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PROBABILISTIC EARTHQUAKE PREDICTION AND ENGINEERING SEISMIC RISK

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ABSTRACT

Computation of engineering seismic risk for a project site in the form of "uniform risk" design response spectra is based on the knowledge of total seismicity expected to occur in the project area, where the seismicity is generally defined from the available past data on earthquake occurrence in the area (Gupta, 1991). This paper presents a formulation to evaluate the effect on these spectra of a probabilistic earthquake prediction in the vicinity of the project. A brief review on the strategy of probabilistic prediction using the data on precursory parameters is presented first. Then, assuming a hypothetical but physically plausible earthquake prediction with different reliabilities, example results are computed to illustrate the modifications in the design spectra evaluated for a typical site in northeast India from past earthquake data only. The amplitudes of the design spectra with lower risk levels are found to increase appreciably for higher reliability values of a probabilistic prediction.

INTRODUCTION

A specific earthquake prediction aims at forecasting precisely the magnitude, location and time of occurrence of the predicted earthquake. But, due to lack of an exact understanding of the earthquake generating forces at great depths inside the earth, such predictions are mostly met with failures and they are far from reality at present. An earthquake prediction is generally based on observing the anomalous behaviour of certain geophysical and geodetic parameters, known as earthquake precursors. Changes in seismic wave-velocities, change in the magnetic, electric and gravitational fields of the Earth, temporal behaviour of b-value, increased radon emission, anomalous uplift or tilt of the ground, abnormal animal behaviour, and the change of water levels in shallow wells are the most commonly monitored precursory parameters (Jachens, 1983; Zhao et al., 1984; Teng, 1980; Zheng, 1981; Gupta and Pagare, 1986; Rikitake, 1976; etc.). However, past experience tells that different earthquakes are not always preceded by exactly the same set of precursors and that the behaviour of the precursors may even be misleading. Thus the task of earthquake prediction is rendered very difficult and large uncertainties are generally associated with a prediction. Therefore, several investigators (e.g. Cao and Aki, 1983; Vere-Jones, 1978; Yamashina, 1981; Hamada, 1984; Grandori et al., 1984; Utsu, 1983; Ferraes, 1985; Collins, 1977; Anderson, 1982; etc.) have proposed to use all the available information to predict an earthquake in a probabilistic way, rather than making a specific prediction.

A probabilistic prediction is defined as the probability of occurrence of an earthquake within specified magnitude, space and time intervals for the given precursory observations which are generally used for a specific prediction. The probability of occurrence of an earthquake estimated from past earthquakes only is usually found to be very small. This probability may be enhanced substantially on the basis of observing certain earthquake precursors. As the enhanced probability reaches nearer one, a probabilistic prediction reaches close to a specific prediction.

This paper presents a formulation to estimate the effect of a probabilistic prediction on the estimation of engineering seismic risk for a project site from the description of seismicity from historical data only. Due to various discrepancies and shortcomings in using the standard spectral shapes (Housner, 1959; Seed et al., 1978; etc.) or the spectra of an actual accelerogram, the engineering seismic risk is specified in terms of 'Uniform Risk Spectra' (Anderson and Trifunac, 1978; Gupta and Ramkrishna, 1986; Lee and Trifunac, 1985; Gupta, 1991; etc.). Example results are presented to illustrate the modifications in the uniform risk response spectra for a typical site in northeast India due to a hypothetical earthquake prediction with different reliabilities. Thus the presented formulation can be used to analyse the design seismic risk by giving due weightage to a prediction. This in turn may be helpful to mitigate the seismic hazards by earthquake resistant design of structures and by strengthening the existing structures.

PROBABILITY OF EARTHQUAKE PREDICTION

Past data on earthquake occurrence in a limited geographical area are generally used to evaluate the probabilities of occurrence of earthquakes of different magnitudes within a specified time interval. For this purpose, a suitable probability function is fitted to the available data assuming that the future earthquakes will also obey this distribution. Though this assumption is not strictly valid due to nonstationary and aperiodic nature of the tectonic stress cycles in a region, it is a reasonably good approximation due to the fact that the time intervals of interest in engineering applications are much smaller than the time periods of tectonic stress cycles. It may be noted that the probabilities obtained from an analysis of past data only gives an idea about the expectancy of earthquakes in the sense of a very long term statistical average, and it cannot be considered as an earthquake prediction. In a statistical earthquake prediction this probability is updated from the observations on earthquake precursors.

