

A MODAL COMBINATION RULE FOR ORDERED PEAK RESPONSE UNDER MULTI-COMPONENT GROUND MOTION

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ABSTRACT

The traditional earthquake-resistant design philosophy involves estimation of the peak elastic response for the specified seismic hazard, which in turn requires a modal combination rule for the multi-degree-of-freedom systems. The typically used rules estimate just the largest response peaks, while the second largest, third largest, ... peaks are assumed to be of no significance. Considering the possibility that structural damage in the post-yield regime can be correlated with these higher-order peaks, a new modal combination rule is developed for the ordered peak response of multistoried buildings excited by the multi-component ground motions. The proposed rule is formulated by using the stationary random vibration theory and by making suitable approximations regarding the peak factors and nonstationarity factors. A numerical study shows that the proposed rule performs better than the CQC3 rule when the building is stiffer to the ground motion and that the level of accuracy for the higher-order peaks up to the 10th largest peak is comparable to that for the largest peak.

KEYWORDS: Multi-Degree-of-Freedom Systems, Multi-component Ground Motions, Higher-Order Peaks, Order Statistics, Modal Combination Rule

INTRODUCTION

The present practice of designing multistoried buildings for seismic resistance involves the use of design (response) spectra as specified for the site under consideration. Design spectra typically specify the maximum elastic response of the single-degree-of-freedom (SDOF) oscillators of different periods and damping ratios under the perceived seismic hazard. Structural response estimated from these spectra is reduced via specified reduction factors in order to take advantage of the energy dissipation during the inelastic response of ductile structural systems. Estimating the (elastic) peak structural response in the case of a multi-degree-of-freedom (MDOF) system involves the use of a modal combination rule, unless the system can be assumed to vibrate as a SDOF system under the earthquake excitation. One can possibly generate time-histories consistent with the design spectra and estimate the peak structural response via numerical integration due to the easy availability of inexpensive and fast computational power. However, practicing engineers find it more convenient to estimate the peak response directly from the response spectra. Furthermore, seismic codes rely on simple and direct procedures, like modal combination rules, wherever possible.

Several researchers (e.g., Goodman et al., 1953; Rosenblueth and Elorduy, 1969; Der Kiureghian, 1981; Wilson et al., 1981; Singh and Mehta, 1983; Der Kiureghian and Nakamura, 1993) have worked on the estimation of the largest peak response in a MDOF system from a prescribed design spectrum and have proposed simple rules of modal combination for different situations. The most popular of these rules, i.e., the SRSS (Square-Root-of-Sum-of-Squares) rule by Goodman et al. (1953), is meant for the structures with well-separated dominant modes and for the ground motions acting like white noise over those modes. Rosenblueth and Elorduy (1969) made the first attempt to account for the correlation in different modes, while Der Kiureghian (1981) and Wilson et al. (1981) proposed the popular CQC (Complete Quadratic Combination) rule without requiring the use of the duration of earthquake excitation. However, both rules are however based on the use of white noise idealization of the excitation and are therefore inappropriate for application when the dominant frequencies of the system are outside the frequency-band of significant energy in the excitation. Singh and Mehta (1983), Der Kiureghian and Nakamura (1993), and Gupta (1994) later proposed more generalized modal combination rules that could account for the narrow-band seismic inputs and effects of high-frequency modes. None of the past efforts,

except that by Gupta (1994), could however be used to estimate the higher-order (second largest, third largest, ...) response peak amplitudes. This is perhaps because higher-order peaks in the linear response were never recognized to constitute an important data for the earthquake-resistant design, even though Amini and Trifunac (1985), Gupta and Trifunac (1987a), and Gupta and Trifunac (1989) made significant attempts to estimate the ordered peak amplitudes in the response of MDOF systems. Recently, Sadhu (2007) showed that higher-order peaks in the linear response may be useful in a simple estimation of a damage measure during the inelastic response. Gupta (1994) developed a modal combination rule that proposed an improvement over the existing rules in a simple manner and also provided for the estimation of the higher-order peaks.

The modal combination rules as mentioned above were proposed for the specific situation of translational ground motion acting along one of the structure axes. However, as shown by Penzien and Watabe (1975), it is important to consider all three translational components of the ground motion acting simultaneously in estimating the structural response. They showed that there exists a set of principal directions along which the ground motion components are uncorrelated. They also observed that these directions remain stable with time during the strong motion phase of the ground motion and that the major principal axis remains horizontal and directed from the epicenter to the site while the minor principal axis is kept vertical.

Development of combination rules for the multi-component ground motions was first attempted by O'Hara and Cunniff (1963). They suggested the NRLS (Naval Research Laboratory Sum) method, in which the resultant response is defined as the maximum of the three components plus the SRSS of the other two. Chu et al. (1972) proposed the use of SRSS method for finding the resultant response. This was later accepted by USNRC (1976). Among the percentage rules, Newmark (1975) suggested the (Max + 40%) rule wherein the maximum of the three components is added to 40% of the other two. In a slightly modified form, Rosenblueth and Contreras (1977) proposed the (Max + 30%) rule, which was later incorporated in the ATC-3 provisions (ATC, 1978). Anagnostopoulos (1981) made a comparative study of different rules and showed that neither of the existing rules properly accounted for the cross-correlation between the ground motion components. Smeby and Der Kiureghian (1985) and Menun and Der Kiureghian (1998) generalized the CQC rule to the CQC3 (Complete Quadratic Combination with three components) rule for application to multi-component excitations, based on the Penzine-Watabe characterization of ground motions and thus accounting for the cross-correlation between the different ground motion components. Hernandez and Lopez (2002) developed a more versatile combination rule, GCQC3 (Generalised Complete Quadratic Combination with three components) that takes into account the quasi-horizontal and quasi-vertical principal components. None of these rules is however meant to estimate the higher-order response peak amplitudes. Further, these rules are developed specifically for those situations when the input ground motion can be assumed to be white noise over the dominant structural frequencies. The response spectrum-based formulation by Gupta and Trifunac (1987b) is the only effort in the direction of estimating the higher-order peaks under multi-component excitations. However, this formulation is for the situation when the structural axes are aligned with the principal axes of the ground motion, and further this does not provide the convenience of a modal combination rule.

