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RESONANT AMPLITUDE RESPOSE OF MACHINE FOUNDATIONS ON COHESIVE SOIL

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SYNOPSIS

Resonant vertical displacement amplitude response of prototype circular footings supported on cohesive soil and subjected to vertical sinusoidal loading of a nature prevalent in machine foundations is presented using the methods of dimensional analysis in conjunction with kinematic and force parameters in phase diagram representation. The response is nonlinear but is presented in terms of a non-dimensional resonant amplitude parameter, dynamic stress amplitude transmitted to the supporting soil, and transmission factor of the soil-foundation system in both graphic and analytic form. The physical variables include the size and mass of the footing, applied dynamic force, eccentricity factor, displacement, and energy dissipation. Size effects are conveniently handled in non-dimensional form. Static stress level was maintained constant at 4·25 psi with footing diameters ranging from 5 ft. 2 in. to 10 ft. 4 in., total weights from 12,820 lbs. to 51,280 lbs., and applied force amplitudes between 525 lbs. and 52,000 lbs. The results may shed insight into other studies of resonant amplitude response of machine foundations.

INTRODUCTION

The major source of objectionable response of machine foundations is the occurrence of resonant phenomena. When the operational speed of the machinery coincides with the natural frequency of the soil-foundation system, resonance occurs and the amplitude of motion of the foundation is greatly amplified. Excessive foundation motion may lead to serious structural damage, excessive wear of the machinery, or objectionable performance. Most work on machine foundations have been directed at the analysis, determination, and prediction of the natural frequencies of the soil-foundation systems. Considerably less attention has been devoted to the consequences of resonance; namely, the resonant amplitudes of the foundation motion. Because of the energy dissipation inherent in any physical system such as a soil-foundation system, the resonant amplitudes have finite values. It is highly desireable to be able to estimate the values of these resonant amplitudes in order to compare them with acceptable displacement tolerances of the foundation.

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The resonant response of a soil-foundation system is a function of many factors, including size and mass of the foundation, mass and distribution of mass of the machinery, operational characteristics of the machinery, properties of the soil supporting the foundation, magnitude and nature of the applied forcing function, frequency of oscillation, applied static stress, etc. The interrelated effects of these factors on the value of the natural frequency have not yet been determined. It is generally recognized that the response of a soil-foundation system is a nonlinear problem of a highly indeterminate nature. However, highly idealized, simplified representations or models of these systems have been utilized to provide the foundation engineer with methods (however limited) for attempting the estimation of prototype response. These expedient solutions are limited with regard to the scope of their applicability to represent actual field conditions. Most experimental studies of machine foundations are on models or relatively small footings with prototype investigations quite limited in scope.

The present paper deals with the analysis and formulation of the vertical mode resonant amplitude response of large scale prototype circular footings supported on cohesive soil and subjected to vertical vibratory loading of a nature prevalent in machine foundations. The physical variables considered include the size and mass of the footing, applied dynamic force, frequency of loading, damping of the system, and footing displacement as well as the phase angle between applied force and displacement. Static stress level was maintained constant at 4·25 psi with footing diameters ranging from 5 ft. 2 in, to 10 ft. 4 in., total weights from 12,820 lbs. to 51,280 lbs., and applied force amplitudes between 525 lbs. and 52,000 lbs. The methods of dimensional analysis are used in setting up functional relationships among the variables. A point by point amplitude linear approximation of assumed harmonic motion is used and kinematic as well as force parameters are represented in phase diagram form.

DIMENSIONAL ANALYSIS

The highly indeterminate nonlinear nature of the response of soil-foundation systems makes dimensional analysis a convenient method of expressing the physical phenomena in functional form in terms of a finite number of physical quantities. Such methods as used to formulate relationships among physical quantities may be briefly summarized as follows. If there are 'm' physical quantities containing 'n' fundamental units, which can be related by an equation, then there are (m—n), and only (m—n). independent, non-dimensional parameters, called π terms, such that the π terms are arguments of some indeterminate, homogeneous function K:

$$K(\pi_1, \pi_2, \pi_3, \ldots \ldots \ldots \pi_{m-n}) = 0 \quad (1)$$

Using a force, length, and time system of fundamental units, the physical quantities given in Table I have been selected considering the steady state vertical oscillation of a circular foundation supported on a cohesive soil and subjected to a sinusoidally applied vertical force.

