ISET Journal of Earthquake Technology, Paper No. 474, Vol. 44, No. 1, March 2007, pp. 1–24

THE EARLY ENTRANCE OF DYNAMICS IN EARTHQUAKE ENGINEERING: ARTURO DANUSSO'S CONTRIBUTION

Luigi Sorrentino
Structural and Geotechnical Engineering Department
Sapienza University, via Antonio Gramsci, 53
00197 Rome, Italy

ABSTRACT

In 1908, a 7.1 equivalent magnitude earthquake struck Southern Italy. During the following year, the scientific community produced a considerable number of contributions, which were also encouraged by two competitions. In both competitions, the highest recognition was given to a Piedmont engineer—Arturo Danusso. Danusso derived the closed form equation of the response of an undamped linear elastic single-degree-of-freedom system to harmonic excitation. Thus, he suggested how to design a one-storey structure in order to minimize the amplification of its ground acceleration. In addition, he studied the case of a two-degree-of-freedom system, determining its two eigenfrequencies and concluding that the motion of each of the two masses can be reduced to the linear combination of the motions of two ideal simple systems subjected to given base motions. Danusso was probably the first to propose a dynamic analysis method rather than static lateral force analysis method and, possibly for the first time in earthquake engineering, he stated that seismic demand does not depend upon the ground motion characteristics alone. Danusso correctly solved the equations, and made some correct statements without writing any equations, as given in this paper. In addition, a brief account on Danusso's life, Italian research after the 1908 earthquake, and Danusso's influence on later Italian earthquake engineering will be presented.

KEYWORDS: Modal Analysis, Southern Calabria-Messina 1908 Earthquake, Early Earthquake Engineering, Arturo Danusso

INTRODUCTION: THE 1908 SOUTHERN CALABRIA-MESSINA EARTHQUAKE AND SUBSEQUENT INITIATIVES

On December 28, 1908, a 7.1 equivalent magnitude earthquake (XI Mercalli Cancani Sieberg Intensity) hit Messina, Reggio Calabria, and Southern Calabria (CPTI04[1]), causing extensive damage and tens of thousands of deaths (Baratta, 1910). On January 12, 1909, the Italian Parliament issued Law No. 12 (Giuffrè, 1987), following which, on 15 January a Royal Decree appointed a panel to single out and suggest earthquake-resistant construction techniques (MLLPP, 1909a). The panel was made up of fourteen engineers, five of which were professors at Italian universities, and an English synopsis of the panel's report can be found on Pages 556–581 of Freeman (1932). It is not possible to follow here all of the committee's work, which spanned damage interpretation, examination of available literature, and, finally and most important, development of one of the first quantitative procedures for the design of earthquake-resistant structures. The procedure became mandatory through the Royal Decree of April 18, 1909 (RI, 1909) and a ministerial memorandum (MLLPP, 1909b).

Concise accounts can be found in Freeman (1932), Housner (1984), Giuffrè (1987), and Bertero and Bozorgnia (2004). It is perhaps significant that this procedure, which was largely a contribution of Modesto Panetti (at that time Professor at the Royal Naval Upper School in Genoa and later key figure at the Technical University in Turin), started from the assumption that any dynamic analysis would be impractical. Therefore, the panel proposed to conventionally substitute dynamic actions with purely static ones in representing seismic effects. This preliminary decision had great impact on subsequent early earthquake engineering in Italy, because it simplified the design procedures but ruled out from the code any dynamic consideration until mid-seventies (MLLPP, 1975), when a design spectrum was introduced (Di Pasquale et al., 2000).

---

[1] Website of CPTI04, http://emidius.mi.ingv.it/CPTI04/

It is perhaps hard to believe that almost contemporary to this original code was an early significant contribution to the use of dynamics for the design of earthquake-resistant structures. As a matter of fact, the Southern Calabria-Messina disaster raised an incredible reaction in public opinion, and specifically in the technical community, and a wealth of contributions was published in scientific journals. For example, whereas in the preceding years only a few communications, notes, or papers followed significant seismic events, in 1909 alone 5 publications were published in "Giornale del Genio Civile", 6 in "Rivista di Ingegneria Sanitaria", 19 in "Annali della Società degli Ingegneri e degli Architetti Italiani", 20 in "Il Cemento", and 26 in "Il Monitore Tecnico". Two competitions also contributed to the flood of publications.

The first competition was called by the Società Cooperativa Lombarda di Lavori Pubblici, under the patronage of the Milan Institution of Engineers and Architects (Manfredini, 1909a, 1909b, 1909c; Anonymous, 1909a, 1909b, 1909c, 1909d, 1909g, 1909h, 1909i). The aim of the contest was to "achieve and apply construction types and systems for civil, rural, and industrial buildings to be adopted in the Italian regions most subjected to seismic shaking and most seismologically dangerous" (Anonymous, 1909g). The deadline for the presentation of the projects was March 31, barely three months after the main shock and prior to the release of the Royal Decree. Two hundred and fourteen proposals were presented (Novelli, 1909), and the panel delivered its decision (Anonymous, 1909h). It is interesting to note that two out of three money prizes went to A. Danusso and G. Revere (the latter together with V. Gianfranceschi), both associate editors of the journal "Il Cemento" (Anonymous, 1909c). The criteria followed by the panel can be found in Manfredini (1909c), Anonymous (1909i), and in Danusso (1960). Danusso's proposal met many of them, with its round raft foundation, centroid kept low by means of lighter infills in the upper storey, and overall good connection through a reinforced concrete two-way frame, which was easier to build in Southern Italy than steel or timber frames. The panel appreciated the calculations accompanying some of the projects because "it is of the outmost interest to give to the builder a guide more rigorous than a simple intuitive criterion, in order to allow the application to this branch of the construction art those calculation and verification methods that form the base of structural mechanics" (Anonymous, 1909i). However, they praised Danusso's contribution much more than any other. Although criticising the modelling of the ground motion through a harmonic function and the small importance attached to the vertical component, the members appreciated the engineer's account of the building deformations under the shaking, something that made him capable of explaining why "an excessive building stiffness does not contribute to its stability" (Anonymous, 1909i). The proposal by Danusso was published in several journal papers (Danusso, 1909c, 1909e).

During the same year, a second competition was called by the Tuscany Institution of Engineers and Architects, on the occasion of the 12th Institution National Conference held in Florence at the beginning of October (Manfredini, 1909d; Losio, 1909; Anonymous, 1909e, 1909f, 1909j, 1909k). The deadline was June 30, but it was later extended to August 31. The impact of this contest was much smaller than the one in Milan, since only 18 projects were submitted. The committee this time was made up by nine engineers designated by local institutions and one delegate of the Agriculture Industry and Trade Ministry. Only two members had been part of the first competition panel. The committee criticized the lack of adequate calculations in many proposals, although the Royal Decree No. 193 was now in force, and the use in a few cases of a steel frame, was considered too expensive to be widely exploited. On this occasion again, Danusso's proposal was the most praised because it was based "on the laws and principles of rational mechanics and theory of elasticity and on the strength of the materials data" (Anonymous, 1909k). During a special "seismological" session, Danusso made also an oral presentation to the conference, "On Earthquake Resistant Constructions", without submitting a written memory (Anonymous, 1909k). This was later published separately in journals (Danusso, 1909d, 1909f, 1910a) and in a 45-page stand-alone volume (Danusso, 1910b). Compared with the Milan proposal, this was widened on the theoretical side (Anonymous, 1909f). Danusso's recommendation of the use of reinforced concrete structures met resistance from many engineers as being a non-local technique that was highly dependent upon the quality of the workmanship (Anonymous, 1909k). However, the Southern Calabria-Messina earthquake and the subsequent Royal Decree, even though it was soon watered down to some extent (RI, 1912; Danusso, 1912a, 1912b; MLLPP, 1921), proved major reasons for the spread of reinforced concrete structures in Italy (Pages 86–93 of Iori (2001)).

ARTURO DANUSSO (1880–1968): BIOGRAPHICAL NOTES

Who was this engineer, whom the Milan Committee addressed with the words: “lucky holder, at the same time, of the most powerful calculation means and of the most effective artifices of the building art” (Anonymous, 1909i)?

Contrary to what is sometimes believed (Page 580 of Freeman (1932); Housner, 1984), when Danusso won the Milan competition, he was not a professor.

Fig. 1 Arturo Danusso (1880–1968) in his office, early 1960s (courtesy: Professor Marco Locatelli)

Born on September 9, 1880 in Priocca d’Alba (Piedmont region, in Northern Italy), Arturo Danusso (Figure 1) lived his first years in Genoa, where his father Ferdinando taught mathematics and physics in a technical high school. At the age of four, he lost his father and moved with his mother, Paolina Dotta, to Turin. He attended Catholic schools (his high school was run by Jesuits), then thanks to a scholarship enrolled in the Civil Engineering College, where he graduated “cum laude” on August 29, 1902. His master, C. Guidi (Vice-President of the Milan competition committee), offered him a position as an assistant. Danusso reluctantly declined because his family’s finances were not flourishing (Danusso, 1978; Cristina Danusso, personal communication). Adhering to his mother’s wishes, he moved to Koblenz for a long stay, thus polishing his German (D’Aquino, 1986; Cristina Danusso, personal communication). His cultural ties with that country always remained strong, even when Italy was at war against it (Danusso, 1916). In 1903, he returned to Turin. After an initial post at the Southern Italy Railway Company in Benevento (Southern Italy), he moved back to Turin in 1905, where he obtained a position in the Porcheddu Enterprise, Italian licensee of the Hennebique reinforced concrete patent (Danusso, 1937). This was the occasion of his first practical experiences and theoretical works, many of which he published in the journal "Il Cemento", established in 1905, where he was associate editor (Anonymous, 1968).

In 1912, Danusso patented the two-way clay reinforced concrete floor "Duplex" (Page 230 of Iori (2001)). In 1915, he won the competition for the chair in Structural Mechanics at the Royal Upper Institute (later Technical University) in Milan, to which he moved (Anonymous, 1915a; Danusso, 1915b). His academic career proceeded jointly with the consulting work. He was also frequently involved in the assessment and the retrofitting of historical constructions, such as the Pisa Tower, the Milan Sant'Ambrogio Belltower, the Turin Mole Antonelliana, the Novara (Piedmont) San Gaudenzio Dome, and the Milan and Pavia cathedrals. He took part also in the design of many large structures such as bridges, dams, power lines, and skyscrapers. Just after the end of World War II, he was elected Milan town councillor and contributed to the reconstruction of the city, and between 1955 and 1959 he was consultant for the Pirelli centre designed by G. Ponti and P.L. Nervi (Page 62 of Desideri et al. (1979); Page 231 of Iori (2001)), carrying out static and dynamic tests on a model.

His research interests were in the field of reinforced concrete structures, investigating their static and dynamic behaviour, which involved testing originally in the Technical University laboratory and then at the Istituto Sperimentale Modelli e Strutture (ISMES, Models and Structures Experimental Institute), which he helped to establish in Bergamo (Lombardy region) in 1951, the year after he retired. He understood the importance of the plastic features of reinforced concrete and exploited them in the design of his buildings (D'Aquino, 1986).

