Paper No. 92. Bulletin of the Indian Society of Earthquake Technology, Vol. VII, No. 2, June 1970, pp. 103–112

VIBRATION TESTS AND EARTHQUAKE BEHAVIOUR OF A CYLINDRICAL SHELL

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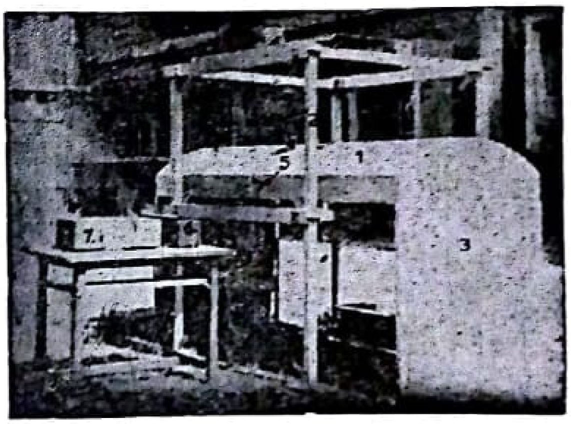
INTRODUCTION

Shell roof structures have received very little attention as regards their characteristics in free or forced vibrations. Few static tests[3] have been performed on cylindrical shell models to determine the resistance of the shell slab against the column top moments which are caused by the earthquake motions. In the present paper free vibration tests as well as static load deflection tests on a single span cylindrical shell model of reinforced concrete are described. The experimental natural frequencies of the shell under rigid and flexible base conditions are compared with the frequencies computed theoretically. The behaviour of the prototype shell roof is then discussed with reference to its frequency and the frequency of the ground motion to which the structure may be subjected.

THE CYLINDRICAL SHELL MODEL

A reduced scale model of a reinforced concrete shell roof was constructed as shown in Fig. 1 for testing under free vibration and lateral loads The dimensions of the model and details of reinforcement are shown in Fig. 2. The model was a scale reduction of a typical 31.0 m span prototype "Long shell" designed for normal vertical loads. The shell slab was cast monolithically with solid traverses on its curved edges and narrow deep edge beams along its longitudinal edges. The complete shell roof rested on four reinforced concrete columns and was monolithic with them. The pertinent dimensions of the model are given below :-



1. Shell model 2. Loading frame 3 Brick Panel Wall 4. Releasing clutch 5. Acceleration pickup 6. Pen recorder 7. Universal amplifier.

Figure 1. Free Vibration Test of Cylindrical Shell Roof Model.

Length of shell	2,480 mm
Radius of shell	0,786 mm
Chord width of shell	1,062 mm
Thickness of shell	0,012 mm
Semicentral angle	39.5°
Depth of edge beam	0,152 mm
Width of edge beam	0,027 mm
Thickness of traverse	0,025 mm
Height of columns above foundation	1,219 mm
Size of reinforced concrete columns	38×38 mm²
Modulus of elasticity $E = 2.1 \times 10^5$ kg/cm²,	
Weight density	2400 kg–m³