TABLE I

	PHYSICAL QUANTITY	SYMBOL	FUNDAMENTAL UNITS
1.	Amplitude of footing displacement	x	L
2.	Total weight of foundation	W	F
3.	Diameter of footing	d	L
4.	Amplitude of applied force	F_D	F
5.	Forcing frequency	ω	T^{-1}
6.	Restoration parameter of soil	q	FL^{-2}
7.	Dissipation parameter of soil	η	$FL^{-2}T$
8.	Natural frequency of system	p	T^{-1}

It is recognized that the stress-strain-time relations for cohesive soils may be complicated nonlinear viscoelastic in nature; however, for simplicity, it is assumed that the soil can be described implicitly in terms of characteristic restoration and dissipation parameters. The characteristic restoration parameter is quite general and may take many forms including a shear or compression modulus, compressive strength, compliance function, or relaxation modulus function, depending upon the circumstances under consideration. The dissipation parameter may represent many forms including a damping coefficient, friction coefficient or viscosity. The natural frequency is not a physical property of a soil but a function of a soil-foundation system and, hence, a function of the other physical quantities of the soil, loading, and foundation. Ordinarily, it might not be included as an independent physical quantity. However, since its expression in terms of the other physical quantities is not known, it is listed separately. Certain operational characteristics of the machinery may be included in the applied force amplitude. The other quantities of Table I are straightforward.

The eight physical quantities and three fundamental units give rise to five independent, non-dimensional π terms. These can be methodically obtained, algebraically transformed, and substituted into Eq. (1) to give

$$K\left[\frac{x}{d}, \frac{F_D}{d^2q}, \frac{W}{d^2q}, \frac{\omega}{p}, \frac{\omega\eta}{q}\right] = 0 \tag{2}$$

Since the amplitude of the footing displacement explicitly appears in only one π term, it may be considered the dependent variable and written as:

$$\frac{x}{d} = \psi\left[\frac{F_D}{d^2q}, \frac{W}{d^2q}, \frac{\omega}{p}, \frac{\omega\eta}{q}\right] \tag{3}$$

In Eq. (3) and hereafter the symbol ψ denotes "some function of" but not necessarily the same function for each equation. This avoids the use of numerous subscripts and superscripts as a means of differentiating between equations.

The non-dimensional π terms may be interpreted in the following manner. Amplitude of footing displacement is given in terms of the footing diameter and expressed as the amplitude parameter, x/d. The term F_D/d^2q may be considered proportional to the ratio of the applied dynamic stress to the resisting stress and, hence, an applied dynamic Cauchy Numder. Analogously, the parameter W/d^2q may be considered proportional to the ratio of the static stress level to the static resisting stress or static Cauchy Number. The term $\omega\eta/q$ may be considered the ratio of the viscous to restoring stresses and related to the familiar dissipation factor of viscoelasticity known as the loss tangent which is associated with the phase angle between the applied force and displacement. The frequency ratio is ω/p.

The prototype test results analyzed in this paper are resonant displacement amplitudes for a constant static stress level; thus, $\omega/p = 1$ and W/d^2q may be considered approximately constant. Subject to the condition on ω/p and W/d^2q, Eq. (3) can be simplified to the form

$$\frac{\Delta}{d} = \psi\left[\frac{F_D}{d^2q}, \frac{\omega\eta}{q}\right] \qquad (4)$$

where Δ is the value of x at resonance. Eq. (4) is the extent to which dimensional analysis alone is helpful. The explicit form of Eq. (4) must be determined experimentally.

THEORETICAL CONSIDERATIONS

As previously noted, the dynamic prototype soil-foundation problem is nonlinear. Since the motion in nonlinear vibration may be periodic but not harmonic, the assumption of a harmonic wave form inherently leads to the concept of linearity. Depending upon the degree of nonlinearity, it is possible to use a harmonic approximation for the response, particularly when only amplitude data are being considered on an individual point by point basis.

Assuming a harmonic wave form for the steady state displacement response of prototype footings under harmonic vibratory loading, the displacement can be written as

$$y = x \sin \omega t \qquad (5)$$

while the velocity, $\dot{y}$, and acceleration, $\ddot{y}$, can be written

$$\dot{y} = x\omega \cos \omega t = \dot{x} \cos \omega t \qquad (6)$$

and

$$\ddot{y} = -x\omega^2 \sin \omega t = \ddot{x} \sin \omega t \qquad (7)$$

Considering the forcing function as a harmonic function of the same frequency as the displacement but out of phase with the displacement by a time interval, Δt, gives

$$F_d = F_D \sin(\omega t + \delta) \tag{8}$$

where $\delta = \omega(\Delta t)$ and F_D is the amplitude of the dynamic force wave.

Since the displacement is a sine function, the velocity is a cosine function and the

