnitude should be normally distrbuted, which is equivalent to a quadratic decay. The double exponential magnitude distribution of equation (5) statistics of earthquake occurrences, and hence. It lacks a physical basis for applying it to earthquake events.

Motivated by its great success for the prediction of floods in United States (Ref. 26), Log-Pearson Type-III Distribution (Pearson, 1930) was also tried to find the maximum magnitude earthquakes for different return periods, Very interestingly, the results from this distribution were found to be in excellent agreement with those from Triple-Exponential distribution (Table II). This suggests that the Log-Pearson Type-III distribution may also be used to find the design earthquake, even though this distribution lacks a physical basis for its application to earthquakes, because unlike the Triple - Exponential distribution it is not based on the statististics of earthquake magnitudes and recurrence times. Before the applicability of Log-Pearson distribution to the earthquakes can be established, it may be necessary to test it for many more cases.

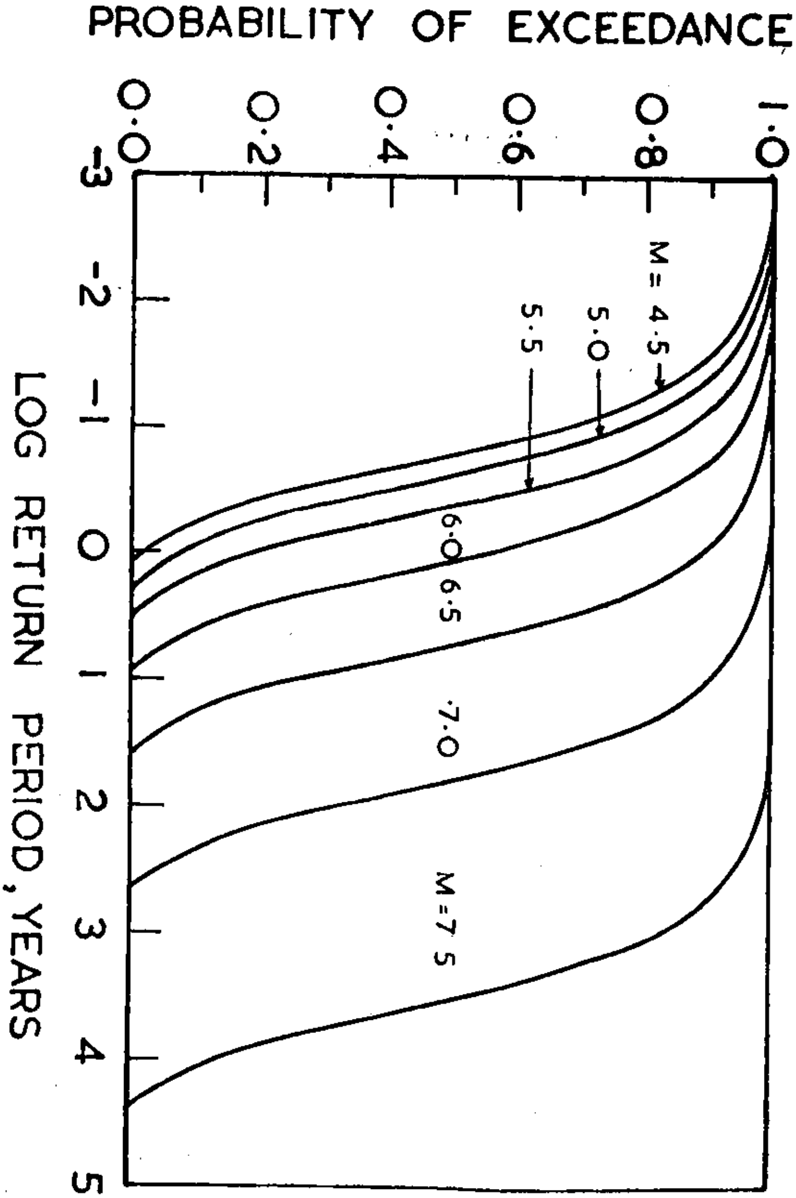

Fig. 2 Probability functions of Triple-Exponential extreme value distribution for different maximum magnitudes in the northwest Himalayan region.