Extreme magnitude distributions are often used to estimate the probability of an earthquake in an area of interest during a specified time interval. Gumbel's Type-I and Type-III distributions have been used by several investigators (e.g., Al-Abbasi and Fahmi, 1985; Burton, 1979; Goswami and Sarmah, 1982; etc.) to study the probabilities of earthquake occurrence in different parts of the World. However, Gupta et al. (1988) found that the Triple-Exponential Distribution (Gan and Tung, 1983) gives more realistic earthquake magnitudes for different return periods in the northwest Himalayan region. This distribution is defined as follows,

$$P(M|Y) = \exp[-\lambda Y \exp(-\exp(\alpha + \beta M))] \tag{1}$$

where $P(M|Y)$ is the probability of magnitude M during Y years since the past occurrence of an earthquake of this magnitude, and λ, α and β are the parameters of the distribution. Using earthquake data for the period 1925-1980, Gupta et al. (1988) have obtained the following least square estimate of this distribution for the northwest Himalayan region.

$$P(M|Y) = \exp[-8.0814Y \exp(-\exp(-4.7231 + 0.9451M))] \tag{2}$$

In absence of the predictive information, this relation is used to find the probabilities of occurrence of different magnitudes during a given time interval.

Let P(E) be the probability of occurrence of an earthquake event E based on the historical data. Then the updated probability $P(E|I)$ for a given predictive information I may be obtained from the Bayes' theorem as

$$P(E|I) = \frac{P(I|E)\,P(E)}{P(I|E)\,P(E) + P(I|\underline{E})\,P(\underline{E})} \tag{3}$$

In this expression, $P(I|E)$ is the probability of getting the precursory information I for event E; i.e., it is the reliability of I, and $P(I|\underline{E})$ is the probability of not occurrence of event E even after getting the precursory information I; i.e., it is the probability of false prediction. The probability $P(\underline{E})$ is given by

$$P(\underline{E}) = 1 - P(E) \tag{4}$$

To illustrate the efficiency of precursory information I in enhancing the probability of occurrence of an event E, in the present study, the updated probability $P(E|I)$ is evaluated for different values of the reliability, $P(I|E)$, of I. Figure 1 shows typical example results on the comparison between the historical probability P(E) of a magnitude 7.0 earthquake as obtained from Eqn. (2) and the updated probabilities $P(E|I)$ for reliability $P(I|E) = .50$, .75 and .90. For all the reliability values, the probability, $P(I|\underline{E})$, of false prediction is assumed to be very small equal to 0.1. From the results in Fig. 1 it is evident that a precursory information I with high value of reliability and with very low rate of false prediction gives great enhancement in the probability of prediction to reach nearer a specific prediction. The following section identifies the conditions on a set of precursors to achieve a high value of their combined reliability.

SELECTION OF EFFECTIVE PRECURSORS

The reliability of a single precursor to predict an earthquake is usually very low. Therefore, the precursory information generally consists of observing simultaneously the anomalous behaviour of several precursors. For n independent precursory informations $I_1, I_2, \ldots, I_n$ with reliabilities $p_1, p_2, \ldots, p_n$; respectively, the total reliability is given by

$$P(I|E) = P(I_1, I_2 \ldots, I_n|E) = 1 - \prod_{i=1}^{n} (1 - p_i) \tag{5}$$

Thus, the combined reliability of a large number of precursors can be increased substantially even if the reliability of each one of them is very small.

The updated probability of event E for a set of n precursors can be written as

$$P(E|I_1, I_2, \ldots, I_n) = \frac{P(I_1, I_2, \ldots, I_n|\,E)\,P(E)}{P(I_i, I_2, \ldots, I_n|E)\,P(E) + P(I_1, I_2, \ldots, I\,|\,\underline{E})\,P(\underline{E})} \tag{6}$$

If one could select a set of precursors which almost always show some anomalous behaviour before an earthquake,

$$P(I_1, I_2, \ldots, I_n|\,E) \simeq 1 \tag{7}$$

Also, for mutually independent precursors, one can write

$$P(I_1, I_2, \ldots, I_n | \underline{E}) = P(I_1) P(I_2) \ldots P(I_n), \tag{8}$$

where $P(I_i)$ is the reliability (probability of getting before an earthquake) of i-th precursory information,

$$P(I_i) = \frac{\text{No. of occurrence of } I_i \times \text{Precursor time}}{\text{Total time of observation}} \tag{9}$$