He also developed an initial interest in the theoretical aspects of pre-stressing, recommending (against the risks of a wrong measuring out of the induced forces and their change through time) the resort to the plasticity of reinforced concrete as a natural resource of statically indeterminate structures (Pages 216, 218, 227 of Iori (2001)). Further references on Danusso' scientific publications can be found in D'Aquino (1986) and on Pages 243–248, 250, 261 of Iori (2001), while a selection of more meditative writings—which give testimony of a deep religious feeling—is collected in Danusso (1978). He was member of the Turin Science Academy, of the Lombardy Institution of Science, Literature, and Arts, of the Italian Research National Council (Danusso, 1957), of the Milan Mathematics and Physics Seminary (Danusso, 1927), within which he helped to establish a course on Mechanics of Vibrations in 1949 (Finzi, 1952; Tibiletti Marchionna, 1997). As a matter of fact, Danusso has had the chance to apply his work in the field of structural dynamics such as machine-induced vibrations and wind-induced vibrations (Danusso, 1919, 1952, 1954a, 1954b). In 1967, when he was seriously ill, the "International Center of Earthquake Engineering" was established by the Technical University of Milan and ISMES, under the auspices of UNESCO (United Nations Educational, Scientific and Cultural Organization) and the National Research Council. The centre was named "Arturo Danusso" (Grandori, 1967). Danusso died in Milan on 5 December 1968.

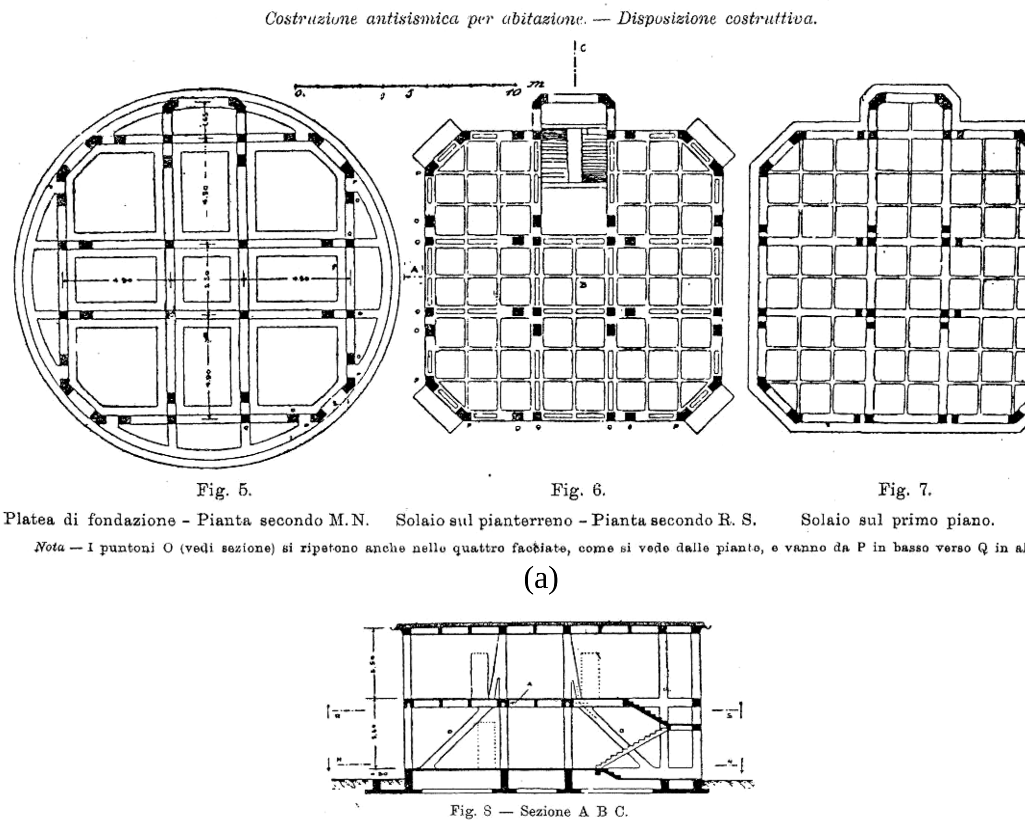
DANUSSO'S 1909–1910 PAPERS

Although the deadline for the Milan competition was barely three months after the Southern Calabria-Messina earthquake, Danusso's proposal was not his first attempt to solve the problem of the "houses that do not collapse" during earthquakes. In a paper with such a title (Danusso, 1909a), published less than twenty days after the shock, he presented many of the ideas that would grant him the highest recognition in the two 1909 contests, and which highlighted the importance of studying the damages caused by severe earthquakes. He had already shown great confidence in two-way reinforced concrete frames, emphasizing which details should be adopted (raft foundation—which he studied in the same year (Danusso, 1909g), columns with increased panel zone height, beams with top and bottom reinforcement, infills growing lighter for the higher storey), and recommending the avoidance of excessive stiffness, a topic on which he would focus later.

The papers related to the two 1909 competitions are Danusso (1909c, 1909d, 1909e, 1909f, 1910a, 1910b). The most comprehensive ones are the Danusso (1910a, 1910b) papers, which are nearly identical to and use almost all of the text from the previous ones.

In Danusso (1909c, 1909e), which followed right after the Milan contest, no equations are to be found. Even so, there must have been some in the submitted contribution, for otherwise the Committee would have complained about the lack of calculations, as they did in other cases. There is, however, a plot (Figure 4(a)), later presented in every one of Danusso's earthquake engineering papers (Danusso, 1909d, 1909f, 1910a, 1910b, 1928, 1931, 1932, 1952, 1954b; on Page 533 of Danusso and Ceruti (1935)) and also to be found, with minor differences, on Page 583 of Freeman (1932). This plot will be discussed later, when the equations at its base are presented. The Danusso (1909c, 1909e) papers conclude with recommendations about buildings' number of storeys and maximum plan size.

The two Danusso (1909c, 1909e) papers are identical, with the latter also having an additional section devoted to the proposed residential building (Figure 2), whose longitudinal cross-section shows the influence of designs by previous practitioners (Danusso, 1909b).



(a)

(b)

Fig. 2 Danusso's earthquake-resistant building, a reinforced concrete frame: (a) Plans at foundation, at first and second level (from Danusso, 1910b); (b) Section

The two Danusso (1909d, 1909f) papers following the Florence competition are again identical. They present the mathematics at the base of Danusso's statements, which will be discussed shortly with reference to the more complete papers published in 1910. The only material not included in the latter is a brief reference to some "in situ" dynamic measurements (Alfani, 1909a, 1909b, 1909c, 1915a), which precede those quoted in Bertero and Bozorgnia (2004).

The two papers published in 1910 are identical, the latter being a reprint of the former in a stand-alone volume. Both are addressed as the "report presented at the 12th Engineers and Architects Conference in Florence in October 1909".

Danusso begins his presentation by stating that he will focus his attention on the ground motions without permanent and severe soil deformation because such ground motions "are beyond the remedies of science. To try to save the building against them would be as to study the safety of trains against the possible collapse of railway bridges" (Danusso, 1910b).

Even without breaking the soil, seismic ground motion is mostly dangerous because it induces vertical and horizontal actions usually not considered, which severely affect the response of columns designed for axial loads only. Already in Danusso (1909c, 1909e) papers there are two key assertions: (1) the intensity of horizontal inertia forces is not governed by ground motion acceleration alone but depends also on the "elastic flexibility of the building skeleton" (Danusso, 1909c); (2) such flexibility strongly influences the intensity of such forces and has some bearing on a rational and cost-effective design. Danusso is aware that "every building is a system of masses more or less stiffly connected" (Danusso, 1909c), with those masses belonging to the superstructure subjected to an acceleration differing from the ground one, due to their inertia and to their connection to the base. It is possible to take advantage of the structure flexibility to attenuate the shaking's sudden effects. "To cut as much as possible this (seismic) energy that will be transferred to the construction—here is, in my opinion, the fundamental standard of seismic building. This will be obeyed by letting the built organ follow the shaking action docilely, not by opposing it stiffly" (Danusso, 1909c).

DYNAMIC CONSIDERATIONS

1. One-Degree-of-Freedom "Elastic Pendulum"

Danusso was keen to leave intuitive suggestions and to compute scientifically the elastic response of a building under a ground shaking. Of the whole edifice, he considers only the structure made up by columns and horizontal floors, constituted by materials reacting to both tension and compression. The columns have no mass, and this is concentrated at floor levels. The vertical component can be separately addressed because, according to Danusso, it induces only a pounding effect that is easy to tackle. The horizontal motion at the base of the building will generate inertia forces at the floor levels.

Danusso's goal is "to determine the laws of motion of the whole superstructure under the combined action of the shake, the inertia of the masses and the elasticity related to the shape of the structure and its materials" (Danusso, 1910b). Once such motion is known it will be possible to look for the "molecular inner stresses in the resisting material" (Danusso, 1910b).

He initially considers a one-storey building, which he assimilates to an "elastic pendulum"—i.e., a mass resting on a vertical massless column clamped to a foundation undergoing horizontal motion.

Following Figure 3(a), he writes (Danusso, 1910b)

$$\mathrm{d}s + \left(f_1 - f \right) = \mathrm{d}x = \mathrm{d}s + \mathrm{d}f \tag{1}$$

with s defined as total displacement of the mass from the initial position, x as ground displacement, $f = kN$ as deflection, k as the flexibility of the pendulum, N as the product of the mass m with its "instantaneous acceleration during its own motion" (Danusso, 1910b).

This leads to

$$N = m\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} \tag{2}$$

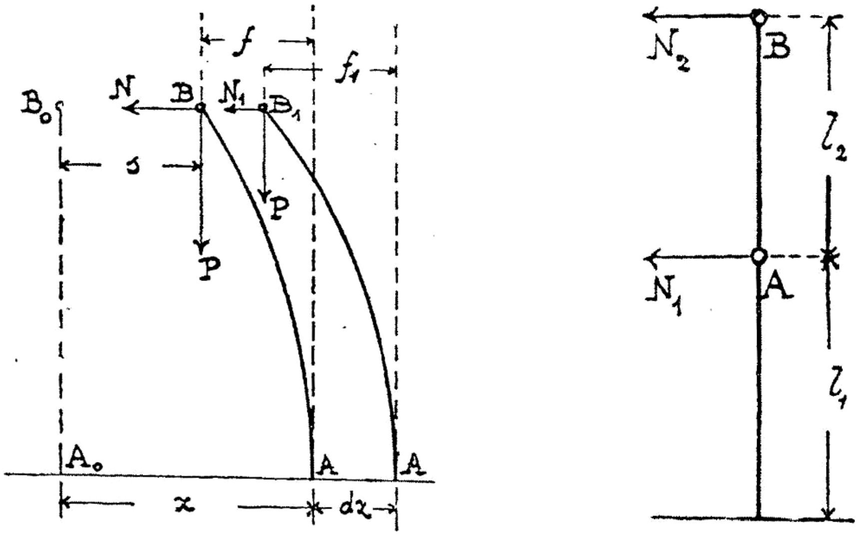
with t being time. Therefore, after integrating once, Danusso gets from Equation (1) the following differential equation (Danusso, 1910b) (also see Appendix II):

$$km\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + s = x\left(t\right) \tag{3}$$

Danusso writes Equation (3) without quoting previous works. He simply states that he made use of "the principle of inertia which dominates the entire dynamics" (Danusso, 1909f), exploited "rational mechanics, and precisely the dynamics of elastic systems" (Danusso, 1909c), and resorted to the "simple combination of D'Alembert's principle with the law of elastic deformations" (Danusso, 1910b). Elsewhere, he states: "the fundamental principle of dynamics informs us that no motion of the building can be thought to be without a system of forces applied to the building elements and such as to resist the motion" (Danusso, 1910b).

No damping is considered. Apparently, this will be considered for the first time in the Italian earthquake engineering literature on Pages 120, 192, 206–207 of Giannelli (1932), who however quotes R. Sano, "On the Vibration of Steel Frame Buildings", without adding a more complete reference. Viscous damping had already been considered on Pages 45–46 of Rayleigh (1877), and possibly even before, who presented the equation of motion in terms of relative displacements. Moreover, Rayleigh (1877) obtained the equation of motion using potential and kinetic energies, in a Lagrange approach (on

Pages 43–45). In contrast, Danusso gets his equation by writing a compatibility equation. Therefore, Danusso probably did not consider this precedent.



