Bull. Ind. Soc. Earth. Tech., Paper No.343, Vol.31, No.3, Sept.1994, pp.141-154

3-D ANALYSIS OF IRREGULAR BUILDINGS WITH RIGID FLOOR DIAPHRAGMS

Satish Kumar * and D.K.Paul**

***Research scholar **Professor**
Department of Earthquake Engineering, University of Roorkee,

Roorkee-India

ABSTRACT

A method of analysis for irregular buildings with rigid floor system where the centre of mass and centre of stiffness of each floor lie on different vertical axes is suggested based on transformation of mass and stiffness matrices about a vertical reference axis. To account for the torsion and translation, each storey of a building is modelled as three degree of freedom per floor. Numerical examples on free vibration characteristics of irregular buildings has been solved and the results are discussed.

INTRODUCTION

Multistoreyed reinforced concerete framed buildings are getting popular in the hilly areas and many of these are constructed on hill slopes. Hill buildings are very irregular and generally forms a combination of stepback and setback. These buildings are generally unsymmetrical in horizontal and vertical planes as shown in Fig. 1 and undergo torsion during earthquakes. Most hilly areas are prone to severe earthquakes specially in Himalayan belt in India. These unsymmetrical buildings require great attention in the analysis and design. Literature on analysis of such buildings is scanty specially where centre of mass and Stiffness of various floors lie on different vertical axes.

A full 3D analysis with 6 degrees of freedom(d.o.f.) per node is required for such buildings for accurate results. Such an analysis procedure is computation intensive and is not advisable for preliminary design. The most simple procedure employed is based on one translational degree of freedom per floor either in x or y direction where only planar model of structure is considered which does not take into account the torsional coupling effects. The torsional analysis is carried out separately based on codal recommendations which may not give adequate representation of the actual behaviour of such buildings.

Buildings with setbacks have been studied by many researchers [1,2,4,5,8,10,14]. Some have proposed complex models needing extensive computing facilities and some have suggested approximate methods for analysis of setback buildings. Experimental studies on seismic response of reinforced concerete setback framed buildings has also been carried out [15].

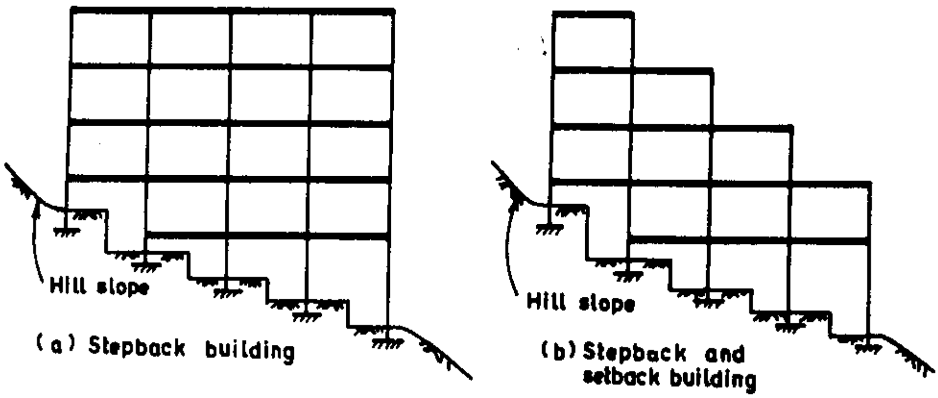


Fig. 1: Stepback and setback building

Any direct effort has not been made to develop an analysis procedure for buildings having both stepback and setback configurations. It necessitates the development of