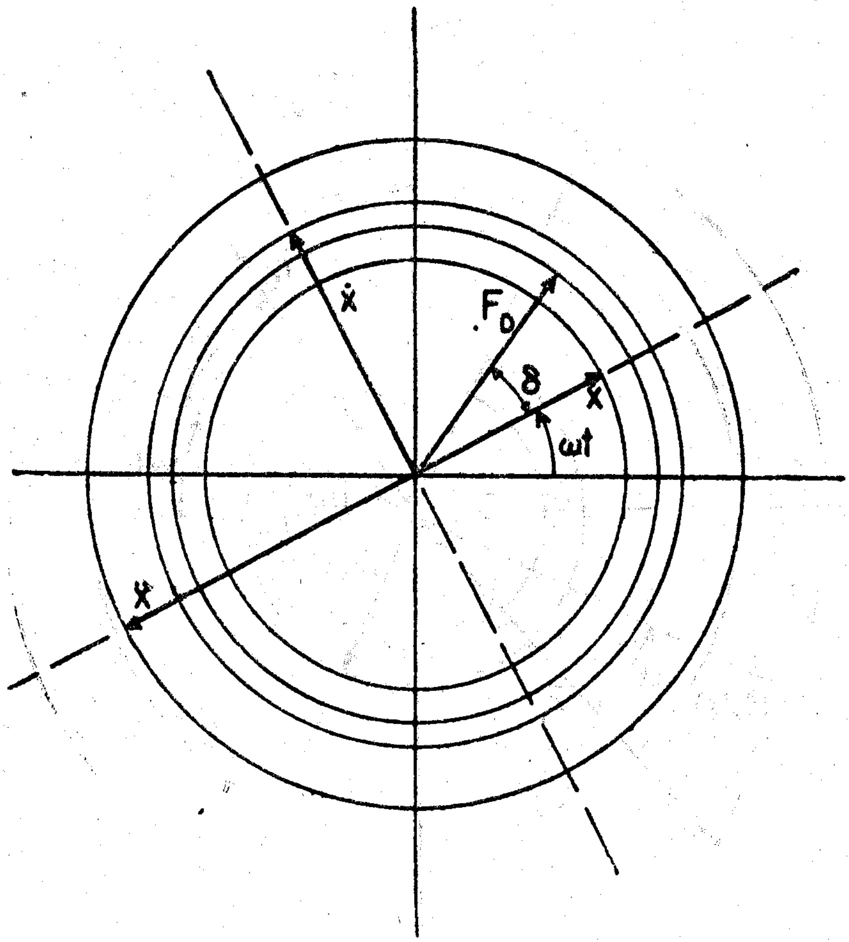

Figure 1 Phase Diagram: Kinematic Parameters and Force

acceleration is a negative sine function; the velocity and acceleration are 90° and 180°, respectively, out of phase with the displacement. Fig. 1 is a vector diagram of the displacement, velocity and acceleration. The angles between the vectors are called phase angles and the diagram itself is called a phase diagram. Since all of the vectors in Fig. 1 are rotated at the same frequency, they may be considered as turning like the spokes of a wheel, preserving their relative positions in the wheel.

D'Alembert's principle utilizes the inertial term $m\ddot{x}$ as a force whose direction is opposite to that of the acceleration vector. The restoring function vector $R_2(x)$ is opposite to that of the displacement and the dissipation function vector $R_1(\dot{x})$ is in the opposite direction of the velocity. Thus, the phase diagram for the force system can be constructed as given in Fig. 2

for the situation in which the forcing function F_D leads the displacement function by the phase angle δ. By resolving the forcing function into two components perpendicular and parallel to the displacement vector and then applying the equilibrium conditions at an instant of time, one obtains the following relations :