(a) (b)

Fig. 3 (a) Simple elastic pendulum; (b) Double elastic pendulum (Danusso, 1910b)

Whereas today it is more common to write the equation of motion using relative displacements and accelerations, a chance that Danusso suggested elsewhere (Danusso, 1928), he preferred to make reference to total (absolute) parameters. The reason for such a choice lies in the fact that Danusso is looking for a force to be used in a static design of the structure. Therefore, inertia force, related to total and not relative accelerations, is what he is interested in. By the way, the advantage of using total quantities has been highlighted also more recently (Uang and Bertero, 1990). Furthermore, at that time only seismograms (i.e., displacement time histories) were available. The first acceleration time history was recorded in 1933, during the Long Beach (California, USA) earthquake (Marletta, 1934; Hudson, 1983).

Danusso (1910b) presents the solution for a generic ground motion as follows (also see Appendix II):

$$s = -\omega \cos \omega t \int_0^t x(t) \sin \omega t \,\mathrm{d}t + \omega \sin \omega t \int_0^t x(t) \cos \omega t \,\mathrm{d}t \tag{4}$$

with $\omega = 1/\sqrt{km}$, a natural circular frequency that he calls “elastic constant”, obtained after setting “at rest” initial conditions. Thus,

$$s(t=0) = \frac{\mathrm{d}s}{\mathrm{d}t}(t=0) = 0 \tag{5}$$

He then considers a ground displacement (Danusso, 1910b)

$$x(t) = r(1 - \cos \alpha t) \tag{6}$$

with r being amplitude and α being the exciting circular frequency. Danusso denotes this ground motion “without initial impact” since initial ground velocity is zero. Probably Danusso selected this ground motion instead of $x(t) = r \cos \alpha t$ so that both ground displacement and velocity are initially equal to zero (compared to Equation (10)).

The correct solution, he wrote, is (Danusso, 1910b)

$$s = r\left[1 + \frac{\cos \omega t - \rho^2 \cos \alpha t}{\rho^2 - 1}\right] \tag{7}$$

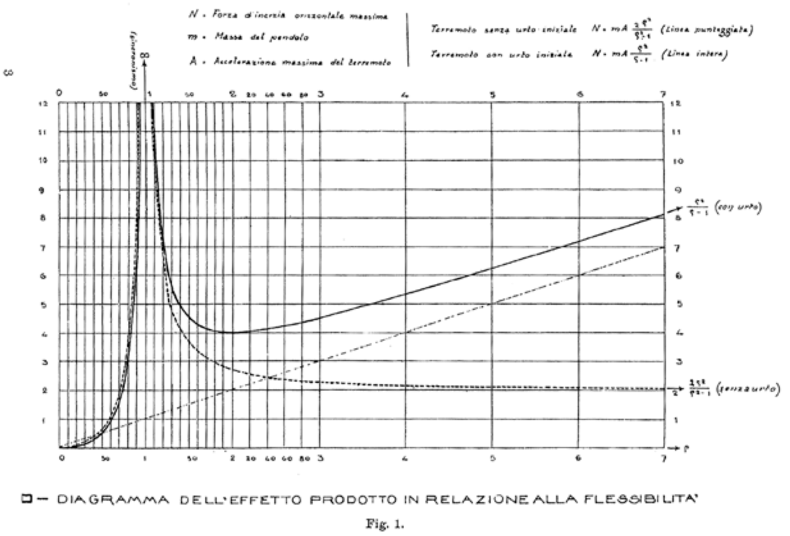
with $\rho = \omega / \alpha$. Therefore, the acceleration undergone by the mass is (Danusso, 1910b)

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = \frac{r\omega^2}{\rho^2 - 1}\left(\cos\alpha t - \cos\omega t\right) \tag{8}$$

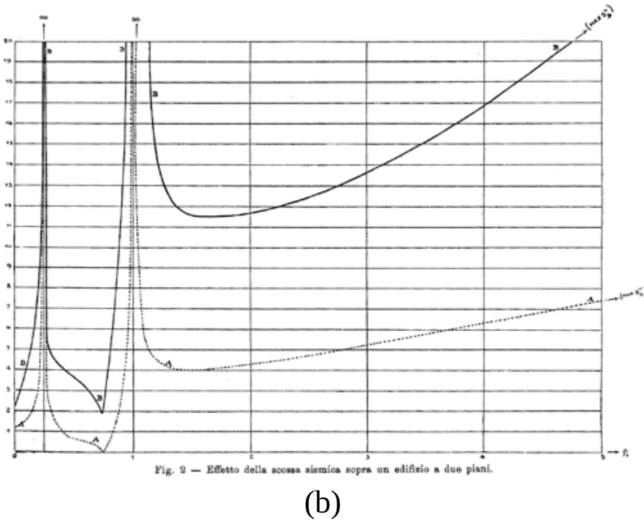
Considering a time t_1 when $\cos\omega t_1 = -1$ and $\cos\alpha t_1 = 1$, the ratio μ of maximum inertia acceleration to the maximum ground acceleration $r\alpha^2$ is indeed (Danusso, 1910b)

$$\mu = \frac{2\rho^2}{\rho^2 - 1} \tag{9}$$

Danusso (1928) will later call the ratio of Equation (9) the “rapporto sismico” (seismic ratio). Because ρ is the only parameter governing μ, Danusso (1910b) calls it “characteristic”. Absolute value of Equation (9) is the one represented in Figure 4(a) with a dashed line.



(a)



(b)

Fig. 4 (a) Seismic ratio under harmonic excitations (Danusso, 1910b) (on the horizontal axis is the “characteristic” ρ, and on the vertical axis is the “seismic ratio” μ; the original caption means: “diagram of the effect [of the ground shaking] in relation to the flexibility”); (b) Shake effect on a two-storey building (Danusso, 1910b)

If the ground displacement is the function

$$x(t) = r\sin\alpha t \tag{10}$$

which Danusso (1910b) denotes as "with an initial maximum impact", the solution he rightly worked out is (Danusso, 1910b)

$$s = \frac{r\omega}{\omega^2 - \alpha^2}\left(-\alpha \sin \omega t + \omega \sin \alpha t\right) \tag{11}$$

Differentiating s two times with respect to time and considering similarly a time t_1 when $\sin \omega t_1 = 1$ and $\sin \alpha t_1 = -1$, μ is indeed (Danusso, 1910b)

$$\mu = \frac{\rho^2}{\rho - 1} \tag{12}$$

Absolute value of Equation (12) is the one represented in Figure 4(a) with a solid line.

To understand why sinusoidal ground displacement induces such amplifications for increasing ρ, let us consider the now customary equation of motion of a mass-spring system to be found in textbooks (e.g., Pages 1–19 of Timoshenko (1928); Pages 113–115, 130–139 of von Kármán and Biot (1940)). Such an equation is

$$\frac{d^2 y}{dt^2} + \omega^2 y = r\alpha^2 \cos \alpha t \tag{13}$$

with $y = s - x$ taken as the relative displacement and with excitation obtained by differentiating two times the ground displacement in Equation (6). In this case, the "at rest" initial conditions for the total mass displacement equation of motion involve the "at rest" initial conditions for the relative mass displacement equation of motion as well. Therefore, it is easy to show that the solution is of the form:

$$y = \frac{r\alpha^2}{\omega^2 - \alpha^2}\left(-\cos \omega t + \cos \alpha t\right) \tag{14}$$

Hence, the "seismic ratio," with symbols consistent with those previously used, is

$$\mu = \frac{\max|\ddot{y}|}{\max|\ddot{x}|} + 1 = \left|\frac{\rho^2 + 1}{\rho^2 - 1}\right| + 1 = \left|\frac{2\rho^2}{\rho^2 - 1}\right| \tag{15}$$

with the dots representing the derivatives with respect to time. This is coincident with Equation (5).

However, if one considers the sinusoidal excitation of Equation (10), the equation of motion becomes

$$\frac{d^2 y}{dt^2} + \omega^2 y = r\alpha^2 \sin \alpha t \tag{16}$$

Anyway, in this case the initial conditions are

$$\begin{aligned} y(0) &= s(0) - x(0) = 0 \\ \dot{y}(0) &= \dot{s}(0) - \dot{x}(0) = -r\alpha \end{aligned} \tag{17}$$

Therefore, the solution is

$$y = \frac{r\alpha^2}{\omega^2 - \alpha^2}\left(-\rho \sin \omega t + \sin \alpha t\right) \tag{18}$$

Hence, the "seismic ratio" is

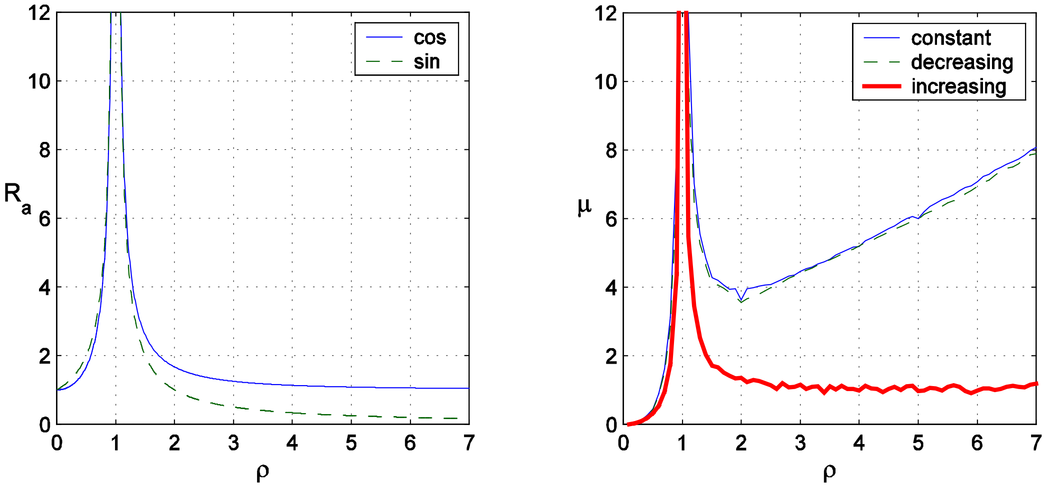
$$\mu = \left|\frac{\rho^3 + 1}{\rho^2 - 1}\right| + 1 = \left|\frac{\rho^2}{\rho - 1}\right| \tag{19}$$

which is coincident with Equation (12).

A mass-spring-damper system, in which transient response is neglected, has the same (relative) acceleration response factors, R_a, whether the excitation is cosinusoidal or sinusoidal (e.g., refer to Pages 75–78 of Chopra (1995) where, with symbols consistent with those here previously used, $\mu = \left|1/\left(\rho^2 - 1\right)\right|$). On the contrary, the mass-spring system considered by Danusso has two different response factors. These are

$$R_{a,\cos} = \left| \frac{\rho^2 + 1}{\rho^2 - 1} \right| \qquad R_{a,\sin} = \left| \frac{1}{\rho - 1} \right| \tag{20}$$

which are plotted in Figure 5(a).



(a) (b)

Fig. 5 (a) Relative acceleration response factor for an undamped linear elastic single-degree-of-freedom oscillator subjected to harmonic excitation; (b) Danusso's "seismic ratio" μ as a function of system natural frequency to excitation frequency ratio ρ, for an undamped linear elastic single-degree-of-freedom oscillator initially at rest subjected to sinusoidal excitations of constant, linearly increasing and linearly decreasing amplitudes

Danusso comments on the plot of Figure 4(a) at length, and it is perhaps noteworthy that no analogous plot is to be found in Rayleigh (1877, 1894). The caption under the plot is "Diagram of the effect [of the ground shaking] in relation to the flexibility". On the horizontal axis is the "characteristic" ρ, and on the vertical axis is the "seismic ratio" μ.