In the present analysis, parameters of the distributions have been evaluated by least squares procedure using Gumbel's plotting rule of equation (9). However, according to some investigators (Knopoff and

Kagan, 1977; Weichert and Milne, 1979; etc.) least squares method leads to large errors and the results are significanly dependent upon the plotting rule. On the other hand, the maximum likelihood method of estimating the parameters does not need any empirical plotting rule for the observed probabilities and its estimators possess many of the desired properties; viz., unbiasedness, consistency, efficiency, and sufficiency. Therefore in various applications, the maximum likelihood method is considered to be superior, in that the parameters estimated by this method has the minimum variance.

CONCLUSIONS

From the present investigation it may be concluded that to have a more realistic estimate of the probabilities of maximum earthquake magnitude in a specified life time, one should describe the frequencies of the large earthquakes by a relation which decays faster than the frequency law of Gutenberg and Richter (1944). The Triple Exponential maximum magnitude distribution, which is based on an exponential decay law, has been found to give more precise estimates of the maximum earthquake magnitudes. Though, it might be possible to have still better frequency laws and the extreme value distributions based on them, the Triple Exponential distribution of epuation (7) seems to be quite adequate to predict the probahilities of maximum earthquake magnitudes in a region of interest, as seen by its application to the northwest Indian region in the present study. By using the maximum likelihood method for determining the parameters of the distributions, the accuracy of the results may perhaps be improved further. The maximum magnitudes from Log Pearson Type-III distribution has been also found to be in excellent agreement with those from Triple Exponential distribution.

ACKNOWLEDGEMENTS

The authors are very much thankful to Dr. Z.S. Tarapore, Director, and Shri S.L. Mokhashi, Additional Director, Central Water and Power Research Station, Khadakwasla, Pune. India for the keen interest shown in the research work presented in this paper and their permission to publish the paper.

REFERENCES

1. Adeli, H., Nemat-Nasser, S. and Rowshandel, B (1978). "A Probabilistic Estimate of Peak Ground Acceleration in Iran", Sixth European Conf. on Earthquake Engineering, Yugoslav Assn. for Earthquake Engineering, Ljubljana, Vol. I, Paper 1-18, 129-134.

2. Adler-Lomnitz, J. and Lomnitz, C. (1978). "A New Magnitude Frequency Relation", Tectonophysics, 49, 237-243.

3. Adler-Lomnitz, J. and Lomnitz, C. (1979). "A Modified Form of the Gutenberg Richter Magnitude-Frequency Relation", Bull. Seism. Soc. Amer, 69, 1209-1214,

4. Al-Abbasi, J.N. and Fahmi, K.J. (1985), "Estimating Maximum Magnitude Earthquakes in Iraq Using Extreme Value Statistics", Geophys. J.R. Astr. Soc., 82, 535-548.

5. Burton. P.W. (1979). "Seismic Risk in Southern Europe Through India Examined Using Gumbel's Third Distribution of Extreme Values Geophys. J.R. Astron. Soc., 59, 249-280.

6. Epstein, B. and Lomnitz, C. (1966). "A Model for Occurrence of Large Earthquakes", Nature, 211, 954-956

7. Gan, Zi Jun and Tung, C.C. (1983). "Extreme Value Distribution of Eearthquake Magnitude", Physics of the Earth and Planetary Interior, 32, 325-330. -

8. Goswami, H.C. and Sarmah, S.K. (1982). "S,K. (1982). "Probilistic Earthquake Expectancy in the Northeast Indian Region", Bull. Seism., Soc. Amer., 72. 999-1010.

9. Goswami, H C. and Sarmah S. K. (1983). "A Comarison of the Estimated Earthquake Probabilistics in the Northeast Indian Region", Tectonophysics, 95, 4/2, 91-99.

10. Gumbel, E.J. (1958). "Statistics of Extremes"; Columbia Univ. Press, New York, 375 pp.

11. Gutenberg, B. and Richter, C.F. (1944) "Frequency of Earthquakes in California", Bull. Seism. Soc. Amer., 34, 185-188.

12, Karnik, V and Schenkova, Z. (1974). "Application of the Theory of Largest Values to the Earthquake Occurrence in the Balkan Region", Stud, Geophys. Geol., 18.

13. Kim, S.G. and Kim, S.J. (1982). "A Seismic Risk Evaluation in Korea" Jour. Phys. Earth". 30,5, 441-450.

14. Kiremidian, A. S. (1982). "Stochastic Models for Seismic Hazard Analysis and their use in Microzonation", Procs. 3rd International Earthquake Microzonation Conf., Univ. of Washington, Seattle, Vol. III, 1205-1214.

