Paper No. 201, Bull. ISET Vol. 17 No. 1, March. 1980, pp 1–10

ON STRESS ACCUMULATION IN A VISCO-ELASTIC LITHOSPHERE CONTAINING A CONTINUOUSLY SLIPPING FAULT[1]

ARABINDA MUKHOPADHYAY[2], B.P. PAL[3] AND SANJAY SEN[4]

Introduction

The problem of earthquake prediction has attracted wide spread attention among seismologists in recent years. The social and economic importance of earthquake prediction has been obvious for a long time, and the steady accumulation of relevant seismological data and improvements in the techniques of analysis have made it possible to hope that effective programmes of earthquake prediction may become feasible in the near future. In this connection, it would be useful to have a better understanding of the process of stress accumulation in the neighbourhood of active faults which may lead to a sudden fault movement generating an earthquake. The quantitative estimation of the stress accumulation would be facilitated if it is possible to devise suitable theoretical models which incorporate the essential features of the mechanism of stress accumulation near seismically active faults. This would enable us to estimate the stress accumulation near the fault below the surface from the observed ground deformation on the surface. Some theoretical models have been developed to explain the accumulation of shear stress near strike slip faults, by Spence and Turcotte (1976) and Budiansky and Amazigo (1976). The stress accumulation in these models is supposed to be due to the relative motion of the parts of the lithosphere on opposite sides of the fault, while the fault itself remains locked. The mechanism of the relative motion is not considered directly in these models, and they require large and steadily increasing stresses in the lithosphere at great distances from the fault. Budiansky and Amazigo (1976) consider a locked strike-slip fault situated in a visco-elastic layer representing the lithosphere, and assume that tectonic forces maintain a constant shear stress in the layer far away from the fault. They show that there would be a steady accumulation of shear stress near the fault leading to fault slip under suitable circumstances. Some work has also been done on post-seismic stress changes in a visco-elastic half-space, following a sudden slip on a fault in the half-space, after which the fault becomes locked, by Randle and Jackson (1977) and others. The stress accumulation near locked strike-slip faults in homogeneous and layered visco-elastic half-spaces have been considered by Mukhopadhyay and Mukherji (1978).

In this connection it has been reported by Spence and Turcotte (1976) and others that continuous aseismic creep has been found to occur in the central section of the San Andreas fault from San Jose to Cholame, and this aseismic creep is reported to range from 1 to 6 Cms/year on the surface in different parts of the fault. This prompted us to consider a model of continuously creeping aseismic strike-slip fault situated in a visco-elastic half space.

---

1 Paper No. 13 presented at Kurukshetra Symposium.

2 Department of Applied Mathematics, University of Calcutta, University College of Science, 92 Acharya Prafulla Chandra Road, Calcutta 700 009.

3 Geophysicist, Geological Survey of India, 15 Park Street, Calcutta 700 016.

4 Research Officer, River Research Institute, IIA, Free School Street, Calcutta - 700016.

Formulation

We consider a long vertical strike slip fault F of depth D, situated in a visco-elastic half-space and reaching upto the surface, as shown in Fig. IIA. 1. We introduce rectangular cartesian coordinates (y_1, y_2, y_3) with the free surface as the plane $y_3 = 0$, the plane of the fault has the plane $y_2 = 0$ and the y_1 – axis along the trace of the fault, so that the visco-elastic half space occupies the region $y_3 > 0$. We consider a fault whose length is large compared to its depth D, so that we have a two-dimensional problem, with the displacements u_i and stresses τ_{ij} $(i, j = 1, 2, 3)$ independent of y_1. For the strike-slip fault movement which we consider, the relevant displacement component is u_1 and the relevant stress components are τ_{12} and τ_{13}. Since u_i, τ_{ij} are independent of y_1, u_1 and (τ_{12}, τ_{13}) are found to be independent of the other components of displacement and stress. We assume that the visco-elastic material is of the Maxwell type, so that the stress-strain relations reduce to the form [Fung (1964)] :

