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ON TWO INTERACTING CREEPING VERTICAL SURFACE BREAKING STRIKE-SLIP FAULTS IN THE LITHOSPHERE***

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ABSTRACT

Two aseismically creeping long vertical surface-breaking parallel strike-slip faults are taken to be situated in a visco-elastic half-space, representing the lithosphere-asthenosphere system. Solutions are obtained for the displacements, stresses and strains, using a technique involving the use of Green's functions and integral transforms, for three possible cases—the case of the absence of any fault creep, the case in which one fault is creeping and the other is locked and the case in which both the faults are creeping, taking into account the displacements and stresses present initially, and assuming that the tectonic forces maintain a constant shear stress far away from the faults. The types of fault creep for which the displacements, stresses and strains are finite everywhere in the model near the faults are identified, and the conditions satisfied by these types of fault creep are determined in a simple form. Fault creep across a fault is generally found to reduce the rate of accumulation of shear stress near itself. The effect of aseismic creep across one fault on the shear stress near the other fault is found to depend on the distance, dimensions, relative position and other characteristics of the two faults. Fault creep across one fault is generally found to reduce the rate of shear stress accumulation near the other neighbouring fault in the theoretical model considered. Under suitable circumstances, aseismic creep across one of the faults is found to result in aseismic release of shear stress near both the faults, thus reducing progressively the possibility of a sudden fault movement, generating an earthquake. The influence of one creeping fault on another is found to decrease quite rapidly with increase in the distance between the faults. The possible uses of the model in the study of the interaction between neighbouring strike-slip faults in the lithosphre and in the estimation of return times of earthquakes is examined.

INTRODUCTION

The problem of earthquake prediction has attached widespread attention among seismologists in recent years, and the steady accumulation of relevant seismological data and improvements in the techniques of their analysis and interpretation, together with the development of relevant theoretical models

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and computer simulation techniques have made it possible to hope that effective programmes of earthquake prediction may become feasible in the future. In this connection, it would be useful to have a better understanding of the process of stress accumulation near active seismic faults, which may lead to sudden fault movement, generating an earthquake.

Observations in seismically active regions in recent years indicate that, during apparently quiet aseismic periods, there are often slow quasistatic aseismic surface movements of the order of a few mms per year, resulting in the accumulation of stress and strain in some cases, which may eventually lead to a sudden fault movement, generating an earthquake, if the stress accumulation reaches sufficiently high levels. The qualitative estimation of this stress accumulation would be facilitated if it is possible to develop suitable theoretical models which incorporate the essential features of the mechanism of stress accumulation in the regions concerned. Such theoretical models would enable us to estimate the stress accumulation below the surface near the fault from the observed aseismic surface movements. In this connection, observational data also indicate continuous, slow, aseismic fault creep across some active faults, including the central part of the San Andreas fault in North America and some other active faults. The effect of this aseismic fault creep on the accumulation and release of stress in the regions concerned is of great interest in the study of the dynamics of the lithosphere-asthenosphere system in seismically active regions during aseismic periods.

In recent years, some theoretical models of the lithosphere-asthenosphere system in seismically active regions during aseismic periods have been developed, with a single locked fault or a single creeping fault in the model, by Nur and Mavko (1974), Rosenman et. al. (1973a, b), Budiansky et. al. (1976). Barker (1976), Spence et. al. (1976), Savage et. al. (1978a, b), Rundle et. al. (1977a, b, c and 1980), Cohen (1978, 1979, 1980a, 1908b, 1984), Mukhopadhyay et. al. (1978a, b, 1979a, b, c, d and 1980a, b), Lehner et. al. (1981), Koseluk et. al. (1981), Yang et. al. (1981) and others. However, active seismic fault systems often consist of several neighbouring faults which may interact when creep or sudden seismic fault movement occurs across one or more of them. For example, in the case of the San Andreas fault system in North America, the Hayward and Calaveras faults are close to and roughly parallel to the main San Andreas fault. Keeping this in view, the authors of this paper are trying to develop theoretical models of interacting faults in the lithosphere-asthenosphere system and one such theoretical model is presented in this paper. It may be noted that such theoretical models, of the lithosphere-asthenosphere system with interacting faults have not generally been developed, except the theoretical models developed by Mukhopadhyay et. al. (1978a, 1979c). However, these two theoretical models were concerned with interaction between two strike-slip faults which undergo sudden seismic movements and become locked subsequently. In this paper, we consider two interacting creeping surface-breaking strike-slip faults in a simple model of the lithosphere-asthenosphere system.

FORMULATION

We consider a simple model of the lithosphere-asthenosphere system consisting of a visco-elastic half-space with its material of the Maxwell type. We consider two long, vertical and interacting strike-slip faults F_1 and F_2 in the half-space across which creep occurs under suitable conditions. We take the faults to be surface-breaking, so that the fault reaches the free surface. We introduce rectangular cartesian coordinates (y_1, y_2, y_3) with the free surface as the plane $y_3 = 0$ and the y_3-axis pointing into the half-space We take the y_1-axis to be parallel to the planes of the faults. Then we can assume that the displacements, stresses and strains will be independent of y_1.

We take the planes of the faults F_1, F_2 to be given by $y_2 = 0$ and $y_2 = D$ respectively. We take D_1 and D_2 to be the depths of the lower edges of the faults below the free surface ($y_3 = 0$).

Fig. 1 shows the section of the model by the plane $y_1 = 0$. For the long faults, since the displacements, stresses and strains are independent of y_1, we

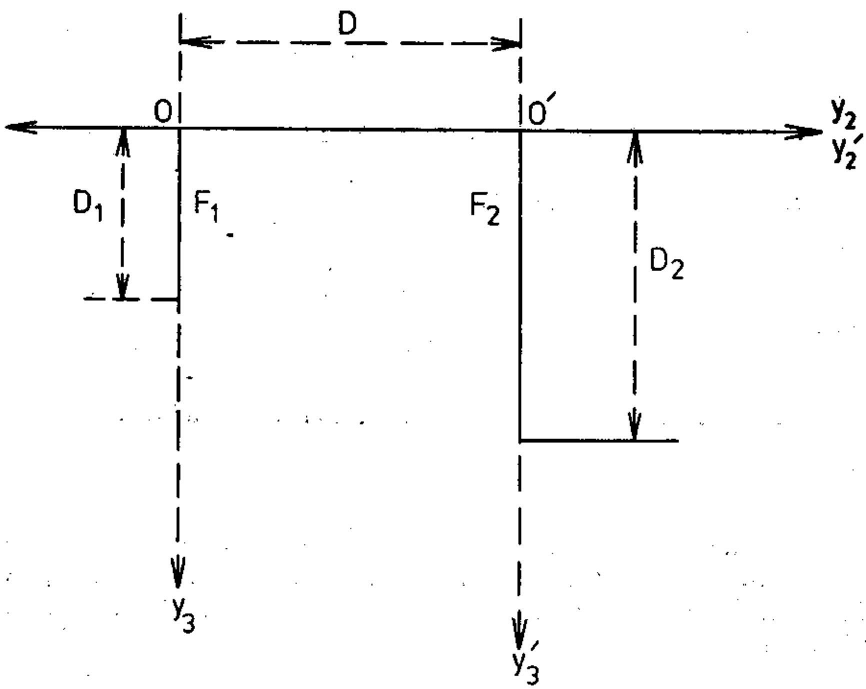

Fig. 1. (Section of the model by the plane $y_1 = 0$).

find that the displacement component along the y_1-axis and the stress components τ_{12} and τ_{13} associated with it are independent of the other components of displacement and stress. For this model, assuming that the material of the half-space is linearly visco-elastic, and of the Maxwell type, the displacement component u_1 and the stress-components τ_{12}, τ_{13} associated with the strike-slip fault movement are connected by the stress-strain relations:

$$\left.\begin{aligned} \left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{12} &= \frac{\partial^2 u_1}{\partial t \partial y_2} \\ \text{and} \quad \left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{13} &= \frac{\partial^2 u_1}{\partial t \partial y_3} \end{aligned}\right\} \tag{1}$$

$$(-\infty < y_2 < \infty,\ y_3 \geqslant 0,\ t \geqslant 0)$$

where μ is the effective rigidity, and η is the effective viscosity,

We consider the model during aseismic periods leaving out the relatively small periods (if any) following sudden fault movements, when seismic disturbances are present in the model. For the slow, aseismic, quasi-static displacements we consider, the inertial forces are very small and are neglected. Hence the relevant stresses satisfy the relation

$$\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0 \tag{2}$$

$$(-\infty < y_2 < \infty,\ y_3 \geqslant 0,\ t \geqslant 0)$$

From (1) and (2), we find that

$$\left.\begin{aligned} \frac{\partial}{\partial t}(\nabla^2 u_1) &= 0 \\ \text{which is satisfied if} \quad \nabla^2 u_1 &= 0 \end{aligned}\right\} \tag{3}$$

At the free surface $y_3 = 0$, we have the boundary condition

$$\tau_{13} = 0 \text{ on } y_3 = 0 \tag{4}$$

We assume that the tectonic forces maintain a constant shear stress τ_∞ far away from the faults, while the stresses near the fault may change with time, due to fault movements (including fault creep). We then have the conditions

$$\tau_{13} \to 0 \text{ as } y_3 \to \infty \text{ (for } -\infty < y_2 < \infty,\ t \geqslant 0) \tag{5}$$

and

$$\tau_{12} \to \tau_\infty \text{ as } |y_2| \to \infty \text{ (for } y_3 \geqslant 0,\ t \geqslant 0) \tag{6}$$

DISPLACEMENTS AND STRESSES IN THE ABSENCE OF FAULT MOVEMENT

In the absence of any movement across the faults, the displacements and stresses are continuous throughout the model. In this case, we measure the time t from any instant after which the relations (1)—(6) become valid for the model. Let $(u_1)_0$, $(\tau_{12})_0$, $(\tau_{13})_0$ which may be functions of (y_2, y_3) be the values of (u_1), (τ_{12}), (τ_{13}) at $t = 0$. $(u_1)_0$, $(\tau_{12})_0$, $(\tau_{13})_0$ also satisfy the relations (1)—(6).