15. Knopoff, L. and Kagan, Y. (1977). "Analysis of the Theory of Extremes as Applied to Earthquake Problem", Jour. Geophys Res., 82, 5647 - 5657.

16. Linsley, Jr., R.K., Kohler, M.A. and Paulhus, J.L.H. (1975). "Hydrology for Engineers, McGrew-Hill International Book Company, Second Ed.

17. Lomnitz, C. (1964). "Estimation Problem in Earthquake Series". Tectonophysics, 2, 193-203.

18. Lomnitz, C. (1966). "Statistical Prediction of Earthquakes", Rev., Geophys., 4, 377-393.

19. MuCue, K. and Papastamatiou. D. (1978) The Value of Extreme Statistics in the Assessment of Sesmic Risk", Sixth European Conf. on earthquake Engineering. Yugoslav Assn. for Earthquake Engineering, Ljubljana, Vol. I, Paper I-07, 47-51.

20. Nordquist, J.M. (1945). Theory of the Largest Values Applied to Earthquake Magnitudes". Trans. Am. Geophys. Union Zes., 26.

21. Pearson, K. (1930), Tables for Statisticians and Biometricians", 3rd Ed., Cambridge University Press, London.

22. Slemmons, D, B, (1982). "Determination of Design Earthquake Magnitudes for Microzonation", Procs. 3rd International Earthquake Microzonation Conf., Univ. of Washington, Seattle, Vol. I, 119-130.

23. Shrivastava, V.K., Chouhan, R.K.S. and Nigam, R. (1976). "Largest Value Theory Applied to Probability of Earthquake Ocurrence". Bull. Ind. Soc. Earthq. Tech. 13, 19-3.

24. Su, S.S. (1978). "Earthquake Risk Analysis for Metro Manila". Procs, of the U.S. Southeast Asia Symposium on Engineering, for Natural Hazards Protection, Dept. of Civil Engineering, Univ. of Illinois, Urbana, ed. by Aug., A.H. - S. 33 - 43.

25. Weichert, D.H. and Milne, W.G, (1979). "On Canadian Methodologies of Probabilistic Seiemic Risk Estimation", Bull. Seism. Soc. Amer., 69. 1549 - 1566.

26. "A Uniform Technique for Determining Flood Flow Frequencies". Hydrol, Comm. Water Resour. Counc., Washington, Bull. 15, 1967.

Table-I

Annual Maximum Magnitudes for 56 Years From 1925-1980 in the Area Between Latitudes 28° and 38° N, and Longitudes 71° and 82° E.

Year.	Magnitude	Year	Magnitude	Year	Magnitude
1925	6.5	1944	5.5	1963	6.0
1926	6.3	1945	6.5	1964	6.3
1927	6.0	1946	5.3	1965	6.0
1928	6.0	1947	6,0	1966	6.1
1929	6.3	1948*	5.0	1967	5.7
1930	5.5	1949*	5.0	1968	5.5
1931	6.5	1950*	5.0	1969	5.6
1932	5.5	1951	6.0	1970	5.4
1933*	5.5	1952	5.0	1971	5.5
1934	5.5	1953	5.8	1972	5.8
1935	6 0	1954	6.5	1973	5.5
1936	6.8	1955	6.0	1974	6.0
1937	6.5	1956	6.5	1975	6.2
1938*	5.5	1957	6.4	1976	6.1
1939	5.5	1958	6.8	1977	5.4
1940	6.3	1959	6.3	1978	5.9
1941	6.0	1960	6.2	1979	5.8
1942	6.0	1961	6.5	1980	6.1
1943	6.8	1962	5.5		

Table-II

Maximum Magnitudes from Various Extreme Value Distributions for Different Return Periods

Return Period (Years)	Mean Maximum Magnitude ± 95% Confidence Interval			Log Pearson Type III (Maximum Magnitude)
	Extreme Value Type I	Extreme Value Type III	Triple Exponential	
2	5.84 ± 0.30	5.88 ± 0.22	5.95 ± 0.20	5.90
5	6.32 ± 0.30	6.33 ± 0.22	6.35 ± 0,20	6.31
10	6.64 ± 0,31	6.59 ± 0.22	6.55 ± 0.21	6.52
25	7.04 ± 0 31	6.88 ± 0.23	6.76 ± 0.21	6.76
50	7.33 ± 0.32	7.08 ± 0.23	6.89 ± 0 21	6.92
100	7.63 ± 0.33	7.26 ± 0.24	7.01 ± 0 22	7.06