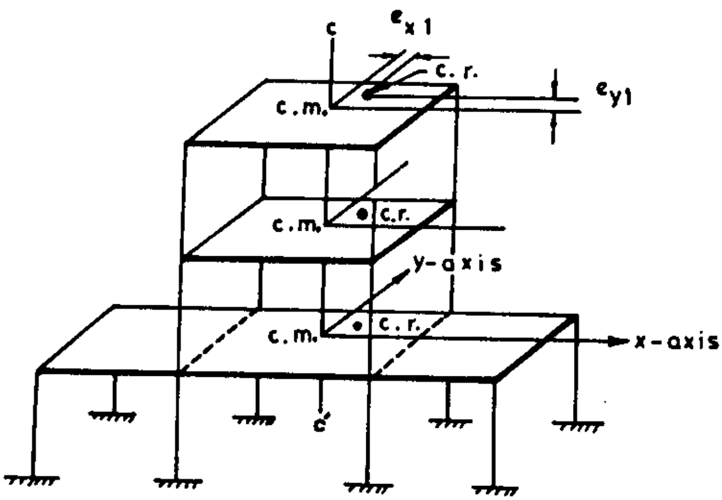


Fig. 2: Idealized symmetr.cal setback building

analysis procedure which takes into account the asymmetry of mass and stiffness distribution at different floor levels of buildings. Rigid floor idealisation with 3 d.o.f.(two translational and one torsional d.o.f.) per floor is considered in the formulation. Idealized multistorey building structure studied by Kan and Chopra is shown in Fig.2, where the centre of mass of all the floors lie on the same vertical axis cc'. This assumption of centres of mass of all the floors to lie on the same vertical axis is true only for buildings which has symmetrical setback on all the sides. Buildings constructed on hill slopes generally do not fall under this category i.e. centre of mass of all floors do not lie on one vertical axis.

In this study, an analysis procedure is presented for irregular buildings on hill slopes which are supported on columns, walls. Floors may be supported by two types of columns (i) interstorey columns i.e. columns between the floors and (ii) columns resting on the ground directly. The floors has been assumed as rigid there by beams at all the floor levels are taken as rigid. The flexibility in the structure is due to columns and walls.

IDEALIZED BUILDING SYSTEM

An idealized multi-storey irregular building structure consisting of rigid floor decks, supported on massless axially inextensible columns and walls is analysed. The centre of mass and centre of resistance of all the floors lie on the different vertical axes on different floors as shown in Fig.3. The kth and lth storey plans are shown separately in Fig.4(a) and (b). The d.o.f. per floor are shown in Fig.5. The centre of mass of the floors are lying on vertical axes such as aa', bb', cc', dd' shown in Fig.3 which shows that the centre of mass of all floors does not lie on one vertical axis. OZ as shown in Fig.3 represents a arbitrary vertical reference axis.

The mass of each floor is lumped at the centre of mass, The mass in X and Y directions is m and the mass moment of inertia of storey in z direction is I_m.

The mass matrix of each storey is known at the storey axis i.e. at the centre of mass. As the structure is unsymmetrical, the c.m. of all the floors lie on different vertical axes and the stiffness and mass matrices of each storey cannot be superimposed as such. A procedure of transformation of the matrices to a same vertical reference axis is suggested. Superposition of these transformed matrices at this reference axis can be made to get the overall stiffness and mass matrices.

TRANSFORMATION OF FORCES AND DISPLACEMENT

Transformation of forces

Let S represent the point of member axis and P_1' , P_2' , P_3' are the three forces acting on it in x, y and z directions respectively which are orthogonal to each other.

Let R represent the point of vertical reference axis, and the effects of forces at member axis on reference axis be P_1 ,P_2 ,P_3 in x, y and z directions respectively and is given by (1). This reference axis can be suitably chosen any where as shown in Fig.6.