To calculate u_1, τ_{12}, τ_{13} for $t \geqslant 0$, we take Laplace transforms of the relations (1)—(6) with respect to t. This gives a boundary value problem for $\bar{u}_1$, $\bar{\tau}_{12}$, $\bar{\tau}_{13}$ (the Laplace transforms of u_1, τ_{12}, τ_{13} with respect to t).

The boundary value problem can be solved easily, as explained in the Appendix, and on inverting the Laplace transforms we have

$$\left.\begin{aligned} u_1\,(y_2, y_3, t) &= (u_1)_0 + \frac{\tau_\infty y_2 t}{\eta} \\ \tau_{12}\,(y_2, y_3, t) &= (\tau_{12})_0 \exp\,(-\mu t/\eta) + \tau_\infty(1-\exp\,(-\mu t/\eta)) \\ \tau_{13}\,(y_2, y_3, t) &= (\tau_{13})_0 \exp\,(-\mu t/\eta) \end{aligned}\right\} \quad (7)$$

So that the shear strain

$$e_{12} = \frac{\partial u_1}{\partial y_2}$$

$$= (e_{12})_0 + \frac{\tau_\infty t}{\eta}$$

where $(e_{12})_0$ is the value of e_{12} at $t = 0$.

It is easily verified by direct substitution, that the solution (7) satisfy (1)—(6).

From these solutions, we find that if the shear stress τ_{12} near the faults tending to cause strike-slip movement, is less than τ_∞ at $t = 0$, there would be a continuous accumulation of the shear stress τ_{12} near the fault for $t > 0$. Ultimately as $t \to \infty$, $\tau_{12} \to \tau_\infty$, in the neighbourhood of the faults. However, if the characteristics of the faults be such that there is a sudden movement or creep across one of the faults, say F_1, when the shear stress τ_{12} near the fault F_1 reaches a critical value $\tau_c < \tau_\infty$, a sudden fault movement or fault creep across F_1 would occur after a finite length of time. We consider here case in which fault creep commence across one of the faults say F_1, when τ_{12} reaches the value $\tau_c < \tau_\infty$ near F_1 at time $t = T_1$.

DISPLACEMENTS AND STRESSES AFTER THE COMMENCEMENT OF FAULT CREEP ACROSS ONE FAULT

If fault creep commences across F_1 while F_2 remains locked, the relations (1)–(6) are satisfied in this case also. In addition, we have the following condition across F_1 ($y_2 = 0, 0 \leqslant y_3 \leqslant D_1$)

$$[u_1] = U_1(t_1)\, f_1(y_3) \qquad (0 \leqslant y_3 \leqslant D_1,\ y_2 = 0) \tag{8}$$

where $t_1 = t - T_1$

$$[u_1] = \underset{y_2 \to 0+0}{Lt} [u_1] - \underset{y_2 \to 0-0}{Lt} [u_1]$$

is the relative displacement across F_1 corresponding to the fault creep and

$$U_1(t_1) = 0 \text{ for } t_1 \leqslant 0$$

i.e. $t \leqslant T_1$. We note that $[u_1] = 0$ for $t \leqslant T_1$.
The velocity of creep

$$\partial/\partial t\, [u_1] = V_1(t_1) \cdot f_1(y_3),$$

where $V_1(t_1) = d/dt_1\, U_1(t_1)$.

We assume that τ_{12}, τ_{13} are continuous everywhere in the model, while u_1 is taken to be continuous everywhere except for the discontinuity across F_1.

To determine u_1, τ_{12}, τ_{13} which satisfy (1) – (6) and (8), we assume that $u_1(t_1)$ and $f_1(y_3)$ are continuous and try to find solutions for u_1, τ_{12}, τ_{13} in the form,

$$u_1 = (u_1)_1 + (u_1)_2$$

$$\tau_{12} = (\tau_{12})_1 + (\tau_{12})_2$$

$$\tau_{13} = (\tau_{13})_1 + (\tau_{13})_2$$

where $(u_1)_1$, $(\tau_{12})_1$, $(\tau_{13})_1$ are continuous everywhere in the model, satisfy (1)–(6) and have the values $(u_1)_0$, $(\tau_{12})_0$, $(\tau_{13})_0$ at $t = 0$, while $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ are zero for $t \leqslant T_1$, satisfy (1)–(5), and the following condition which replaces (6):

$$(\tau_{12})_2 \to 0 \text{ as } |\, y_2 \,| \to \infty \tag{9}$$

$$(y_3 \geqslant 0,\ t \geqslant T_1)$$

The solutions for $(u_1)_1$, $(\tau_{12})_1$, $(\tau_{13})_1$ are for the same as solutions (7) for u_1, τ_{12}, τ_{13} since they satisfy exactly the same conditions.

On substituting $t_1 = t - T_1$, we find that $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ which are functions of (t_1, y_2, y_3) satisfy the following relations, obtained from (1)–(5), (9) and (8):

$$\left.\begin{aligned}\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t_1}\right)(\tau_{12})_2 &= \frac{\partial^2 (u_1)_2}{\partial t_1\,\partial y_2}\\ \left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t_1}\right)(\tau_{13})_2 &= \frac{\partial^2 (u_1)_2}{\partial t_1\,\partial y_3}\end{aligned}\right\} \tag{1a}$$

$$\frac{\partial}{\partial y_2}(\tau_{12})_2 + \frac{\partial}{\partial y_3}(\tau_{13})_2 = 0 \tag{2a}$$

$$\nabla^2 (u_1)_2 = 0 \tag{3a}$$

[(1a), (2a), (3a) being valid for $-\infty < y_2 < \infty$, $y_3 \geqslant 0$, $t \geqslant 0$]

$$(\tau_{13})_2 = 0 \text{ on } y_3 = 0 \quad (-\infty < y_2 < \infty,\ t_1 \geqslant 0) \tag{4a}$$

$$(\tau_{13})_2 \to 0 \text{ as } y_3 \to \infty \quad (-\infty < y_2 < \infty,\ t_1 > 0) \tag{5a}$$

$$(\tau_{12})_2 \to 0 \text{ as } y_2 \to \infty \quad (y_3 \geqslant 0,\ t_1 \leqslant 0) \tag{6a}$$

$$\left.\begin{aligned}&[(u_1)_2] = U_1(t_1)\, f_1(y_3) \text{ across } F_1\\ &(y_2 = 0,\ 0 \leqslant y_3 \leqslant D_1,\ t_1 \leqslant 0)\\ \text{with}\quad &U_1(0) = 0\\ \text{and}\quad &(u_1)_2, (\tau_{12})_2, (\tau_{13})_2 = 0 \text{ for } t_1 \leqslant 0\end{aligned}\right\} \tag{7a}$$

To obtain solutions for $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ for $t_1 > 0$, we take Laplace transforms of (1a)–7(a) with respect to t_1.

This gives a boundary value problem which can be solved by using a suitably modified form of a Green's function technique developed by Maruyama (1966), as explained in the Appendix.

On inverting the Laplace transforms, we obtain solutions for $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ for $t_1 \geqslant 0$.

Finally we have, for $t \geqslant T_1$, $y_2 \neq 0$

$$\left.\begin{aligned} u_1 &= (u_1)_1 + (u_1)_2 \\ \tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2, \\ \tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 \end{aligned}\right\} \tag{10}$$

where

$$\left.\begin{aligned} (u_1)_1 &= (u_1)_0 + \frac{\tau_\infty y_2 t}{\eta}, \\ (\tau_{12})_1 &= (\tau_{12})_0 \exp(-\mu t/\eta) + \tau_\infty (1 - \exp(-\mu t/\eta)) \\ (\tau_{13})_1 &= (\tau_{13})_0 \exp(-\mu t/\eta) \end{aligned}\right\} \tag{11}$$

and

$$(u_1)_2 = H(t - T_1) \cdot \frac{U_1(t_1)}{2\pi} \psi_{11}(y_2, y_3) \tag{12}$$

$$(\tau_{12})_2 = H(t - T_1) \times \left(\frac{\mu}{2\pi} \int_0^{t_1} V_1(\tau) \exp\left(\frac{-\mu(t_1 - \tau)}{\eta}\right) d\tau\right) \times \phi_{11}(y_2, y_3) \tag{13}$$

$$(\tau_{13})_2 = H(t - T_1) \times \left(\frac{\mu}{2\pi} \int_0^{t_1} V_1(\tau) \exp(-\mu/\eta\,(t_1 - \tau))\, d\tau\right) \times \phi_{21}(y_2, y_3) \tag{14}$$

where, $t_1 = t - T_1$, $H(t - T_1)$ is the Heaviside function, so that $H(t - T_1) = 0$ for $t \leqslant T_1$ and $H(t - T_1) = 1$ for $t > T_1$,

$$\psi_{11}(y_2, y_3) = \int_0^{D_1} f_1(x_3) \left[\frac{y_2}{(x_3 - y_3)^2 + y_2^2} + \frac{y_2}{(x_3 + y_3)^2 + y_2^2}\right] dx_3 \tag{15}$$

$$\phi_{11}(y_2, y_3) = \frac{\partial \psi_{11}}{\partial y_2} = \int_0^{D_1} f_1(x_3) \left[\frac{(x_3 - y_3)^2 - y_2^2}{\{(x_3 - y_3)^2 + y_2^2\}^2} + \frac{(x_3 + y_3)^2 - y_2^2}{\{(x_3 + y_3)^2 + y_2^2\}^2}\right] dx_3 \tag{16}$$

and

$$\phi_{21}(y_2, y_3) = \frac{\partial \psi_{11}}{\partial y_3} = \int_0^{D_1} 2 f_1(x_3) \left[\frac{(x_3 - y_3)\, y_2}{\{(x_3 - y_3)^2 + y_2^2\}^2} - \frac{(x_3 + y_3)\, y_2}{\{(x_3 + y_3)^2 + y_2^2\}^2}\right] dx_3 \tag{17}$$

$(y_2 \neq 0)$

Hence $e_{12} = \frac{\partial u_1}{\partial y_2}$, the shear strain is given by

$$e_{12} = (e_{12})_0 + \frac{\tau_\infty t}{\eta} + H(t - T_1)\,\frac{u_1(t_1)}{2\pi}\,\phi_{11}(y_2, y_3) \tag{18}$$

In (12)–(17), we note that $U_1(t_1)$ and $V_1(t_1)$ vanish for $t_1 \leqslant 0$.