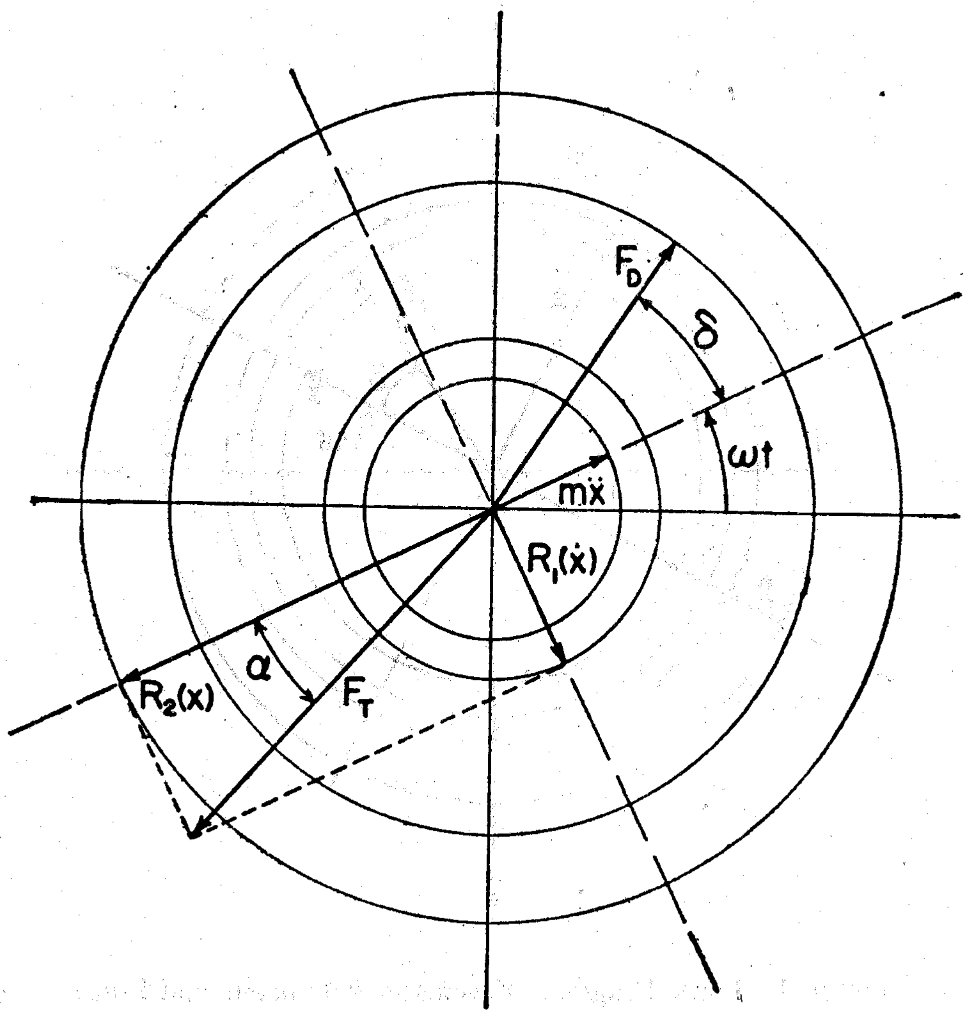

Figure 2 Phase Diagram: Force Parameters

$$R_1(\dot{x}) - F_D \sin \delta = 0 \tag{9}$$

and

$$m\ddot{x} - R_2(x) + F_D \cos \delta = 0 \tag{10}$$

Eqs. (9) and (10) give the amplitude of the dissipation function as

$$R_1(\dot{x}) = F_D \sin \delta \tag{11}$$

and the restoring function amplitude as

$$R_2(x) = m\ddot{x} + F_D \cos \delta \tag{12}$$

The same relations can be obtained directly from the equation of motion using Eqs. (7) and

(8) and evaluating the results at $\omega t=0$ and $\omega t=\pi/2$.

The amount of displacement of the footing is a function of the amount of force transmitted to the cohesive soil supporting the footing. This tranmsitted force, F_T, is made up of the two components R_1 and R_2, as indicated in Fig. 2; that is, the transmitted force is the vector sum of the force transmitted through the dissipation mechanism or damper and the force transmitted through the restoring mechanism. The amplitude or modulus of the transmitted force vector is denoted as F_T. Fig. 3 is a vector force polygon for the response

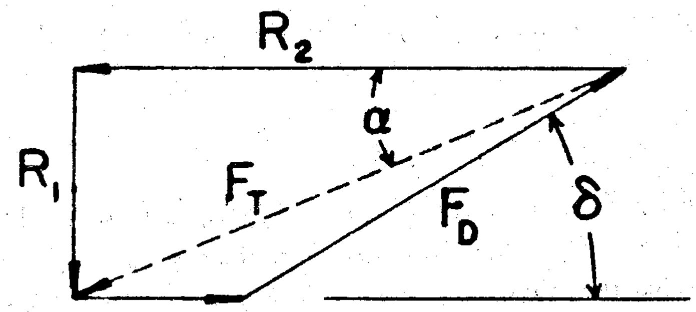

Figure 3 Force Vector Polygon

given in Fig. 2. All force vectors are shown in Fig. 3. However, F_T is equivalent to the vector sum of R_1 and R_2 and can be combined with the applied force amplitude, F_D, and the inertial force amplitude to form a force triangle. From Newton's second law, the resultant of all the external forces acting on the footing system must be exactly equal to the product of the mass and acceleration with the direction of the acceleration. Thus, the external resultant force vector connot have a component normal to the acceleration vector. This leads to the relation

$$F_T \sin \alpha = F_D \sin \delta \tag{13}$$

and the transmitted force amplitude is

$$F_T = F_D \frac{\sin \delta}{\sin \alpha} \tag{14}$$

As indicated in Fig. 3,

$$\alpha = \tan^{-1}\left(\frac{R_1}{R_2}\right) \tag{15}$$

Substitution of Eqs. (11) and (12) into Eq. (15) gives

$$\alpha = \tan^{-1}\left[\frac{F_D \sin \delta}{m\ddot{x} + F_D \cos \delta}\right] \tag{16}$$

and sin α can be conveniently obtained from a table of the values of the trigonometric functions.