Based on the above, there exists a clear need to develop a simple and more versatile modal combination rule that can estimate not just the largest peak but also the second largest, third largest, ... peaks in the response of a multi-storied building under the excitation of multi-component ground motion with arbitrary characteristics. The present study aims to develop such a rule for a fixed-base, MDOF system excited by a multi-component ground motion. The proposed rule is developed by broadly following the procedure adopted by Gupta (1994) under the framework of stationary random vibration theory. Performance of the proposed rule is evaluated through a numerical study based on six recorded ground motions with wide variety in their characteristics and for a 5-story building with seven different sets of floor mass and story stiffness properties.

FORMULATION OF THE PROPOSED RULE

1. PSDF of a Typical Response

Let us consider a linear, classically damped, lumped-mass system having n degrees of freedom (DOFs). The system is fixed-base and is subjected to three translational ground accelerations at its base:

$\ddot{u}_{g1}(t)$ and $\ddot{u}_{g2}(t)$ along two mutually perpendicular horizontal directions (X_s and Y_s) aligned with the structure axes, and $\ddot{u}_{g3}(t)$ along the vertical direction. On expanding the response of the system in terms of the normal coordinates and undamped mode shapes of the system, let ω_j and ζ_j respectively denote the natural frequency and damping ratio in the j th mode. Further, let $\gamma_1^{(j)}$, $\gamma_2^{(j)}$, and $\gamma_3^{(j)}$ respectively denote the participation factors with respect to $\ddot{u}_{g1}(t)$, $\ddot{u}_{g2}(t)$, and $\ddot{u}_{g3}(t)$ in the j th mode.

On assuming stationarity in the excitation and in the response, PSDF of the response $r(t)$ of the system may be expressed as (Sadhu, 2007)

$$S_r(\omega) = \sum_{k=1}^3 \sum_{l=1}^3 S_z^{kl}(\omega) \sum_{j=1}^n \sum_{q=1}^n r_j r_q \gamma_k^{(j)} \gamma_l^{(q)} \text{Re}(H_j(\omega) H_q^*(\omega)) \quad (1)$$

where $S_z^{kl}(\omega)$ is the cross-PSDF of the k th and l th components of the base acceleration for $k \neq l$, and is the PSDF of the k th component for $k = l$; r_j is the normalized amplitude of the response $r(t)$ in the j th mode of vibration and is expressed as a linear combination of the elements of the j th mode shape (e.g., it is equal to the i th element of the j th mode shape for the displacement response at the i th DOF); and

$$H_j(\omega) = \frac{-1}{\omega_j^2 - \omega^2 + 2i\zeta_j \omega_j \omega} \quad (2)$$

(with $i = \sqrt{-1}$) is the transfer function relating the relative displacement of the equivalent SDOF oscillator in the j th mode to the input base acceleration.

On using the partial fractions for $\text{Re}(H_j(\omega) H_q^*(\omega))$ as in Gupta and Trifunac (1990), Equation (1) leads to

$$S_r(\omega) = \sum_{k=1}^3 \sum_{l=1}^3 S_z^{kl}(\omega) \sum_{j=1}^n \sum_{q=1}^n \left\{ r_j r_q \gamma_k^{(j)} \gamma_l^{(q)} (C_{jq} + D_{jq}) |H_j(\omega)|^2 - r_j r_q \gamma_k^{(j)} \gamma_l^{(q)} D_{jq} \frac{|\omega H_j(\omega)|^2}{\omega_j^2} \right\} \quad (3)$$

where C_{jq} and D_{jq} are the coefficients given in terms of ζ_j , ζ_q , and $\varrho = \omega_q/\omega_j$ as

$$C_{jq} = \frac{1}{B_{jq}} \left[8\zeta_j (\zeta_j + \zeta_q \varrho) \{ (1 - \varrho^2)^2 - 4\varrho (\zeta_j - \zeta_q \varrho) (\zeta_q - \zeta_j \varrho) \} \right] \quad (4)$$

$$D_{jq} = \frac{1}{B_{jq}} \left[2(1 - \varrho^2) \{ 4\varrho (\zeta_j - \zeta_q \varrho) (\zeta_q - \zeta_j \varrho) - (1 - \varrho^2)^2 \} \right] \quad (5)$$

with

$$B_{jq} = 8\varrho^2 \left[(\zeta_j^2 + \zeta_q^2) (1 - \varrho^2)^2 - 2(\zeta_q^2 - \zeta_j^2 \varrho^2) (\zeta_j^2 - \zeta_q^2 \varrho^2) \right] + (1 - \varrho^2)^4 \quad (6)$$

C_{jq} becomes maximum at $\varrho = 1$, while D_{jq} becomes maximum near $\varrho = 1$ (it is equal to zero at $\varrho = 1$). Both sharply fall off to small values as $\varrho \ll 1$ and $\varrho \gg 1$.

In general, the translational components $\ddot{u}_{g1}(t)$, $\ddot{u}_{g2}(t)$, and $\ddot{u}_{g3}(t)$ are correlated processes. However, Penzien and Watabe (1975) have shown that there exists a set of (orthogonal) principal directions, along which the components of ground acceleration are uncorrelated. The orientation of these axes remains approximately constant with time during the strong motion phase of the ground motion. During this phase, the major principal axis is horizontal and directed from the epicenter to the site, the intermediate principal axis is horizontal and perpendicular to the major axis, and the minor principal axis is nearly vertical. Since the axes X_s and Y_s of a structure in plan may not always align with the major and intermediate principal axes, X_p and Y_p , as shown in Figure 1, components of the motion along the axes of the structure are usually correlated. The degree of this correlation depends on the relative orientation of the structure axes with respect to the principal directions of the excitation.