These results for $t \geqslant T_1$, $y_2 \neq 0$ remain valid as long as F_2 remains locked.

The integrals for ψ_{11}, ϕ_{11} and ϕ_{21} can be calculated in closed form if $f(y_3)$ is a polynomial. In particular, if the relative displacement due to creep is independent of depth, we have

$$f(y_3) = \text{constant}$$

$$= K \text{ (say)},$$

$$(0 \leqslant y_3 \leqslant D_1).$$

We find easily that, in this case,

$$\left.\begin{aligned}
\psi_{11}(y_2, y_3) &= K\left[\tan^{-1}\left(\frac{D_1 - y_3}{y_2}\right) + \tan^{-1}\left(\frac{D_1 + y_3}{y_2}\right)\right] \\
\phi_{11}(y_2, y_3) &= -K\left[\frac{D_1 - y_3}{(D_1 - y_3)^2 + y_2^2} + \frac{D_1 + y_3}{(D_1 + y_3)^2 + y_2^2}\right] \\
\phi_{21}(y_2, y_3) &= K\left[\frac{-y_2}{(D_1 - y_3)^2 + y_2^2} + \frac{y_2}{(D_1 + y_3)^2 + y_2^2}\right]
\end{aligned}\right\} \tag{19}$$

We note that, ϕ_{11} and ϕ_{21} (and hence τ_{12} and τ_{13}) have singularities at ($y_2 = 0$, $y_3 = D_1$), which is the lower edge of the fault. Singularities of shear stress of this type have been obtained earlier for static models of locked faults in elastic materials by Chinnery (1961, 1963), Maruyama (1966), Rybicki (1971) and others.

To investigate the conditions under which the displacements and stresses are bounded everywhere in the model including the edge of the fault, we consider the integrals for ψ_{11}, ϕ_{11}, ϕ_{21} given by (12), (13), (14). Integrating these integrals by parts assuming that $f(y_3)$ and $f'(y_3)$ are continuous in $0 \leqslant y_3 \leqslant D_1$, so that integration by parts is valid, we find after some calculation that the displacements and stresses will be bounded near the edge of the fault and else where in the model if the following sufficient conditions are all satisfied simultaneously (as explained in the Appendix):

(i) $f_1(y_3)$ and $f'(y_3)$ are continuous in $0 \leqslant y_3 \leqslant D_1$

(ii) $f_1''(y_3)$ is either continuous in $0 < y_3 < D_1$, or has a finite number of points of finite discontinuity in $0 < y_3 < D_1$,

(iii) *either* $f_1''(y_3)$ is finite and continuous at $y_3 = 0$ and $y_3 = D_1$, or there exist constants m and n, both < 1, such that $(y_3)^m f_1''(y_3) \to 0$ or to a finite limit as $y_3 \to 0+0$ and $(D_1 - y_3)^n f_1''(y_3) \to 0$ or to a finite limit as $y_3 \to D_1 - 0$.

(iv) $f_1(D_1) = 0, f_1'(D_1) = 0 = f_1'(0)$.

(c1)

These conditions imply that the magnitude of the relative displacement across the fault varies smoothly with depth and approaches zero with sufficient rapidity as

The integrals in (12), (13) and (14) can be evaluated in closed form if $f_1(y_3)$ is a polynomial satisfying the four conditions (c1) given above. We find that, the simplest polynomial of this type (of the lowest degree in y_3) would be

$$f_1(y_3) = 1 - \frac{3y_3^2}{D_1^2} + \frac{2y_3^3}{D_1^3}$$

It is easily verified that $f_1(y_3)$ satisfies (i)–(iv).

In this case,

$$[u_1] = U_1(t)\left\{1 - \frac{3y_3^2}{D_1^2} + \frac{2y_3^3}{D_1^3}\right\} \qquad \ldots(20)$$

and the creep velocity $V_1(t_1) f_1(y_3)$

$$= V_1(t_1)\left\{1 - \frac{3y_3^2}{D_1^2} + \frac{2y_3^3}{D_1^3}\right\}$$

$$\left[V_1(t_1) = \frac{d}{dt} u_1(t_1),\ t_1 = t - T_1\right]$$

The integrals in (15)–(17) are evaluated in closed form and we obtain the displacement, stresses and strains given by (10)–(14) where we now have the following expressions for $y_3 \neq 0$;

$$\psi_{11}(y_2, y_3) = \left[1 - \frac{3y_2^2}{D_1^2} - \frac{2y_3^3}{D_1^3} + \frac{3y_3^2}{D_1^2} - \frac{6y_2^2 y_3}{D_1^3}\right]$$

$$\times \left[\tan^{-1}\left(\frac{D_1 - y_3}{y_2}\right) + \tan^{-1}\left(\frac{y_3}{y_2}\right)\right]$$

$$-\left[1-\frac{3y_3^2}{D_1^2}-\frac{2y_3^3}{D_1^3}+\frac{3y_2^2}{D_1^2}+\frac{6y_2^2y_3}{D_1^3}\right]$$

$$\times\left[\tan^{-1}\left(\frac{D_1+y_3}{y_2}\right)-\tan^{-1}\left(\frac{y_3}{y_2}\right)\right]$$

$$+\,y_2\left[-\frac{3y_3}{D_1^3}(D_1-y_3)-\frac{y_2^2}{D_1^3}\right]\log_e\left[(D_1-y_3)^2+y_2^2\right]$$

$$+\,y_2\left[\frac{3y_3}{D_1^3}(D_1+y_3)-\frac{y_2^2}{D_1^3}\right]\log_e\left[(D_1+y_3)^2+y_2^2\right]$$

$$+\,y_2\left(\frac{2y_2^2}{D_1^3}-\frac{6y_3^2}{D_1^3}\right)\log_e(y_2^2+y_3^2)-\frac{4}{D_1}y_2 \qquad \ldots(21)$$

$$\phi_{11}(y_2,y_3)=\frac{\partial\psi_{11}}{\partial y_2}=\left[\frac{6y_3}{D_1^2}-\frac{12y_2y_3}{D_1^3}\right]\tan^{-1}\left(\frac{D_1-y_3}{y_2}\right)$$

$$-\left[\frac{6y_3}{D_1^2}+\frac{12y_2y_3}{D_1^3}\right]\tan^{-1}\left(\frac{D_1+y_3}{y_2}\right)$$

$$-\frac{24y_2y_3}{D_1^3}\tan^{-1}\left(\frac{y_3}{y_2}\right)$$

$$-\left[\frac{(D_1-y_3)^2(D_1-2y_3)}{D_1^3}+\frac{3y_2^2}{D_1^2}-\frac{6y_2^2y_3}{D_1^3}\right]$$

$$\times\frac{(D_1-y_3)}{[y_2^2+(D_1-y_3)^2]}$$

$$-\left[\frac{(D_1+y_3)^2(D_1-2y_3)}{D_1^3}+\frac{3y_2^2}{D_1^2}+\frac{6y_2^2y_3}{D_1^3}\right]$$

$$\times\frac{(D_1+y_3)}{[y_2^2+(D_1+y_3)^2]}+\frac{4y_2^4}{D_1^3(y_2^2+y_3^2)}-\frac{4}{D_1}$$

$$-\frac{1}{D_1^3}\left[3y_3(D_1-y_3)+3y_2^2\right]\log_e\left[(D_1-y_3)^2+y_2^2\right]$$

$$-\frac{2y_2^2}{D_1^3}\frac{[3y_3(D_1-y_3)+y_2^2]}{(D_1-y_3)^2+y_2^2}\log_e\left[(D_1+y_3)^2+y_2^2\right]$$

$$+\frac{6}{D_1^3}(y_2^2-y_3^2)\log_e(y_2^2+y_3^2) \qquad \ldots(22)$$

$$\phi_{21}(y_2,y_3)=\frac{\partial\psi_{11}}{\partial y_3}$$

$$= \left[-\frac{6y_3}{D_1^2} + \frac{6y_3^2}{D_1^3} - \frac{6y_3^3}{D_1^3}\right] \tan^{-1}\left(\frac{D_1 - y_3}{y_2}\right)$$

$$- \left[-\frac{6y_3}{D_1^2} - \frac{6y_3^2}{D_1^3} + \frac{6y_3^3}{D_1^3}\right] \tan^{-1}\left(\frac{D_1 + y_3}{y_2}\right)$$