Since $P(E)$ is generally very small, $P(\underline{E}) \approx 1$. Thus, Eqn. (6) gives

$$P(E| I_1, I_2, \ldots, I_n) \approx \frac{P(E)}{P(E) + P(I_1) P(I_2) \ldots\ldots P(I_n)} \tag{10}$$

From Eqn. (10) it is clear that the updated probability of E can be made very high by having a small value of the product of the probabilities $P(I_i)$. To get a small product one should select the precursors with very short precursory times or which are very rarely observed. Thus, it may be concluded that for getting higher probabilities of prediction, one should select mutually independent precursors with very low probabilities of occurrence and with very low failure rates.

ENGINEERING SEISMIC RISK USING PREDICTION

Seismic risk for earthquake resistant design of a structure is defined as the probability of experiencing a specified level of earthquake ground motion during an estimated economic life of the structure. For this purpose, the ground motion is commonly specified in terms of the design response spectra, because the spectra superposition method (Goodman et al., 1958; Gupta and Trifunac, 1987; Wilson et al., 1981) provides a simple and efficient way to analyse the seismic response of structures. Though the normalized standard spectral shapes (Housner, 1959; Seed et al., 1978; etc.) are used to define the design spectrum in many applications, several discrepancies and shortcomings are associated with their use (Anderson and Trifunac, 1978; Der-Kiureghian and Ang., 1977). Therefore, it is recommended that 'Uniform Risk Response Spectra', which are evaluated using a detailed description of the seismicity of a project area of interest and the frequency dependent attenuation relations for the spectral amplitudes should be used for design purposes. The definition and the method of computing such spectra is described in detail at several places (Anderson and Trifunac, 1978; Gupta and ramkrishna, 1986; Lee and Trifunac, 1985) and it will not be repeated here. The computation of uniform risk spectra takes into consideration the random nature of earthquake magnitude, location and time of occurrence and hence, these spectra provide a more realistic design basis compared to the spectra evaluated using a fixed design earthquake magnitude and distance. The results of a single design earthquake are very sensitive to the magnitude and epicentral distance of the earthquake, which generally have large uncertainties and biases associated with them.

Let n_{ij} be the mean number of earthquakes per year with magnitudes in a small interval around magnitude M_i expected to occur in an area element at a distance R_j from a project site. Then the probability distribution of the response spectrum amplitudes, RSA(T), for a wave-period T can be defined as (Gupta and Ramkrishna, 1986; Gupta, 1991).

$$P[RSA(T)] = \exp[-Y \sum_{i,j} q_{ij}\, n_{ij}] \tag{11}$$

The summations i and j in this equation are taken respectively over all the area elements into which the area of the project is divided and over all the magnitude intervals into which the interval between the minimum and the maximum magnitudes is partitioned to cover the entire magnitude range. Y is the life time in years for which the design spectra are to be evaluated and q_{ij} is the probability of occurrence of RSA(T) from an earthquake of magnitude M_j at an epicentral distance R_i. Values of q_{ij} for different M_j and R_i can be obtained from the probabilistic empirical scaling relations similar to those given by Trifunac and Lee (1979), as described in detail by Gupta (1991). The occurrence rates n_{ij} of earthquakes of different magnitudes are obtained from the past data by using a magnitude-frequency distribution law (e.g. Gutenberg and Richter, 1956). From the knowledge of n_{ij} and q_{ij}, the distribution function of Eqn. (11) is computed for several different wave-periods, using which the spectral amplitudes at all the periods are evaluated for a constant level of risk (probability of exceedance) to obtain a uniform risk design spectrum (Anderson and Trifunac, 1978). Thus, the uniform risk spectra are generally computed from a description of seismicity from the past data on earthquake occurrences. However, if a probabilistic prediction is made in the project area, it may be used as described below to update the design spectra evaluated from historical data only.

