Paper No. 204, Bull. ISET, Vol. 17 No. 2, June 1980 pp. 5–10

ON ORIENATIONS FOR LINEAR AND CROSS-SHAPED SEISMIC ARRAYS[1]

J. R. Arora[2]

Introduction

Since early sixties, a wealth of literature has appeared on the subject of seismic arrays, but most of the work is devoted, essentially to the processing of seismic array data. Denham (1963) dealt with the problem of construction of small arrays for refraction shooting. Birtill and whiteway (1965), among others, considered the problem of array design relying primarily on empirical consideration regarding the signals and the noise. Burg (1964), Capon (1969) and some others considered the array design problem mathematically and derived their conclusions on the basis of frequency wavenumber spectra of the expected signals and the noise.

Hartenberger and Van Nostrand (1972) point out the fact that comparatively lesser attention has been given to, what they call, the rational design of a seismic array. They demonstrate the way in which noise and signal characteristics impose limits on an array and show clearly that not all the sensors of LASA are really essential. Arora (1970) considers the problem of array design mathematically in spece domain rather than in wavenumber domain for immediate interpretation of the results. Basing his considerations on the form of noise correlation as a decreasing function of the sensor separation, he tries to obtain the optimum number of geophones in linear seismic array of fixed length and obtains results similar to those of Hartenberger and Van Nostrand (1972). Since he takes the noise to be isotropic, the directionality of the array does not enter into his consideration. The ambient noise, however, as it is well known, has in many cases directional properties and such as pointed out by Hartenberger and Van Nostrand, among others, the design of a seismic array is a function of the direction and of the velocity of noise in the band pass of the signals required to be detected or extracted. Here we shall be concerned with the effect of the directional properties of the noise on the orientations of linear and cross-shaped arrays and shall try to obtain optimal orientations for these types of arrays as function of the noise correlation.

The noise is taken to be space-time stationary. Also suppose that the correlation of the noise samples from two sensors is a function of the direction of the line joining the sensor in addition to its being dependent upon their separation. First an M element non-linear array is considered, results about the directionality of a linear array is derived and then these results are extended to a cross-shaped array.

Noise-to-signal-ratio of a seismic array

Let $\vec{r}_j$, $1 \leqslant j \leqslant M$, denote the position vector of the jth sensor P_j of an M element array. Supposing that the array data processing consists of beam forming, the recording of the jth

1. Paper No. 15 presented at Kurukshetra Symposium.
2. Department of Mathematics, Kurukshetra University, Kurukshetra (Haryana) – 132119

element can be written as the sum

$$s(t) + n(t, \vec{r}_j) \tag{1}$$

of the signal component s(t) and noise component $n(t, \vec{r}_j)$ and it is supposed that through proper phase shifting the signal components at all the geophones are in phase. Let s(t) and $n(t, \vec{r}_j)$ be orthogonal, so that the (power) signal-to noise-ratio can be written as

$$1/SNR = D/M^2S \quad = NSR, \tag{2}$$

where S denotes the signal power for each of the sensors and may be defined in the manner it has been done, among others, by the author (1970)

$$D = \overline{\left[\sum_{j=1}^{M} n(t, \vec{r}_j) \right]^2}$$

$$= \sum_{i,j=1}^{M} \overline{n(t, \vec{r}_i) \, n(t, \vec{r}_j)} \tag{3}$$

bar denoting the statistical averaging. Now suppose that the noise is stationary ergodic process, as is usually the case in respect of ambient seismic noise so that we may write

$$P_{ij} = \overline{n(t, r_i) \, n(t, r_j)} = f(r_{ij}, a_{ij}) \; i \neq j; \; g_0, \text{ a constant, } i = j \tag{4}$$

where $r_{ij} = | r_i - r_j | = r_{ji}$, denotes the distance P_i P_j and a_{ij} denotes the angle which the join of P_i to P_j makes with some fixed direction. It may be noted that P_{ij} denotes the correlation of the noise traces from P_i and P_j, and hence $P_{ij} = P_{ji}$.