Danusso notes that the system, due to its proper characteristics, can reduce or increase the ground acceleration. This is of the utmost interest—possibly for the first time in earthquake engineering it is stated, based on a mathematical demonstration, that the seismic demand does not depend upon the ground motion characteristics alone but also on those of the structure. For Danusso, this is also the explanation of the very wide differences observed in the behaviour of buildings after severe earthquakes. He discusses the "synchronism" (Danusso, 1909c) condition $\rho = 1$, stating that although it is theoretically very worrying it is actually rather unlikely because earthquake "shakings are all short and moreover changing" (Danusso, 1909c).

As a practical example, Danusso then assumes an amplitude of 2.5 m/s^2 and a period of 1/3 s. He also states that the range of ρ from 0 to 1/2 is virtually not obtainable because this would be valid for a steel frame with columns that are designed for buckling but therefore are unable to carry any load when displaced from rest. Therefore, only values greater than 1 should be considered, and only such values are plotted in the figure to be found on Page 583 of Freeman (1932), and the case $\rho = 2$ should be preferred because a local minimum is located in the "impact" case. Higher values of the "characteristic" would be theoretically better for the ground motion "without impact", but they would involve significant amplifications in the other case which, even if resisted by means of the higher resistance, can involve global lack of equilibrium of the system. Based on this plot, Danusso maintains that an excessively stiff building "would be truncated by the actions of an impact", and that he does "not believe that absolute stiffness is the goal toward which every earthquake-resistant construction should tend" (Danusso, 1909c). Moreover, such a solution would bear unnecessarily higher costs. Values around 2 can be gained even with the more (compared to steel) stiff reinforced concrete frame, which is Danusso's preferred solution. The recommended design procedure is to assume an acceleration that is four times the ground ones, to dimension the structure, and then to verify whether the value of ρ is reasonably close to 2. Otherwise, a second attempt should be sufficient.

Danusso (1910b) says that he also considered the case of "an initial impact with given velocity and a steady-state motion continuing with the same velocity. The pendulum oscillates regularly with a constant period, depending not upon the impact velocity (on this only the amplitude is dependent) but uniquely on the elastic properties of the column—i.e., on the material nature, its shape, and the borne load. And if, while pursuing the analogy with an ordinary compound pendulum, the reduced length of an elastic pendulum is sought, it is easy to prove that this is equal to the elastic deflection that the column would undergo when the top load is applied horizontally".

Danusso (1910b) also studies the response to a generic ground motion displacement time history, recorded by a seismograph. He is persuaded that "any motion, even if complex, can be reduced to the superposition of simpler elementary motions". Considering the time history as a series of sinusoidal "waves", the response to each can be obtained by means of Equation (11) and by determining the values of the constants of integration in order to match the position and velocity condition at the end of the previous wave. For example, at the end of the first full sine "wave", the position, S_1, and velocity, V_1, of the system are (Danusso, 1910b)

$$\begin{aligned} S_1 &= -\frac{r_1\omega^2}{\omega^2-\alpha_1^2}\frac{\alpha_1}{\omega}\sin\left(2\pi\frac{\omega}{\alpha_1}\right) \\ V_1 &= \frac{r_1\omega^2\alpha_1}{\omega^2-\alpha_1^2}\left[1-\cos\left(2\pi\frac{\omega}{\alpha_1}\right)\right] \end{aligned} \tag{21}$$

Then Danusso says: "In order to simplify the expressions, it is convenient to take a new time origin at the beginning of every wave".

Danusso (1910b) writes also that he considered the response of the system to sinusoidal motions of increasing or decreasing amplitudes, and he states that in the first case he found "a lesser effect" compared with that of a constant amplitude, while in the second he found "a not greater and sometimes equal effect". He does not add any equation. However, if the ground motions are

$$\begin{aligned} x(t) &= \frac{t}{t_2}r\sin\alpha t \\ x(t) &= \left(1-\frac{t}{t_2}\right)r\sin\alpha t \end{aligned} \tag{22}$$

with t_2 being the end of the time interval considered and with "at rest" initial conditions being considered, it is possible to prove that the solutions are

$$\begin{aligned} s &= \frac{r\omega^2\left\{2\alpha\left(\cos\omega t-\cos\alpha t\right)+\left(\omega^2-\alpha^2\right)t\sin\alpha t\right\}}{t_2(\alpha-\omega)^2(\alpha+\omega)^2} \\ s &= \frac{-r\omega^2\left\{2\alpha\left(\cos\omega t-\cos\alpha t\right)+\left(\omega^2-\alpha^2\right)\left[(t-t_2)\sin\alpha t+\dfrac{\alpha}{\omega}t_2\sin\omega t\right]\right\}}{t_2(\alpha-\omega)^2(\alpha+\omega)^2} \end{aligned} \tag{23}$$

respectively. By deriving the expressions in Equation (23) two times and by normalizing with respect to ground acceleration amplitude, the "seismic ratios" are obtained as

$$\mu_{\text{incr}} = \left| \frac{\rho^2 \left\{ \frac{4}{n}\rho^2 + \left(1-\rho^2\right) \right\}}{\left(1-\rho\right)^2 \left(1+\rho\right)^2} \right|$$
$$\mu_{\text{decr}} = \left| \frac{\rho^2 \left\{ \frac{4}{n}\rho^2 + \left(1-\rho^2\right) \left[\left(1-\frac{t_1}{t_2}\right) + \rho \right] \right\}}{\left(1-\rho\right)^2 \left(1+\rho\right)^2} \right| \tag{24}$$

with $n = \alpha t_2$. Hence, for $t_2 >> 1/\alpha$ the first term in the curly braces vanishes, and for $t_2 >> t_1$ the "seismic ratios" reduce to

$$\mu_{\text{incr}} \cong \left| \frac{\rho^2}{\rho^2 - 1} \right|$$
$$\mu_{\text{decr}} \cong \left| \frac{\rho^2}{\rho - 1} \right| \tag{25}$$

which are indeed minor or equal, respectively, to Equation (12). The "seismic ratios" have also been computed numerically, again verifying Danusso's statement (Figure 5(b)).

2. Two-Degree-of-Freedom, Two-Storey Building

Probably in the framework of the newly issued seismic code, which limited to two, the number of stories of any building, Danusso (1910b) then considers a two-storey structure as a two-degree-of-freedom system (Figure 3(b)). He writes two coupled differential equations, analogous to Equation (3) (also see Appendix I and Appendix II):

$$k_1\, m_1 \frac{d^2 s_A}{dt^2} + k_2\, m_2 \frac{d^2 s_B}{dt^2} + s_A = x(t)$$
$$k_2\, m_1 \frac{d^2 s_A}{dt^2} + k_3\, m_2 \frac{d^2 s_B}{dt^2} + s_B = x(t) \tag{26}$$

with k_1, k_2 and k_3 being constants to be determined by means of "Virtual Works Theory" or "Elasticity Ellipse" and the "Reciprocity Law, whatever the degree of restraint granted by the column in *A* and *B*", so that (Danusso, 1910b)

$$f_A = k_1 N_1 + k_2 N_2$$
$$f_B = k_2 N_1 + k_3 N_2 \tag{27}$$

Danusso (1910b) then presents, without intermediate development, the two "elastic constants (similar to ω)" ξ and η (also see Appendix I):

$$\xi^2 = \frac{B - \sqrt{B^2 - 4A}}{2A}$$
$$\eta^2 = \frac{B + \sqrt{B^2 - 4A}}{2A} \tag{28}$$

with

$$A = m_1 m_2 \left(k_1 k_3 - k_2^2\right)$$
$$B = k_1 m_1 + k_3 m_2 \tag{29}$$

ξ and η are the roots of the polynomial, quadratic in ω_n^2, of the eigenvalue problem:

$$\left([km]-\frac{1}{\xi_n^2}[I]\right)\{\phi_n\}=0 \tag{30}$$

with ξ_n and $\{\phi_n\}$ being the *n*th eigenvalue and (right) eigenvector, respectively (in today's, not Danusso's, terminology); square brackets being used for the square matrices; [*I*] as the identity matrix; and curly braces indicating a column vector. The [*km*] matrix is equal to

$$[km]=\begin{bmatrix} k_1m_1 & k_2m_2 \\ k_2m_1 & k_3m_2 \end{bmatrix} \tag{31}$$

It is important to stress that the orthogonality of modes does not hold with respect to the [*km*] matrix. This is due to the fact that the problem is not self-adjoint. Danusso (1910b) does not present the matrix of (right) eigenvectors. However, he writes that the two functions s_A and s_B are expressed in finite terms as follows:

$$s_A=\frac{1}{\eta^2-\xi^2}\left[\frac{(R_A)}{\xi^2}-\frac{(S_A)}{\eta^2}\right] \tag{32}$$

$$s_B=\frac{1}{k_2m_2\left(\eta^2-\xi^2\right)}\left[\left(m_2k_3-\frac{1}{\eta^2}\right)\frac{(R_A)}{\xi^2}+\left(\frac{1}{\xi^2}-m_2k_3\right)\frac{(S_A)}{\eta^2}\right] \tag{33}$$

with (R_A) and (S_A), respectively, being the expressions of the distance of two equivalent pendulum masses from the initial position in any instant. Therefore, from the last two equations, Danusso's matrix $[\phi]$ of right eigenvectors, never explicitly presented, is

$$[\phi]=\frac{1}{\eta^2-\xi^2}\begin{bmatrix} \dfrac{1}{\xi^2} & -\dfrac{1}{\eta^2} \\ \dfrac{k_3m_2-\dfrac{1}{\eta^2}}{k_2m_2\xi^2} & -\dfrac{k_3m_2-\dfrac{1}{\xi^2}}{k_2m_2\eta^2} \end{bmatrix} \tag{34}$$

which is indeed the eigenvector matrix associated with the matrix in round brackets in Equation (30). According to Danusso (1910b), (R_A) and (S_A) can be determined as the total displacements of two simple pendulums with "elastic constants (similar to ω)" ξ and η, dragged at the foot, respectively, by the motions (also see Appendix I and Appendix II):

$$\begin{aligned} F_1(t)&=\frac{1}{A}\left[1-\xi^2m_2\left(k_3-k_2\right)\right]x(t) \\ F_2(t)&=\frac{1}{A}\left[1-\eta^2m_2\left(k_3-k_2\right)\right]x(t) \end{aligned} \tag{35}$$

Therefore, Danusso (1910b) clearly uncoupled the two expressions in Equation (26), without explaining how he acted. In order to uncouple the problem and diagonalize [*km*], it is necessary to compute the matrix of the left eigenvectors—i.e., the eigenvectors of the transpose of the matrix in round brackets in Equation (30), or, equivalently, the columns of $[\phi]^{-1}$. Therefore, the matrix $[\psi]$ of left eigenvectors is equal to

$$[\psi]=\eta^2\xi^2\begin{bmatrix} -\left(\xi^2k_3m_2-1\right) & -\left(\eta^2k_3m_2-1\right) \\ \xi^2k_2m_2 & \eta^2k_2m_2 \end{bmatrix} \tag{36}$$

Let us consider the variable transformation

$$\{s\}=[\phi]\{R\} \tag{37}$$

with $\{R\}$ being the vector of new coordinates, and then pre-multiply by $[\psi]^T$ each term in Equation (26). Thus,

$$[\psi]^T [km]\ [\phi]\frac{d^2R}{dt^2} + [\psi]^T [I]\ [\phi]R = [\psi]^T \{1\} x(t) \tag{38}$$

with {1} being the vector of ones. The expressions in Equation (26) thus become

$$\begin{aligned} \frac{d^2R_A}{dt^2} + \xi^2 R_A &= \xi^2\eta^2\xi^2\left[1-\xi^2 m_2(k_3-k_2)\right]\ x(t) \\ \frac{d^2S_A}{dt^2} + \eta^2 S_A &= \eta^2\eta^2\xi^2\left[1-\eta^2 m_2(k_3-k_2)\right]\ x(t) \end{aligned} \tag{39}$$

Because $\eta^2\xi^2 = 1/A$, the manipulation presented in Equation (35) is correct, and indeed Equation (39) can be regarded as equivalent to the already solved Equation (3). That is why Danusso (1910b) concluded that "the motion of each of the two masses of the double pendulum can be reduced to the linear combination of the motions of two ideal simple pendulums subjected to given base motions and characterised by given elastic constants". Such a procedure falls fully within what is defined as modal analysis in standard textbooks (Page 159 of Meirovitch (1986)).