The transmission factor, T.F., is the ratio of the transmitted force to applied force. At resonance, the transmission factor can be written as

$$(T.F.)_r = \frac{F_T}{F_D} = \frac{\sin \delta}{\sin \alpha} \tag{17}$$

and the resonant transmitted force amplitude can be written

$$F_T = F_D\,(T.F.)_r \tag{18}$$

It is interesting to note that the transmission factor is both non-dimensional and a measure of the energy dissipation parameter of the system. Hence, the transmission factor can be considered related to $\omega\eta/q$. Since Eq. (4) is in functional form, the transmission factor may be substituted for $\omega\eta/q$ to give

$$\frac{\Delta}{d} = \psi\left[\frac{F_D}{d^2 q},\ (T.F.)_r\right] \tag{19}$$

Since the arguments of Eq. (14) are independent and non-dimensional, they can be algebraically transformed as desired provided the final two arguments are also independent and non-dimensional. Multiplying the two arguments together and elimination of F_D/d^2q gives the parameters $[F_D\,(T.F.)_r/d^2q]$ and $(T.F.)_r$. Dividing the first of these by $\pi/4$ and using Eq. (18) gives

$$\frac{F_D\,(T.F.)}{\frac{\pi d^2}{4}\,q} = \frac{F_T}{Aq} = \frac{\sigma_{DT}}{q} \tag{20}$$

where A is the cross sectional area of the footing and σ_{DT} is the dynamic stress amplitude transmitted to the supporting soil at resonance. Thus, Eq. (19) can be written

$$\frac{\Delta}{d} = \psi\left[\frac{\sigma_{DT}}{q},\ (T.F.)_r\right] \tag{21}$$

The explicit form of the functional relation of Eq. (21) must be determined by experiment.

PROTOTYPE TEST RESULTS

The author has been involved in the analysis of the results of a number of prototype footing tests conducted using vertical sinusoidal forces generated by the centrifugal force due to a rotating eccentrically mounted mass. The test results analyzed in this paper were obtained from reinforced concrete circular footings with diameters of 5 ft., 2 in., 7 ft. 4 in., 9 ft. 2 in, and 10 ft. 4 in. supported on the surface of a relatively uniform silty clay. Unforturately, extensive soil test data were not available from the test area. Each footing was loaded to a static pressure of 4·25 psi with ballast symmetrically placed and secured to the footing. The static pressure included the weight of the footing, weight of the vibrator, and ballast load. The areas included were 20·97, 41·94. 62·92 and 83·89, sq. ft. while the static weights included 12,820, 25,640, 38,460, and 51,280 lbs., respectively. For a footing test a particular eccentricity was selected for a constant magnitude of eccentric mass and steady state conditions were obtained for various values of frequency. Four values of eccentricity were used for each footing. Sinusoidal forces were applied for frequencies ranging from approximetely 6 cps. to 30 cps., subject to the limitations of the vibrator. This corresponds to force amplitude, F_D, ranging from approxiamtely 525 lbs. to 52,000 lbs., depending upon the magnitude of the eccentric mass, the eccentricity, and frequency of oscillation. All footings were carefully instrumented with various configurations of transducers and pick ups for both test control and displacement measurement. Special instrumentation was used to measure the phase angle, δ, between the applied force and the footing displacement. Thus, for each frequency of oscillation, the force amplitude, vertical displacement amplitude, and phase angle between force and displacement were obtained.