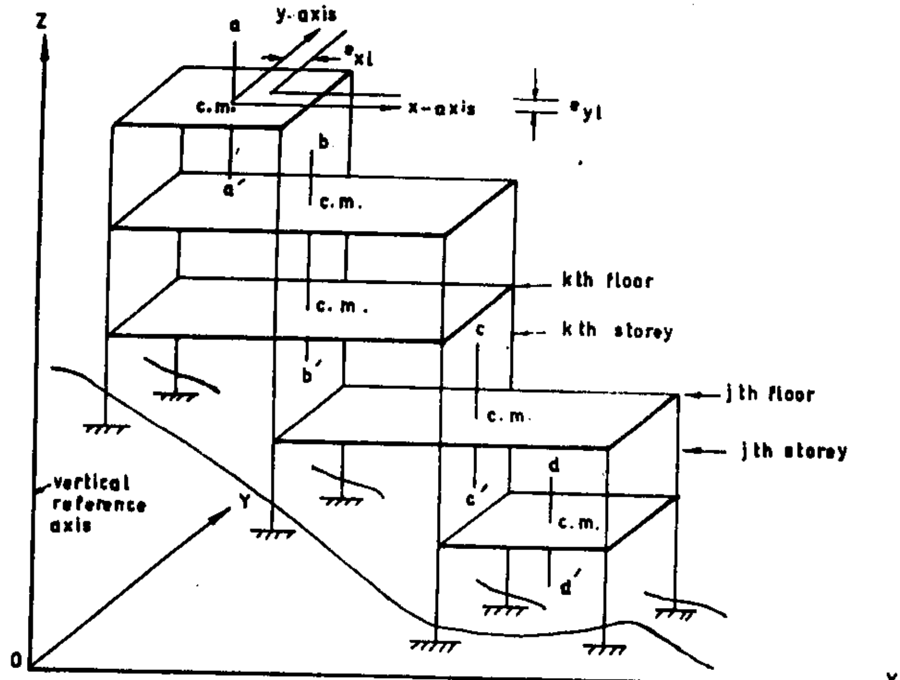


Fig. 3 : Idealized multistorey stepback and setback building

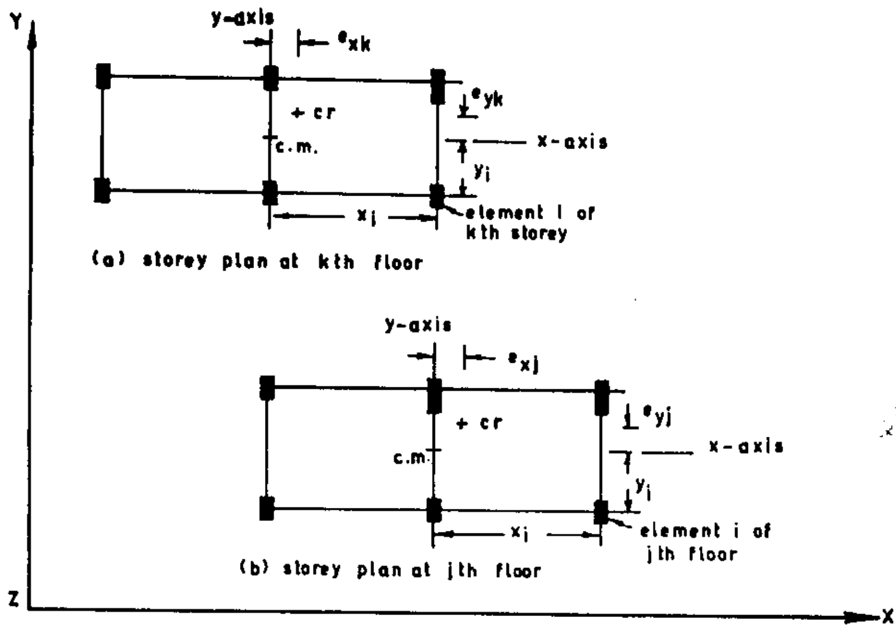


Fig. 4 : Storey plan at kth and jth floor

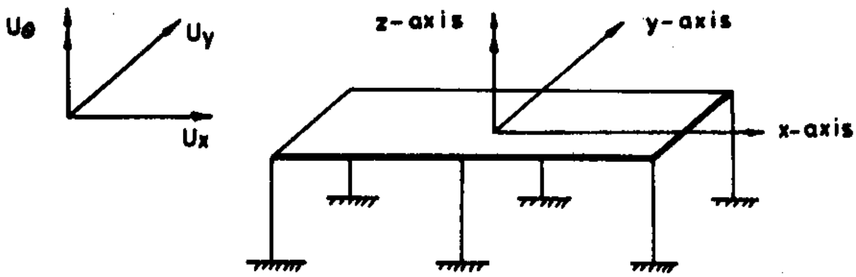


Fig. 5: 3 d.o.f. per floor

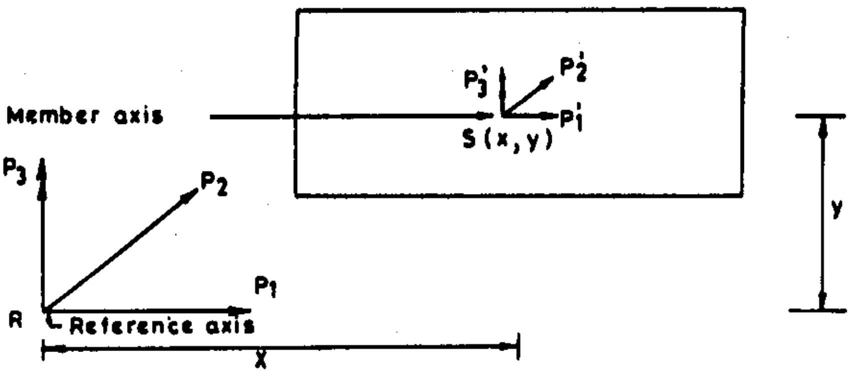


Fig.6: Reference and Member axes

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -y & x & 1 \end{bmatrix} \begin{bmatrix} P_1' \\ P_2' \\ P_3' \end{bmatrix} \qquad \text{or } p = Rp' \tag{1}$$

$$\begin{bmatrix} P_j \\ P_k \end{bmatrix} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} P_j' \\ P_k' \end{bmatrix} \qquad \text{or } P = R_T P'$$

where p_j , p_k and $p_{j'}$, $p_{k'}$ are the member end actions at reference axis and member axis respectively, x and y are the distances between member and reference axes in x and y directions respectively.