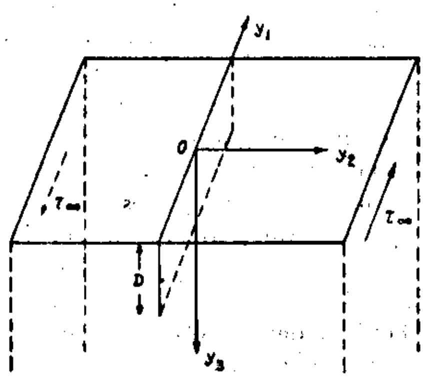

Fig. IIA 1.

$$\left.\begin{aligned} \left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{13} &= \frac{\partial^2 u_1}{\partial t\,\partial y_3} \\ \text{and} \quad \left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{12} &= \frac{\partial^2 u_1}{\partial t\,\partial y_2} \end{aligned}\right\} \qquad \text{(IIA. 1)}$$

where μ is the effective rigidity and η is the effective viscosity.

For the slow aseismic creeping displacements we are condsidering, the inertial forces are very small and are neglected. For such quasi-static, aseismic displacements, the stresses satisfy the condition,

$$\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0 \qquad \text{(IIA. 2)}$$

We assume that the shear stress τ_{12} has a constant value τ_∞ far away from the fault, maintained by tectonic forces. Near the fault, the stress system would be altered from time to time due to fault creep. Thus, the stresses would satisfy the following boundary conditions :

$$\left.\begin{aligned} \tau_{13} &= 0 \quad \text{on} \quad y_3 = 0, \\ \tau_{13} &\to 0 \quad \text{as} \quad y_3 \to \infty \end{aligned}\right\} \qquad \text{(IIA. 3)}$$

and $\tau_{12} \rightarrow \tau_\infty$ as $y_2 \rightarrow \infty$ for $y_3 \geq 0$ (IIA. 4)

From (IIA. 1) and (IIA. 2) we easily find that $\frac{\partial}{\partial t}(\nabla^2 u_1) = 0$

which is satsified if $\nabla^2 u_1 = 0$ (IIA. 5)

Displacements and Stresses in the absence of fault slip

We first consider the deformation of the medium in the absence of fault slip, when the displacements and stresses would be continuous throughout the medium. We take Laplace transforms of (IIA. 1) to (IIA. 5) with respect to t. The resulting boundary value problem can be solved without much difficulty, and on inverting the laplace transforms, we find that,

$$\left.\begin{aligned} u_1(y_2, y_3, t) &= (u_1)_0 + \frac{\tau_\infty \cdot y_2 \cdot t}{\eta} \\ \tau_{12}(y_2, y_3, t) &= (\tau_{12})_0\, e^{-\mu t/\eta} + \tau_\infty \cdot (1 - e^{-\mu t/\eta}) \\ \tau_{13}(y_2, y_3, t) &= (\tau_{13})_0\, e^{-\mu t/\eta} \end{aligned}\right\} \quad \text{(IIA. 5A)}$$

where $(u_1)_0$, $(\tau_{12})_0$, $(\tau_{13})_0$, which may be functions of (y_2, y_3), are the values of u_1, τ_{12}, τ_{13} at time $t = 0$, when (IIA. 1) to (IIA. 5) become valid for the system. We note that $\tau_{13} \rightarrow 0$ as $t \rightarrow \infty$. Assuming that $(\tau_{12})_0 < \tau_\infty$, near the fault, we find that the value of τ_{12} near the fault increases steadily from $(\tau_{12})_0$ and $\rightarrow \tau_\infty$ as $t \rightarrow \infty$. If the characteristics of the fault F be such that it starts creeping continuously when the shear stress τ_{12} near it reaches a critical value $\tau_c < \tau_\infty$, the fault creep would commence after sometime, when τ_{12} reaches this value τ_c.

Displacements and Stresses after the commencement of fault creep

We now measure the time t from the instant at which fault creep commences. For the slow aseismic fault creep we are considering equations (IIA. 1) to (IIA. 5) are valid. In addition, there is a dislocation across the fault F which changes with time as long as the creep continues. The boundary conditions across the creeping fault are taken to be

$$[u_1] = f(y_3) \cdot U(t) \quad \text{(IIA. 6)}$$

and $[\tau_{12}] = 0 = [\tau_{13}]$

across F : $y_2 = 0,\ 0 < y_3 < D.$ (IIA. 7)