Let $P(E|I)$ be the probability of occurrence of an earthquake event E for a given precursory information I in a project area and let $P[RSA(T)|E]$ be the probability of having the spectral amplitude RSA(T) at the project site from event E. If $P(RSA(T)]$ is the probability of not exceeding the amplitude RSA(T) from the historical seismicity as evaluated using Eqn. (11), the total probability, $P_t[RSA(T)]$, of not exceeding RSA(T) from both the historical seismicity and the predicted earthquake can be obtained as follows :

$P_t[RSA(T)]$ =

Prob. of not exceeding RSA(T) from historical seismicity
x Prob. of not exceeding RSA(T) from predicted earthquake (12)

The second probability on the right hand side is given by

1 - Prob. of occurrence of RSA(T) from the predicted earthquake
x Prob. of occurrence of the predicted earthquake

$$= 1 - P[RSA(T)|\, E]\, P(E|I)$$

Thus, Eqn. 912) gives

$$P_t[RSA(T)] = P[RSA(T)]\,(1 - P[RSA(T)|\, E]\, P(E|I)) \tag{13}$$

This probability distribution function for the response spectral amplitudes RSA(T) at different periods can be used to obtain the uniform risk design spectra due to historical seismicity as well as an earthquake prediction in the project area of interest.

SAMPLE RESULTS

The ideas presented above are applied to compute the example response spectra with different confidence levels for a typical site in the highly seismic northeast Indian region. The location of the site and the distribution of epicenters of past earthquakes are shown in Fig. 2. First, the probability distribution functions of the spectral amplitude at different wave-periods are computed from Eqn. (11) for a life period of 100 years, by describing the expected seismicity in the area around the site on the basis of available past data only. The seismicity is defined in terms of mean annual number of earthquakes, n_{ij}, expected to occur in the ith source element of size 0.5^o Lat x 0.5^o Long and magnitude interval ($M_j - \delta M_j$, $M_j + \delta M_j$), with M_j = 4.1, 4.5, 4.9, 5.3, 5.7, 6.1, 6.5, 6.9, 7.3, 7.7, 8.1 and 8.5, and δM_j equal to 0.2 for all j. The distance R_i refers to the distance between the site and the center of the ith source element. Using the probability distribution P[RSA(T)] computed as above, uniform risk spectra are constructed for a damping value of 5% and confidence levels equal to .10, .50 and .90. Such spectra computed using historical data only generally form the basis of engineering design.

Now, if an earthquake prediction is made in the project area with a specific reliability value, the expression of Eqn. (13) can be used to update the spectra computed above. Such updated spectra have been computed for the purpose of illustration for a hypothetical prediction of magnitude 7.0 earthquake with a uniform probability of occurrence everywhere within 50 km of the example site. The results have been obtained for three different reliabilities of prediction, P(E |I), equal to 0.1, 0.5 and 0.9. Figures 3a to 3c show the comparisons of these spectra with the corresponding spectra based only on the historical seismicity data. It is seen that a probabilistic prediction with high reliability may significantly increase the amplitudes of the design spectra with higher confidence levels. But the rate of increase in the present example is found to be rather slow for P(E I) greater than 0.5.

CONCLUSIONS

Because a specific earthquake prediction is generally met with a failure, it may be more rational to use all the precursory information to get the probability of occurrence of the predicted earthquake within specified magnitude, space and time intervals. The reliability of a statistical prediction can be increased by selecting the precursors such that they have very low failure rates; that they are mutually independent, i.e. they have distinctly different precursor times; and that their probabilities of occurrence are very low, i.e. they have very short precursor times.

A formulation has bee presented in this paper to utilise a probabilistic earthquake prediction to update the design response spectra derived from a description of seismicity based on historical data only. Sample results computed for the purpose of illustration show that a prediction may significantly enhance the amplitudes of the design spectra with low levels of risk (higher confidence levels). Because a prediction is generally made on a very short term basis compared to the life of a project, it is not possible to consider its effect in the estimation of design seismic risk at the initial stage. The present formulation, however, provides a way to update the design spectra in the light of a probabilistic prediction. This may be very useful to ensure the safety of structures and to mitigate the possible hazards by strengthening, if necessary.