If the forces P_1' , P_2' , P_3' are known at the member axis, then P_1 , P_2 , P_3 can be calculated with the help of transformation matrix R_T

Transformation of displacement

In the same manner, if u, v and θ are the three displacements of a storey i at reference axis, then u', v' and θ' are the three corresponding displacements of that storey at member axis. It is expressed as (2).

$$\begin{bmatrix} u' \\ v' \\ \theta' \end{bmatrix} = \begin{bmatrix} 1 & 0 & -y \\ 0 & 1 & x \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ \theta \end{bmatrix} \qquad \text{or } u' = R^T u \tag{2}$$

$$\begin{bmatrix} u_j' \\ u_k' \end{bmatrix} = \begin{bmatrix} R^T & 0 \\ 0 & R^T \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \qquad \text{or } U' = R_T^T U$$

where u_j , u_k and $u_{j'}$, $u_{k'}$ are the member end displacement at reference and member axes respectively.

Transformation of Stiffness Matrix from member axis to reference axis

Let P_{cm} is the force vector, K_{cm} is the stiffness matrix, U_{cm} is the displacement vector of member i about member axis then

$$P_{cm} = K_{cm} U_{cm} \tag{3}$$

P_{cm} and U_{cm} has been transferred to a reference axis with the help of transformation matrices, it will lead to

$$P_r = K_r U_r$$

where K_r is the stiffness matrix at reference axis defined as

$$K_r = R_T K_{cm} R_T^T \tag{4}$$

The stiffness matrices for all the members can be worked out about the vertical reference axis. Then the overall stiffness matrix of the structure can be obtained by superposition method.

Transformation of mass matrix from storey axis to reference axis

Transformation of mass matrix from storey axis to reference axis can be carried out in the same manner as has been done for stiffness matrix. Let M_r represent the mass matrix of floor i about the reference axis, M_{cm} represent the mass matrix of floor i about the storey axis then

$$M_r = R\, M_{cm}\, R^T \tag{5}$$

$$M_r = \begin{bmatrix} m & 0 & -my_i \\ 0 & m & mx_i \\ -my_i & mx_i & \left(mx_i^2 + my_i^2 + I_m\right) \end{bmatrix} \tag{6}$$

Where x_i and y_i are the distances between centre of mass and the vertical reference axes in x and y directions respectively. The mass matrix of the overall structure can be worked out about the same reference axis by superposition method.

EQUATION OF MOTION

The free vibration equation of motion for the building can be written as

$$M\ddot{U} + C\dot{U} + KU = 0$$

$$\ddot{U} = [\ddot{U}_1 \;\; \ddot{U}_2 \;\; \ddot{U}_3 \;\; \ddot{U}_4 \text{-------} \;\; \ddot{U}_n]^T$$

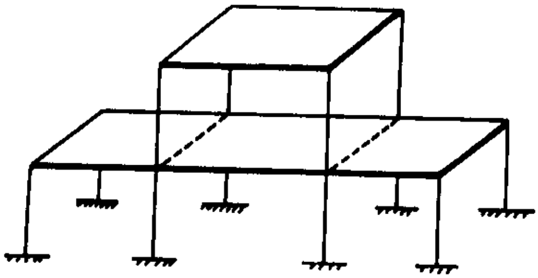
$$\ddot{U}_1 = [\ddot{u}_1 \;\; \ddot{v}_1 \;\; \ddot{\theta}_1]^T$$

M and K are the overall mass and stiffness matrices about reference axis. C is the damping coefficient matrix.

The eigen value problem is solved to get the frequencies of vibration and mode shapes at the reference axis.

Three different problems has been solved with the proposed method to show that the results obtained by the proposed method compares exactly with the results of Kan and Chopra's method for buildings having center of mass lying on the same vertical axis, the reference axis can be chosen anywhere wiithout affecting the results and to study the seismic response of buildings with unsymmetrical setbacks.

Figure 7 shows the two storey r.c. frame with stiff floor system. The complete data is indicated in the figure itself. This problem has been solved by Kan and Chopra's method and the proposed method.