$$- \left[\frac{12y_3^3}{D_1^3} - \frac{12y_3^2}{D_1^3}\right] \tan^{-1}\left(\frac{y_3}{y_2}\right)$$

$$- \left[\frac{(D_1 - y_3)^2 (D_1 + 2y_3)}{D_1^3} + \frac{3y_2^2}{D_1^2} - \frac{6y_2^2 y_3}{D_1^3}\right] \frac{y_2}{y_2^2 + (D_1 - y_3)^2}$$

$$- \left[\frac{(D_1 + y_3)^2 (D_1 - 2y_3)}{D_1^3} + \frac{3y_2^2}{D_1^2} + \frac{6y_2^2 y_3}{D_1^3}\right] \frac{y_2}{y_2^2 + (D_1 + y_3)^2}$$

$$+ \left[\frac{4y_2^2}{D_1^2} - \frac{12y_2^2 y_3}{D_1^3}\right] \frac{y_2}{y_2^2 + y_3^2}$$

$$- \frac{3y_2 (D_1 - 2y_3)}{D_1^3} \log_e [(D_1 - y_3)^2 + y_2^2]$$

$$- \frac{y_2}{D_1^3} [3y_3 (D_1 - y_3) + y_2^2] \frac{2 (D_1 - y_3)}{(D_1 - y_3)^2 + y_2^2}$$

$$+ \frac{3y_2 (D_1 - 2y_3)}{D_1^3} \log_e [(D_1 + y_3)^2 + y_2^2]$$

$$+ \frac{y_2}{D_1^3} [3y_3 (D_1 + y_3) - y_2^2] \frac{2 (D_1 + y_3)}{(D_1 + y_3)^2 + y_2^2}$$

$$- \frac{12y_2 y_3}{D_1^3} \log_e (y_2^2 + y_3^2)$$

$$+ \frac{2y_2 y_3}{D_1^3} \frac{(y_2^2 - 3y_3^2)}{y_2^2 + y_3^2} \qquad \ldots(23)$$

In particular, if

$$u_1(t_1) = V_1 t_1,$$

where V_1 is a constant, so that, across F_1,

$$[u_1] = V_1 t_1 \left(1 - \frac{3y_3^2}{D_1^2} + \frac{2y_3^3}{D_1^3}\right) H(t_1)$$

for $\quad 0 \leqslant y_3 \leqslant D_1,\ t_1 \geqslant 0$ (i.e., $t > T_1$)

and $[u_1] = 0$ for $t_1 \leqslant 0$ (i.e. $t \leqslant T_1$),

we obtain from (10)–(14), for

$$t > T_1, \quad y_2 \neq 0,$$

$$u_1 = (u_1)_0 (y_2, y_3) + \frac{\tau_\infty y_2 t}{\eta} + H(t-T_1) \frac{V_1 t_1}{2\pi} \psi_{11}(y_2, y_3)$$

$$e_{12} = \frac{\partial u_1}{\partial y_2} = (e_{12})_0 (y_2, y_3) + \frac{\tau_\infty t}{\eta} + H(t-T_1) \times \frac{V_1 t_1}{2\pi} \phi_{11} (y_2, y_3)$$

$$\tau_{12} = (\tau_{12})_0 \exp(-\mu t/\eta) + \tau_\infty [1-\exp(-\mu t/\eta)] + H(t-T_1) \frac{\eta V_1}{2\pi} [1-\exp(-\mu t_1/\eta)] \phi_{11}(y_2, y_3)$$

and

$$\tau_{13} = (\tau_{13})_0 \exp(-\mu t/\eta) + H(t-T_1) \frac{\eta V_1}{2\pi} [1-\exp(-\mu t_1/\eta)] \phi_{21}(y_2, y_3)$$

where ψ_{11}, ϕ_{11}, ϕ_{21} are given by (17), (18) and (19). Here the velocity of creep across F_1 is

$$V_1 f_1(y_3) \; (0 \leqslant y_3 \leqslant D_1)$$

The results (10)–(24) remain valid for $t \geqslant T_1$ as long as F_2 remains locked.

DISPLACEMENTS AND STRESSES AFTER COMMENCEMENT OF CREEP ACROSS THE SECOND FAULT

If fault creep commences across the second fault F_2 also at $t = T_2$ ($T_2 \geqslant T_1 \geqslant 0$), we have the following conditions across F_1 and F_2 for $t \geqslant T_2$:

$$\left.\begin{array}{ll} \text{(a)} & [u_1] = u_1(t_1) f_1(y_3) H(t_1) \text{ across } F_1 \\ & (0 \leqslant y_3 \leqslant D_1, \; y_2 = 0) \\ \text{(b)} & [u_1] = u_2(t_2) f_2(y_3) H(t_2) \text{ across } F_2 \\ & (0 \leqslant y_3 \leqslant D_2, \; y_2 = D) \end{array}\right\} \qquad (5)$$

when $t_2 = t - T_2$,

$$u_1(t_1) = 0 \text{ for } t_1 \leqslant 0,$$

$$u_2(t_2) = 0 \text{ for } t_2 \leqslant 0.$$

and $[u_1]$ is the discontinuity in u_1 across F_2, i.e.

$$[u_1] = \underset{y_2 \to D+0}{\text{Lt}} (u_1) - \underset{y_2 \to D-0}{\text{Lt}} (u_1)$$
$$(0 \leqslant y_3 \leqslant D_2)$$

For $t > T_2$, we try to find solutions for u_1, τ_{12}, τ_{13} in the form

$$\left.\begin{aligned} u_1 &= (u_1)_1 + (u_1)_2 + (u_1)_3 \\ \tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2 + (\tau_{12})_3 \\ \text{and} \quad \tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 + (\tau_{13})_3 \end{aligned}\right\} \tag{26}$$

where $(u_1)_1$, $(\tau_{12})_1$, $(\tau_{13})_1$ are continuous everywhere in the model and satisfy the relation (1)–(6); $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ vanish for $t_1 \leqslant 0$, satisfy (1)–(5) and (9) for $t_1 \geqslant 0$, and are continuous everywhere in the model except for the discontinuity of $(u_1)_2$ across F_1, where we have,

$$\left.\begin{aligned} &[(u_1)_2] = u_1(t_1) f_1(y_3) H(t_1) \\ &(t_1 = t - T_1)\,[0 \leqslant y_3 \leqslant D_1,\ y_2 = 0], \end{aligned}\right\} \tag{27}$$

$(u_1)_3$, $(\tau_{12})_3$, $(\tau_{13})_3$ vanish for $t_2 \leqslant 0$, i.e, $t \leqslant T_2$, satisfy (1)–(5) and the condition

$$(\tau_{12})_3 \to 0 \text{ as } |y_2| \to \infty \ (y_3 \geqslant 0) \tag{28}$$

while $(u_1)_3$ is continuous everywhere in the model except across F_2, where we have

$$[(u_1)_3] = u_2(t_2)\, H(t_2)\, f_2(y_3)$$

$$[y_2 = D,\ 0 \leqslant y_3 \leqslant D_2] \tag{29}$$

and $(\tau_{12})_3$, $(\tau_{13})_3$ are continuous everywhere in the model. We note that the solutions for $(u_1)_1$, $(\tau_{12})_1$, $(\tau_{13})_1$ will again be given by (11), and the solutions for $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ will be given by (12)-(24) as before, in appropriate cases.

For $(u_1)_3$, $(\tau_{12})_3$, $(\tau_{13})_3$, on writing

$$y_2' = y_2 - D,$$

$$y_3' = y_3,$$

$$t_2 = t - T_2,$$

we find that they satisfy the following conditions for $t_2 \geqslant 0$, obtained from (1)–(5), (28) and (29)

$$\left.\begin{aligned}\left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t_2}\right)(\tau_{12})_3 &= \frac{\partial^2(u_1)_3}{\partial t_1\,\partial y_2'}\\ \left(\frac{1}{\eta}+\frac{1}{\mu}\frac{\partial}{\partial t_2}\right)(\tau_{13})_3 &= \frac{\partial^2(u_1)_3}{\partial t_1\,\partial y_3'}\end{aligned}\right\} \tag{1b}$$

$$\frac{\partial}{\partial y_2'}(\tau_{12})_3 + \frac{\partial}{\partial y_3'}(\tau_{13})_3 = 0 \tag{2b}$$

$$\nabla^2(u_1)_3 = 0 \tag{3b}$$

[(1b), (2b), (3b) are valid for $y_3' > 0,\ -\infty < y_2 < \infty,\ t_2 \geqslant 0$]

$$(\tau_{13})_3 = 0 \text{ on } y_3' = 0 \tag{4b}$$

$$(\tau_{13})_3 \to 0 \text{ as } y_3' \to \infty\ (-\infty < y_2' < \infty) \tag{5b}$$

$$(\tau_{12})_3 \to 0 \text{ as } |y_2'| \to \infty\ (y_3' \geqslant 0) \tag{6b}$$

and $[(u_1)_3] = u_2(t_2)f_2(y_3')$ across F_2

$$(y_2' = 0,\ 0 \leqslant y_3' \leqslant D_2,\ t_2 \geqslant 0) \tag{7b}$$

while $(u_1)_3$ is continuous everywhere except across F_2, $u_2(0) = 0$, and $(u_1)_3$, $(\tau_{12})_3$, $(\tau_{13})_3$ vanish for $t_2 \leqslant 0$.