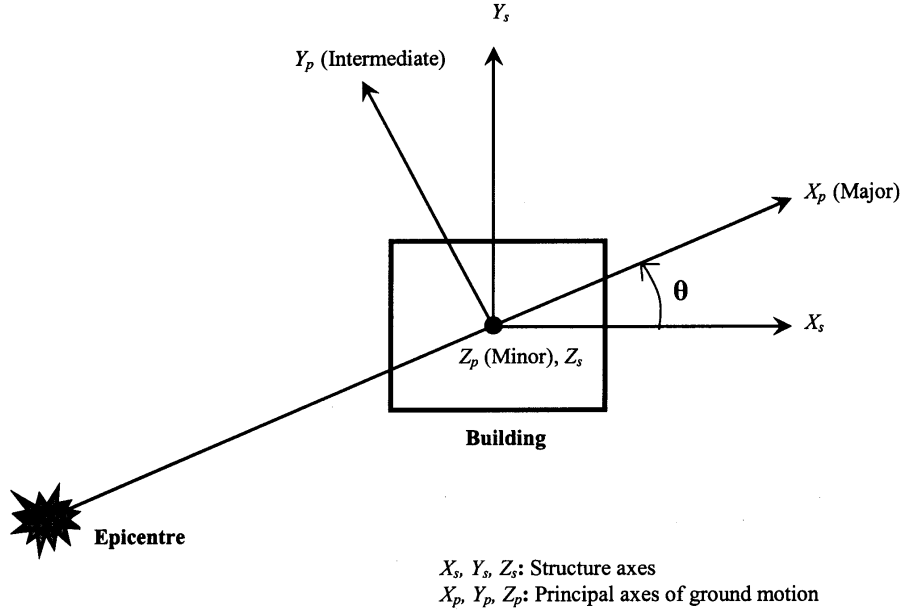


Fig. 1 Illustration of structure axes and principal axes of ground motion

It is possible to express the PSDF matrix $[S_z(\omega)]$ of the ground acceleration vector $\{\ddot{u}_g(t)\}$ along the structure axes in terms of the PSDF matrix $[S_z^p(\omega)]$ of the acceleration vector $\{\ddot{u}_g^p(t)\}$ along the principal directions as (Smeby and Der Kiureghian, 1985)

$$[S_z(\omega)] = [R]^T [S_z^p(\omega)] [R] \quad (7)$$

where $[R]$ is the transformation matrix given by

$$[R] = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

in terms of the relative orientation θ of the major principal axis X_p with respect to the structure axis X_s (see Figure 1). The off-diagonal elements of $[S_z(\omega)]$ and $[S_z^p(\omega)]$ denote the cross-PSDFs of the corresponding ground acceleration components. Since the off-diagonal elements of $[S_z^p(\omega)]$ are zero, i.e., $S_z^{kl,p} = 0$ for $k \neq l$, the off-diagonal elements of $[S_z(\omega)]$ are real quantities. Also, the cross-PSDFs of the vertical component ($\ddot{u}_{g_3}(t)$) with the two horizontal components ($\ddot{u}_{g_1}(t)$ and $\ddot{u}_{g_2}(t)$) are zero.

On substituting the expressions of PSDFs and cross-PSDFs of the accelerations along the structure axes from Equation (7), Equation (3) becomes

$$S_r(\omega) = \sum_{j=1}^n \sum_{q=1}^n r_j r_q \left\{ (C_{jq} + D_{jq}) |H_j(\omega)|^2 - D_{jq} \frac{|\omega H_j(\omega)|^2}{\omega_j^2} \right\} \times \left[\sum_{k=1}^3 \gamma_k^{(j)} \gamma_k^{(q)} S_z^{kk,p}(\omega) - \sum_{k=1}^2 \sum_{l=1}^2 (-1)^{k+l} \gamma_l^{(j)} \gamma_l^{(q)} S_z^{kk,p}(\omega) \sin^2 \theta - \sum_{k=1}^2 (-1)^k (\gamma_1^{(j)} \gamma_2^{(q)} + \gamma_2^{(j)} \gamma_1^{(q)}) S_z^{kk,p}(\omega) \sin \theta \cos \theta \right] \quad (9)$$

This expression can be used to obtain the response PSDF of the system from the (three) PSDFs of principal ground accelerations and orientation of X_p with respect to X_s (instead of PSDFs and cross-PSDFs for the ground accelerations along the structure axes).

2. Ordered Peak Response Amplitudes

In stationary random vibration theory, ordered peak amplitudes of any response are estimated by computing moments of the PSDF of the response process and by multiplying the peak factor computed from these moments with the root-mean-square (r.m.s.) value of the process. This procedure is followed in this section to formulate the expression for an ordered peak of a typical response $r(t)$.