For the sake of simplicity, now assume

$$f(x,b) = g(x) \; h(b) \tag{5}$$

$$\text{Therefore, } h(a_{ij}) = h(a_{ji}) \tag{6}$$

h(a) is a periodic function of 'a' with period π, and hence has the Fourier series consisting of cosine terms only i.e.

$$h(a) = c_0/2 + c_2 \cos 2a + c_4 \cos 4a + \ldots \tag{7}$$

If h(a) be approximated by its first (m + 1) terms,

$$\text{Then} \quad h(a) \simeq \bar{h}(a); \tag{8}$$

$$\text{Where} \quad \bar{h}(a) = c_0/2 + c_2 \cos 2a + c_4 \cos 4a + \ldots + c_{2m} \cos (2ma) \tag{9}$$

From elementary Trigonomentry,

$$\bar{h}(a) = A_0 + A_2 \cos^2 a + A_4 \cos^4 a + \ldots + A_{2m} \cos^{2m} a \tag{10}$$

This involves only even powers of cosines thus enabling to carry on the analysis in terms of coefficients A_i's.

In practical situations, h(a) is not given. The set of triplets (a_i, b_j, f_{ij}) from the observational data are known, where f_{ij} denotes the value of the correlation between a pair of geophones distant a_i and inclined at an angle b_j to the fixed direction. Thus the surface

$$\begin{aligned} f &= f(a,b) \\ &= g(a)\ h(b), \{\text{from (7)}\} \end{aligned} \tag{11}$$

has to be determined which gives the best fit, in some sense, to the set of triplets (a_i, b_j, f_{ij}) (Schumaker, 1976).

Noise-to-signal-ratio of a linear array

It is evident that for a linear array a_{ij} for all i, j with i<j (i>j) is the same and, therefore,

$$a_{ij} = \begin{cases} a, & i<j, \\ +\,a, & i>j, \end{cases} \tag{12}$$

and hence from (3), (4), (5) and (6),

$$D = M\,g_0 + h(a) \sum_{i,j=1}^{M} g(r_{ij}) \tag{13}$$

In case of a uniform array of fixed length R having distance r between the adjacent geophones,

$$\left|\vec{r}_i - \vec{r}_j\right| = \left|i-j\right| r = \left|i-j\right| \quad R/(M-1)$$

Therefore from (4) and (13),

$$D = M\,g_0 + N\,h(a), \tag{14}$$

$$N = 2 \sum_{p=1}^{M} (M-p)\,g_p \tag{15}$$

From (2), (14) and (15) it is found that SNR (or NSR) of a linear array depends upon: (a) the length R of the array, (b) the number M of the geophones in it, (c) the functional dependence g(y) of the noise correlation on the geophone separation y, (d) the nature of the expected signals, (e) the behaviour h(a), a factor in the noise correlation with the direction a of the array, which we shall call the direction factor (of the array). Except (e), which is being discussed here, the other points have been covered by the author in his work of 1970.

Determination of the best direction for a linear array

As is known that D stands for the noise power of the array and g_0 for the noise power of each of the geophone of the array & using the fact that SNR of an M element array cannot exceed M, it is found from (2), (3) and (14) that either h(a) and N are both positive or both negative. By definition g(a) is positive from all a, and hence from (10) and (14)

$$A_0 > 0,\ A_0 + A_2 + \dots + A_{2m} > 0 \tag{16}$$

Remembering that we are concerned with the direction factor only, the maximum value of SNR (or the minimum value of the NSR), will be determined by the value of minimising h(a) Differentiating h(a) twice with respect to a. We see from (9) that the best direction for the array will be given by

$$\sum_{r=1}^{m} r\, c_{2r} \sin (2ra) = 0$$

$$\sum_{r=1}^{m} r^2 \cos (2ra) > 0 \tag{17}$$

It may be noted from (6) that the period of h(a) is equal to $\pi/2$ and from (7) it is sufficient to consider

$$a \in [0, \pi/2] \tag{18}$$

For want of observational data regarding noise correlation, c_i's can not be determined. Instead of (17), therefore, (10) is easier to analyse and is being considered. The folliwng two cases which, of course satisfy (16) are given below :

(i) All the A_i's are non-negative.