The conditions satisfied by $(u_1)_3$, $(\tau_{12})_3$, $(\tau_{13})_3$ are exactly similar to the conditions (1a)–(6a) and (8a) satisfied by $(\tau_{11})_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ with y_2, y_3, t_1, $V_1(t_1)$, $f_1(y_3)$ replaced by y_2', y_3', t_2, $V_2(t_2)$, $f_2(y_3')$. Proceeding exactly as before, we obtain corresponding to the results (12)–(14),

$$(u_1)_3 = H(t - T_2)\frac{u_2(t_2)}{2\pi}\,\psi_{11}'(y_2', y_3') \tag{30}$$

$$(\tau_{12})_3 = H(t - T_2)\left(\frac{\mu}{2\pi}\int_0^{t_2} V_2(\tau)\exp\left\{\frac{-\mu(t_2-\tau)}{\eta}\right\}d\tau\right)$$

$$\times\ \phi_{11}'(y_2', y_3') \tag{31}$$

$$(\tau_{12})_3 = H(t - T_2)\left(\frac{\mu}{2\pi}\int_0^{t_2} V_2(\tau)\exp\left\{-\frac{\mu}{\eta}(t_2 - \tau)\right\}d\tau\right)\times$$

$$\phi_{21}'(y_2', y_3') \qquad (32)$$

$$(y_2' \neq 0 \text{ i.e. } y_2 \neq D)$$

where $t_2 = t - T_2,\ y_2' = y_2 - D,\ y_3' = y_3$

$$V_2(t_2) = \frac{d}{dt_2} V_2(t_2)$$

and $\psi_{11}', \phi_{11}', \phi_{21}'$ are obtained from $\psi_{11}, \phi_{11}, \phi_{21}$ on replacing $y_2, y_3, D_1, f_1(x_3)$ in $\psi_{11}, \phi_{11}, \phi_{21}$ by $y_2', y_3', D_2, f_2(x_3)$ respectively, where $\psi_{11}, \phi_{11}, \phi_{21}$ are given by (15)–(24) in the appropriate cases: Thus, finally, we have for $t \geqslant T_2$, $y_2 \neq 0$,

$$\left.\begin{aligned} u_1 &= (u_1)_1 + (u_1)_2 + (u_1)_3 \\ \tau_{12} &= (\tau_{12})_1 + (\tau_{12})_2 + (\tau_{12})_3 \\ \tau_{13} &= (\tau_{13})_1 + (\tau_{13})_2 + (\tau_{13})_3 \end{aligned}\right\} \qquad (33)$$

where

$(u_1)_1, (\tau_{12})_1, (\tau_{13})_1$ are given by (11),

$(u_1)_2, (\tau_{12})_2, (\tau_{13})_2$ are given by (12)–(14)

and

$(u_1)_3, (\tau_{12})_3, (\tau_{13})_3$ are given by (30)–(32)

Hence we have, for $t > T_2$

$$e_{12} = (e_{12})_0 + \frac{\tau_\infty t}{\eta} + H(t - T_1)\frac{u_1(t_1)}{2\pi}\phi_{11}(y_2, y_3)$$

$$+ H(t - T_2)\frac{u_2(t_2)}{2\pi}\phi_{11}'(y_2', y_3') \qquad (34)$$

$$(t_1 = t - T_1,\ t_2 = t - T_2,\ y_2' = y_2 - D,\ y_3' = y_3)$$

For boundedness of displacements and stresses everywhere in the medium for $t_2 \geqslant 0$ (i.e. $t \geqslant T_2$), both $f_1(y_3)$ and $f_2(y_3')$ have to satisfy the four conditions (c1) stated earlier for $f_1(y_3)$, where, for $f_2'(y_3')$, we replace (y_2, y_3, D_1) by (y_2', y_3', D_2) in these conditions.

In particular, if we have, for $t \geqslant T_2$ the conditions of discontinuity of u_1 across F_1 and F_2 given by (25), where

$$u_1(t_1) = V_1 t_1 \quad \text{and} \quad u_2(t_2) = V_2 t_2$$

where V_1, V_2 are constants, we have, for $t \geqslant T_2$, $y_3 \neq 0$, $y_3 \neq D$,

$$\left.\begin{aligned}
u_1 &= (u_1)_0 + \frac{\tau_\infty y_2 t}{\eta} + \frac{V_1 t_1\, H(t - T_1)}{2\pi}\, \psi_{11}(y_2, y_3) \\
&\quad + \frac{V_2 t_2\, H(t - T_2)}{2\pi}\, \psi_{11}'(y_2', y_3') \\
e_{12} &= (e_{12})_0 + \frac{\tau_\infty t}{\eta} + \frac{V_1 t_1}{2\pi}\, H(t - T_1)\, \phi_{11}(y_2, y_3) \\
&\quad + \frac{V_2 t_2\, H(t - T_2)}{2\pi}\, \phi_{11}'(y_2', y_3') \\
\tau_{12} &= (\tau_{12})_0 \exp(-\mu t/\eta) + \tau_\infty(1 - \exp(-\mu t/\eta)) \\
&\quad + \frac{\eta V_1}{2\pi}\, H(t - T_1)(1 - \exp(-\mu t_1/\eta))\, \phi_{11}(y_2, y_3) \\
&\quad + \frac{\eta V_2}{2\pi}\, H(t - T_2)(1 - \exp(-\mu t/\eta)\, \phi_{11}'(y_2', y_3') \\
&\text{and} \\
\tau_{13} &= (\tau_{13})_0 \exp(-\mu t/\eta) + \frac{\eta V_1}{2\pi}\, H(t - T_1)(1 - \exp-(\mu t_1/\eta))\, \phi_{21}(y_2, y_3) \\
&\quad + \frac{\eta V_2}{2\pi}\, H(t - T_2)(1 - \exp(-\mu t_2/\eta))\, \phi_{21}'(y_2', y_3') \\
&(t_1 = t - T_1,\ t_2 = t - T_2,\ y_2' = y_2 - D,\ y_3' = y)
\end{aligned}\right\} \quad (35)$$

If creep commences simultaneously across F_1 and F_2 the solutions remain valid, with $T_1 = T_2$ so that $t_1 = t_2$, and u_1, τ_{12}, τ_{13} for $t \geqslant T_1 (= T_2)$ are then given by (33).

If the fault creep across F_1 or F_2 (or both) stops after some time (say $t = T_3$), the results remain valid, if we take $u_1(t_1)$ or $u_2(t_2) = 0$ for $t > T_3$ in the appropriate cases. The results also remain valid if the fault creep across F_1 or F_2 starts, stops after some time, and starts again, provided we take the appropriate forms of $V_1(t_1)$ or $V_2(t_1)$. In such cases, we assume, however, that the dependence of the fault creep on the depth y_3 does not change, so that $f_1(y_3)$ and $f_2(y_3)$ remain the same. We also note that the solutions (33)–(35) are actually valid for all $t > 0$, since $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ vanish for $t \leqslant T_1$ and $(u_1)_3$, $(\tau_{12})_3$, $(\tau_{13})_3$ vanish for $t \leqslant T_2$. We note that, in the solutions (10) and (33), $(u_1)_1$, $(\tau_{12})_1$, $(\tau_{13})_1$ can be interpreted as the displacements and stresses associated with the displacements and stresses present initially at $t=0$ and the shear stress τ_∞ maintained by tectonic forces far away from the fault, while $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ represent the displacements and stresses due to fault creep across F_1 and $(u_1)_3$, $(\tau_{12})_3$, $(\tau_{13})_3$ represent those due to fault creep across F_2.

We also note that, to obtain $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ in the neighbourhood of F_1, we make $y_2 \to 0 + 0$ and $y_2 \to 0 - 0$ to obtain the limiting values on the right and left of F_1 in Fig. 1. Similarly, to obtain $(u_1)_3$, $(\tau_{12})_3$, $(\tau_{13})_3$ in the neighbourhood of F_2, we make $y_2 \to D + 0$ and $y_2 \to D - 0$ to get the limiting values of the right and left of F_2 in Fig. 1.

DISCUSSION OF THE RESULTS AND CONCLUSIONS

To study in greater detail the changes of the displacements stresses and strains in the model near the faults with time, and specially the influence of fault creep, we compule the changes of the surface displacement u_1 and surface shear strain e_{12} near the faults, as well as the shear stress τ_{12} near the faults, tending to cause strike-slip movement, for relevant values of the model parameters μ, η, D_1, D_2, D, τ_∞ and for relevant types of creep velocities across the faults. Keeping in view the case of shallow strike-slip faults in the lithosphere, we take values for μ in the range $(3 \text{ to } 4) \times 10^{11}$ dynes/cm^2, while D_1, D_2, D are taken to have values in the range 5 to 20 kms. We have carried out computations for some simple types of fault creep, satisfying the conditions (c1) for the boundedness of displacements, stresses and strains for all finite (y_2, y_3, t) so that the creep displacements across F_1 and F_2, commencing at times $t = T_1$, and $t = T_2(T_2 > T_1 > 0)$ have the following form:

$$[u_1] = V_1 t_1 f_1(y_3)$$

across F_1,

and

$$[u_1] = V_2 t_2 f_2(y_3)$$

across F_2,

where V_1, V_2 are constants, representing the creep velocities on the free surface $y_3 = 0$, and $f_1(y_3)$, $f_2(y_3)$ are polynomials, satisfying the conditions (c1) for boundedness of displacements, stresses and strains. In particular we consider the case in which

$$f_1(y_3) = 1 - \frac{3y_3^2}{D_1^2} + \frac{2y_3^3}{D_1^3}$$

$$f_2(y_3) = 1 - \frac{3y_3^2}{D_2^2} + \frac{2y_3^3}{D_2^3}$$

For V_1 and V_2, we consider values in the range to 5 cms/year, which is the range of observed creep velocities on the surface across creeping strike-slip faults in North America. For η, we consider values in the range $10^{21}-10^{22}$ poise, keeping in view the fact that Cathles (1976) has obtained reasonably good agreement between observational results on post-glacial uplift and corresponding theoretical results for theoretical models of the lithosphere-asthenosphere system with values of η in this range. For τ_∞, we consider values in the range 50 to 200 bars, and for $(\tau_{12})_0$ near the faults we consider values in the range 0 to 100 bars; with $(\tau_{12})_0 < \tau_\infty$ in all cases.