where $[u_1]$ is the discontinuity in u_1 across the fault F, defined by

$$[u_1] = \mathop{\mathrm{Lt}}_{y_2 \rightarrow 0+0} (u_1) - \mathop{\mathrm{Lt}}_{y_2 \rightarrow 0-0} (u_1)$$

and $[\tau_{12}]$, $[\tau_{13}]$ are the discontinuities in τ_{12}, τ_{13} across the fault. We take $U(0) = 0$, and note that the velocity of creep across F is

$$\frac{\partial}{\partial t}[u_1] = f(y_3)\, V(t)$$

where $$V(t) = \frac{d}{dt} U(t)$$

We assume $V(t)$ and $f(y_3)$ to be continuous functions. To determine the displacements and stresses we take Laplace transforms of both sides of (IIA. 1) to (IIA. 7). The resulting boundary value problem is solved by using the Green's function technique developed by Maruyama (1966), as explained in the Appendix. It is found that exact solutions in closed form can be obtained if $f(y_3)$ is a constant or a polynomial in y_3. The solutions are found to be given by

$$u_1 = (u_1)_0 + \frac{\tau_\infty \, y_2 \, t}{\eta} + \frac{U(t)}{2\pi} \phi_1 (y_2, y_3) \tag{IIA. 8}$$

$$\tau_{12} = (\tau_{12})_0 \, e^{-\mu t/\eta} + \tau_\infty \cdot (1 - e^{-\mu t/\eta}) - \frac{\mu}{2\pi} \cdot \phi_2 (y_2, y_3) \cdot \int_0^t V(\tau) . \, e^{-\mu(t-\tau)/\eta} \, d\tau \tag{IIA. 9}$$

and

$$\tau_{13} = (\tau_{13})_0 . \, e^{-\mu t/\eta} - \frac{\mu}{2\pi} \cdot \phi_3 (y_2, y_3) \int_0^t V(\tau) . \, e^{-\mu(t-\tau)/\eta} \, d\tau \tag{IIA. 10}$$

where $(u_1)_0$, $(\tau_{12})_0$, $(\tau_{13})_0$ are the values of u_1, τ_{12}, τ_{13} at $t = 0$ when the fault creep just commences.

For creep dislocation independent of depth, corresponding to $f(y_3) = 1$, we have

$$\left.\begin{aligned} \phi_1 (y_2, y_3) &= \tan^{-1}\left(\frac{D + y_3}{y_2}\right) + \tan^{-1}\left(\frac{D - y_3}{y_2}\right) \\ \phi_2 (y_2, y_3) &= \frac{(D+y_3)}{(D+y_3)^2 + y_2^2} + \frac{D - y_3}{(D-y_3)^2 + y_2^2} \\ \phi_3 (y_2, y_3) &= \frac{y_2}{(D-y_3)^2 + y_2^2} - \frac{y_2}{(D+y_3)^2 + y_2^2} \end{aligned}\right\} \tag{IIA. 11}$$

For other forms of $f(y_3)$ we have

$$\left.\begin{aligned} \phi_1 (y_2, y_3) &= \int_0^D f(x_3)\left[\frac{y_2}{(x_3-y_3)^2 + y_2^2} + \frac{y_2}{(x_3+y_3)^2 + y_2^2}\right] dx_3 \\ \phi_2 (y_2, y_3) &= \int_0^D f(x_3)\left[\frac{y_2^2-(x_3-y_3)^2}{\{y_2^2+(x_3-y_3)^2\}^2} + \frac{y_2^2-(x_3+y_3)^2}{\{y_2^2+(x_3+y_3)^2\}^2}\right] dx_3 \\ \phi_3 (y_2, y_3) &= \int_0^D f(x_3)\left[\frac{2\,(x_3+y_3)\,y_2}{\{(x_3+y_3)^2+y_2^2\}^2} - \frac{2\,(x_3-y_3)\,y_2}{\{(x_3-y_3)^2+y_2^2\}^2}\right] dx_3 \end{aligned}\right\} \tag{IIA12}$$

From (IIA. 11) we find that for $f(y_3) = 1$, there is a singularity of the stresses at the lower tip of the fault : $y_2 = 0$, $y_3 = D$. Similar singularities in shear stress at the tip of the fault are present in the solutions obtained by Rybicki (1971) for static or quasi-static faults in elastic media. On investigating analytically, the integrals in (IIA. 12) we find that the displacements and stresses will be bounded everywhere including the tip of the fault if the following conditions are satisfied :

(a) $f(y_3), f'(y_3)$ are continuous in $0 \leqslant y_3 \leqslant D$,

(b) $f''(y_3)$ is continuous in $0 < y_3 < D$, or has a finite number of points of finite discontinuity in $0 < y_3 < D$.