On taking the p th moment of $S_r(\omega)$ about the origin,

$$\lambda_p^r = \int_0^\infty \omega^p S_r(\omega) d\omega \quad (10)$$

Equation (9) leads to

$$\lambda_p^r = \sum_{j=1}^n \sum_{q=1}^n r_j r_q \left[\sum_{k=1}^3 \gamma_k^{(j)} \gamma_k^{(q)} \delta_{p,jq}^k \lambda_{p,j}^{D,k} - \sum_{k=1}^2 \sum_{l=1}^2 (-1)^{k+l} \gamma_l^{(j)} \gamma_l^{(q)} \delta_{p,jq}^k \lambda_{p,j}^{D,k} \sin^2 \theta \right. \\ \left. - \sum_{k=1}^2 (-1)^k (\gamma_1^{(j)} \gamma_2^{(q)} + \gamma_2^{(j)} \gamma_1^{(q)}) \delta_{p,jq}^k \lambda_{p,j}^{D,k} \sin \theta \cos \theta \right] \quad (11)$$

where,

$$\delta_{p,jq}^k = C_{jq} + D_{jq} v_{p,j}^k \quad (12)$$

with

$$v_{p,j}^k = 1 - \frac{\lambda_{p,j}^{V,k}}{\omega_j^2 \lambda_{p,j}^{D,k}} \quad (13)$$

is the term determining the extent of cross-correlation of the j th and q th modes in the p th moment during the excitation by the k th principal component of the ground acceleration. Further, in Equations (11) and (13),

$$\lambda_{p,j}^{D,k} = \int_0^\infty \omega^p |H_j(\omega)|^2 S_z^{kk,p}(\omega) d\omega \quad (14)$$

is the p th moment of the PSDF of the relative displacement response of a SDOF oscillator with ω_j frequency and ζ_j damping ratio, and subjected to the base acceleration $\ddot{u}_{gk}^p(t)$, and

$$\lambda_{p,j}^{V,k} = \int_0^\infty \omega^p |\omega H_j(\omega)|^2 S_z^{kk,p}(\omega) d\omega \quad (15)$$

is the p th moment of the PSDF of the relative velocity response of this oscillator. It may be mentioned that for $p = 0$, $v_{p,j}^k$ is a measure of the deviation of the rate of zero crossings of the displacement response of the same SDOF oscillator from that in the case of an ideal white noise excitation (Gupta, 2002). This factor decreases with the increasing natural period and becomes zero near the dominant period of the ground motion. Further, $v_{p,j}^k$ together with D_{jk} becomes an important component of cross-correlation when ω_j is not close to the dominant frequency of the ground motion and therefore may be ignored when the excitation acts like white noise over the frequencies of interest.

Equation (11) may be used to calculate the moments, λ_0^r , λ_2^r , and λ_4^r , of the response PSDF by taking $p = 0, 2, 4$, respectively. The r.m.s. value of the response process may be estimated by taking the square-root of λ_0^r , and the peak factor (for the desired order and level of confidence) may be estimated by using all three moments along with the strong motion duration of excitation (Gupta, 2002). On multiplication of the r.m.s. value with the peak factors for different orders, estimates of the largest, second largest, third largest, ... peaks may be obtained. In order to include the effects of inherent nonstationarity in response, the (stationary) r.m.s. value may be modified by multiplying it with a nonstationarity factor. We assume that this factor is known. It is further assumed that the peak factors would remain affected due to the nonstationarity, as those depend on the ratios of the moments of response PSDF, not on the moments *per se*. The nonstationarity factor may be close to unity provided the excitation PSDF $S_z^{kk,p}(\omega)$

is compatible with a response spectrum (for the k th principal component of the ground acceleration) and thus includes the effects of nonstationarity indirectly (see, for example, Kaul, 1978; Unruh and Kana, 1981; Christian, 1989).

In view of the above discussion, the s th ordered peak amplitude of the response process may be expressed as

$$r_{\text{peak}}^{(s)} = \eta_r^{(s)} \beta_r^{(s)} \left[\sum_{j=1}^n \sum_{q=1}^n r_j r_q \left\{ \sum_{k=1}^3 \gamma_k^{(j)} \gamma_k^{(q)} \delta_{0,jq}^k \lambda_{0,j}^{D,k} - \sum_{k=1}^2 \sum_{l=1}^2 (-1)^{k+l} \gamma_l^{(j)} \gamma_l^{(q)} \delta_{0,jq}^k \lambda_{0,j}^{D,k} \sin^2 \theta - \sum_{k=1}^2 (-1)^k (\gamma_1^{(j)} \gamma_2^{(q)} + \gamma_2^{(j)} \gamma_1^{(q)}) \delta_{0,jq}^k \lambda_{0,j}^{D,k} \sin \theta \cos \theta \right\} \right]^{1/2} \quad (16)$$

where $\eta_r^{(s)}$ is the corresponding peak factor and $\beta_r^{(s)}$ is the nonstationarity factor.

Continuing with the logic of relating the ordered peak response with the r.m.s. response via nonstationarity factor and peak factor, $\lambda_{0,j}^{D,k}$ may be expressed as $(\text{SD}_j^k / \eta_{j,(1)}^{D,k} \beta_j^{D,k})^2$, where SD_j^k is the largest peak amplitude of the relative displacement response of the SDOF oscillator with ω_j frequency and ζ_j damping ratio in response to the base acceleration, $\ddot{u}_{gk}^p(t)$, for the same level of confidence to which $\eta_r^{(s)}$ corresponds; $\eta_{j,(1)}^{D,k}$ is the corresponding peak factor; and $\beta_j^{D,k}$ is the nonstationarity factor associated with the response process. In the same way, $\lambda_{0,j}^{V,k}$ may be expressed as $(\text{SV}_j^k / \eta_{j,(1)}^{V,k} \beta_j^{V,k})^2$, where SV_j^k is the largest peak amplitude of the relative velocity response; $\eta_{j,(1)}^{V,k}$ is the corresponding peak factor; and $\beta_j^{V,k}$ is the nonstationarity factor. Equation (16) may thus be expressed as