The computed values of displacements, stresses and strains show that, in the absence of fault creep, there is generally a gradual accumulation of surface shear strain e_{12} and the shear stress τ_{12} near the faults, tending to cause strike-slip movement, if $(\tau_{12})_0 < \tau_\infty$, The rate of accumulation of shear stress decreases slowly with time, and $\tau_{12} \rightarrow \tau_\infty$ ultimately if there is no fault creep. However, if creep across F_1 or F_2 commences, it influences significantly the subsequent displacements, stresses and strains near the faults. The magnitude of this effect is found to depend on:—

(a) the velocity and spatial distribution (across the faults) of the relative aseismic creep displacements

$$[\text{i.e. } V_1, V_2' \text{ and } f_1(y_2), f_2(y_2)]$$

(b) The depths D_1, D_2 of the faults and the distance D between them.

(c) The shear stress τ_∞ far away from the fault, maintained by tectonic forces.

(d) The displacements, stresses and strains present at $t = 0$.

(e) The values of the model parameters μ, η relates to the material rheology.

It is found that, when creep across a fault commences, it reduces the rate of accumulation of surface shear strain and shear stress near itself, and also, to a smaller extent, near the other neighbouring fault. Simultaneous creep across both the faults reduces further the rate of accumulation of shear stress and surface shear strain near the faults. Greater creep velocities result in greater reduction in the rate of accumulation of shear stress and surface shear strain, and for sufficiently large creep velocities across the faults, there is a continuous aseismic release of shear stress near the faults, so that the possibility of a sudden fault movement, generating an earthquake, becomes remote. This is consistent with the observed relative absence of earthquakes near the central part of the San Andreas fault in North America, where aseismic creep has been observed across the strike-slip fault. The influence of one fault on another is found to decrease quite rapidly with increase in the distance between them, and if $D \gg (D_1, D_2)$, the effect of creep across one fault on the stresses and strains near the other becomes very small.

However, in this case also, creep across a fault reduces the rate of accumulation of shear stress and surface shear strain near itself. Another interesting result is that, in the absence of fault creep, the rate of accumulation of surface shear strain is of the order of 10^{-7} per year, which is of the same order of magnitude as the rate of surface shear strain accumulation near the locked northern part of the San Andreas fault, where there is no fault creep on the surface.

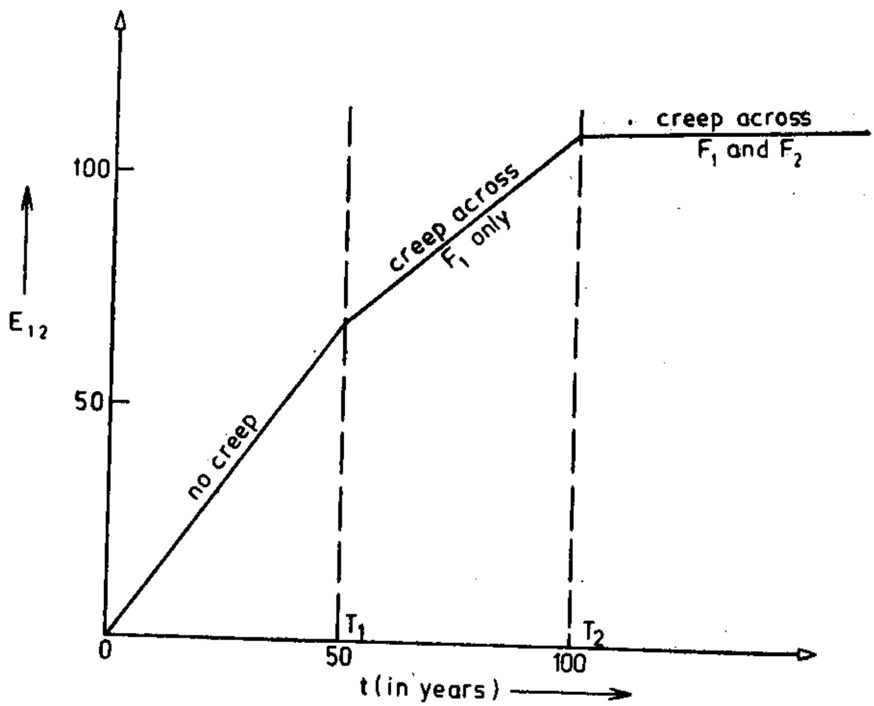

Fig. 2 (Changes in the surface shear strain near F_2)

Fig. 2 shows the changes of the surface shear strain with time for a typical set of values of the model parameters

Here
$$E_{12} = [e_{12} - (e_{12})_0] \times 10^7$$
$$y_3 = 0$$
$$y_2 \to D$$

is proportional to the increase of surface shear strain near the fault F_2, and the time t is in years. Fig. 2 corresponds to the following values of the model parameters :

$$D_1 = D_2 = D = 10 \text{ kms},$$

$$\eta = 10^{21} \text{ poise}, \quad \mu = 3 \times 10^{11} \text{ dynes/cm}^2,$$

$$\tau_\infty = 100 \text{ bars}, \quad T_1 = 50 \text{ years},$$

$$T_2 = 100 \text{ year}, (\tau_{12})_0 = 0,$$

$$V_1 = V_2 = 2 \text{ cms/year}$$

$$f_1(y_3) = 1 - \frac{3y_3^2}{D_1^2} + \frac{2y_3^3}{D_1^3}$$

$$f_2(y_3) = 1 - \frac{3y_3^2}{D_2^2} + \frac{2y_3^3}{D_2^3}$$

From Fig. 2, we find that E_{12} increases steadily upto $t = T_1$, when creep across F_1 commences. For $t > T_1$, the creep across F_1 reduces the rate of increase of E_{12} near F_2. After $t = T_2$, when creep across F_2 also commences,

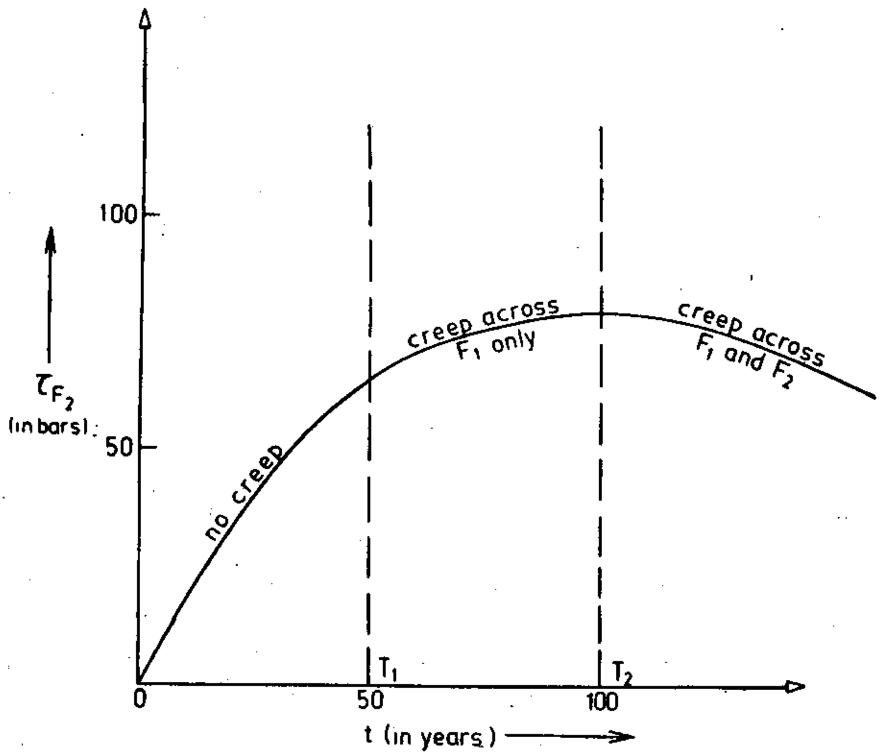

Fig. 3 (Changes in the shear stress τ_{12} near F_2)

the rate of accumulation of E_{12} near F_2 is reduced to a greater extent, and becomes nearly zero.

Fig. 3 shows the changes with time of the shear stress τ_{12} near the mid-point of the fault

$$F_2\left(\text{i.e. } y_3 \to \frac{D_3}{2}, y_2 \to D\right)$$

Hence
$$\tau_{F_2} = [(\tau_{12}) - (\tau_{12})_0]$$
$$y_3 \to \frac{D_3}{2}$$
$$y_2 \to D$$

For Fig. 3, the model parameters have the same values as those corresponding to Fig. 2, and the time t is in years, while τ_{F_2} is in bars. Fig. 3 shows that τ_{F_2} increases gradually with time, upto $t = T_1$, and the rate of increase decreases very slowly with time. After $t = T_1$, when creep across F_1 commences, there is a significant reduction in the rate of increase of τ_{F_2}. Finally, after $t = T_2$, when creep across F_2 also commences, there is a gradual and continuous aseismic release of τ_{F_2} instead of an increase.