Here h(a) is a monotonically decreasing function of a [from (10) & (18)] and hence the best direction for the array is given by

$$a = \pi/2 \tag{19}$$

This special case corresponds to a hypothetical model of the ambient noise in a coastal region where the sea waves are supposed to generate (noise) wave motion with wavefronts parallel to the coast. This clearly means that the noise correlation would be maximum for an array parallel to the coast. Evidently, therefore, the best direction for the array would be the perpendicular direction to the coast, as given by (19). This noise model is clearly too simple to be of any practical significance. In general, there are more than one such directions which correspond to the minimum value (s) of the direction factor.

A simple illustration of the fact that there can be more than one preferred directions which minimise the direction factor is given by the following special case ;

(ii) One of the A_i's $(i > 0)$ is $-$ve, (16) is satisfied.

Consider the direction factor given by

$$1 - \tfrac{1}{2} \cos^2 a + \tfrac{1}{4} \cos^4 a$$

It is easy to see that there are two preferred directions given by $a = 0$ and $a = \pi/6$. However it can be immediately seen that

$$a = \pi/6 \tag{20}$$

gives the best direction for the array.

Determination of the best direction for a cross-shaped array

Consider a cross array having M elements in each of the two arms of the array. From (2), (3) and (14), the sum of the noise-to-signal-ratio for the two arms can be written as

$$\frac{2M\, g_0 + N\, [h\,(a) + h\,(a + \pi/2)]}{M^2\, S} \tag{21}$$

so that the direction factor for such an array can be written as $F(a) = \bar{h}(a) + \bar{h}\,(a + \pi/2)$

$$= c_0 + c_4 \cos 4\,a + c_8 \cos 8\,a + \ldots + c_{4p} \cos (4\,pa) \text{ from (9)} \tag{22}$$

$$= (A_0 + A_2) + A_4 (\cos^4 a + \sin^4 a) + A_6 (\cos^6 a) + \dots A_{2p} (\cos^{4p} a + \sin^{4p} a) \quad (23)$$

where $p = [m/2]$ denotes the greatest integer not greater than m/2. From (22), we see that

$$\sum_{r=1}^{p} r\, c_{4r} \sin (4ra) = 0 \quad (24)$$

and

$$\sum_{r=1}^{p} r^2 c_{4r} \cos (4ra) > 0$$

will determine the value of 'a' which minimises the direction factor. However due to the lack of knowledge of c_i's for want of data regarding noise correlations, instead of analysing (24) two special cases from (23) are considered :

(1) All the A_i's are non-negative

From (23), in the interval $[0, \pi/4]$ the direction factor is monotonically increasing and in the remaining subinterval $\pi/4$, $\pi/2$ of the interval of length $\pi/2$, which is equal to the period of the direction factor, it is monotonically decreasing. Thus

$$a = 0, \quad a = \pi/2 \quad (25)$$

minimise the direction factor. It may, however, be noted that in so far as the directionality is concerned, (25) refer to the same cross-shaped array.

(2) $A_i = 0, \quad i = 4, 6, \dots \dots, p$ (26)

Equation (26) implies that we have approximated h(a) by (9) with $m = 1$.

From (23), in this case the direction factor is independent of a. Thus in case (26) holds, an array oriented in one direction is as good as an array oriented in another direction.