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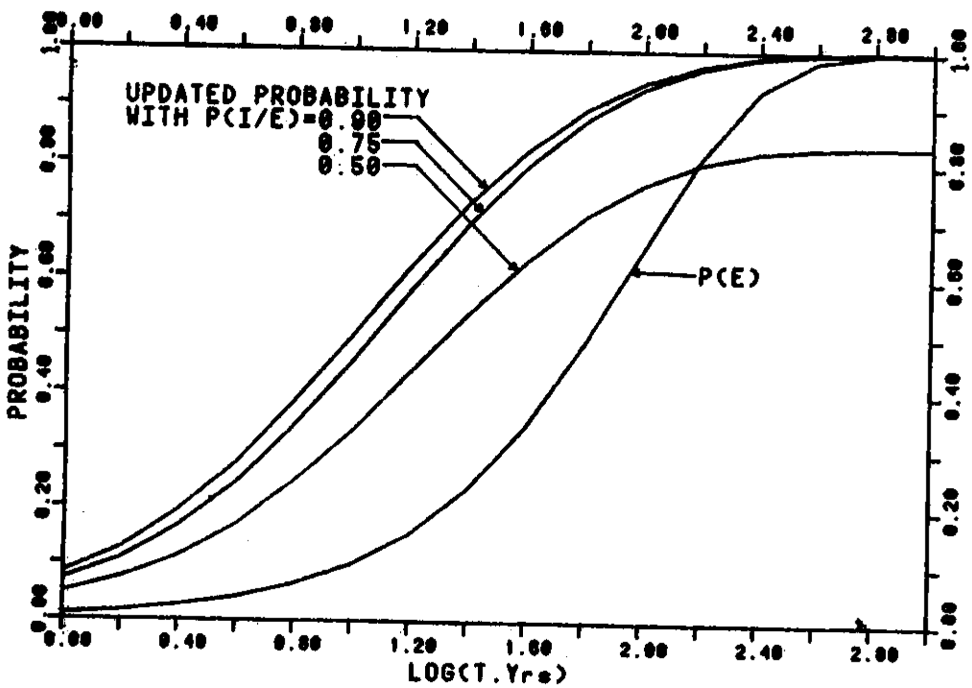

Fig. 1 : Comparison of the probability of occurrence, P(E), of a magnitude 7.0 earthquake as evaluated from historical data with the updated probability, P(E|I), for different values of the reliability, P(I|E), of the precursory information I. The probability P(I|E) of false prediction is assumed as 0.1.

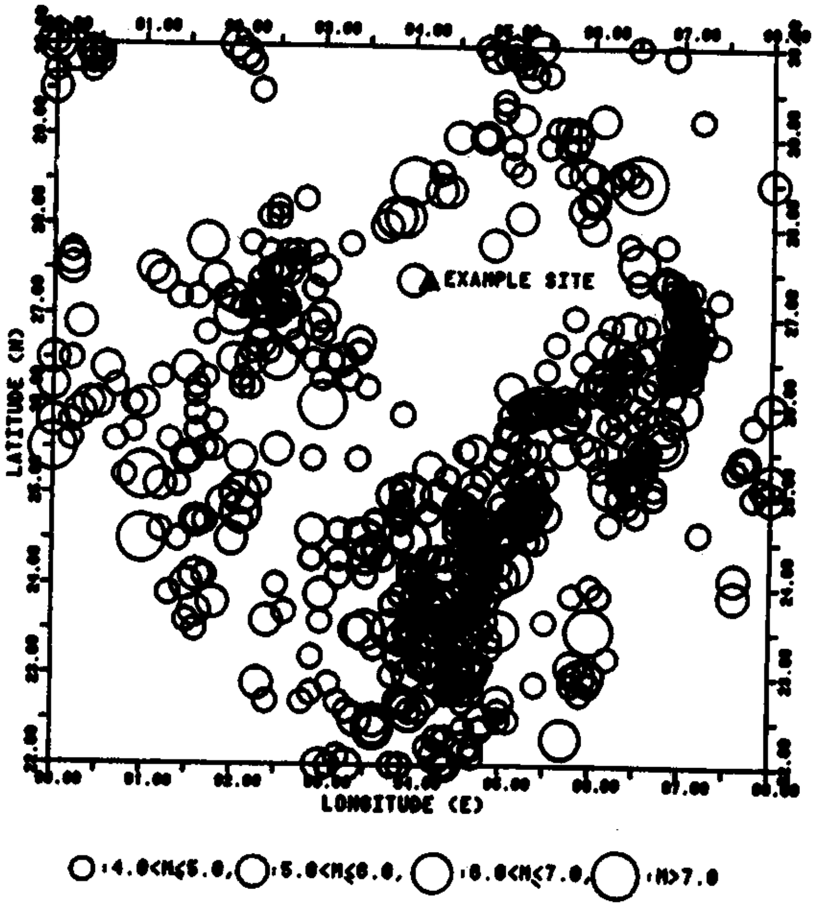

Fig. 2 : Distribution of the epicentres of the earthquake data used in the present study with respect to the example site in the Northeast-Indian region.