(c) $(y_3)^m . f''(y_3) \rightarrow 0$ or to a finite limit as $y_3 \rightarrow 0+0$

and $(D-y_3)^n . f''(y_3) \rightarrow 0$ or to a finite limit as $y_3 \rightarrow D-0$,

where $m < 1, n < 1$ are constants

and (d) $f(D) = 0 = f'(D), \quad f'(0) = 0.$

These conditions imply that the magnitude of the dislocation varies smoothly over the fault and approaches the value zero with sufficient rapidity near the tip of the fault. The integrals in (IIA. 12) can be evaluated in closed form if $f(y_3)$ is a polynomial. In particular, if

$f(y_3) = (y_3^2 - D^2)^2/D^4$, we find that

$$\phi_1(y_2, y_3) = \frac{2y_2 . (3y_3^2 - y_2^2)}{D^3} - \frac{10 y_2}{3D} + \frac{2 y_2 y_3 (y_3^2 - y_2^2 - D^2)}{D^4} \times \ln \frac{(D-y_3)^2 + y_2^2}{(D+y_3)^2 + y_2^2} + \frac{y_2^4 - y_2^2 (6 y_3^2 - 2 D^2) + (y_3^2 - D^2)^2}{D^4} \times \left\{ \tan^{-1}\left(\frac{D-y_3}{y_2}\right) + \tan^{-1}\left(\frac{D+y_3}{y_2}\right) \right\} \quad \text{(IIA. 13)}$$

$$\begin{aligned}\phi_2(y_2, y_3) = -\Bigg[& \frac{6(y_3^2 - y_2^2)}{D^3} - \frac{10}{3D} + \frac{2 y_3 (y_3^2 - 3y_2^2 - D^2)}{D^4} \ln \frac{(D-y_3)^2 + y_2^2}{(D+y_3)^2 + y_2^2} \\ & + \frac{4 y_2^2 y_3 (y_3^2 - y_2^2 - D^2)}{D^4} \left\{ \frac{1}{(D-y_3)^2 + y_2^2} - \frac{1}{(D+y_3)^2 + y_2^2} \right\} \\ & + \frac{4y_2^3 - 2y_2 (6y_3^2 - 2D^2)}{D^4} \left\{ \tan^{-1}\left(\frac{D-y_3}{y_2}\right) + \tan^{-1}\left(\frac{D+y_3}{y_2}\right) \right\} \\ & - \frac{y_2^4 - y_2 (6 y_3^2 - 2D^2) + (y_3^2 - D^2)^2}{D^4} \times \\ & \left\{ \frac{(D-y_3)}{(D-y_3)^2 + y_2^2} + \frac{(D+y_3)}{(D+y_3)^2 + y_2^2} \right\} \Bigg] \end{aligned} \quad \text{(IIA. 14)}$$

The expression for $\phi_3(y_2, y_3)$ is similar, and we find that, in this case (ϕ_1, ϕ_2, ϕ_3) all remain bounded near the tip of the fault.

From the solutions given above, we calculate the rate of accumulation of shear strain near the surface, given by

$$R = \left[\frac{\partial}{\partial t} (e_{12}) \right]_{\substack{y_2 = 0 \\ y_3 = 0}} = \left(\frac{\partial^2 u_1}{\partial t\, \partial y_2} \right)_{\substack{y_2 = 0 \\ y_3 = 0}} \quad \text{(IIA. 15)}$$

and the average value of τ_{12} over the fault F, given by

$$(\tau_{12})_A = (1/D) \int_0^{y_3} (\tau_{12})_F \, dy_3 \qquad \text{(IIA. 16)}$$

We find that $R = \frac{\tau_\infty}{\eta} - \frac{V(t)}{\pi D}$, if $f(y_3) = 1$

and $R = \frac{\tau_\infty}{\eta} - \frac{16\,V(t)}{6\pi D}$, if $f(y_3) = \frac{(y_3^2 - D^2)^2}{D^4}$ (IIA. 17)