Danusso (1910b) did not quote any reference about how he solved the problem. The solution for self-adjoint problems had already been presented on Pages 360–365 of Kelvin and Tait (1867) and on Page 107 of Rayleigh (1877). Moreover, a German translation of both books, probably easier for Danusso to understand, was available (Kelvin and Tait, 1871; Rayleigh, 1879). Besides, the study of a three-degree-of-freedom system was already available in the Italian literature (Levi-Civita, 1896). As for problems that are not self-adjoint, at least from a mathematical point of view, the solution must have been known. Probably Danusso was more aware of the mathematics of the problem (Pages 327–366 of Lie (1888)) than of the previous treatments by Kelvin and Rayleigh; otherwise, he would have adopted a more synthetic and efficient symbolism, already present in the two Britons' very comprehensive treatises. He also did not develop any further a method in the direction of the response spectrum method. In any case, the methodology of Danusso was probably the first application of modal analysis to earthquake engineering.

Danusso (1910b) did not study explicitly the response of the double pendulum to the kind of ground motions previously considered. However, he presented a plot (Figure 4(b)), without specifying the values of the parameters and the ground motion he assumed. He stressed that "synchronism" happened for two different values of the abscissa and that "the curves of maximum accelerations of masses m_1 and m_2 reach a secondary minimum and then climb up again slowly, leaving to the designer of two-storey houses that freedom that we asked for since the beginning".

Several attempts have been made to reproduce Figure 4(b). Defining

$$\rho_1 = \frac{\xi}{\alpha}\ ;\ \rho_2 = \frac{\eta}{\alpha} \tag{40}$$

and

$$\mu_1 = \left|\frac{\ddot{s}_{A,\max}}{r\alpha^2}\right|\ ;\ \mu_2 = \left|\frac{\ddot{s}_{B,\max}}{r\alpha^2}\right| \tag{41}$$

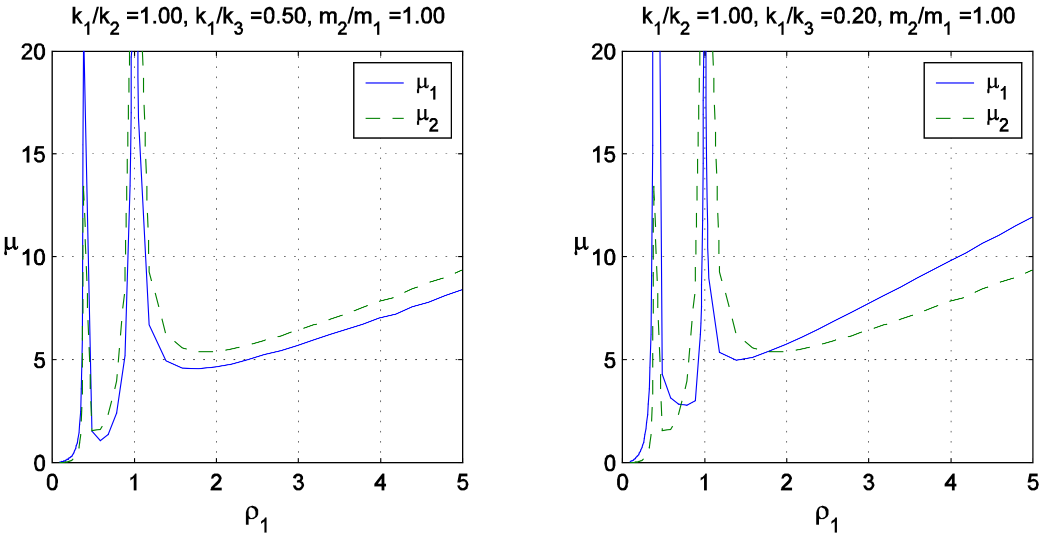
and considering a ground displacement time history $x(t) = r\sin\alpha t$, plots of Equation (41) have been computed, and some are presented in Figure 6. Although similar to Figure 4(b), they do not match it. These curves have been numerically worked out because the problem is undamped and seemingly it is impossible to calculate in closed form the response factors. It is perhaps noteworthy that Danusso did not present this plot again in his later papers, as he did with that in Figure 4(a).

DANUSSO'S LATER PAPERS AND HIS INFLUENCE

During September 1–6, 1930, Danusso attended the first international conference on concrete and reinforced concrete in Liege, Belgium. At the conference, he presented a paper on earthquake-resistant constructions. A brief paper in French is included in the proceedings (Danusso, 1932), while a longer one was published in a volume containing the full version of all the Italian manuscripts (Danusso, 1931), which is the same as a previously published paper (Danusso, 1928) quoted on Page 580 of Freeman (1932).

In the 1928 paper, Danusso presents again the results obtained in 1909–1910 using a more effective symbolism and adding new results. In particular, he studies the case of a mass-spring system under an excitation that is the product of two sines or of a sine and a cosine function, showing that they are less dangerous than the ones previously studied. He then considers a system with *n* degrees of freedom, generalising the solution already found for the two degrees of freedom already shown and stressing again that "it is easy to recognise in the motion of any mass of an *n*-tuple pendulum a linear combination of the motions of *n* simple pendulums" (Danusso, 1928). Finally, he studies the case of an elastic prismatic and homogenous tower, which was later developed by Bertolini (1935).

In his last paper on earthquake-resistant constructions (Danusso, 1946), Danusso reconsiders the results previously obtained, uses a terminology more customary today when he writes of "eigenvalues", and makes references to the works by T. Levi-Civita, Rayleigh, and G. Krall, but there are no significant new results. Apparently Danusso was not aware of the research developed in the meantime by Biot (1932, 1933, 1934) (Trifunac, 2003). This is proved also by his last study on building vibrations induced by machines, wind and earthquakes (Danusso, 1952), where he has summarized his and other researchers' results.



(a) (b)

Fig. 6 Plot of Equation (41) for a shear-type frame with (a) floors with the same height, mass, and columns, and (b) with the lower floor stiffer than the other (in comparison with Figure 3(b))

All these papers show the continuous recurrence of a sincere interest in the topic, which was still something episodic in the Italian landscape of those years.

Danusso's proposal, backed at the 12th Conference of Italian Engineers and Architects, to establish a permanent institute on "seismotecnic" studies (Anonymous, 1909f, 1909k) was not listened to (Danusso, 1912a, 1912b, 1915a). Similarly, his proposal to visit Yokohama after the 1923 Kanto earthquake was not funded (Cristina Danusso, personal communication). In contrast, after the 1923 event the Earthquake Research Institute was established in Japan (Reitherman, 2006a, 2006b). This partially explains the decay of studies on earthquake engineering in Italy for many decades. Other reasons were

- The death of some key figures: in 1914, Italo Maganzini (Luiggi, 1914), President of the Committee appointed to write what then became the 1909 Royal Decree No. 193; and in 1918, Silvio Canevazzi (Revere, 1918a, 1918b), who worked with Modesto Panetti on that Decree (MLLPP, 1909a) and then developed a simplified design method (Canevazzi, 1913a, 1913b; Page 579 of Freeman (1932)).

- The switch of interest of some others: Panetti was more and more attracted by aeronautics—probably he found that there was more glory there (Marzolo, 1957).
- The occurrence of the large January 15, 1915 Avezzano (Central Italy) earthquake when World War I had already started and a few months before Italy joined it. This event again raised some interest, resulting in some bulky publications (Masciari-Genoese, 1915; Alfani, 1915b; Anonymous, 1915b), but without the scientific impetus observed in 1909 (Danusso, 1915c, 1915d, 1915e, 1915f) and without the same extension as in 1909. As a matter of fact, in 1915 two papers were published in "Giornale del Genio Civile", two in "Rivista di Ingegneria Sanitaria", four in "Annali della Società degli Ingegneri e degli Architetti Italiani", three in "Il Cemento", and four in "Il Monitore Tecnico".
- The appointment of a committee after that 1915 event with only one academic, Roman Cesare Ceradini, who was a member also of the 1909 committee (MLLPP, 1915).

Isolated papers or textbooks on earthquake engineering can be found during the 1920s and 1930s. For example, the July 23, 1930 Vulture earthquake (Pages 513–555 of Freeman (1932)) helped to revive attention to the subject.

Camillo Guidi showed a constant interest in earthquake-resistant construction, both studying earthquake effects (Guidi, 1925, 1926a), including applications to elastic frames in his textbooks (Guidi, 1926b, 1930), and performing possibly the first experimental tests in Italy on a reinforced concrete frame (Guidi, 1927), subjected to horizontal forces.

Danusso's work was certainly known. In his very successful reinforced concrete textbook, Santarella (1926) made reference on Pages 398–400 to Danusso's 1909 papers, although the examples presented in the case studies used the approach proposed in the code developed by Panetti.

Panetti (Page 157 of MLLPP (1913); Panetti, 1914, 1916) praises the achievements the Piedmont engineer made by means of dynamics. Panetti, commenting on a lightly amended version of Royal Decree 1909 (RI, 1913), suggested assuming a higher seismic coefficient for higher stiffness (Page 163 of MLLPP (1913)), but such an indication did not become part of the code and, therefore, its influence on practical design was negligible.

Priolo (1930, 1931) was aware of Danusso's 1909 papers and quotes his conclusions, although he states that at that time there was no experimental definition of the inertia acceleration as a function of the ratio between building frequency and ground motion frequency. Therefore, in the design examples in his works he follows the equivalent static procedure of the Italian seismic code. It was only after World War II that Priolo (1951) made reference to the dynamic behaviour of structures to guide their earthquake-resistant design. However, his work was then influenced by American literature.

Giannelli, in his textbook, briefly quotes Danusso's research and notes the importance of the dynamic issues in earthquake-resistant design (Page 120 of Giannelli (1932)). Although he presents the study of multi-degrees-of-freedom plane frames with an approach similar to that in Danusso (1928) and discusses the role of damping on the building periods, finding also the solution for damped vibrations under harmonic excitation, he does not present any example among the many numerical exercises he carries out.

Krall (1940) mentioned Danusso's work on Page xvi of Volume 1 and Page 422 of Volume 2, and considered explicitly earthquake-induced inertia forces in his proposed design of two steel power line pylons on the Messina strait (Krall, 1947, 1948; Priolo, 1948). However, the open competition for the final design and the construction of those towers was won by the Società Anonima di Elettrificazione, whose leading consultant was Danusso (Anonymous, 1952, 1955, 1956; Toscano, 1958, 1960). The towers, built between 1952 and 1955, have a height of 224 m and a span of 3646 m, which was the world's largest at the time. In 1958 the public work was awarded by the Italian National Institution of Engineers and Architects (Anonymous, 1958).