$$r_{\text{peak}}^{(s)} = \left[\sum_{j=1}^n \sum_{q=1}^n r_j r_q \left\{ \sum_{k=1}^3 \gamma_k^{(j)} \gamma_k^{(q)} \delta_{0,jq}^k \left(\frac{\text{SD}_j^k}{\bar{\eta}_{j,(1)}^{D,k} \bar{\beta}_j^{D,k}} \right)^2 - \sum_{k=1}^2 \sum_{l=1}^2 (-1)^{k+l} \gamma_l^{(j)} \gamma_l^{(q)} \delta_{0,jq}^k \left(\frac{\text{SD}_j^k}{\bar{\eta}_{j,(1)}^{D,k} \bar{\beta}_j^{D,k}} \right)^2 \sin^2 \theta - \sum_{k=1}^2 (-1)^k (\gamma_1^{(j)} \gamma_2^{(q)} + \gamma_2^{(j)} \gamma_1^{(q)}) \delta_{0,jq}^k \left(\frac{\text{SD}_j^k}{\bar{\eta}_{j,(1)}^{D,k} \bar{\beta}_j^{D,k}} \right)^2 \sin \theta \cos \theta \right\} \right]^{1/2} \quad (17)$$

where, $\bar{\eta}_{j,(1)}^{D,k}$ is the peak factor $\eta_{j,(1)}^{D,k}$ normalized by $\eta_r^{(s)}$ and $\bar{\beta}_j^{D,k}$ is the nonstationarity factor $\beta_j^{D,k}$ normalized by $\beta_r^{(s)}$. Further, $\delta_{0,jq}^k$ may be expressed as

$$\delta_{0,jq}^k = C_{jq} + D_{jq} \left[1 - \left(\frac{\eta_{j,(1)}^{D,k}}{\eta_{j,(1)}^{V,k}} \right)^2 \left(\frac{\beta_j^{D,k}}{\beta_j^{V,k}} \right)^2 \left(\frac{\text{SV}_j^k}{\text{PSV}_j^k} \right)^2 \right] \quad (18)$$

where $\text{PSV}_j^k (= \omega_j \text{SD}_j^k)$ is the largest peak amplitude of the pseudo-velocity response.

In the next sub-section, suitable approximations will be made to develop a modal combination rule from Equation (17).

3. Approximations for the Proposed Rule

Equation (17) may be used to estimate the ordered response peak amplitudes for the same level of confidence for which SD_j^k and SV_j^k have been estimated. This may also be used to estimate the peak amplitudes consistent with the seismic hazard at a site, which is characterized by certain spectral displacement (SD) and spectral velocity (SV) curves for the three components of the ground motion. There is a need, however, to have reasonable estimates of normalized nonstationarity and peak factors (in Equation (17)) and nonstationarity and peak factors for the displacement and velocity responses (in Equation (18)).

It is proposed to carry out two types of simplifications: one relating to the factors in Equation (17) and another to the factors in Equation (18). The normalized nonstationarity and peak factors refer to how different these factors are in the largest modal response and in a higher-order system response. The normalized nonstationarity factors may be assumed equal to unity provided (i) the system response and the modal response for any order of peak are affected by comparable amounts due to nonstationarity, and (ii) a higher-order response is affected as much by the nonstationarity as the largest response in the modal and system responses. The former is strictly not true as the rate of convergence to the state of stationarity by a modal response depends on the number of cycles per unit time in response (or the modal frequency) whereas this rate in the system response is governed by the natural frequencies of the dominating modes. Due to this, the normalized nonstationarity factor is likely to be less than unity in the case of lower modes, and greater than unity for the higher modes. Similarly, normalized nonstationarity factors may be more in the case of higher-order peaks as nonstationarity affects a higher-order response more than a lower order response (Gupta and Trifunac, 1987a). For simplicity, however, $\bar{\beta}_j^{D,k}$ is uniformly assumed equal to unity. The normalized peak factor $\bar{\eta}_{j,(1)}^{D,k}$ also involves the effects of (i) the peak factors being different for the system response and the modal response (for the same order of peak), and (ii) the peak factors being different for different orders of peaks (for the system or modal response). The effect of the former is negligible due to little sensitivity of the peak factor to the governing statistical parameters (i.e., band-width and number of peaks) within the range anticipated for both system and modal responses. The effect of the order of peak can be approximated by a simple expression proposed by Gupta (1994), and therefore, the normalized peak factor is proposed to be

$$\bar{\eta}_{j,(1)}^{D,k} \equiv \frac{\eta_{j,(1)}^{D,k}}{\eta_r^{(s)}} = 1 \quad ; \quad s = 1$$

$$= \frac{1}{0.4e^{-0.25s} + 0.67} \quad ; \quad s = 2, 3, \dots \tag{19}$$

The nonstationarity factor for the modal displacement response in Equation (18) is likely to be greater than that for the modal velocity response due to domination by the longer periods. For simplicity, however, this discrepancy between the two factors is proposed to be neglected. The peak factors for the modal displacement and velocity responses are anyway expected to be very close as both refer to the largest peak.

In view of the above approximations, the proposed modal combination rule may be expressed as

$$r_{\text{peak}}^{(s)} = \left[\sum_{j=1}^n \sum_{q=1}^n r_j r_q \left\{ \sum_{k=1}^3 \gamma_k^{(j)} \gamma_k^{(q)} \delta_{0,jq}^k \left(\frac{SD_j^k}{\bar{\eta}_{j,(1)}^{D,k}} \right)^2 - \sum_{k=1}^2 \sum_{l=1}^2 (-1)^{k+l} \gamma_l^{(j)} \gamma_l^{(q)} \delta_{0,jq}^k \left(\frac{SD_j^k}{\bar{\eta}_{j,(1)}^{D,k}} \right)^2 \sin^2 \theta \right. \right. \\ \left. \left. - \sum_{k=1}^2 (-1)^k (\gamma_1^{(j)} \gamma_2^{(q)} + \gamma_2^{(j)} \gamma_1^{(q)}) \delta_{0,jq}^k \left(\frac{SD_j^k}{\bar{\eta}_{j,(1)}^{D,k}} \right)^2 \sin \theta \cos \theta \right\} \right]^{1/2} \tag{20}$$

with

$$\delta_{0,jq}^k = C_{jq} + D_{jq} \left[1 - \left(\frac{SV_j^k}{PSV_j^k} \right)^2 \right] \tag{21}$$

and $\bar{\eta}_{j,(1)}^{D,k}$ as in Equation (19). It may be mentioned that the proposed rule becomes same as the CQC3 rule (Menun and Der Kiureghian, 1998) for $\delta_{0,jq}^k = C_{jq}$ and for $s = 1$. The former condition is effectively obtained by assuming $v_{0,j}^k$ as zero, which, as discussed earlier, is strictly true only in the case of white-noise excitations. Further, $\delta_{0,jq}^k$, as in Equation (21), is similar to the cross-correlation term used in the formulation of Singh and Mehta (1983).