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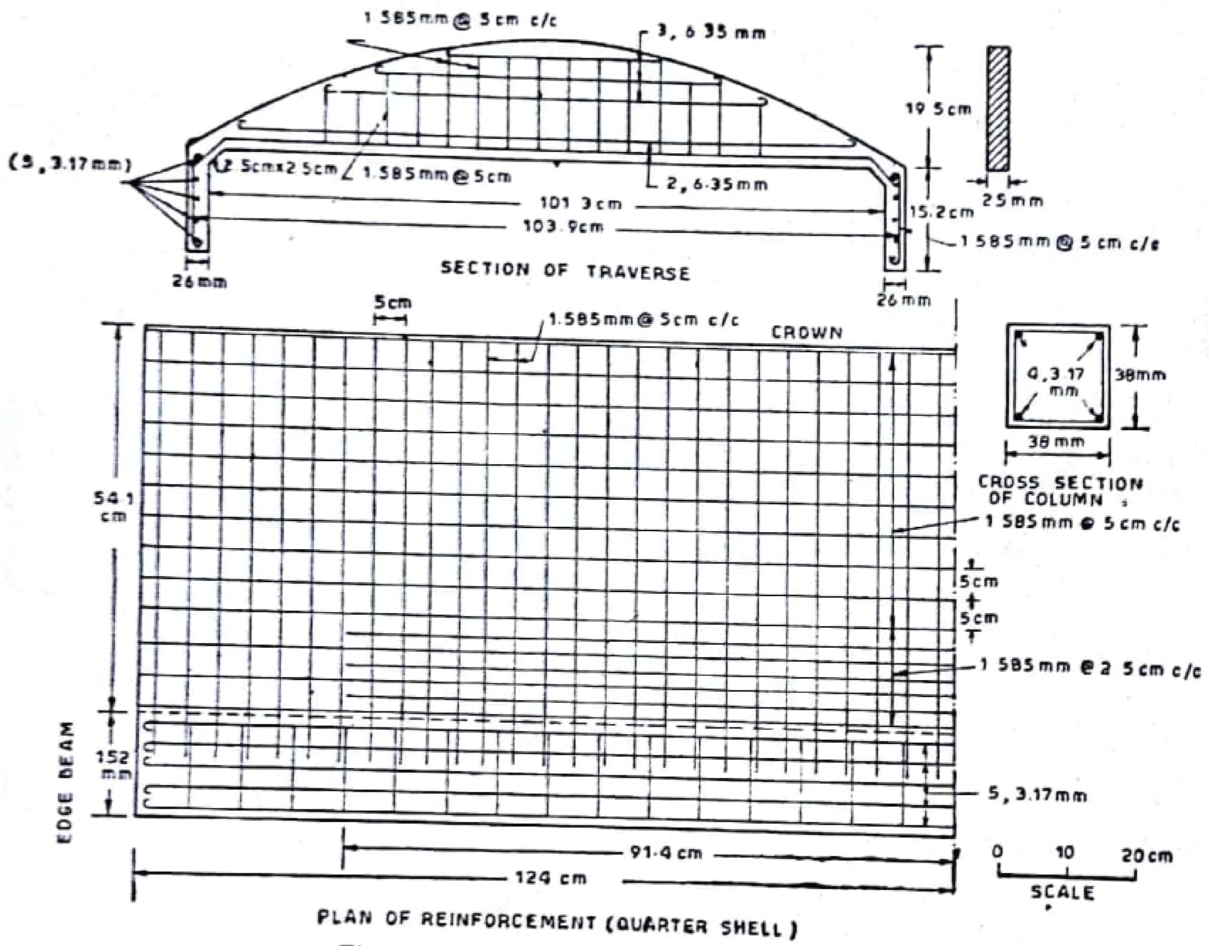


Figure 2. Cylindrical Shell Roof Model

For applying radial or vertical loads on the shell surface, a few circular holes with adequate reinforcement around the opening were left in the shell slab so that a pulling hook could be attached to it at these points (see Figure 1). A brick panel wall was inserted between the columns in the transverse direction to make the sub-structure rigid. It was removed later for making the substructure flexible. Similarly, diagonal braces were introduced in the panels in the longitudinal direction and the structure tested with and without them.

EXPERIMENTAL TESTS ON SHELL MODEL

The shell model was tested in two different stages. In the first stage the supporting structure of columns was made rigid in transverse direction by constructing a brickwall (see Figure 1), so that the shell could be excited alone in free vibration without the vibration of supporting structure. In the second stage the wall was removed and the whole structure of shell and columns was tested under conditions of free vibration as well as static horizontal load.