$E = 25491\ \text{N/mm}^2$

$\nu = 0.15$

First storey:

$I_x = 0.275754 \times 10^{11}\ \text{mm}^4$

$I_y = 0.275754 \times 10^{11}\ \text{mm}^4$

$K_t = 2.5570 \times 10^{12}$ N-mm/rotation

$e_x = 188$ mm

$e_y = 282$ mm

Second storey:

$I_x = 0.1904217 \times 10^{11}\ \text{mm}^4$

$I_y = 0.1904217 \times 10^{11}\ \text{mm}^4$

$K_t = 1.7666 \times 10^{12}$ N-mm/rotation

$e_x = 188$ mm

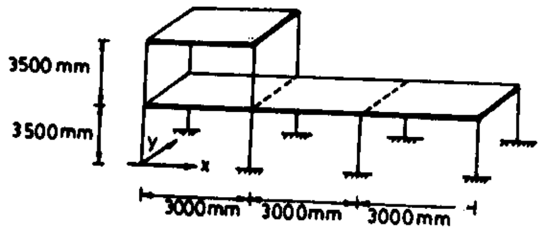
$e_y = 282$ mm

Fig. 7: Two storey r. c. setback frame

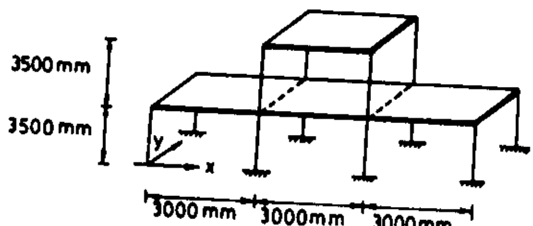
The time period of vibration in the first six modes are worked out as 0.13555, 0.11098,0.11030, 0.05114, 0.05081, 0.03492 secs by both the methods of analysis.

From the solution of the above example, in which the centre of mass of floors lie on the same vertical axis, it is observed that the results of the natural time periods obtained by the present study agrees exactly with the results obtained by the method suggested by Kan and Chopra.

Figure 8 shows the two storey r.c. symmetrical and unsymmetrical setback frame. Size of columns is 500 mmx500 mm. Given E=25491 N/mm^2, υ=0.15.



(a) Unsymmetrical setback building



(b) Symmetrical setback building

Fig. 8: Two storey r. c. setback buildings

The time periods of vibration and mode shapes for the above cases are shown in Figs. 9 and 10. It shows that the symmetrical setback frame is subjected to pure translation or torsional motion whereas the unsymmetrical setback frame is subjected to coupled translational and torsional motions.

A multistoreyed frame structure located on hill slope as shown in Fig. 11 is analysed by taking vertical reference axis at four different locations.

(i) Along the centre of mass of roof.
(ii) Along the centre of mass of IInd floor.
(iii) Along the centre of mass of Ist floor.
(iv) Along the centre of mass of the whole structure which is located at 900 mm from centre of mass of roof along x-axis.