Orientations for the array when there are two or more preferred directions

While considering special case (i) in respect of the determination of the best direction for linear array it is noted that, h(a) was found to be monotonically decreasing function of a. on the interval $[0, \pi/2]$ equal to the period of h(a). Consequently there was found to be one preferred direction for the array and the period of h(a) is equal to π. Now consider the behaviour of h(a) in an interval I of the length π. In this case h(a) will be monotonically decreasing function on some sub-interval of I and monotonically increasing on the remaining sub-interval (s) of I. Thus in general, we shall get more than one preferred directions for a linear array. As an illustration let us suppose that the direction factor for a linear array is given by

$$\begin{aligned} H(a) = A_0 + A_2 \cos^2 a + A_4 \cos^4 a + \dots A_{2m} \cos^{2m} a \\ + B_0 + B_2 \cos^2 (a+b) + B_4 \cos^4 (a+b) + \dots B_{2m} \cos^{2m} (a+b) \end{aligned} \quad (27)$$

where b is a constant given by $| b | < \pi/2$ and A_i's and B_i's are non-negative constants. From the foregoing it should be clear that $a = \pi/2$ and $a = \pi/2 - b$ are two of the preferred direction in this case. Substituting for the power of cos a) and of cos (a+b) in terms of sines and cosines of multiples of 'a' and of (a+b) and simplifying, (27) can be written as

$$\begin{aligned} H(a) = C_0/2 + C_2 \cos 2a + C_4 \cos 4a + \dots + C_{2m} \cos (2ma) \\ + D_2 \sin 2a + D_4 \sin 4a + \dots + D_{2m} \sin (2ma), \end{aligned} \quad (28)$$

where C_i's and D_i's are determined by A_i's and B_i's. That the direction factor for a linear array in the general case, is given by (28) should not be surprising.

From (28), the direction factor for a cross shaped array is given by,

$$H(a) + H(a + \pi/2) = C_0 + C_4 \cos 4a + C_8 \cos 8a + \ldots + C_{4p} \cos(4pa)$$
$$+ D_4 \sin 4a + D_8 \sin 8a + \ldots + D_{4p} \sin(4pa), \qquad (29)$$

Thus for the general case, the best orientations for the linear and cross shaped arrays are to be determined from (28) and (29) respectively by minimising these expressions. The coefficients C_i's and D_i's are to be determined from the observational data as already indicated.

Concluding remarks

From what has been said in the foregoing, it should become clear that at the sites where noise is suspected to possess directional properties, it might be fruitful to explore the behaviour of the noise correlation with different directions and to make the consequential choice for the orientation of the array. Since the direction factor of a cross shaped array involves higher powers of the trigonometric ratios than the direction factor for a linear array, it is evident that for a cross shaped array, the array orientation is not as important as for a linear array. In fact from special case (2) we see that in certain cases the array orientation might not matter at all for a cross array.

From special case (i), it should be clear that due to the presence of signal-generated noise, the directions from which signal are more frequently expected might be preferred directions, an example being the direction of the (known) sites of nuclear explosions. In such cases, however the correlation of the signal-generated noise has to be taken account of. It is evident that the behaviour of this type of noise cannot be determined by the usual methods and hence some other method for finding the functional between the noise correlation and direction has to be found.

Acknowledgement

The author expresses his thanks to ISET for help in preparation of the paper.

References

Arora, J.R. (1970), Unpublished Ph.D. Thesis, Kurukshetra University, Kurukshetra.

Birtill, J.W. and Whiteway, F.E. (1965), 'The application of phased arrays to the analysis of seismic body waves', Phil. Trans. Roy. Soc. London. Series A. Vol. 258, p. 421-493.

Burg. J.P. (1964), 'Three dimensional filtering with array of seismometers,' Geophysics, Vol. 29, p. 693-713.

Capon, J. (1969), 'High resolution frequency wavenumber spectrum analysis', Proc. IEEE. Vol. 56 p. 1408-1418.

Denham, D. (1963), 'Use of group of geophones to improve signal-noise-ratio of first arrival in refraction shooting', Geo. Pros. Vol. 11, p. 389-408.

Hartenberger, R.A. and Van Nostrand, R.G. (1972), 'Influence of the number and spacing of sensors on the effectiveness of a seismic array', Geo. Pros. Vol. 20, p. 771-784.

Schumaker, L.L. (1976), 'Fitting surfaces to scattered data' (in) Lorentz, G.G. (edted), Approximation Theory Vol. II, p. 203, Academic Press, New York.