The free vibration tests on the shell were performed giving initial displacement to the shell surface by a clutch arrangement shown in Figure 1, and also by tamping by hand on the shell surface. The shell was excited at different points and vibrations were also picked up at different points on the shell with the idea of obtaining clearest possible records. Miller type accelerometers with Brush Universal Amplifiers and Ink writing oscillographs were used for picking, amplifying and recording the vibrations. The following types of tests were conducted both with and without the wall :—

1. Free vibration test to excite antisymmetrical mode of shell.
2. Free vibration test to excite symmetrical mode of shell.
3. Free vibration test by pulling the shell at the traverse in the longitudinal direction.
4. Static test by pulling the shell at the traverse in the longitudinal direction.
5. Static test by pulling the shell in transverse direction at the middle of edge beam.

In order to obtain the frequency of the shell in antisymmetrical mode, the shell was pulled from one side asymmetrically and vibrations were picked up such that the symmetrical component of mode of vibration was eliminated. This was achieved by pulling the shell radially at quarter point of curved surface (Figure 3a) and recording the horizontal vibration on edge beam. Pulling the shell horizontally from edge beam (Figure 3b) in the transverse direction also gave an uncoupled vibration record when picked up at the edge beam itself. A typical vibration record obtained by radial pulling and releasing is given in Fig. 4a. The observed frequencies of vibration are given in Table 1.

For exciting symmetrical mode of vibration, the shell was pulled vertically upwards at its central point and the vertical vibrations were picked up at a point on the longitudinal axis of symmetry of shell (Fig. 3 c). A typical free vibration record obtained in this case is shown in Fig. 4b and the observed frequencies are given in Table 1 for various pulling loads.

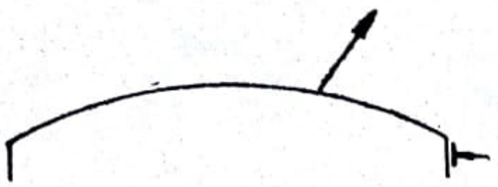
The frequencies of vibration obtained after removing the wall by these two methods of excitation are also given in Table 1.

The frequency of vibration of the shell column system that is the structure as a whole was obtained by pulling the shell in the longitudinal direction (Fig. 3d). The observed frequency values are given in Table 2.

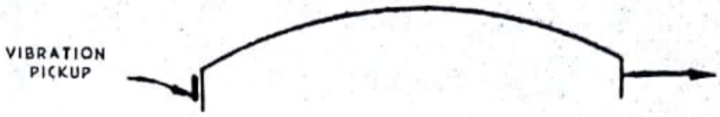
To obtain the static load-deflection relations, a horizontal load was applied to pull the shell in the transverse and longitudinal directions separately (Fig. 3b and 3d) and the deflections were recorded along the height of columns. The observed load-deflection curves are given in Fig. 5a and 5b. The stiffness of substructure obtained from these tests is given in Table 3, together with the theoretical values of stiffness.

THEORETICAL ANALYSIS

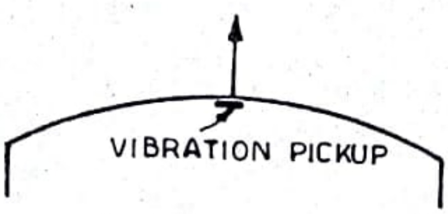
The theoretical frequencies and mode shapes of the shell roof resting on rigid supports have been found from the solution of the eighth order partial differential equation of vibration of shell. The details of the method of computation have been described and the frequencies and mode shapes of shells having various length parameters and depths of edge beams have been presented by the authors earlier[1,2]. An out-line of the method is presented here.