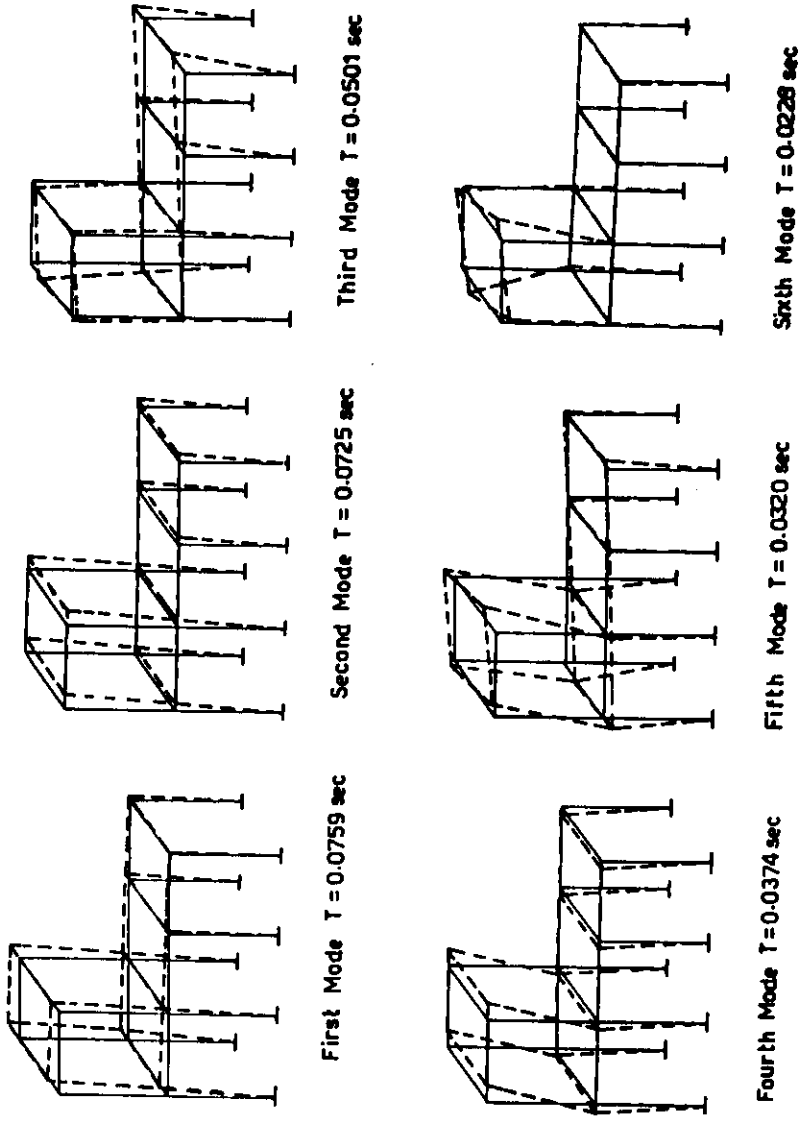


Fig. 9: Mode Shapes of two storey r.c. setback building

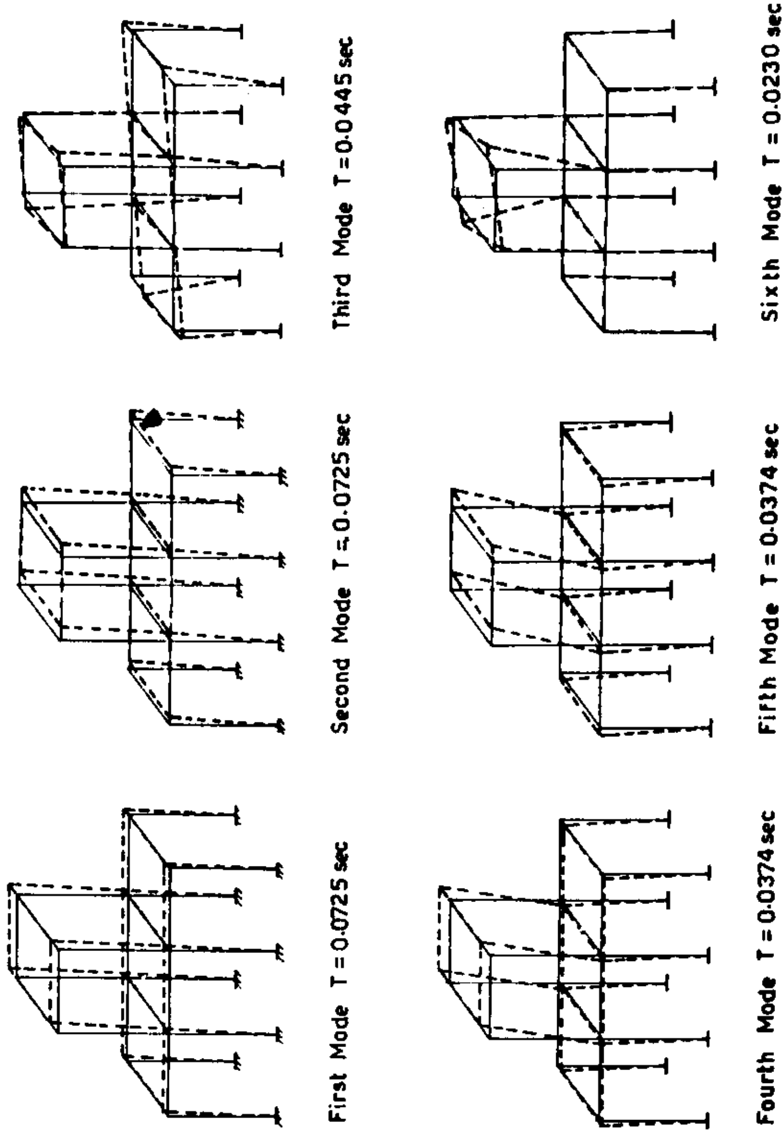


Fig. 10: Mode Shapes of two storey r. c. setback building

ISSN 0019-6371

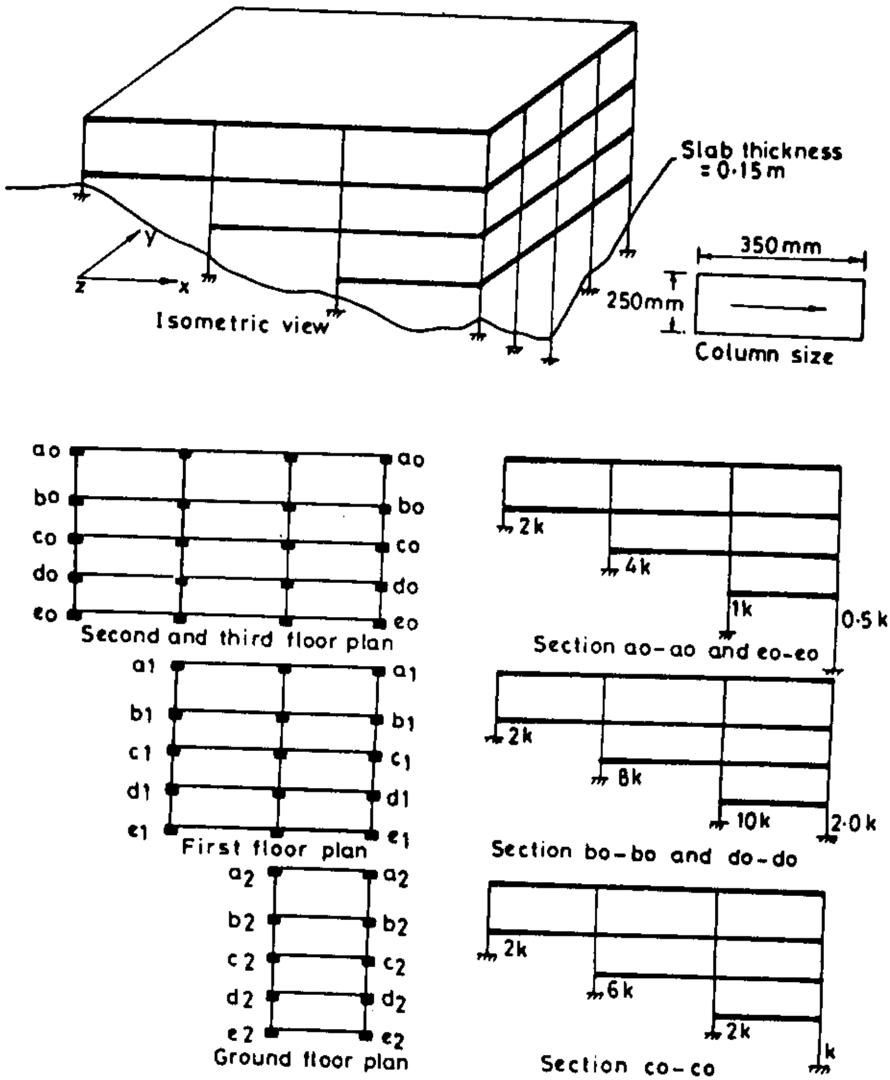


Fig.11: Multistorey r.c. building on hill slope

Given E = 25491 N/mm^2, γ =2.4 t/m^2 , L =3500 mm, mass of 3.5mx4.0m slab = 5.14 N-s^2 /mm , size of column = 350 mmx250 mm

The results are shown in Table 1.

Table 1. - Natural Time Periods of Vibration

MODE	Time periods with reference axis at			
	i	ii	iii	iv
1	0.33722	0.33722	0.33722	0.33722
2	0.22838	0.22838	0.22838	0.22838
3	0.20466	0.20466	0.20466	0.20466
4	0.13019	0.13019	0.13019	0.13019
5	0.10328	0.10328	0.10328	0.10328
6	0.09204	0.09204	0.09204	0.09204
7	0.07893	0.07893	0.07893	0.07893
8	0.07828	0.07828	0.07828	0.07828
9	0.07092	0.07092	0.07092	0.07092
10	0.06284	0.06284	0.06284	0.06284
11	0.05643	0.05643	0.05643	0.05643
12	0.05091	0.05091	0.05091	0.05091

Results of time periods of vibration shown in Table 1 indicates that these are exactly the same. It shows that the vertical reference axis can be suitably chosen anywhere.