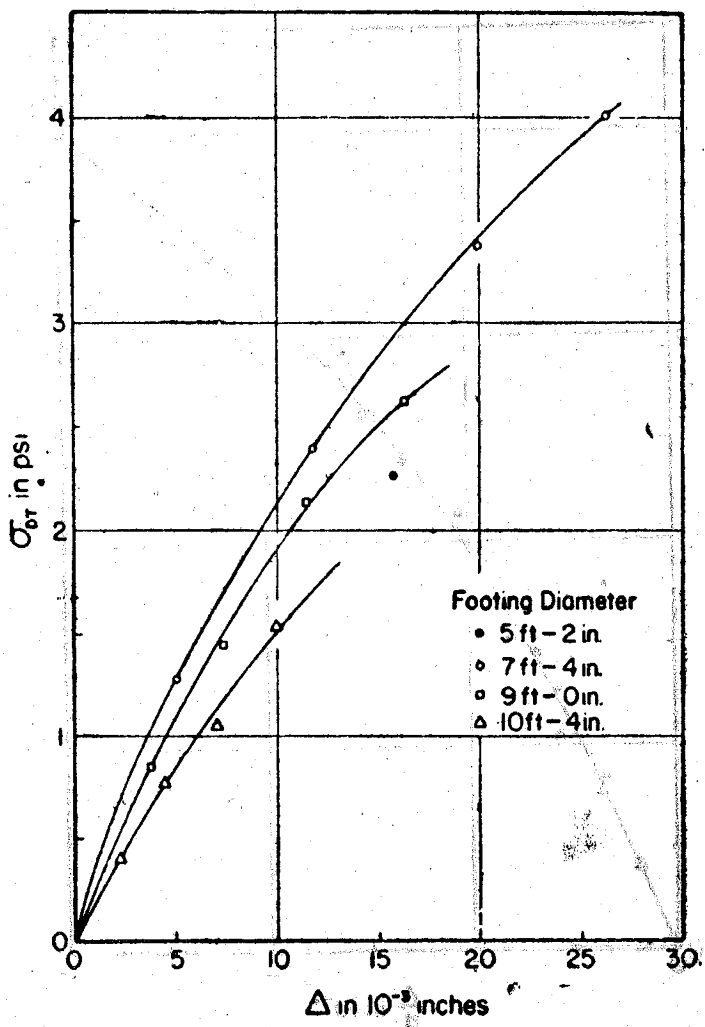

Figure 4 Transmitted Stress versus Displacement : Resonant Amplitude Response

Fig. 4 is analogous to a conventional static type of stress-deflection plot for the footing tested. In this case, the stress considered is the resonant dynamic stress amplitude transmitted to the supporting cohesive soil and the deflection used is the resonant amplitude of footing displacement. The response given in Fig. 4 is similar to that obtained from conventional static loadings; namely, for a constant value of transmitted dynamic stress amplitude the deflection amplitude is larger for the larger size footing. The results of the single test for the 5 ft. 2 in. diameter footing do not fit in with the trends of the other 12 tests. However, the reliability of the test on the small footing is highly questionable because of improper functioning of the phase angle instrumentation, leading to doubtful values of the phase angles for this particular test. Thus, one must disregard that point on Fig. 4.

Because the footings were all supported on the same cohesive soil, the restoration parameter of the soil may be considered constant. Thus, the non-dimensional parameter σ_{DT}/q is proportional to the resonant stress amplitude transmitted, σ_{DT}. Fig. 5 is a plot of the non-

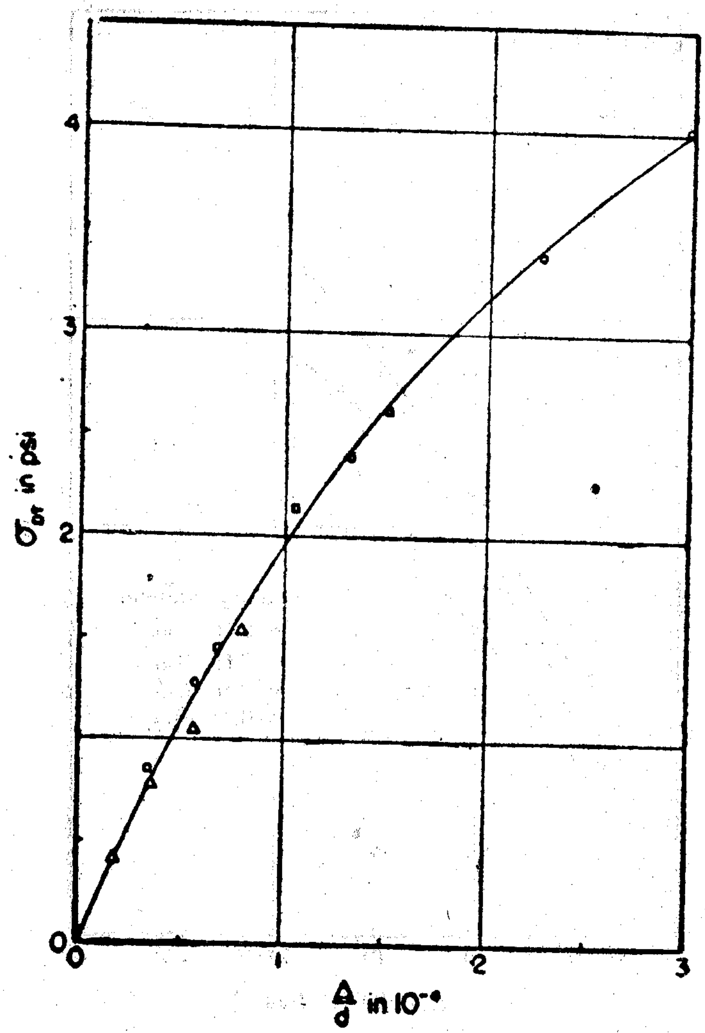

Figure 5 Transmitted Stresss versus Dimensionless Displacement Parameter : Resonance Amplitude Response

dimensional resonant amplitude parameter, Δ/d, as a function of σ_{DT}. The response of the three large footings given in Fig. 4 seems to collapse in a single band or curve in Fig. 5 with no apparent phenomenological order. The advantage of the non-dimensional formulation is quite apparent. Since σ_{DT} is proportional to σ_{DT}/q, Fig. 5 is essentially a plot of σ_{DT}/q versus Δ/d for a normalized value of q. Although the value of the resonant transmission factor, $(T.F.)_r$, varies from 1·03 to 1·40, the lack of any apparent effect of $(T.F.)_r$ in Fig. 5 indicates that it can be considered negligible and Eq. (21) simplifies to

$$\frac{\Delta}{d} = \psi\left[\frac{\sigma_{DT}}{q}\right] \tag{22}$$

It must be emphasized that the effect of $(T.F.)_r$ is already included in σ_{DT} as indicated in Eqs. (17), (18), and (20).