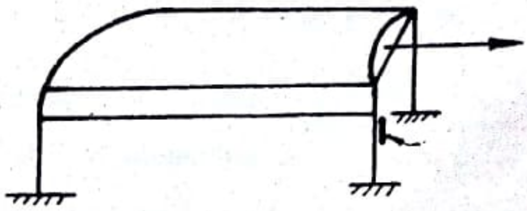
(a) Radial Pull at Quarter Point



(b) Horizontal Pull of Shell at Edge Beam

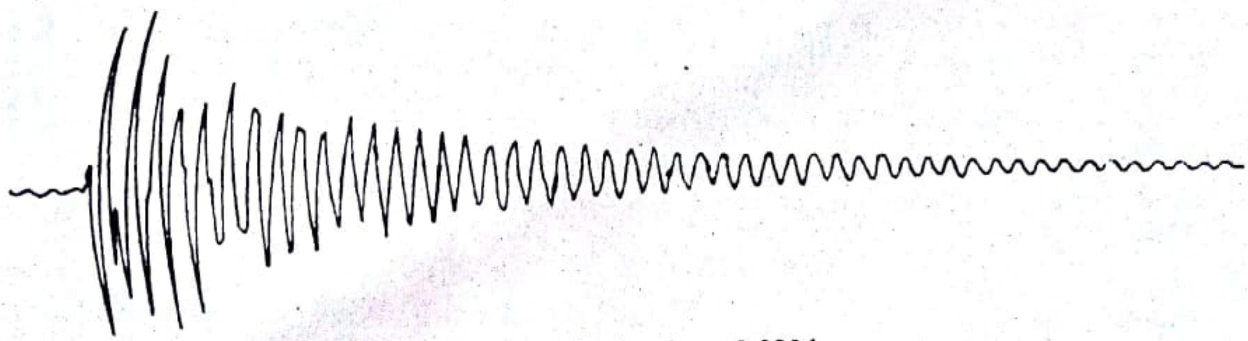


(c) Vertical Pull of Shell from Central Point



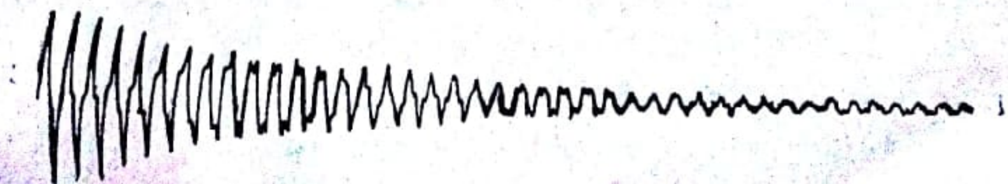
(d) Longitudinal Pull of Shell

Figure 3



f = 44.7 c/s, $\zeta = 0.0284$

(a) Radial Pull at Quarter Point



f = 46 3 c/s, $\zeta = 0.0211$

(b) Vertical Pull at Central Point

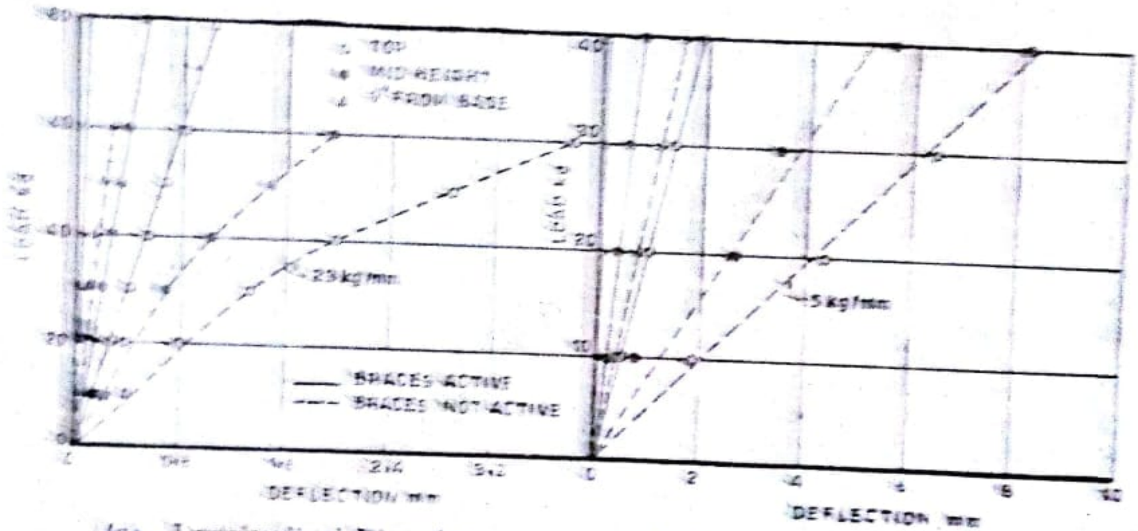
Figure 4 Free Vibration Record of R C. Cylindrical Shell Model

... dimensionless partial differential operator given by ... along the length of shell equal to x/r and ∇^2

$$\nabla^2 = \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \phi^2}$$

where ϕ is the angular coordinate measured from the vertical axis of symmetry of the shell.