For $f(y_3) = (y_3^2 - D^2)^2/D^4$, we have

$$(\tau_{12})_A = (\tau_{12})_{0A} \cdot \exp(-\mu t/\eta) + \tau_\infty \cdot (1 - e^{-\mu t/\eta}) - \phi(t) \qquad \text{(IIA. 18)}$$

where

$$\phi(t) = (4/3D) \cdot \frac{\mu}{2\pi} \int_0^t V(\tau) \cdot e^{-\mu(t-\tau)/\eta} \, d\tau \qquad \text{(IIA. 19)}$$

and $(\tau_{12})_{0A}$ is the average value of $(\tau_{12})_0$ over F. If V(t) = a constant = V,

we have

$$\phi(t) = \frac{4\eta \cdot V}{6\pi D} \cdot (1 - e^{-\mu t/\eta}) \qquad \text{(IIA. 20)}$$

for constant creep velocity V.

Discussion of the results and conclusions

We now study the influence of continuous fault creep in our model on the strain accumulation on the surface near the fault, and on the changes in the shear stress τ_{12} near the fault, which is expected to control the strike-slip fault movement. In assigning suitable values to the parameters in our model, viz., τ_∞, η, D, V(t), μ and $f(y_3)$, we keep in view the San Andreas fault, the most well-known and widely studied active strike-slip fault, and take D = 10 kms. and $\mu = 3.78 \times 10^{11}$ dynes/cm^2, given by Aki (1967) for the lithosphere. We choose $\eta = 3 \times 10^{21}$ poise in our model.

We note that estimates for the stress drops near the San Andreas fault for the largest well-recorded earthquake, the San Francisco earthquake of 1906, are in the range 50-100 bars. We make the reasonable conjecture that the stress drop in a very large earthquake is nearly equal to τ_∞, and take $\tau_\infty = 50$ bars in our model. We consider the case V(t)=V, a constant. Noting that the aseismic creep velocity on the surface of our model is V, and that aseismic creep velocities on the surface in the central section of the San Andreas fault have been reported to be in the range of 1 to 6 cms. per year [Spence and Turcotte (1976)], we take values of V from V = 0 to V = 6 cms. per year. We consider the dislocation

$$[u] = V.t. (y_3^2 - D^2)^2/D^4,$$

so that the displacements and stresses are bounded everywhere, and compute $E_{12} = [e_{12} - (e_{12})_0] \times 10^6$ near the fault on the surface ($y_2 = 0$, $y_3 = 0$) and $(\tau_{12})_A$, the average value of τ_{12} over the fault. Fig. IIA. 2 shows the variation of E_{12} with the time t. It is found that the rate of surface strain accumulation decreases as V increases, from about 0.5×10^{-6}/year for V = 0, to almost zero for V = 0.6 cms./year. Fig. IIA. 3 and Fig.

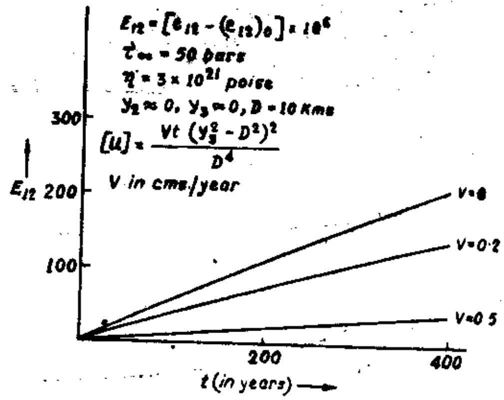

Fig. IIA 2

IIA. 4 show the variation of $(\tau_{12})_A$ with time, where $(\tau_{12})_{0A}$, the average value of τ_{12} over the fault at $t = 0$ (when the continuous fault slip commences) has the value 20 bars and 40 bars in Fig. IIA. 3 and Fig. IIA. 4. In both cases, it is found that the rate of accumulation of the average shear stress $(\tau_{12})_A$ decreases as V increases, and if V is sufficiently large, $(\tau_{12})_A$ decreases with time, so that there is aseismic release of shear stress. In each case, τ_{12} approaches a finite limit as t becomes large, and if V has the critical value of about 2.5 cms/year, $(\tau_{12})_A \rightarrow 0$ as $t \rightarrow \infty$, so that there is complete aseismic release of shear stress by continuous fault creep. Thust, the possibility of a sudden and rapid fault movement, causing an earthquake, is reduced continuously with time, and becomes very small for large t. The critical value V mentioned above lies in the range of creep velocities reported for the central part of the San Andreas fault [Spence and Turcotte (1976)]. Of course, this critical velocity changes significantly on changing the model parameters.