NUMERICAL ILLUSTRATION OF THE PROPOSED RULE

1. Example Building and Excitations

In order to illustrate the proposed rule, two horizontal components of six ground motion records as in Table 1 are considered. Vertical component is not considered, and thus $k = 2$ for each of these cases. Principal directions (major and intermediate) for each record are obtained through eigenvalue analysis of the 2×2 (temporal) covariance matrix of the common strong motion phase of the two horizontal components, as identified by the duration definition of Trifunac and Brady (1975). Orientation of the major principal direction with respect to the first component of each example motion in Table 1 (see the 4th column) is given in Table 2. For example, orientation of the major principal axis with respect to the S04E component in the case of the Borrego Mountain motion is 23.5° (clockwise). Using these values of orientation, principal components of the example motions ($\{\ddot{u}_g^p(t)\}$) are obtained as $[R]\{\ddot{u}_g(t)\}$, with θ in Equation (8) taken as the orientation of the major principal direction (from Table 2).

Table 1: Details of the Example Ground Motions

Record No.	Earthquake	Site	Components
1	Borrego Mountain Earthquake, 1968	Engineering Building, Santa Ana, Orange County, California	S04E S86W
2	Imperial Valley Earthquake, 1940	El Centro Site, Imperial Valley Irrigation District, California	S00E S90W
3	Kern County Earthquake, 1952	Taft Lincoln School Tunnel, California	N21E S69E
4	Michoacan Earthquake, 1985	Av. University Centre, Mexico City	N00E N90E
5	Parkfield Earthquake, 1966	Array No. 5, Cholame, Shandon, California	N05W N85E
6	San Fernando Earthquake, 1971	Utilities Building, 215 West Broadway, Long Beach, California	N90E N00E

Table 2: Details of the Principal Components of Example Ground Motions

Earthquake	Orientation of the Major Component (degree)	Characteristics	Major Component	Intermediate Component
Borrego Mountain Earthquake, 1968	-23.5	T_g (s)	5.6	1.9
		PGA (g)	0.014	0.011
Imperial Valley Earthquake, 1940	-21.6	T_g (s)	0.68	0.51
		PGA (g)	0.35	0.19
Kern County Earthquake, 1952	34.4	T_g (s)	0.65	0.42
		PGA (g)	0.21	0.15
Michoacan Earthquake, 1985	26.8	T_g (s)	2.10	2.07
		PGA (g)	0.19	0.08
Parkfield Earthquake, 1966	-18.4	T_g (s)	0.32	0.30
		PGA (g)	0.42	0.39
San Fernando Earthquake, 1971	-8.2	T_g (s)	5.82	4.91
		PGA (g)	0.03	0.02

The Fourier and pseudo spectral acceleration (PSA) spectra of the principal components of the six example motions are shown in Figures 2(a)–2(f) and Figures 3(a)–3(f). Figures 2(a)–2(f) show these spectra for the major principal component of the Borrego Mountain, Imperial Valley, Kern County, Michoacan, Parkfield, and San Fernando motions respectively, while Figures 3(a)–3(f) show these spectra for the intermediate principal component of these motions. In each figure, the two spectra are normalized to their respective maximum values. Table 2 gives the values of dominant period T_g (the period corresponding to the maximum of the Fourier spectrum) and peak ground acceleration (PGA) for both principal components of each of the six example motions. It may be observed that all the six motions cover a wide range of energy distributions, with dominant periods as 5.82 s at one end for the San Fernando motion and 0.3 s on the other for the Parkfield motion. In terms of the band of significant energy, the Michoacan motion is at one extreme with significant energy over a narrow band of 1.8–3 s, while the Kern County motion is at the other with significant energy over a wide band of 0.2–6 s.