Assuming that the ends $x = 0$ and $x = L$ (Fig. 5) are simply supported, Levy type solution can be used by taking the radial deflection in the form.



(a)—Longitudinal Direction

b—Transverse Direction

Figure 5. Load Deflection Curves

$$w = e^{m\phi} \sin \frac{n \pi x}{L} \sin pt \tag{2}$$

where m is a characteristic root, n the number half waves in the longitudinal direction and p the circular natural frequency in radians per second. Substituting equation (2) in equation (1) the characteristic equation is obtained as

$$\left[m^2 - \left(\frac{n \pi r}{L}\right)^2 \right]^4 - \frac{12\, m_1\, r^4\, p^2}{E t^2_1} \left[m^2 - \left(\frac{n \pi r}{L}\right)^2 \right]^2 + \frac{12\, r^2}{t^2_1} \left(\frac{n \pi r}{L}\right)^4 = 0 \tag{3}$$

which will give eight values of m. Substituting the notation

$$Z = \left[m^2 - \left(\frac{n \pi r}{L}\right)^2 \right]^2$$

in equation (3) and solving the resulting quadratic equation, Z is obtained as,

$$Z = \frac{6\, m_1\, r^4}{E\, t^2_1}\left[p^2 + \sqrt{p^4 - \frac{E^2\, t^4_1}{3m^2_1\, r^6}\left(\frac{n\,\pi\, r}{L}\right)^4}\,\right] \tag{4}$$

Depending upon the quantity under radial sign in (4), it being positive, *zero or negative*, Z will have two unequal real, two real equal or two *unequal complex roots respectively*. Accordingly the deflection function will have different forms. Since the quantity under the square root sign is a function of the frequency, it turns out that the deflected shape function will vary with the frequency under consideration. In any case the radial deflection w of the shell can in general be expressed in terms of eight undetermined coefficients, A1, A2, A3 ... ... A8 in the following form

$$w = [A_1\phi_1 + A_2\phi_2 + A_3\phi_3 + A_4\phi_4 + A_5\phi_5 + A_6\phi_6 + A_7\phi_7 + A_8\phi_8] \sin \frac{n\,\pi\, x}{L} \sin pt. \tag{6}$$

The functions ϕ_1, ϕ_2, ϕ_3 ... ... ϕ_8 depend upon the variable angle ϕ and take different values[2], corresponding to each case of the roots of auxiliary equation (3). In every case four of these functions are symmetrical and four antisymmetrical with respect to the angle ϕ.

Applying the four boundary conditions for forces and displacements of the shell at the straight edges and considering the symmetry of the shell, four equations can be obtained in terms of the undetermined coefficients. The determinant of these four equations should vanish for a natural frequency of the shell. The mode shape may then be determined by setting one of the undetermined coefficients equal to unity and solving remaining three simultaneous equations from which modal deflections can be worked out. Thus symmetrical and antisymmetrical modes are determined separately.

The computed mode shapes and frequencies for first symmetrical and antisymmetrical mode of vibration for the shell tested here are given in Fig. 7. The theoretical and experimental frequencies are compared with each other in Table 4. It may be seen that the theoretical and experimental values are in agreement within 4%.