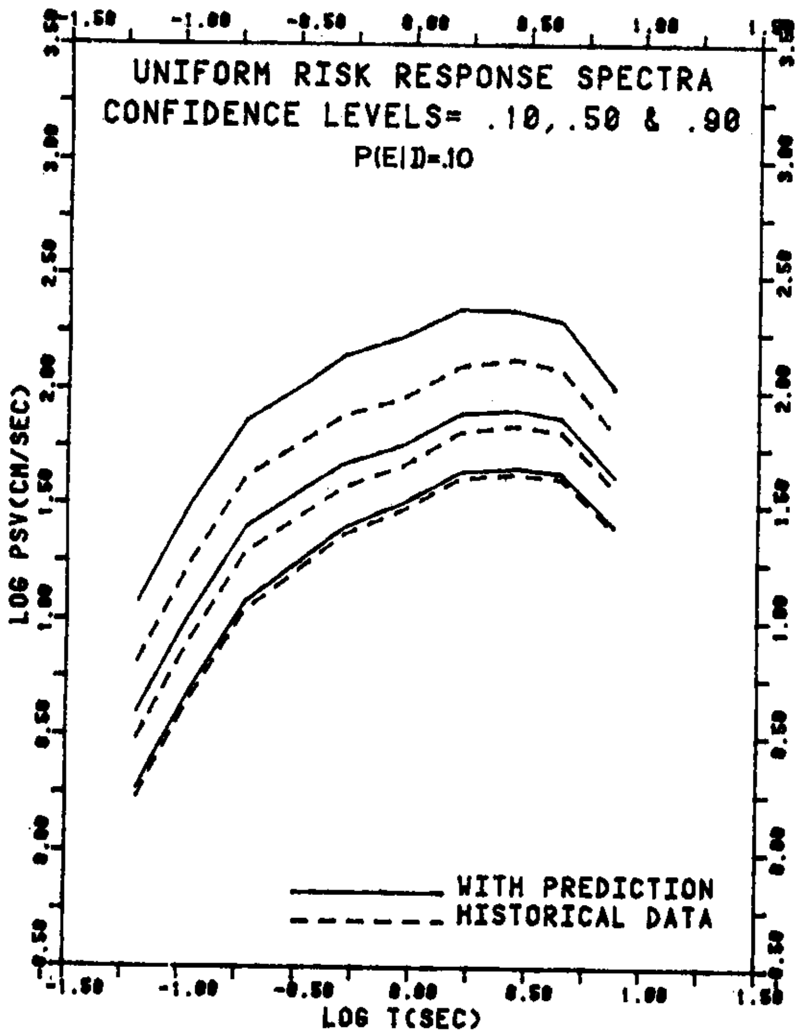

Fig.3a : Comparison of the uniform risk PSV spectra for different confidence levels computed from historical data with the corresponding spectra based on a probabilistic prediction of a magnitude 7.0 earthquake with reliability of prediction, $P(E|I) = 0.10$.