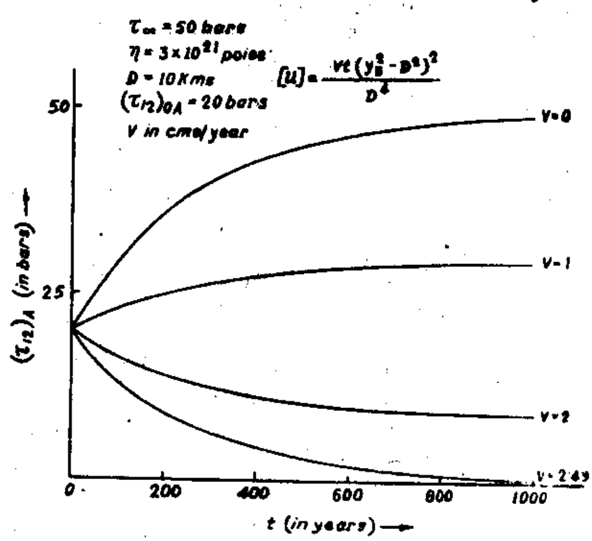

Fig. IIA 3

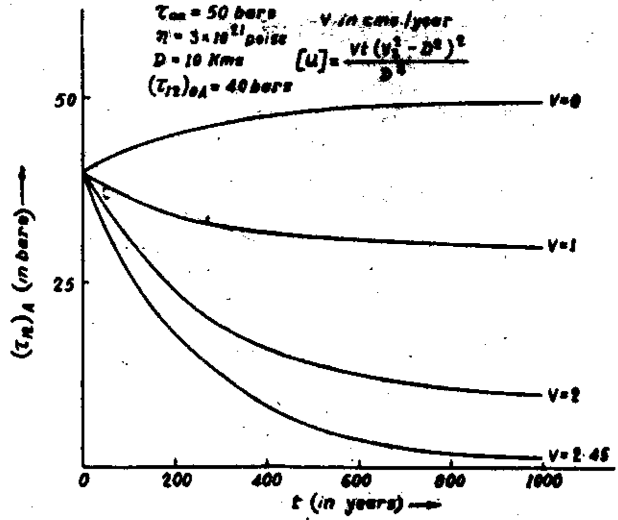

Fig. IIA. 4

We note that, if detailed and reliable data on the ground deformation on the surface and its changes with time near creeping strike slip faults are available over a sufficiently long period, comparison of the observational data with the theoretically calculated ground displacement may enable us to obtain more reliable estimates for the parameters of our model. It may then be possible to obtain reliable estimates of the variation of the shear stress τ_{12} near the fault, so that the probability of a rapid and sudden fault movement, causing earthquake, may be assessed, and the maximum probable magnitude of such an earthquake, if it does occur, can be estimated roughly. Finally, we note that the simple model we consider can be expected to represent only approximately the complex tectonic processes occuring near actual creeping strike-slip faults. However, it is hoped that these results may give some insight into the nature of the process of aseismic fault creep and the resulting release of stress and strain.

Appendix

We take Laplace transforms of (IIA. 1)-(IIA. 7) with respect to t, and obtain a boundary value problem, characterised by the relations,

$$\left.\begin{aligned} \bar{\tau}_{12} &= \frac{p}{p/\mu + 1/\eta}\frac{\partial \bar{u}_1}{\partial y_2} + \frac{(\tau_{12})_0/\mu - (\partial u_1/\partial y_2)_0}{p/\mu + 1/\eta} \\ \bar{\tau}_{13} &= \frac{p}{p/\mu + 1/\eta}\frac{\partial \bar{u}_1}{\partial y_3} + \frac{(\tau_{13})_0/\mu - (\partial u_1/\partial y_3)_0}{p/\mu + 1/\eta} \end{aligned}\right\} \qquad \text{(IIA. 1a)}$$