The explicit form of the functional relation of Eq. (22) is presented graphically in Fig. 5. By plotting $1/\sigma_{DT}$ versus $d/\Delta\ \sigma_{DT}$ on log scales, as given in Fig. 6, and fitting the plot with a straight line approximation leads to an expression of the form

$$\frac{\Delta}{d} = B\ (\sigma_{DT})C \tag{23}$$

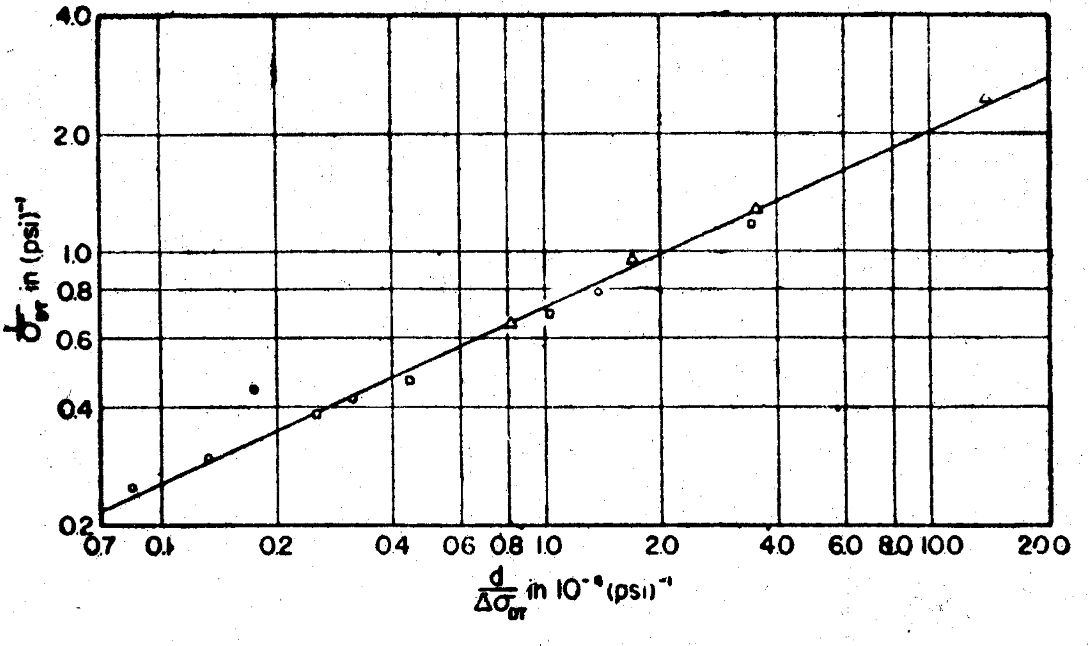

Figure 6 Resonant Amplitude Response : Measured Phase Angles

For the particular cohesive soil and footing systems tested, the values of the coefficients are $B = 4{\cdot}81 \times 10^{-5}$ and $C = 1{\cdot}21$ with normalized value of q = 1 psi.

In order to use the results given in Figs. 5 and 6 or Eq. (23) for a particular vibratory loading on a footing with the static stress level considered and supported on the cohesive soil tested, it is necessary to know the value of the resonant transmission factor in order to calculate the value of σ_{DT}. However, the resonant transmission factor is a function of the soil-foundation system and the manner of loading.

From the conventional analysis of the forced vibration of a linear damped spring-mass system, it can be shown that the resonant value of the transmission factor can be written

$$[T.F.]_{r-L} = \sqrt{1+\left(\frac{\Delta}{\varepsilon}\right)^2} \quad (24)$$

where ε is the eccentricity factor defined as

$$\varepsilon = \frac{M_0 e}{M} \quad (25)$$

with M_0 the eccentric mass, e the eccentricity, and M the total mass of the footing-machine system. Since Δ is an expression of the soil-foundation system and ε relates to the footing-machine loading system, Eq. (24) may be related to the actual resonant transmission factor associated with the prototype dynamic soil-foundation system. Comparison of the resonant