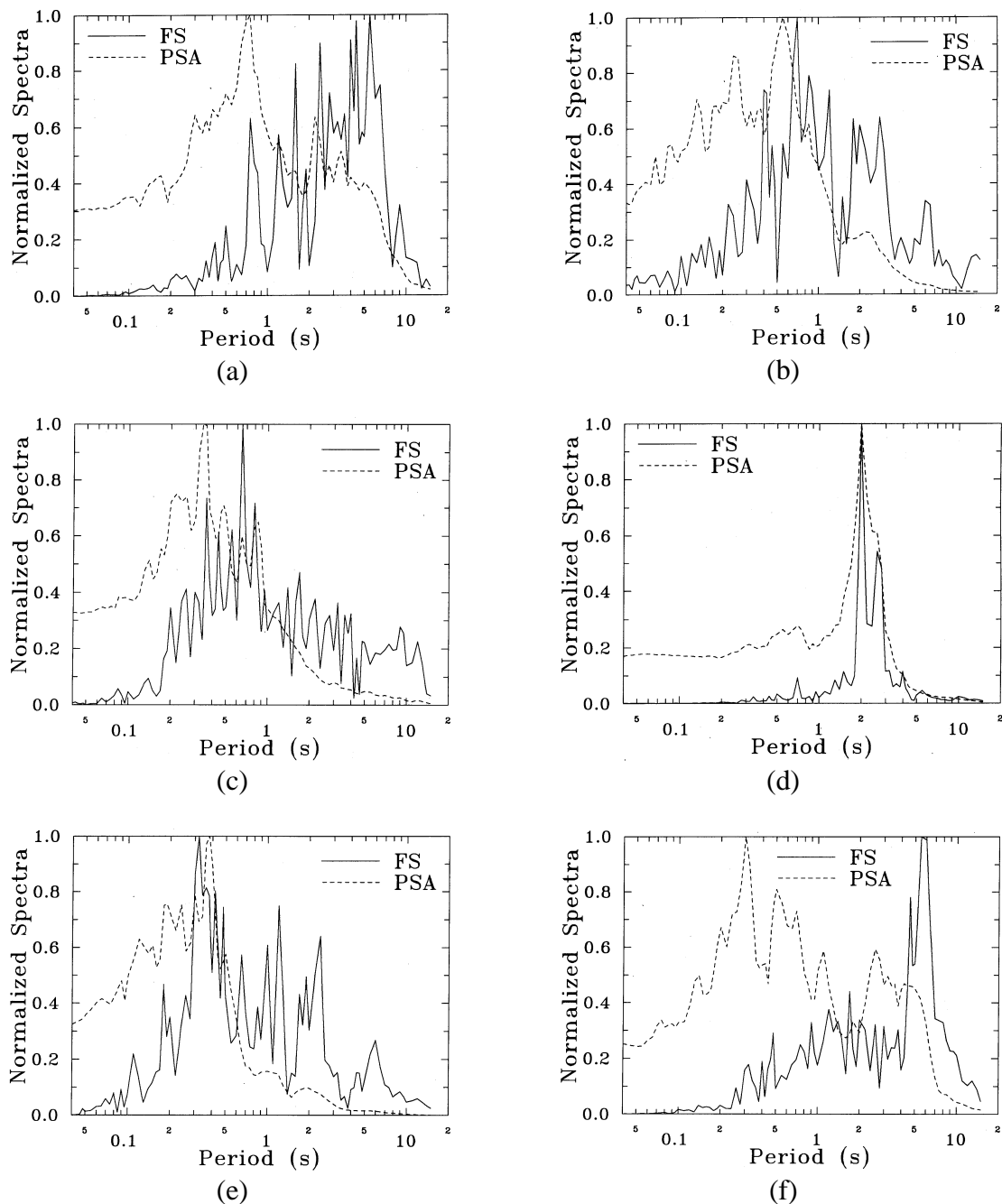


Fig. 2 Normalized Fourier amplitude and PSA spectra for the major principal component of (a) Borrego Mountain, (b) Imperial Valley, (c) Kern County, (d) Michoacan, (e) Parkfield, and (f) San Fernando earthquake motions