$$\frac{\partial \bar{\tau}_{12}}{\partial y_2} + \frac{\partial \bar{\tau}_{13}}{\partial y_3} = 0 \qquad \text{(IIA. 2a)}$$

$$\left.\begin{aligned} \bar{\tau}_{13} &= 0 \quad \text{on} \quad y_3 = 0 \\ \bar{\tau}_{13} &\to 0 \quad \text{as} \quad y_3 \to \infty \end{aligned}\right\} \qquad \text{(IIA. 3a)}$$

$$\bar{\tau}_{12} \rightarrow \tau_\infty/p, \quad \text{as} \quad y_2 \rightarrow \infty \tag{IIA. 4a}$$

$$\nabla^2 \bar{u}_1 = 0 \tag{IIA. 5a}$$

$$[\bar{u}_1] = f(y_3)\,\bar{U}(p) \ \text{across} \ F: y_2 = 0,\ 0 \leqslant y_3 < D \tag{IIA. 6a}$$

and
$$[\bar{\tau}_{12}] = 0 = [\bar{\tau}_{13}] \ \text{across} \ F \tag{IIA. 7a}$$

where p is the Laplace transform variable, $\bar{u}_1$, $\bar{\tau}_{12}$, $\bar{\tau}_{13}$, $\bar{U}$ are the Laplace transforms of u_1, τ_{12}, τ_{13}, U(t) with respect to t; the suffix zero in $(u_1)_0$, $(\tau_{12})_0$ etc. denotes the value at t = 0, and $[\bar{u}_1]$ is the discontinuity in $\bar{u}_1$ across F. We try to find $\bar{u}_1$, $\bar{\tau}_{12}$, $\bar{\tau}_{13}$ in the form,

$$\bar{u}_1 = (\bar{u}_1)_1 + (\bar{u}_1)_2,\ \bar{\tau}_{12} = (\bar{\tau}_{12})_1 + (\bar{\tau}_{12})_2,\ \bar{\tau}_{13} = (\bar{\tau}_{13})_1 + (\bar{\tau}_{13})_2,$$

where $(\bar{u}_1)_1$, $(\bar{\tau}_{12})_1$, $(\bar{\tau}_{13})_1$ satisfy (IIA. 1a) - (IIA. 5a) and are continuous throughout $y_2 > 0$, while $(\bar{u}_1)_2$, $(\bar{\tau}_{12})_2$, $(\bar{\tau}_{13})_2$ satisfy

$$\left.\begin{aligned} (\bar{\tau}_{12})_2 &= \bar{\mu}\frac{\partial(\bar{u}_1)_2}{\partial y_2} \\ (\bar{\tau}_{13})_2 &= \bar{\mu}\frac{\partial(\bar{u}_1)_2}{\partial y_3} \end{aligned}\right\} \tag{IIA. 1b}$$

$$\left(\bar{\mu} = \frac{p}{p/\mu + 1/\eta}\right)$$

$$\frac{\partial}{\partial y_2}(\bar{\tau}_{12})_2 + \frac{\partial}{\partial y_3}(\bar{\tau}_{13})_2 = 0 \tag{IIA. 2b}$$

$$\left.\begin{aligned} (\bar{\tau}_{13})_2 &= 0 \quad \text{at} \quad y_3 = 0 \\ (\bar{\tau}_{13})_2 &\rightarrow 0 \quad \text{as} \quad y_3 = \infty \end{aligned}\right\} \tag{IIA. 3b}$$

$$(\bar{\tau}_{12})_2 \rightarrow 0 \quad \text{as} \quad y_2 \rightarrow \infty \quad \text{for } y_3 > 0 \tag{IIA. 4b}$$

$$\nabla^2 (\bar{u}_1)_2 = 0 \tag{IIA. 5b}$$

$$[(\bar{u}_1)_2] = f(y_3)\,\bar{U}(p) \ \text{across F} \tag{IIA. 6b}$$

and
$$[(\bar{\tau}_{12})_2] = 0 = [(\bar{\tau}_{13})_2] \tag{IIA. 7b}$$

The solution for $(\bar{u}_1)_1$, $(\bar{\tau}_{12})_1$, $(\bar{\tau}_{13})_1$ is easily found to be

$$\left.\begin{aligned} (\bar{u}_1)_1 &= \frac{\tau_\infty \cdot y_2}{\eta \cdot p^2} + \frac{(u_1)_0}{p} \\ (\bar{\tau}_{12})_1 &= \tau_\infty\left(\frac{1}{p} - \frac{1}{p+\mu/\eta}\right) + \frac{(\tau_{12})_0}{p+\mu/\eta} \\ \text{and} \quad (\bar{\tau}_{13})_1 &= (\tau_{13})_0/(p + \mu/\eta) \end{aligned}\right\} \tag{IIA. 8a}$$