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REFERENCES

1. Barker, T.G. (1976): Quasi-steady motions near San Andreas fault zone, Geophys. J.R. Astr. Soc., **45**, 689–705.
2. Budiansky, B. et. al. (1976): Interaction of fault slip and lithospheric creep: J. Geophys. Res., **81**, 4897–4900.
3. Carslaw, H.S. (1950): Introduction to the Theory of Fourier Series and Integrals: (3rd Ed.) Dover Publications.
4. Cathles, L.M. (1976): The Viscosity of the Earth's Mantle; Princeton Univ. Press.
5. Cohen, S.C. (1978): The visco-elastic stiffness model of seismicity. J. Geophys. Res., **83**, No. B11, Nov. 10, 1978, 5425–5432.

6. Cohen, S.C. (1979): Post-Seismic Surface Deformations due to Lithospheric and Asthenospheric Visco-Elasticity. Geophys. Res. Letters., 6, 129-137.

7. Cohen, S.C. (1980a): Post-Seismic Visco-elastic surface deformation and stress, 1, Theoretical Considerations. Displacement and Strain Calculations. J. Geophys. Res., 85, No. B 6, 3131–3150.

8. Cohen, S.C. (1980b): Post-Seismic Visco-Elastic deformation and Stress, 2 Stress Theory and Computation, Dependence of displacement, Strain and stress on fault parameters. J. Geophys. Res. Vol. 85, No. B6, 3151–3158.

9. Cohen, S.C. et. al. (1984): Crustal deformation, the earthquake cycle and models of visco-elastic flow in the asthenosphere, Geophys. J. Roy. Astr. Soc., Vol. 78, p. 735–750.

10. Koseluk, R.A. et. al. (1981): An elastic Rebound Model for normal fault earthquakes. J. Geophys. Res., 86, 1081–1090.

11. Lehner, F. K. et. al. (1981): Stress diffusion along Rupturing plate boundaries. Journal of Geophysical Research, 86, No. B7, 6155-6170.

12. Mukhopadhyay, A. et. al. (1978 a); On stress Accumulation in a Visco-Elastic lithosphere. Proceedings of the sixth International Symposium on Earthquake Engineering, Roorkee, Vol. 1, p. 71–76.

13. Mukhopadhyay, A. et. al. (1978 b): On the Interaction between Two locked strike-slip faults. Proceedings of the sixth International Symposium on Earthquake Engineering. Roorkee, Vol. 1, p. 77–82.

14. Mukhopadhyay, A et. al. (1979 a): On stress Accumulation near a Finite Rectangular Fault. Indian Journal of Meterology, Hydrology and Geophysics (Mausam), Vol. 30, p. 347–352.

15. Mukhopadhyay, A. et. al. (1979 b): On stress Accumulation and Fault slip in the Lithosphere. Indian Journal of Meterology, Hydrology and Geophysics. (Mausam), Vol. 30, p. 353–358.

16. Mukhopadhyay, A. et. al. (1979 c): On stress Accumulation in the Lithosphere and Interaction between Two strike-slip faults. Indian Journal of Meterology, Hydrology and Geophysics (Mausam). Vol. 30. p. 359-363.

17. Mukhopadhyay, A. et. al. (1979 d): A Mechanism of Stress Accumulation near a strike-slip fault. Indian Journal of Meterology, Hydrology and Geophysics (Mausam), Vol. 30, p. 367–372.

18. Mukhopadhyay, A. (1980 a): On Stress Accumulation in a Visco-elastic Lithosphere Containing a Continuously sliping fault; Bull. Soc. Earthquake Technology, Vol. 17, No. 1. p. 1–10,

19. Mukhopadhyay, A. (1980 b): On Stress Accumulation near a Continously slipping Fault in a Two-layer model of the Lithosphere, Bull. Soc. Earthquake Technology, Vol. 17 No. 4, p. 29–38.

20. Nur, A. and Mavko, G. (1974): Post-Seismic Visco-Elastic Rebound, Science, **183**, 204–206.

21. Rosenman, M. et. al. (1973): Stress Relaxation in a semi-infinite Visco-elastic Earth Model. Bull. Seis. Soc. Am, **63**, 2145–2154.

22. Rundle, J.B. et. al. (1977 a): A Visco-elastic Relaxation Model for Post-Seismic Deformation from the San Fransisco Earthquake of 1906; Pure and Applied Geophysics, **115**, 401–411.

23. Rundle. J.B. et. al. (1977 b): A three-dimensional Visco-Elastic Model of a Strike-slip fault. Geophys. J. Roy. Astr. Soc, **49**, 575–591.

24. Rundle, J. B. et. al. (1977 c): A Kinematic Visco-elastic model of the San Francisco Earthquake of 1906. Geophys. J. Roy. Astrn. Soc.. **50**, 441–458.

25. Rundle, J. B. et. al. (1980): Speculations on the nature of the Southern California (Palmdale) uplift. Bull. Seis. Soc. Am., **70**, 1869–1886.

26. Savage, J. C. et. al. (1978a): Asthenospheric Readjustment and the Earthquake Cycle. J. Geophys. Res. **83**, 3369–3376.

27. Savage, J. C. et. al. (1978 b): Comment on "Non-linear stress Propagation-H. J. Melosh"., J. Geophys. Res.. **83**, 5005-5008.

28. Spence, D. A. et. al. (1976): An Elasto-Static Model of Stress Accumulation on the San Andreas fault. Proc. Roy. Soc. Lond., A, **349**, 319–341.

29. Yang, M. et. al. (1981): Time-dependent Deformation and Stress Relaxation after strike-slip Earthquakes. J. Geophys. Res., **86**, 2889-2901.

APPENDIX

DISPLACEMENTS AND STRESSES IN THE ABSENCE OF FAULT MOVEMENT—METHOD OF DETERMINATION:

In this case u, τ_{12}, τ_{13} satisfy (1)–(6) and are continuous everywhere. On taking Laplace transforms of (1)–(6) w.r. to, we obtain a boundary value problem for $(\bar{u}_1, \bar{\tau}_{12}, \bar{\tau}_{13})$, the Laplace transforms of $(u_1, \tau_{12}, \tau_{13})$ w.r. to t, characterised by the relations:

$$\left.\begin{aligned}
\bar{\tau}_{12} &= \frac{p}{\dfrac{p}{\mu}+\dfrac{1}{\eta}}\cdot\frac{\partial \bar{u}_1}{\partial y_2} + \frac{\dfrac{(\tau_{12})_0}{\mu} - \dfrac{\partial (u_1)_0}{\partial y_2}}{\dfrac{p}{\mu}+\dfrac{1}{\eta}} \\
\bar{\tau}_{13} &= \frac{p}{\dfrac{p}{\mu}+\dfrac{1}{\eta}}\cdot\frac{\partial \bar{u}_1}{\partial y_3} + \frac{\dfrac{(\tau_{13})_0}{\mu} - \dfrac{\partial (u_1)_0}{\partial y_3}}{\dfrac{p}{\mu}+\dfrac{1}{\eta}}
\end{aligned}\right\} \qquad \text{(A1)}$$

$$\frac{\partial \bar{\tau}_{12}}{\partial y_2} + \frac{\partial \bar{\tau}_{13}}{\partial y_3} = 0 \qquad \text{...(A2)}$$

$$\nabla^2 \bar{u}_1 = 0 \qquad \text{...(A3)}$$

$$\bar{\tau}_{13} = 0 \quad \text{on} \quad y_3 = 0 \ (-\infty < y_2 < \infty) \qquad \text{...(A4)}$$

$$\bar{\tau}_{13} \to 0 \quad \text{as} \quad y_3 \to \infty \ (-\infty < y_2 < \infty) \qquad \text{...(A5)}$$

$$\bar{\tau}_{12} \to \frac{\tau_\infty}{p} \quad \text{as} \quad |y_2| \to \infty \ (y_3 \geq 0) \qquad \text{...(A6)}$$

where p is the Laplace transform variable. Starting with a trial solution for $\bar{u}_1$ of the form,

$$\bar{u}_1 = A(p)\, y_2 + B(p)$$

where A, B are independent of (y_2, y_3), we easily find that the solution for $\bar{u}_1$, satisfying (A1)–(A6) is given by

$$\bar{u}_1 = \frac{(u_1)_0}{p} + \frac{\tau_\infty y_2}{\eta p^2}$$

So that, by (A1),

$$\bar{\tau}_{12} = \frac{(\tau_{12})_0}{\left(p + \dfrac{\mu}{\eta}\right)} + \tau_\infty \left(\frac{1}{p} - \frac{1}{p + \dfrac{\mu}{\eta}}\right) \qquad \text{(A7)}$$

and

$$\bar{\tau}_{13} = \frac{(\tau_{13})_0}{\left(p + \dfrac{\mu}{\eta}\right)}$$

Inverting the Laplace transform w.r. to t, (A7) gives *us* the solution (7).