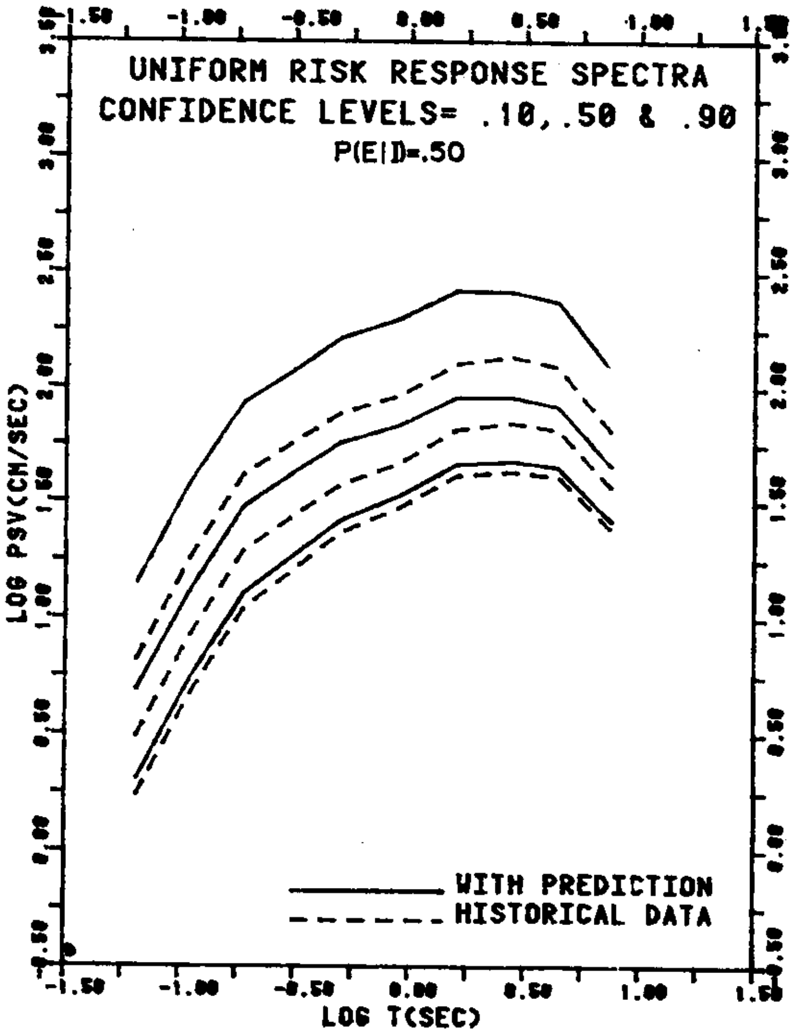

Fig.3b : Comparison of the uniform risk PSV spectra for different confidence levels computed from historical data with the corresponding spectra based on a probabilistic prediction of a magnitude 7.0 earthquake with reliability of prediction, P(E|I) = 0.50.

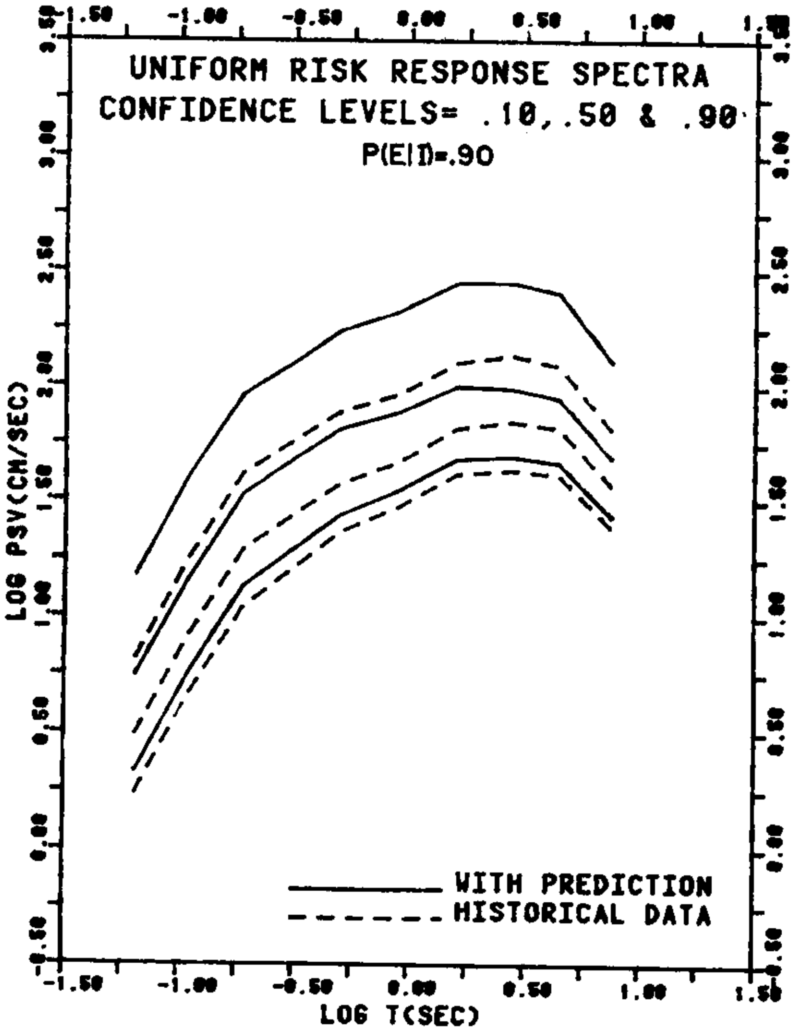

Fig.3c : Comparison of the uniform risk PSV spectra for different confidence levels computed from historical data with the corresponding spectra based on a probabilistic prediction of a magnitude 7.0 earthquake with reliability of prediction, P(E|I) = 0.90.