Danusso started the design process in 1945. As for earthquake related issues, he modelled the structure as a two lumped masses system. Although a dominant period between 0.25 and 1.5 s was expected, Danusso designed the structure to withstand a transient harmonic excitation, lasting five pulses, with same period as that of the tower (2.5 s), and amplitude of 2.5 m/s^2. He also performed shake-table tests on a 1:25 scale model.

As a recognition to Danusso's work in the field, he was asked to deliver the introductive keynote lecture in the Conference held in Messina for the 50th anniversary of the 1908 earthquake (Danusso, 1960; Anonymous, 1960a, 1960b). The conference was attended, among others, by G.W. Housner, L. Jacobsen, H. Kawasumi, K. Kubo, and K. Takeyama.

Danusso also used to hold lectures on earthquake-resistant constructions for a wider, not necessarily technical, audience, such as at Rotary meetings (Cristina Danusso, personal communication). However, as already mentioned, it was only in the mid-1970s that the stiffness of the structure was taken into account in the Italian seismic code, when a simplified design spectrum was introduced.

Even smaller was probably Danusso's international influence. Although Freeman was aware of the outline of his research, there is no evidence that M.A. Biot, at the time of Liege 1930 conference studying at Louvain (Belgium), had any knowledge (Biot, 1932, 1933, 1934). During the discussion following the Liege conference session in which Danusso's paper was presented, no reference was made to it, and apparently the only U.S. participant at the conference was C.C. Fishburn from the Bureau of Standards, Department of Commerce, Washington, D.C.

CONCLUSIONS

In the introduction to his 1910 papers, Arturo Danusso writes that "the studies on earthquake-resistant buildings until now, and with very few exceptions, produced a series of personal criteria, more or less empirical, which deserve nonetheless great respect, since a good building relies for three quarters on the designer's common sense and experience, but they do not suffice to solve scientifically the problem that nowadays interests all the Italians" (Danusso, 1910b). The research conducted by him after the 1908 Southern Calabria-Messina earthquake represents a major turning point in the field of earthquake engineering: Danusso was probably the first to propose a dynamic analysis method rather than static lateral force analysis method and to apply modal analysis. Moreover, he proved scientifically that seismic demand does not depend upon the ground motion characteristics alone. He showed that he possessed the mathematical knowledge to solve differential equations and their linear systems. His considerations about the need to account for the dynamic properties of a building in its design, and the fact that a linear elastic *n*-degrees-of-freedom system can be considered as equivalent to *n* single-degree-of-freedom oscillators, are right and of the greatest interest. However, his quantitative design recommendation suffered from the lack of reliable, strong ground motion recordings. Danusso's consideration of a ground motion "with an initial impact" perhaps looks as too severe, unless it is very close to the fault where the detailed nature of the ground motion cannot be predicted and large velocities can develop. Thus, his design recommendations already seemed exaggerated to Panetti (MLLPP, 1913) and Bertolini (Bertolini, 1935). Apparently the lack of systematic experimental research, both on buildings and on seismic ground motion, as well as the absence of interest within the academic community and the lack of practical applications of his mathematical results, caused the most advanced results of his research to be forgotten. All of these aspects show the importance of a constant and widespread effort to gain higher results, and how isolated, although excellent, achievements can be forgotten.

ACKNOWLEDGEMENTS

I would like to thank Professor Cristina Danusso De Pol, who kindly has given me access to her grandfather's biographical notes ("A Little History of Arturo"), the 45 pages typewritten by his daughter Eleonora, probably around 1963 and 1966. She also brought to my attention some of Danusso's late works I was not aware of. Moreover, I am deeply indebted to Professor Mihailo D. Trifunac, who kindly encouraged me to write this paper.

APPENDIX I: POSSIBLE MISPRINTS IN DANUSSO'S PAPERS

In the following are reported actual equations found in Danusso's papers. In this paper they have been interpreted as affected by misprints. The first expression in Equation (28) (also in Danusso, 1910a) was actually written as

$$\xi^2 = B - \frac{\sqrt{B^2 - 4A}}{2A} \tag{A.1}$$

In Equation (26) (also in Danusso, 1910a) Danusso actually wrote

$$k_2\, m_1 \frac{d^2 s_A}{dt_2} + \ldots \tag{A.2}$$

In Equation (35) (contrary to Danusso, 1910a) the second expression actually is

$$F_2(t) = \frac{1}{A}\left[1 - \eta^2 m_2 (k_3 - k_2) x(t)\right] \tag{A.3}$$

APPENDIX II: CHANGES IN DANUSSO'S ORIGINAL SYMBOLS

Danusso's presentation has been retained herein almost every time, in order to make a complete assessment of his work possible. In particular, this makes it apparent how sometimes a much simpler and concise symbolism was still lacking.

However, in a few cases presented in this appendix his original symbols have been changed in order to avoid confusion or to translate them into English style.

On the right-hand side of Equation (3), Danusso writes $f(t)$ instead of $x(t)$. However, since he previously used the symbol f to indicate the deflection and previously referred to the ground displacement by means of symbol x, the latter has been preferred.

In Equations (4), (26), and (35), Danusso actually writes "sen" instead of "sin", as is customary in the Italian language. In the spirit of an English presentation of his work, this has been translated as the other text quotations presented.

REFERENCES

1. Alfani, G. (1909a). "Riassunto delle Comunicazioni fatte al Congresso degli Ingegneri ed Architetti Italiani, Tenutosi a Firenze nell'Ottobre 1909", Giornale dei Lavori Pubblici e delle Strade Ferrate, Vol. 36, No. 47, pp. 773–778 (in Italian).
2. Alfani, G. (1909b). "Le Vibrazioni e le Oscillazioni nei Fabbricati. Studi e Ricerche", Il Monitore Tecnico, Vol. 15, No. 35, pp. 683–686; No. 36, pp. 704–707 (in Italian).
3. Alfani, G. (1909c). "Alcuni Studj sulle Vibrazioni Meccaniche dei Fabbricati", Proceedings of XII Congresso degli Ingegneri e Architetti Italiani, Firenze, Italy, Vol. 1, p. 37, p. 131; Vol. 2, pp. 344–385 (in Italian).
4. Alfani, G. (1915a). "Le Vibrazioni nei Fabbricati Prodotte da Macchine in Movimento", Il Monitore Tecnico, Vol. 21, No. 14, pp. 238–241 (in Italian).
5. Alfani, G. (1915b). "Ing. F. Masciari-Genoese—Trattato di Costruzioni Antisismiche, Preceduto da un Corso di Sismologia. (Ulrico Hoepli, Milano, 1915, Prezzo L. 24)", Il Monitore Tecnico, Vol. 21, No. 28, p. 434 (in Italian).
6. Anonymous (1909a). "Programma del Concorso per Costruzioni Edilizie nelle Regioni Italiane Soggette a Movimenti Sismici", Il Cemento, Vol. 5, No. 2, pp. 17–18 (in Italian).
7. Anonymous (1909b). "Concorso per Costruzioni Edilizie nelle Regioni Italiane Soggette a Movimenti Sismici", Il Cemento, Vol. 5, No. 10, p. 158 (in Italian).
8. Anonymous (1909c). "L'esito del Concorso di Costruzioni per Regioni Sismiche", Il Cemento, Vol. 5, No. 12, pp. 181–182 (in Italian).
9. Anonymous (1909d). "Relazione della Giuria del Concorso per Costruzioni Edilizie nelle Regioni Italiane Soggette a Movimenti Sismici Indetto dalla Società Cooperativa Lombarda di Lavori Pubblici sotto gli Auspici del Collegio degli Ingegneri ed Architetti di Milano", Il Cemento, Vol. 5, No. 18, pp. 273–277; No. 19, pp. 292–297 (in Italian).
10. Anonymous (1909e). "XII Congresso degli Ingegneri e degli Architetti in Firenze nel 1909", Il Cemento, Vol. 5, No. 10, pp. 158–160 (in Italian).
11. Anonymous (1909f). "Il XII Congresso degli Ingegneri e degli Architetti in Firenze (Ottobre 1909)", Il Cemento, Vol. 6, No. 21, pp. 333–335 (in Italian).
12. Anonymous (1909g). "Programma del Concorso per Costruzioni Edilizie nelle Regioni Italiane Soggette a Movimenti Sismici", Il Monitore Tecnico, Vol. 15, No. 2, p. 36 (in Italian).

13. Anonymous (1909h). “Le Deliberazioni della Giuria del Concorso per le Costruzioni Antisismiche Bandito dalla Cooperativa Lombarda dei Lavori Pubblici”, Il Monitore Tecnico, Vol. 15, No. 18, p. 348 (in Italian).
14. Anonymous (1909i). “Relazione della Giuria del Concorso per Costruzioni Edilizie nelle Regioni Italiane Soggette a Movimenti Sismici Indetto dalla Società Cooperativa Lombarda di Lavori Pubblici sotto gli Auspici del Collegio degli Ingegneri ed Architetti di Milano”, Il Monitore Tecnico, Vol. 15, No. 26, pp. 501–504; No. 27, pp. 522–526; No. 28, pp. 543–546 (in Italian).
15. Anonymous (1909j). “XII Congresso degli Ingegneri e Architetti Italiani, Firenze 1909. Concorso per Costruzioni Antisismiche”, Il Monitore Tecnico, Vol. 15, No. 36, pp. 709–711 (in Italian).
16. Anonymous (1909k). “Concorso per Costruzioni Antisismiche Indetto dal Collegio Toscano degli Ingegneri e Architetti”, Proceedings of XII Congresso degli Ingegneri e Architetti Italiani, Firenze, Italy, Vol. 1, pp. 9–10, p. 37, pp. 131–134; Vol. 2, p. 344, pp. 450–455 (in Italian).
17. Anonymous (1915a). “Congratulazioni a Danusso”, Il Cemento, Vol. 12, No. 5, p. 65 (in Italian).
18. Anonymous (1915b). “Rivista di Libri. F. Masciari-Genoese. Trattato di Costruzioni Antisismiche”, Annali della Società degli Ingegneri e degli Architetti Italiani, Vol. 30, No. 10, p. 234 (in Italian).
19. Anonymous (1952). “L’Inizio dei Lavori per l’Attraversamento dello Stretto di Messina con Elettrodotto a 220 kV”, L’Ingegnere, Vol. 26, No. 3, pp. 269–277 (in Italian).
20. Anonymous (1955). “L’attraversamento dello Stretto di Messina di una linea a 220 kW”, Acciaio e Costruzioni Metalliche, Vol. 7, No. 4, pp. 45 (in Italian).
21. Anonymous (1956). “L’attraversamento elettrico dello Stretto di Messina”, Acciaio e Costruzioni Metalliche, Vol. 8, No. 3, pp. 118–122 (in Italian).
22. Anonymous (1958). “Le Giornate Siciliane dell’ANIAI. Il Premio ANIAI 1957 per l’Ingegneria Elettrotecnica Assegnato all’Elettrodotto sullo Stretto di Messina”, L’Ingegnere, Vol. 32, No. 5, pp. 445–446 (in Italian).
23. Anonymous (1960a). “Convegno di Studi di Ingegneria Sismica Messina”, Costruzioni Metalliche, Vol. 12, No. 1, p. 31 (in Italian).
24. Anonymous (1960b). “Resoconto del Convegno di Studi di Ingegneria Sismica (Messina, 10–13 Dicembre 1959)”, Il Cemento, Vol. 57, No. 2, pp. 27–29 (in Italian).
25. Anonymous (1968). “Arturo Danusso Ingegnere”, Il Cemento, Vol. 65, No. 11–12, p. 242 (in Italian).
26. Baratta, M. (1910). “La Catastrofe Sismica Calabro Messinese (28 Dicembre 1908)”, Società Geografica Italiana, Rome, Italy (in Italian).
27. Bertero, V.V. and Bozorgnia, Y. (2004). “The Early Years of Earthquake Engineering and Its Modern Goal” in “Earthquake Engineering: From Engineering Seismology to Performance-Based Engineering (edited by Y. Bozorgnia and V.V. Bertero)”, CRC Press, London, U.K.
28. Bertolini, I. (1935). “Sulla Statica Sismica delle Torri. Degli Spostamenti Sismici nelle Torri a Sezione Costante”, Il Politecnico, Vol. 83, No. 6, pp. 341–352; No. 7, pp. 407–416; No. 10, pp. 569–580 (in Italian).
29. Biot, M.A. (1932). “Transient Oscillations in Elastic System”, Ph.D. Thesis No. 259, Aeronautics Department, California Institute of Technology, Pasadena, U.S.A.
30. Biot, M.A. (1933). “Theory of Elastic Systems Vibrating under Transient Impulse with an Application to Earthquake-Proof Buildings”, Proceedings of the National Academy of Sciences of the United States of America, Vol. 19, No. 2, pp. 262–268.
31. Biot, M.A. (1934). “Theory of Vibration of Buildings during Earthquakes”, Zeitschrift für Angewandte Matematik und Mechanik, Vol. 14, No. 4, pp. 213–223.
32. Canevazzi, S. (1913a). “Note di Edilizia Antisismica”, Il Cemento, Vol. 10, No. 13, pp. 195–197; No. 14, pp. 211–215; No. 15, pp. 228–231; No. 17, pp. 260–262; No. 18, pp. 275–277 (in Italian).
33. Canevazzi, S. (1913b). “Note di Edilizia Antisismica”, Gamberini e Parmeggiani, Bologna, Italy (in Italian).
34. Chopra, A.K. (1995). “Dynamics of Structures: Theory and Applications to Earthquake Engineering”, Prentice Hall, Upper Saddle River, U.S.A.
35. Danusso, A. (1909a). “Le Case che non Crollano (A Proposito del Terremoto Calabro-Siculo)”, Il Cemento, Vol. 5, No. 1, pp. 4–9 (in Italian).