DISPLACEMENTS AND STRESSES FOR $t > T_1$, AFTER FAULT CREEP ACROSS F_1 COMMENCES: METHOD OF DETERMINATION

To obtain solutions in this case for $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ we take Laplace transforms of (1a)–(7a) with respect to t_1, noting that $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ vanish for $t_1 \leqslant 0$. This gives a boundary value problem for $\overline{(u_1)_2}$, $\overline{(\tau_{12})_2}$, $\overline{(\tau_{13})_2}$ the Laplace transforms of $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ w.r. to t_1, characterised by the relations:

$$\left.\begin{aligned} \overline{(\tau_{12})_2} &= \bar{\mu}\frac{\partial(\bar{u}_1)_2}{\partial y_2} \\ \overline{(\tau_{13})_2} &= \bar{\mu}\frac{\partial(\bar{u}_1)_2}{\partial y_3} \end{aligned}\right\} \tag{A8}$$

$$\frac{\partial}{\partial y_2}\overline{(\tau_{12})_2} + \frac{\partial}{\partial y_3}\overline{(\tau_{13})_2} = 0 \tag{A9}$$

$$\nabla^2\overline{(u_1)_2} = 0 \tag{A10}$$

$$\overline{(\tau_{13})_2} = 0 \quad \text{on} \quad y_3 = 0(-\infty < y_2 < \infty) \tag{A11}$$

$$\overline{(\tau_{13})_2} \to 0 \quad \text{on} \quad y_3 \to \infty(-\infty < y_2 < \infty) \tag{A12}$$

$$\overline{(\tau_{12})_2} \to 0 \quad \text{as} \quad |\, y_2 \,| \to \infty(y_3 > 0) \tag{A13}$$

$$\overline{[(u_1)_2]} = \bar{u}_1(p) f_1(y_3)$$

$$\text{across} \quad F_1(y_2 = 0,\ 0 \leqslant y_3 \leqslant D_1) \tag{A14}$$

while $\overline{(\tau_{12})_2}$, $\overline{(\tau_{13})_2}$ are continuous across F_1.

Here

$$\bar{\mu} = \frac{p}{\dfrac{p}{\mu} + \dfrac{1}{\eta}} \tag{A15}$$

This boundary value problem for $\overline{(u_1)_2}$, $\overline{(\tau_{12})_2}$, $\overline{(\tau_{13})_2}$ can be solved by using a suitable modification of the Green's function technique developed by Maruyama (1966) and Rybicki (1971) for static dislocations in elastic media and it is easily shown that

$$\overline{(u_1)_2(Q)} = \int_{F_1} \overline{[(u_1)_2]}(P)\cdot G(P, Q)\cdot dX_3 \tag{A16}$$

where $Q(y_1, y_2, y_3)$ is any point in the model not lying on F_1, $P(x_1, x_2, x_3)$ is any point on F_1 and

$$[\overline{(u_1)_2}](P) = \bar{u}_1(p) f_1(x_3) \tag{A17}$$

is the discontinuity in $\overline{(u_1)_2}$ across F_1 at $P \cdot G(P, Q)$ is a Green's function, given in this case by

$$\left.\begin{aligned} G(P, Q) &= \bar{\mu}\frac{\partial}{\partial x_2} G_1(P, Q) \\ \text{where} \quad & \\ G_1(P, Q) &= \frac{1}{4\pi\bar{\mu}}[\log_e\{(x_2-y_2)^2 + (x_3-y_3)^2\} \\ &\quad + \log_e\{(x_2 + y_2)^2 + (x_3 + y_3)^2\}] \end{aligned}\right\} \tag{A18}$$

Noting that the integrand over F_1 in (A16) is over the range 0 to D_1 for x_3, we obtain $\overline{(u_1)_2}$ from (A16), $\overline{(\tau_{12})_2}$ and $\overline{(\tau_{13})_2}$ are then obtained from (A8). Finally, on inverting the Laplace transform with respect to t_1 and using the convolution theorem, we obtain $(u_1)_2$, $(\tau_{12})_2$, $(\tau_{13})_2$ in the form (12)–(17).

SUFFICIENT CONDITIONS FOR THE FINITENESS OF THE DISPLACEMENTS AND STRESSES

In considering the boundedness of displacements and stresses in the model for finite y_2, y_3, t, we note that, if the creep velocities are finite across F_1 and F_2 then $u_1(t_1) f_1(y_3)$ and $u_2(t_2) f_2(y_2')$, the relative creep displacements across F_1 and F_2, assumed in (8) and (25), will be finite. In this case, from the solutions for displacements and stresses valid for all $t > 0$, given by (33), (11), (12)—(14) and (30)—(32), we find that the displacements and stresses are finite for all finite (y_2, y_3, t) in the model (for $t \geqslant 0$), provided $(\psi_{11}, \phi_{11}, \phi_{21})$, given by (15)—(17), as well as $(\psi_{11}', \phi_{11}', \phi_{21}')$, derived from them, as explained earlier, are finite for all points (y_2, y_3, t) in the model.

Now $(\psi_{11}, \phi_{11}, \phi_{21})$ will be finite provided the integrals in (15)—(17) converge and are finite. From the properties of integrals, discussed by Carslaw (1950), it follows that this is ensured if $f_1(y_3)$ is continuous in $0 \leqslant y_3 \leqslant D_1$, as assumed in the first of the conditions (Cl) stated earlier, provided $y_2 \neq 0$ and (y_2, y_3) are finite, However, as $y_2 \to 0$ the integral in (15)—(17) are not always convergent or finite, and limit $y_2 \to 0$ and the integral over x_3 are not necessarily interchangeable. To study the behaviour of these integrals as $y_2 \to 0$, we assume that $f_1(y_3)$ satisfies the first three conditions in (Cl).

Then, for $y_2 \neq 0$, integration by parts is justified for the integrals in (15)—(17), and integration by parts of the resulting integrals is also justified.

Assuming the validity of the conditions (Cl), integrating by parts once for ψ_{11} and twice for ϕ_{11}, ϕ_{21}, and simplifying we have, from (15)–(17), for $y_2 \neq 0$,

$$\frac{\psi_{11}(y_2, y_3)}{K_1} = f_1(D_1)\left[\tan^{-1}\left(\frac{D_1-y_3}{y_2}\right) + \tan^{-1}\left(\frac{D_1+y_3}{y_2}\right)\right]$$

$$-\int_{-y_3}^{D_1-y_3} f_1'(z+y_3)\tan^{-1}(z/y_2)\,dz$$

$$-\int_{y_3}^{D_1+y_3} f_1'(z-y_3)\tan^{-1}(z/y^2)\,dz$$

and

$$\frac{\phi_{11}(y_2, y_3)}{K_2} = -f_1(D_1)\left[\frac{D_1-y_3}{y_2^2+(D_1-y_3)^2} + \frac{D_1+y_3}{y_2^2+(D_1+y_3)^2}\right]$$

$$+\frac{f_1'(D_1)}{2}[\log_e\{y_2^2+(D_1-y_3)^2\} + \log_e\{y_2^2+(D_1+y_3)^2\}]$$

$$-f_1'(0)[\log_e(y_2^2+y_3^2)]$$

$$-\tfrac{1}{2}\int_{-y_3}^{D_1-y_3} f_1''(z+y_3)\log_e(z^2+y_2^2)\,dz$$

$$-\tfrac{1}{2}\int_{y_3}^{D_1+y_3} f_1''(z-y_3)\log_e(z^2+y_2^2)\,dz$$

where K_1, K_2 are finite non-zero constants and primes in $f_1'(z+y_3)$, $f_1''(z+y_3)$ etc. denote differentiation with respect to the argument.

For ϕ_{21}, we get a similar expression on applying the last condition (iv) in (Cl), viz., $f_1(D_1) = 0, f_1'(D_1) = 0 = f_1'(0)$, the expressions for ψ_{11}, ϕ_{11}, ϕ_{21} reduce to the following form for $y_2 \neq 0$:

$$\frac{\psi_{11}(y_2, y_3)}{K_3} = \int_{-y_3}^{D_1-y_3} f_1'(z+y_3)\tan^{-1}(z/y_2)\,dz$$

$$+\int_{y_3}^{D_1+y_3} f_1'(z-y_3)\tan^{-1}(z/y_2)\,dz \qquad \text{(A19)}$$

$$\frac{\phi_{11}(y_2, y_3)}{K_4} = \int_{-y_3}^{D_1-y_3} f_1''(z+y_3)\log_e(z^2+y_2^2)\,dz$$

$$-\int_{y_3}^{D_1+y_3} f_1''(z-y_3)\log_e(z^2+y_2^2)\,dz \qquad \text{(A20)}$$

$$\frac{\phi_{31}(y_2, y_3)}{K_5} \int_{-y_3}^{D_1-y_3} f_1''(z + y_3) \tan^{-1}(z/y_2)\, dz$$

$$+ \int_{y_3}^{D_1+y_3} f_1''(z + y_3) \tan^{-1}(z/y_2)\, dz \qquad \text{(A21)}$$

where K_3, K_4, K_5 are finite non-zero constants. Using the conditions (Cl) satisfied by $f_1(y_3)$ and the properties of integrals [Carslaw (1950)], we find that the integrals in (A19)–(A21) and converge and are finite for $y_2 \neq 0$, and they remain bounded as $y_2 \to 0$. Moreover the limit $y_2 \to 0$ and the integrals with respect to z in (A19)–(A21) are interchangeable if (Cl) are satisfied. Thus, if $f_1(y_3)$ satisfies the conditions (Cl), ψ_{11}, ϕ_{11}, ϕ_{31} remains finite for all points (y_2, y_3, t), including the neighbourhood of the fault F_1 ($y_2 \to 0$, $0 \leqslant y_3 \leqslant D_1$).

Similarly ψ_{11}', ϕ_{11}', ϕ_{31}' remain finite for all points (y_2, y_3, t), including the neighbourhood of F_2 ($y_2 \to D$, $0 \leqslant y_3 \leqslant D_2$) provided satisfies four conditions exactly similar to (Cl), with y_2, y_3, D_1 replaced by y_2', y_3', D_2.

Hence, finally, we conclude that the displacements and stresses are finite for all finite (y_2, y_3, t), including the neighbourhood of F_2 and F_1 (where y_2 and $y_2' \to 0$) provided $f_1(y_3)$ satisfies the four conditions in (Cl) and $f_2(y_3')$ satisfies exactly similar conditions, with (y_2, y_3, D_1) replaced by (y_2', y_3', D_2).