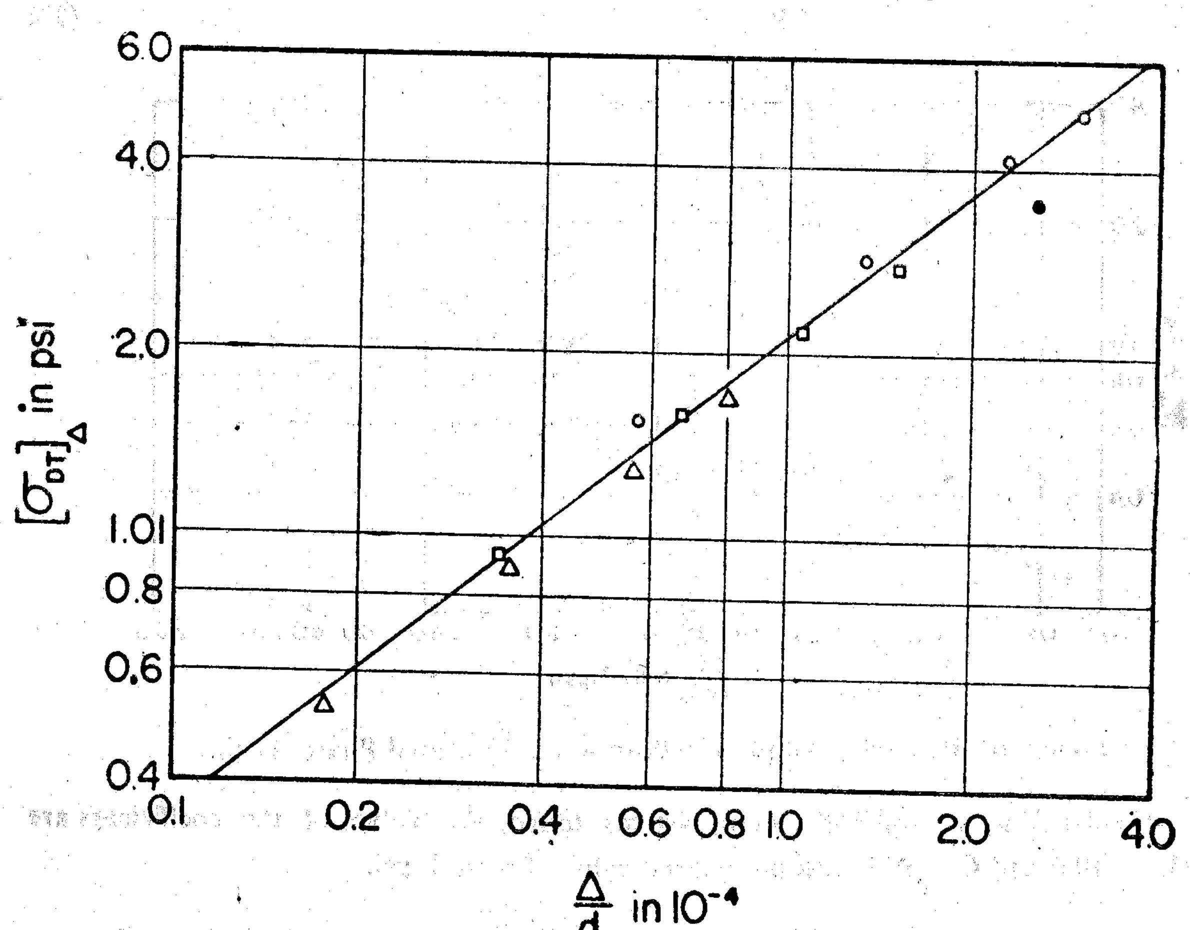

Figure 7 Resonant Amplitude Response : Transmission Factor By Equation 24

transmission factors determined by Eqs. (17) and (24) indicate that those determined by Eq. (24) are approximately 89 per cent higher than those given by Eq. (17). The values determined with Eq. (24) range from 1·28 to 1·74 while those determined by Eq. (17) range from 1·03 to 1·40. If the resonant transmission factors calculated with Eq. (24) are used in Eq. (20) to determine the transmitted dynamic stress amplitudes, σ_{DT}, the results given in Fig. 7 are obtained. Approximation of the results of Fig. 7 with a straight line representation leads to an equation of the form of Eq. (23) with the values of the coefficients $B = 3{\cdot}79 \times 10^{-5}$ and $C = 1{\cdot}287$. In Fig. 7, the response of the 5 ft. 2 in. diameter footing is more compatible with the results of the other footing tests. This would seem to indicate that the phase angle measurements for the small footing were indeed in error.

For the prototype study conducted, the response can be represented by Fig. 7 and Eq. (24). It can also be represented by Figs 5 and 6 or Eq. (23) along with the values of $(T.F.)_r$ determined from the phase angle response of the soil-foundation system. It is felt that these analyses and representations shed insight on the estimation of resonant displacement amplitude response of machine foundations supported on cohesive soil and subjected to steady state vibratory loading. The effects of footing size and total mass are included; however, the possible effects of the static stress level are not necessarily included as the static stress was maintained constant for the present study. Additional aspects such as the effects of resonant frequency, static stress level, and response at frequencies other than resonance remain to be investigated.

CONCLUSIONS

Resonant vertical displacement amplitude response of prototype circular footings supported on cohesive soil and subjected to vertical sinusoidal loading of a nature prevalent in machine foundations can be presented using the methods of dimensional analysis in conjunction with kinematic and force parameters in phase diagram representation. The response is nonlinear but can be presented in terms of a non-dimensional resonant amplitude parameter, dynamic stress amplitude transmitted to the supporting soil, and transmission factor of the soil-foundation system in both graphic and analytic form. The physical variables include the size and mass of the footing, applied dynamic force, eccentricity factor, displacement and energy dissipation. Size effects can be conveniently handled in non-dimensional form. Static stress level was maintained constant at 4·25 psi with footing diameters ranging from 5 ft.2 in. to 10 ft. 4 in., total weights from 12,820 lbs. to 51,820 lbs., and applied force amplitudes between 525 lbs. and 52,000 lbs. The results can shed insight into other studies of resonant amplitude response of machine foundations.