DISCUSSION OF RESULTS

The analysis of building when the centre of mass of the floors lie on the same vertical axis indicates that the results obtained by the proposed method and Kan and Chopra's method are exactly the same. Results indicate that the regular setback structures are not torsionally coupled whereas the irregular setback structures are torsionally coupled as evident from the mode shapes. The reference axis can be suitably chosen anywhere specially for those irregular structures where the centre of mass axes of all the floors do not lie on the same vertical axis. Results shown in Table 1 indicate that time periods are exactly the same for different location of reference axes there by suggesting that the method can be used for the analysis of such buildings. It demonstrates that the transfer of stiffness and mass to any convenient vertical reference axis can be successfully applied for analysis of irregular buildings on hill slopes or any other similar structural problem.

CONCLUSIONS

The methodology for the analysis of irregular buildings with rigid floor system on hill slopes where the centre of mass of the floors do not lie on the same vertical axis is presented here. The analysis using this simplified method takes care of the torsional coupling effect in such buildings. The method does not require the computation of centre of rigidity of the storey. This methodology requires much less computation effort as compared to the 6 d.o.f/node analysis procedure. This method can be employed in design offices for buildings with rigid floor system having centre of mass lying on different vertical axes as there is no other simplified straight forward method available for such irregular buildings.

REFERENCES

1. Blume, J.A. and D.P. Jhaveri (1969), Time history response of buildings with unusual configurations, Proc. IVth WCEE, vol iii pp(155-170)
2. Cheung, V.W.T. and W.K. Tso (1987), Lateral load analysis for buildings with setback, J. of Structural Engg. ASCE, vol. 113, no. 2 pp(209-227)
3. Clough, R.W. and J. Penzien, Dynamics of Structures(1975), Mcgraw Hill, New York.
4. Corderoy, H.J.B. and D.P. Thambiratnam(1993), Microcomputer analysis of torsionally coupled multistorey buildings for earthquakes. Int. J. of Computers and structures vol. 46 no. 4 pp(593-602)
5. Fukada, Y., T. Kobayashi, M. Adachi, M. Nagate and Y. Hayashi (1992), Torsional vibration of a building with stepped heights, Proceedings of 10th WCEE, vol. 7 pp(3939-3944)
6. Goel, R.K. and A.K. Chopra (1993), Seismic code analysis of buildings without locating centre of rigidity. J. of Structural Engg. ASCE, vol. 119, no. 10 pp(3039-3055)
7. Hejal, R. and A.K. Chopra(1989), Lateral torsional coupling in earthquake response of frame buildings, J. of Structural Engg. ASCE, vol. 115, no. 4 pp(852-867)
8. Jain, S.K. and U.K. Mandal(1992), Seismic response of horizontal setback buildings, Proc. xth WCEE, vol. 7 pp(3853-3858)
9. Kan, C.L. and A.K. Chopra (1976), Coupled lateral torsional response of buildings to ground shaking, Report No. EERC 76-13, EERC, University of California, Berkeley.
10. Kan, C.L. and A.K. Chopra (1977), Elastic earthquake analysis of torsionally coupled multistorey buildings, J. of EESD, Vol. V, PP (395-412).
11. Pachuau, L.Z. (1992), Seismic response of R.C. framed buildings on hill slopes, M.E. Thesis University of Roorkee, Roorkee.
12. Paul, D.K. (1993), Simplified seismic analysis of buildings on hill slopes, Bull. Indian Society of Earthquake Technology, vol. 30, no. 4, pp(113-124)
13. Penzien, J. (1969), Earthquake response of irregularly shaped buildings, Proc. 4th WCEE, vol 2 pp(75-89)
14. Satake, N. and A. Shibata (1988), Dynamic inelastic analysis for torsional behaviour of a setback type building, Proceeding of the 9th WCEE, vol v pp(49-54)
15. Shahrooz, B.M. and J.P. Moehle (1990), Seismic response and design of setback buildings, J. of structural Engg. ASCE, vol 116(5) pp(1423-1439)
16. Sternik, M. and J. Gluck (1993), Decoupling of lateral equilibrium equations for asymmetric multistorey structures. J. of Structural Engg. ASCE, vol. 119 no. 10 pp(2871-2882)
17. Wood, S.L. (1992), Seismic response of r.c. frames with irregular profiles, J. of Structural Engg. ASCE, vol. 118, no. 2 pp(545-566)
18. Yang, Y.B. and W.H.Huang (1993), Seismic response of light equipment in torsional buildings. J. of EESD, vol. 22 pp(113-128)