36. Danusso, A. (1909b). "Il Problema delle Case nei Paesi del Terremoto", Il Cemento, Vol. 5, No. 2, pp. 18–19 (in Italian).
37. Danusso, A. (1909c). "I Progetti Premiati al Concorso per Costruzioni Edilizie nelle Regioni Italiane Soggette a Movimenti Sismici. Le Costruzioni Antisismiche (Ing. A. Danusso)", Il Monitore Tecnico, Vol. 15, No. 22, pp. 423–428 (in Italian).
38. Danusso, A. (1909d). "La Statica delle Costruzioni Antisismiche", Il Monitore Tecnico, Vol. 15, No. 33, pp. 641–645 (in Italian).
39. Danusso, A. (1909e). "I Progetti Premiati al Concorso per Costruzioni Edilizie nelle Regioni Italiane Soggette a Movimenti Sismici. Le Costruzioni Antisismiche", Il Cemento, Vol. 6, No. 16, pp. 241–249 (in Italian).
40. Danusso, A. (1909f). "La Statica delle Costruzioni Antisismiche", Il Cemento, Vol. 6, No. 24, pp. 369–373 (in Italian).
41. Danusso, A. (1909g). "Contributo al Calcolo delle Fondazioni Continue in Cemento Armato", Il Cemento, Vol. 5, No. 3, pp. 36–41; No. 4, pp. 51–55; No. 5, pp. 68–71; No. 7, pp. 100–103; No. 8, pp. 116–119; No. 10, pp. 145–147 (in Italian).
42. Danusso, A. (1910a). "La Statica delle Costruzioni Antisismiche", Giornale dei Lavori Pubblici e delle Strade Ferrate, Vol. 37, No. 6, pp. 94–97; No. 7, pp. 113–114; No. 8, pp. 131–132; No. 9, pp. 148–151 (in Italian).
43. Danusso, A. (1910b). "La Statica delle Costruzioni Antisismiche: Comunicazione Svolta nel 12 Congresso degli Ingegneri ed Architetti in Firenze nell'Ottobre del 1909", Tipografia dell'Unione Editrice, Rome, Italy (in Italian).
44. Danusso, A. (1912a). "La Revisione delle Norme Asismiche Ministeriali", Il Monitore Tecnico, Vol. 18, No. 10, pp. 183–184 (in Italian).
45. Danusso, A. (1912b). "La Revisione delle Norme Asismiche Ministeriali", Il Cemento, Vol. 9, No. 8, pp. 114–115 (in Italian).
46. Danusso, A. (1915a). "Il Terremoto", Il Cemento, Vol. 12, No. 2, pp. 17–19 (in Italian).
47. Danusso, A. (1915b). "Lettera al Direttore", Il Cemento, Vol. 12, No. 6, p. 81 (in Italian).
48. Danusso, A. (1915c). "F. Masciari-Genoese. Trattato di Costruzioni Antisismiche. Preceduto da un Corso di Sismologia—con 900 Figure nel Testo e 1002 Pagine. U. Hoepli—Milano", Il Cemento, Vol. 12, No. 6, pp. 92–93 (in Italian).
49. Danusso, A. (1915d). "Bibliografia", Il Monitore Tecnico, Vol. 21, No. 30, pp. 463–464 (in Italian).
50. Danusso, A. (1915e). "Per la Dignità della Letteratura Scientifica", Giornale del Genio Civile, Vol. 53, pp. 488–489 (in Italian).
51. Danusso, A. (1915f). "Per la Dignità della Letteratura Scientifica", Il Cemento, Vol. 12, No. 11, pp. 145–146 (in Italian).
52. Danusso, A. (1916). "Noi e i Tedeschi", Il Monitore Tecnico, Vol. 22, No. 9, pp. 125–127 (in Italian).
53. Danusso, A. (1919). "Sul Calcolo delle Ossature che Sopportano Macchinari e Trasmissioni. Un Caso di Sincronismo", Il Cemento, Vol. 16, No. 2, pp. 8–11 (in Italian).
54. Danusso, A. (1927). "Notizie Sull'Impiego della Matematica negli Ordinari Problemi della Scienza delle Costruzioni", Rendiconti del Seminario Matematico e Fisico di Milano, Vol. 1, pp. 91–116 (in Italian).
55. Danusso, A. (1928). "Sulla Statica delle Costruzioni Antisismiche", Rendiconti del Seminario Matematico e Fisico di Milano, Vol. 2, pp. 175–199 (in Italian).
56. Danusso, A. (1931). "Sulla Statica delle Costruzioni Asismiche", Proceedings of La Partecipazione Italiana al Primo Congresso Internazionale del Beton Semplice ed Armato, Liegi settembre 1930, Tipografia del Senato, Rome, Italy, pp. 55–78 (in Italian).
57. Danusso, A. (1932). "Le Calcul Statique des Constructions Asismiques", Proceedings of Premier Congrès International du Béton et du Béton Armé, Liege, Belgium, Vol. 1, pp. 310–312 (in French).
58. Danusso, A. (1937). "In Memoria di G.A. Porcheddu", Il Cemento Armato, Vol. 34, No. 11, p. 185 (in Italian).
59. Danusso, A. (1946). "Costruzioni Asismiche" in "Tecnica del Cemento Armato (edited by G. Albenga)", Edizioni della Bussola, Rome, Italy (in Italian).

60. Danusso, A. (1952). “Vibrazioni degli Edifici” in “Lezioni sulle Vibrazioni Meccaniche (edited by B. Finzi)”, Tamburini, Milan, Italy, pp. 133–152 (in Italian).
61. Danusso, A. (1954a). “Sul Calcolo delle Ossature che Sopportano Macchinari e Trasmissioni. Un Caso di Sincronismo”, Il Cemento, Vol. 51, No. 10, pp. 2–5 (in Italian).
62. Danusso, A. (1954b). “Vibrazioni degli Edifici”, Il Cemento, Vol. 51, No. 12, pp. 2–11 (in Italian).
63. Danusso, A. (1957). “Inaugurazione del Symposium su la Plasticità nella Scienza delle Costruzioni in Onore di Arturo Danusso. Milano, 25 Settembre 1956”, Zanichelli, Bologna, Italy (in Italian).
64. Danusso, A. (1960). “Nel Cinquantenario del Terremoto di Messina”, Il Cemento Armato, Vol. 57, No. 1, pp. 3–6 (in Italian).
65. Danusso, A. (1978). “La Scienza e lo Spirito negli Scritti di Arturo Danusso”, Morcelliana, Brescia, Italy (in Italian).
66. Danusso, A. and Ceruti, G. (1935). “Costruzioni: Concetti Teorici Fondamentali e Criteri di Pratica Applicazione”, Principato, Messina, Italy (in Italian).
67. D’Aquino, U. (1986). “Danusso Arturo” in “Dizionario Biografico degli Italiani (edited by G. Alessi)”, Istituto dell’Enciclopedia Italiana, Rome, Italy (in Italian).
68. Desideri, P., Nervi, P., Jr. and Positano, G. (editors) (1979). “Pier Luigi Nervi”, Zanichelli, Bologna, Italy (in Italian).
69. Di Pasquale, G., Fralleone, A., Pizza, A.G. and Serra, C. (2000). “L'evoluzione della Normativa Sismica” in “La Classificazione e la Normativa Sismica dal 1909 al 1984 (edited by R. De Marco)”, Istituto Poligrafico e Zecca dello Stato, Rome, Italy (in Italian).
70. Finzi, B. (editor) (1952). “Lezioni sulle Vibrazioni Meccaniche”, Tamburini, Milano, Italy (in Italian).
71. Freeman, J.R. (1932). “Earthquake Damage and Earthquake Insurance”, McGraw-Hill, New York, U.S.A.
72. Giannelli, A. (1932). “Lezioni sui Telai Elastici Piani”, Tipografia del Senato, Rome, Italy (in Italian).
73. Giuffrè, A. (1987). “Cento Anni di Norme Sismiche Italiane”, Ingegneria Sismica, Vol. 4, No. 2, pp. 13–18 (in Italian).
74. Grandori, G. (1967). “Theory of Earthquake Engineering”, International Center of Earthquake Engineering, Milan, Italy; ISMES, Bergamo, Italy.
75. Guidi, C. (1925). “Costruzioni Antisismiche”, Annali dei Lavori Pubblici, Vol. 63, No. 3, pp. 263–264 (in Italian).
76. Guidi, C. (1926a). “Costruzioni Antisismiche”, Il Cemento Armato, Vol. 23, No. 3, pp. 32–33 (in Italian).
77. Guidi, C. (1926b). “Esercizi Sulla Scienza delle Costruzioni. Quarta Edizione con Appendice”, Tip. V. Bona, Turin, Italy (in Italian).
78. Guidi, C. (1927). “Studi Sperimentali su Costruzioni in Cemento Armato”, Il Cemento Armato, Vol. 24, No. 2, pp. 9–14; No. 3, pp. 23–30 (in Italian).
79. Guidi, C. (1930). “Telai Antisismici”, Tipografia del Senato, Rome, Italy (in Italian).
80. Housner, G.W. (1984). “Historical View of Earthquake Engineering”, Proceedings of the Eigth World Conference on Earthquake Engineering, San Francisco, U.S.A., Post-Conference Volume, pp. 25–39.
81. Hudson, D.E. (1983). “Golden Anniversary Workshop on Strong-motion Seismometry, March 30–31”, Department of Civil Engineering Report, University of Southern California, Los Angeles, U.S.A.
82. Iori, T. (2001). “Il Cemento Armato in Italia: dalle Origini alla Seconda Guerra Mondiale”, Edilstampa, Rome, Italy (in Italian).
83. Kelvin, W.T. and Tait, P.G. (1867). “Treatise on Natural Philosophy”, Oxford University Press, Oxford, U.K.
84. Kelvin, W.T. and Tait, P.G. (1871). “Handbuch der Theoretischen Physik”, Vieweg, Braunschweig, Germany (in German).
85. Krall, G. (1940). “Meccanica Tecnica delle Vibrazioni”, Eredi V. Veschi, Rome, Italy (in Italian).