The theoretical stiffness of the substructure in the flexible condition, that is when the diagonal braces and walls are removed, is calculated in the longitudinal direction by assuming the moment of inertia of the shell roof as infinite in comparison to that of the columns. Therefore, the lateral load required at top level of columns in the longitudinal direction of of the shell to cause a unit displacement there will be 48 EI/h³ where I is the gross transformed moment of inertia of the cross section of a column and h its clear height between the foundation and the edge beam. In the transverse direction this stiffness will be less because the solid traverse has a depth equal to that of the shell and does not extend upto the depth of the edge beams. The narrow deep edge beams will provide only a partial restraint to the rotation of the top ends of columns. Consequently the points of contraflexure in columns will shift upwards from their mid-points and the lateral load stiffness will reduce as compared with fixed ended columns. In the limiting case if the top end becomes rotationally flexible like a hinge, the lateral load stiffness will reduce to 3EI/h³ per column, that is 12 EI/h³ for the four columns.

The theoretical frequency of the shell-column system is based on the assumption that the shell roof is a rigid mass attached to the columns acting like shear springs having their stiffness as given in the above paragraph.

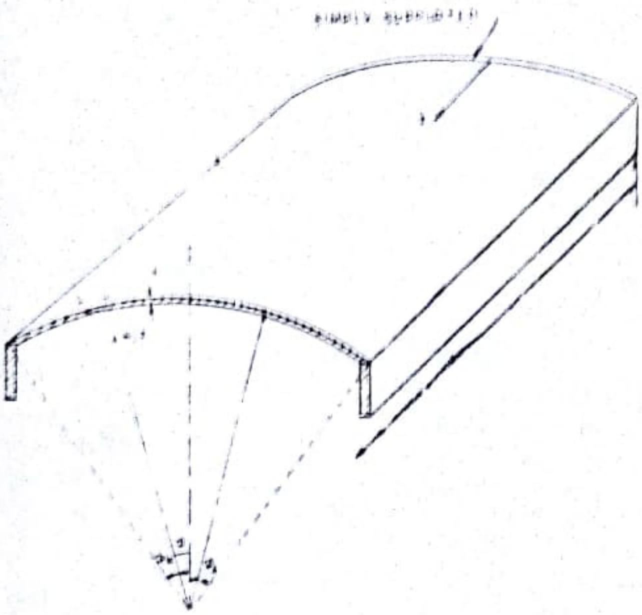
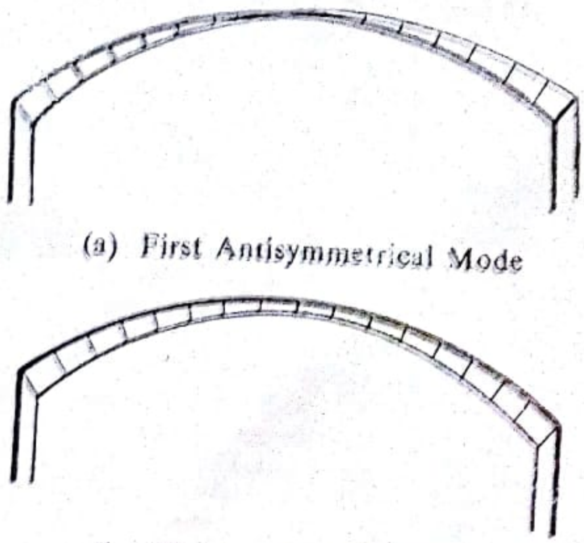


Figure 6. Coordinate System and Shell Geometry



(a) First Antisymmetrical Mode

(b) First Symmetrical Mode

Figure 7. Theoretical Frequency and Mode Shape of Shell Model

DISCUSSION OF RESULTS

1. From Table 4, it is seen that the frequencies of first symmetrical and first antisymmetrical modes of shell as obtained from tests are in close agreement with theory, the difference being less than 4%. It is of course not possible to obtain frequencies in higher modes by free vibration test and a forced resonant type excitation is necessary in that case.

2. The action of the shell-column structure in the longitudinal direction is like that of a rigid mass supported on columns. This may be seen from Table 2 by the close agreement between theoretical and experimental frequency in this direction where the theoretical frequeney was derived by treating the shell as a rigid mass and no joint rotation at the ends of columns was considered. The observed stiffness of the substructure below a load of 40 kg is, however, higher than that indicated by the observed frequency. The observed and theoretical frequency and observed and theoretical stiffness are compareable when the lateral load exceeds 40 kg. At this load the columns seem to have developed hair cracks resulting in reduction of stiffness.