The boundary value problem for $(\bar{u}_1)_2$, $(\bar{\tau}_{12})_2$ $(\bar{\tau}_{13})_2$ can be solved by using a suitable modification of the Green's function technique developed by Maruyama (1966) and Rybicki (1971) for static dislocations in elastic media. Following Maruyama (1966) and Rybicki (1971), it is easily shown that

$$(\bar{u}_1)_2(Q) = \int_F [(\bar{u}_1)_2(P)]\, G(P, Q)\, dx_3$$

where $Q(y_1, y_2, y_3)$ is any point in the medium, $P(x_1, x_2, x_3)$ is any point on the fault, and

$$[(\bar{u}_1)_2(P)] = f(x_3)\,\bar{U}(p)$$

is the discontinuity in $(\bar{u}_1)_2$ across F at P. G(P, Q) is a Green's function, given by

$$G(P, Q) = \mu \frac{\partial}{\partial x_2} G_1(P, Q)$$

where
$$G_1(P, Q) = \frac{1}{4\pi\mu}\left[\ln\{(x_2-y_2)^2 + (x_3-y_3)^2\} + \ln\{(x_2+y_2)^2 + (x_3+y_3)^2\}\right]$$

The integral for $(\bar{u}_1)_2$ can be evaluated in closed form if $f(y_3)$ is a polynomial in y_3. After obtaining $(\bar{u}_1)_2$, we obtain $(\bar{\tau}_{12})_2$, $(\bar{\tau}_{13})_2$ from (IIA. 1b). Finally, using (IIA. 8a), we obtain

$$\bar{u}_1 = (\bar{u}_1)_1 + (\bar{u}_1)_2$$

$$\bar{\tau}_{12} = (\bar{\tau}_{12})_1 + (\bar{\tau}_{12})_2 \quad \text{and} \quad \bar{\tau}_{13} = (\bar{\tau}_{13})_1 + (\bar{\tau}_{13})_2$$

and verify that they satisfy (IIA. 1a) — (IIA. 7a). The Laplace transforms are then inverted, using the convolution theorem, to give u_1, τ_{12}, τ_{13} as functions of y_2, y_3, t.

Acknowledgement

One of the authors, Arabinda Mukhopadhyay, thanks, the Department of Applied Mathematics and Theoretical Physics, University of Cambridge, where a part of this work was carried out, for the facilities extended to him. He also thanks the Association of Commonwealth Universities, U.K. and the British Council, for financial assistance which enable him to work in Cambridge as a Seniar Visitor Barun Prasad Pal thanks the Director General and Chief Geophysicist, G.S.I., for their kind interest and encouragement. Sanjay Sen thanks the Director, River Research Institute, West Bengal for his kind interest and cooperation.

References

Aki, K. (1967) : J. Geophys. Res., 72, 1217-1231.

Budiansky, B. and Amazigo, J.C. (1976) : J. Geophys. Res., 81, 4897-4900,

Fung, Y.C. (1964) : Foundations of Solid Mechanics, Prentice-Hall.

Maruyama, T (1966) : Bull. Earthquake Res. Inst., Tokyo Univ., 44, 811-871.

Mukhopadhyay, A. and Mukherji, P. (1978) : Proc. Sixth Symposium on Earthquake Engineering. Roorkee, Vol. 1 71-76.

Rundle, J.B. and Jackson, D.D. (1977) ; Geophys. J.R. Astr. Soc., 49, 575-592.

Rybicki, K. (1971) : Bull. Seis. Soc. Amer., 61, 79-92.

Spence, D.A. anu Turcotte, D.L. (1976) : Proc. Roy. Soc. Lond., A, 349, 319-341.