86. Krall, G. (1947). "Meccanica Asismica delle Torri", Rendiconti dell'Accademia Nazionale dei Lincei. Classe di Scienze Fisiche, Matematiche e Naturali, Ser. 8, Vol. 3, No. 2, pp. 3–11 (in Italian).
87. Krall, G. (1948). "Un Progetto di Torri-Piloni alte 200 m. per l'Attraversamento dello Stretto di Messina con Linea Aerea a 200 kV. e Questioni Generali di Asismica", Ingegneria Ferroviaria, Vol. 3, No. 2, pp. 89–100 (in Italian).
88. Levi-Civita, T. (1896). "Sul Moto dei Sistemi con Tre Gradi di Libertà", Rendiconti dell'Accademia dei Lincei, Ser. 5, Vol. 5, No. 2, pp. 164–171 (in Italian).
89. Lie, S. (1888). "Theorie der Transformationsgruppen", BG Teubner, Leipzig, Germany (in German).
90. Losio, C. (1909). "XII Congresso Degli Ingegneri e Architetti in Firenze", Rivista di Ingegneria Sanitaria, Vol. 5, No. 20, pp. 319–325 (in Italian).
91. Luiggi, L. (1914). "Necrologio del Prof. Ing. Comm. Italo Maganzini", Annali della Società degli Ingegneri e degli Architetti Italiani, Vol. 29, No. 9, pp. 201–206 (in Italian).
92. Manfredini, A. (1909a). "I Danni del Terremoto e le Nuove Costruzioni nelle Zone Colpite. Una Lodevolissima Iniziativa della Cooperativa Lombarda delle Opere Pubbliche", Il Monitore Tecnico, Vol. 15, No. 1, pp. 1–2 (in Italian).
93. Manfredini, A. (1909b). "Due Concorsi Tecnici a Milano: L'Esposizione dei Modelli di Agganciatore Automatico ai Veicoli Ferroviari e l'Esposizione dei Progetti per le Costruzioni Antisismiche—Il Concorso per il Nuovo Palazzo Municipale di Messina e quello per la Nuova Facciata della Stazione di Milano", Il Monitore Tecnico, Vol. 15, No. 12, pp. 221–223 (in Italian).
94. Manfredini, A. (1909c). "Deliberazioni della Giuria nel Concorso delle Costruzioni Antisismiche Bandito dalla Cooperativa Lombarda dei Lavori Pubblici", Il Monitore Tecnico, Vol. 15, No. 18, pp. 341–342 (in Italian).
95. Manfredini, A. (1909d). "XII Congresso degli Ingegneri e Architetti in Firenze", Il Monitore Tecnico, Vol. 15, No. 28, pp. 542–543 (in Italian).
96. Marletta, R. (1934). "Il Terremoto del 10 Marzo 1933 a Long Beach", Annali dei Lavori Pubblici, Vol. 12, No. 9, pp. 745–778 (in Italian).
97. Marzolo, F. (1957). "Modesto Panetti: Commemorazione", Ministero della Pubblica Istruzione, Consiglio Superiore della P.I., Tip. S. Pancrazio, Rome, Italy (in Italian).
98. Masciari-Genoese, F. (1915). "Trattato di Costruzioni Antisismiche Preceduto da un Corso di Sismologia", Hoepli, Milan, Italy (in Italian).
99. Meirovitch, L. (1986). "Elements of Vibration Analysis", McGraw-Hill, New York, U.S.A.
100. MLLPP (1909a). "Relazione della Commissione Incaricata di Studiare e Proporre Norme Edilizie Obbligatorie per i Comuni Colpiti dal Terremoto del 28 Dicembre 1908 e da Altri Anteriori", Report, Ministero dei Lavori Pubblici, Rome, Italy; Giornale del Genio Civile, Vol. 47, No. 4, pp. 197–294 (in Italian).
101. MLLPP (1909b). "Circolare Esplicativa delle Norme Inviata ai Prefetti, ai Sindaci, ai Presidenti delle Deputazioni Provinciali, agli Ingegneri del Genio Civile e agli Uffici Tecnici di Finanza", Circolare No. 2664, Ministero dei Lavori Pubblici, Direzione Generale dei Servizi Speciali—Sezione Terremoto, Div. 13, Rome, Italy; Bollettino Ufficiale del Ministero dei Lavori Pubblici, Vol. 10, No. 13, pp. 593–623; L'Ingegneria Ferroviaria, Vol. 6, No. 9, pp. 12–14 (in Italian).
102. MLLPP (1913). "Seconda Relazione della Commissione Incaricata di Rivedere le Norme Edilizie Obbligatorie per i Comuni Colpiti dal Terremoto del 28 Dicembre 1908 e da Altri Anteriori: Istruzioni Tecniche, Metodi di Calcolo, Applicazioni", Report, Ministero dei Lavori Pubblici, Rome, Italy; Giornale del Genio Civile, Vol. 51, No. 10, pp. 545–697 (in Italian).
103. MLLPP (1915). "Relazione del Comitato Speciale del Consiglio dei Lavori Pubblici Incaricato di Studiare e Proporre Norme Edilizie Obbligatorie per i Comuni Colpiti dal Terremoto del 13 Gennaio 1915", Report, Ministero dei Lavori Pubblici, Rome, Italy (in Italian).
104. MLLPP (1921). "Comitato Speciale per le Costruzioni nei Paesi Colpiti dal Terremoto. Esempi di Calcolo di Edifici a Tre Piani nelle Regioni Sismiche", Report, Ministero dei Lavori Pubblici, Rome, Italy; Bollettino Ufficiale del Ministero dei Lavori Pubblici, Vol. 22, No. 25, pp. 1596–1608 (in Italian).

105. MLLPP (1975). “Approvazione delle Norme Tecniche per le Costruzioni in Zone Sismiche”, Decreto Ministeriale No. 39, Ministero dei Lavori Pubblici, Rome, Italy; Gazzetta Ufficiale della Repubblica Italiana, Vol. 116, No. 93, pp. 1–7 (in Italian).

106. Novelli, L. (1909). “Impressioni sul Concorso per Costruzioni Antisismiche di Milano”, L’Ingegneria Ferroviaria, Vol. 6, No. 20, pp. 344–347 (in Italian).

107. Panetti, M. (1914). “Sulla Tecnica delle Costruzioni Asismiche. Comunicazione dell’ing. Modesto Panetti Fatta nella Seduta del 26 Giugno 1914. Estratto dagli Atti della Società degli Ingegneri e degli Architetti di Torino”, Tip. P. Celanza e C., Turin, Italy (in Italian).

108. Panetti, M. (1916). “I Progressi della Dinamica nella Tecnica dell’Ingegnere. Estratto dagli Atti della Società Italiana per il Progresso delle Scienze. VIII Riunione—Roma, Marzo 1916”, Società Italiana per il Progresso delle Scienze, Rome, Italy (in Italian).

109. Priolo, D.A. (1930). “Le Costruzioni Antisismiche: Manuale Pratico con Esempi Numerici Completi di Calcolo per Edifici Ordinari, Chiese ad Una e Tre Navate”, G. Principato, Messina, Italy (in Italian).

110. Priolo, D.A. (1931). “Cenni sulle Costruzioni Antisismiche”, Il Politecnico, Vol. 79, No. 10, pp. 601–610 (in Italian).

111. Priolo, D.A. (1948). “Sui Criteri più Idonei per il Progetto di Alti Piloni Metallici in Zone Sismiche. Strutture Geodetiche”, Giornale Genio Civile, Vol. 86, No. 9, pp. 488–501 (in Italian).

112. Priolo, D.A. (1951). “Dinamica delle Costruzioni Antisismiche”, Ed. del Giornale del Genio Civile, Rome, Italy (in Italian).

113. Rayleigh, J.W.S. (1877). “The Theory of Sound”, MacMillan, London, U.K.

114. Rayleigh, J.W.S. (1879). “Die Theorie des Schalles”, Vieweg, Braunschweig, Germany (in German).

115. Rayleigh, J.W.S. (1894). “The Theory of Sound”, MacMillan, London, U.K.

116. Reitherman, R. (2006a). “The Effects of the 1906 Earthquake in California on Research and Education”, Earthquake Spectra, Vol. 22, No. S2, pp. S207–S236.

117. Reitherman, R. (2006b). “Earthquakes That Have Initiated the Development of Earthquake Engineering”, Bulletin of the New Zealand Society for Earthquake Engineering, Vol. 39, No. 3, pp. 145–147.

118. Revere, G. (1918a). “Prof. Ing. Silvio Canevazzi”, Il Monitore Tecnico, Vol. 24, No. 8, p. 63 (in Italian).

119. Revere, G. (1918b). “Necrologio [Silvio Canevazzi]”, Il Cemento, Vol. 15, No. 4, pp. 37–38 (in Italian).

120. RI (1909). “Norme Tecniche ed Igieniche Obbligatorie per le Riparazioni, Ricostruzioni e Nuove Costruzioni degli Edifici Pubblici e Privati nei Comuni Colpiti dal Terremoto del 28 Dicembre 1908 e da Altri Precedenti”, Regio Decreto No. 193, Regno d’Italia, Rome, Italy; Gazzetta Ufficiale del Regno d’Italia, Vol. 50, No. 95, pp. 1957–1962 (in Italian).

121. RI (1912). “Norme Obbligatorie per le Riparazioni, Ricostruzioni e Nuove Costruzioni degli Edifici nei Comuni Colpiti dal Terremoto, in Sostituzione di Quelle Approvate col R. D. 18 Aprile 1909, N. 193”, Regio Decreto No. 1080, Regno d’Italia, Rome, Italy; Gazzetta Ufficiale del Regno d’Italia, Vol. 53, No. 247, pp. 6037–6042 (in Italian).

122. RI (1913). “Testo Unico delle Leggi Emanate in Conseguenza del Terremoto del 28 Dicembre 1908”, Regio Decreto No. 1261, Regno d’Italia, Rome, Italy; Gazzetta Ufficiale del Regno d’Italia, Vol. 54, No. 290, pp. 1–33 (in Italian).

123. Santarella, L. (1926). “Il Cemento Armato nelle Costruzioni Civili ed Industriali”, Hoepli, Milan, Italy (in Italian).

124. Tibiletti Marchionna, C. (1997). “Il Seminario Matematico e Fisico di Milano (1927–1996)”, Rendiconti del Seminario Matematico e Fisico di Milano, Vol. 67, pp. 27–47 (in Italian).

125. Timoshenko, S.P. (1928). “Vibration Problems in Engineering”, Van Nostrand, New York, U.S.A.

126. Toscano, A.M. (1958). “L'Attraversamento Elettrico dello Stretto di Messina”, Dedalo, Rome, Italy (in Italian).

127. Toscano, A.M. (1960). “Le Torri di Messina”, Costruzioni Metalliche, Vol. 12, No. 5, pp. 205–225 (in Italian).

128. Trifunac, M.D. (2003). "23rd ISET Annual Lecture: 70-th Anniversary of Biot Spectrum", ISET Journal of Earthquake Technology, Vol. 40, No. 1, pp. 19–50.
129. Uang, C.M. and Bertero, V.V. (1990). "Evaluation of Seismic Energy in Structures", Earthquake Engineering & Structural Dynamics, Vol. 19, No. 1, pp. 77–90.
130. von Kármán, T. and Biot, M.A. (1940). "Mathematical Methods in Engineering", McGraw-Hill, New York, U.S.A.