3. The action of the shell and column in transverse direction is somewhat similar to that in longitudinal direction but the stiffness of columns is lower in transverse direction because of rather imperfect rotational restraint at the top.

4. Testing under steady state forced vibrations is warranted for distinguishing between symmetrical and antisymmetrical mode shapes as it is difficult to obtain mode shape from free vibration test.

EXPECTED DYNAMIC BEHAVIOUR OF SHELL ROOF DURING EARTHQUAKES

Projecting the frequencies of vibration of the model to a prototype shell having a span length of about 30m, the natural frequencies of the first antisymmetrical and first symmetrical modes are found to be of the order of 3 and 5 cycles per second or the periods of 0.33 second and 0.2 second respectively. Now the acceleration response spectra of the various earthquakes recorded on rock or firm ground show the maximum peaks between the periods 0.2 second to 0.5 sec. Therefore, the two lowest periods of a prototype long shell lie in the critical range of periods. Such a shell, if supported by rigid walls at curved ends and subjected to ground motion transverse to the span, may develop a large response and the stresses induced in the shell may even be larger than what could be resisted by its material.

On the other hand if the shell is supported on columns without having any rigid elements like wall or diagonal braces, the estimated period of the prototype shell is about 3 seconds. This is a very long period for any recorded earthquake and the forces developing will be rather small. The shell will just act as a rigid body supported by flexible column supports. However, two points will need attention in this case. First, the dynamic deflections in such a flexible structure will be large at the top of columns. This will result in large eccentricity of load on the columns. Therefore columns must be checked against the moments due to the eccentrialy of load as well as the buckling effect. Second factor is the transmission of stresses in the shell due to the moment produced at the end of the columns. Such moments may be avoided by making hinged instead of monolithic connection between. This will however increase the effective length of the columns, decrease their stiffness, elongate the time period of the structure, increase the lateral deflections and eccentricity moments and increase the moments at base of columns. The problem of distribution of stresses in the shell slab due to column top moment has been studied by Tsuboi and Kawaguchi[3]. But suitable design criteria are yet to be developed in this respect by further analysis and testing. Similarly response of shells resting on rigid support subjected to ground motions has also to be investigated.

CONCLUSIONS

On the basis of experimental and theoretical results presented in the paper the following conclusions may be drown :—

1. The first symmetrical and the first antisymmetrical modes of the frequencies in a shell can be obtained from its free vibration tests. The results obtained in the case studied are in close agreement with theory.

2. The first two frequencies of a prototype shell are likely to lie in the critical range for earthquake. The vibration of the shell slab will be of concern during an earthquake when its supporting structure is rigid against lateral deflection.

3. The behaviour of the shell and column in the longitudinal and transverse direction under horizontal load can be determined by considering the shell as rigid and columns as shear springs with proper boundary conditions. Under earthquake condition, the additional eccentricity due to deflections and buckling tendency of columns has to be taken care of.

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TABLE 1

Natural Frequency and Damping of Shell Model
(Rigid Support)

Pulling Load kg.	Frequency c/s				Damping Ratio
	Radial pull at quarter point		Vertical pull at quarter point		
	With wall	Without wall	With wall	Without wall	
20	44.7	41.66	46.3	40.3	Radial pull 0.028
40	44.7	41.66	46.3	40.3	Vertical pull 0.021
60	44.7	41 66	46 3	40.3	
80	44.7	41.66	46.3	40.3	
100	44.7	41.66	46.3	40.3	

TABLE 2

Natural Frequency and Damping of Shell-Column System
(Braces and Walls Removed)

Pulling Load kg.	Frequency in c/s	
	Longitudinal pulling of shell	Theoretical† Frequency c/s
20	3.33	
40	3.33	3.97
60	3.33	

† Based on theoretical stiffness.