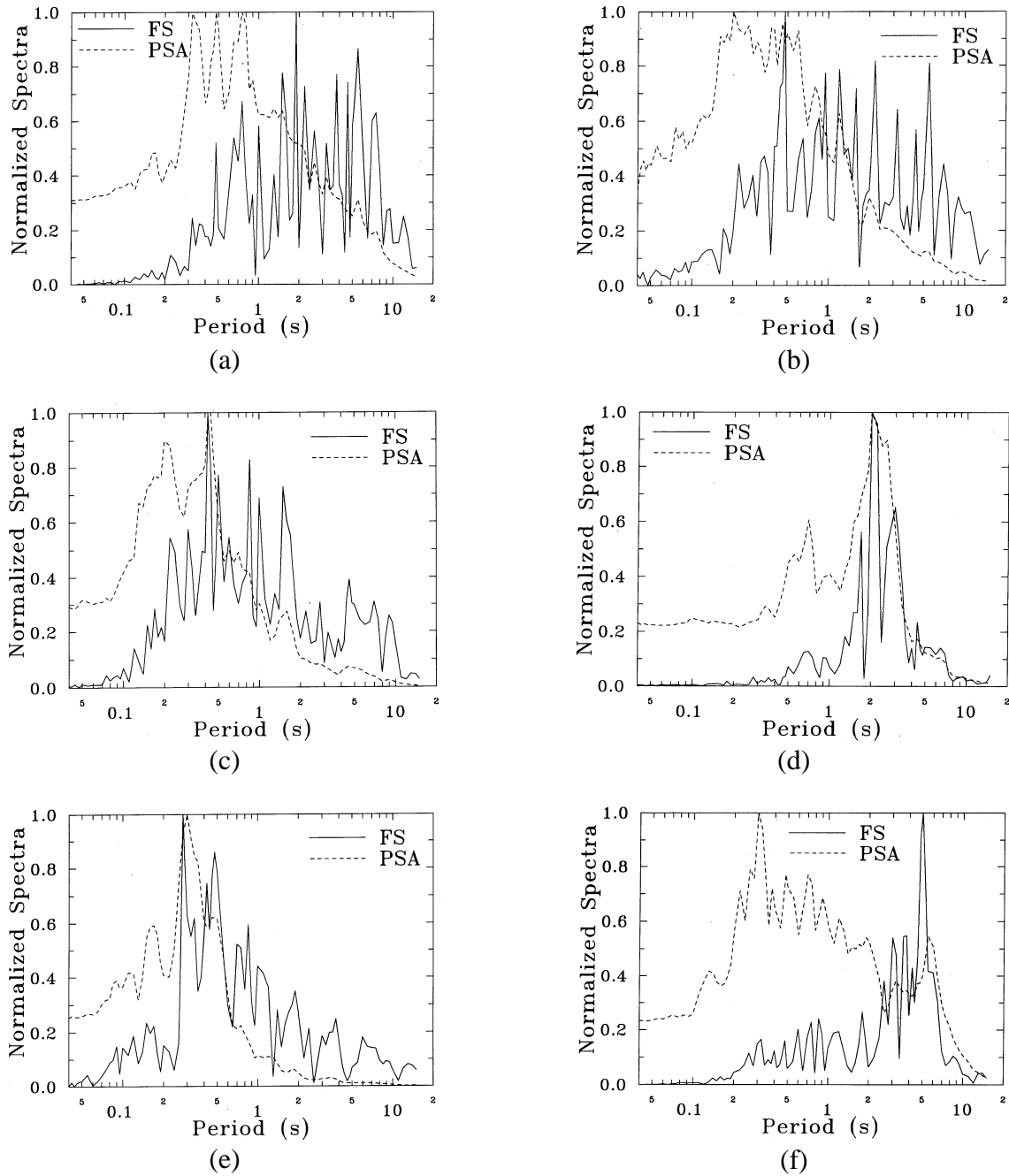


Fig. 3 Normalized Fourier amplitude and PSA spectra for the intermediate principal component of (a) Borrego Mountain, (b) Imperial Valley, (c) Kern County, (d) Michoacan, (e) Parkfield, and (f) San Fernando earthquake motions

A 5-story symmetric building having rigid floor masses supported by massless, inextensible columns is considered for the numerical study. Two translational DOFs are considered at each floor, and therefore, the example building is a 10-DOF system ($n = 10$). Seven different cases of this building involving different proportions in floor masses and story stiffnesses are considered. Table 3 shows the values of the reference floor masses and story stiffnesses (in the X_s - and Y_s -directions), and Table 4 shows seven different sets of factors α and β that are multiplied with the reference masses and stiffnesses, respectively, for the seven example cases of the building. Table 4 also shows the corresponding fundamental periods of the building in the X_s - and Y_s -directions. It may be observed that a wide range of fundamental periods of the multistoried buildings is covered by these example cases (0.03–2.0 s in the X_s -direction). The example building is assumed to be classically damped with modal damping ratio as 0.05. It may be noted that the pairs of the closely spaced modes in the example building are uncoupled,

and therefore, the illustration would not include the contribution of modal cross-correlation due to the closeness of frequencies for a single-component excitation. This is not a serious limitation though, because the proposed rule accounts for this contribution on well-established lines and thus the focus here is indeed not on examining the proposed rule against this contribution.

Table 3: Details of the Reference Floor Masses and Story Stiffnesses

Floor Level from Top	Floor Mass (t)	Story Stiffness in the X_s -Direction (kN/m)	Story Stiffness in the Y_s -Direction (kN/m)
1	800	2212000	1392000
2	800	2696000	2179000
3	800	3100000	2697000
4	800	3424000	3112000
5	1150	4998000	4792000

Table 4: Properties of the Example Building for Different Cases of Mass and Stiffness Properties

Case	α	β	Fundamental Period in the X_s -Direction (s)	Fundamental Period in the Y_s -Direction (s)
I	0.125	16	0.031	0.029
II	0.25	8	0.063	0.058
III	0.5	4	0.125	0.117
IV	1	2	0.254	0.234
V	2	1	0.53	0.467
VI	4	0.5	1.1	0.938
VII	8	0.25	2	1.87

2. Results and Discussion

To illustrate and evaluate the performance of the proposed modal combination rule, the example building is subjected, in all seven cases (of mass and stiffness properties), to each of the six pairs of horizontal principal excitations at its base, and the estimates of the largest base shear in the X_s -direction are obtained from (i) the (exact) time-history analysis, (ii) the proposed rule, and (iii) the CQC3 rule (Menun and Der Kiureghian, 1998). Since the orientation θ of the principal axis X_p of the ground motion (with respect to the structure axis X_s) is an input parameter, the estimates of the largest base shear are obtained for the entire range of θ from 0° to 180° .

Figures 4(a)–4(f) show the comparisons of the largest peaks of base shears in the X_s -direction for the exact, proposed, and CQC3 analyses in the cases of Borrego Mountain, Imperial Valley, Kern County, Michoacan, Parkfield, and San Fernando motions, respectively. Each figure shows the comparisons for the entire range of θ values. For these results, the example building is assumed to have mass and stiffness properties for Case IV, with fundamental periods equal to 0.254 and 0.234 s in the X_s - and Y_s -directions, respectively. All six figures show that the largest base shear in the X_s -direction is symmetric about $\theta = 90^\circ$ in the cases of proposed and CQC3 rules. This is due to the fact that the example building is symmetric and therefore there is no coupling between the modes of the building in the X_s - and Y_s -directions. The curves for the time-history results are however asymmetric, since $\ddot{u}_{g1}^p(t)$ and $\ddot{u}_{g2}^p(t)$ are not exactly uncorrelated (because the orientation of the principal directions has been determined based on

the strong motion segment of the ground acceleration). Due to this, accelerations along the X_s - and Y_s -directions in the case of $\theta = \alpha$ become different from those in the case of $\theta = 180^\circ - \alpha$, in terms of both frequency content and cross-correlation. In the case of this “residual cross-correlation” between the two principal components becoming zero, the two acceleration components (in the X_s - and Y_s -directions) for $\theta = \alpha$ would be different from those for $\theta = 180^\circ - \alpha$ only in the sense of the sign of the cross-PSDF between them, and this will not make any difference in the base-shear results due to symmetry of the structure. It may be observed from Figures 4(a)–4(f) that the estimates of base-shear from both rules (i.e., proposed and CQC3) are in reasonably good agreement with those from the time-history analysis results at all values of θ , except in the case of the Kern County motion. For this motion, both rules lead to large errors at around $\theta = 60^\circ$. This is possibly due to significant “residual correlation” between the two principal components.

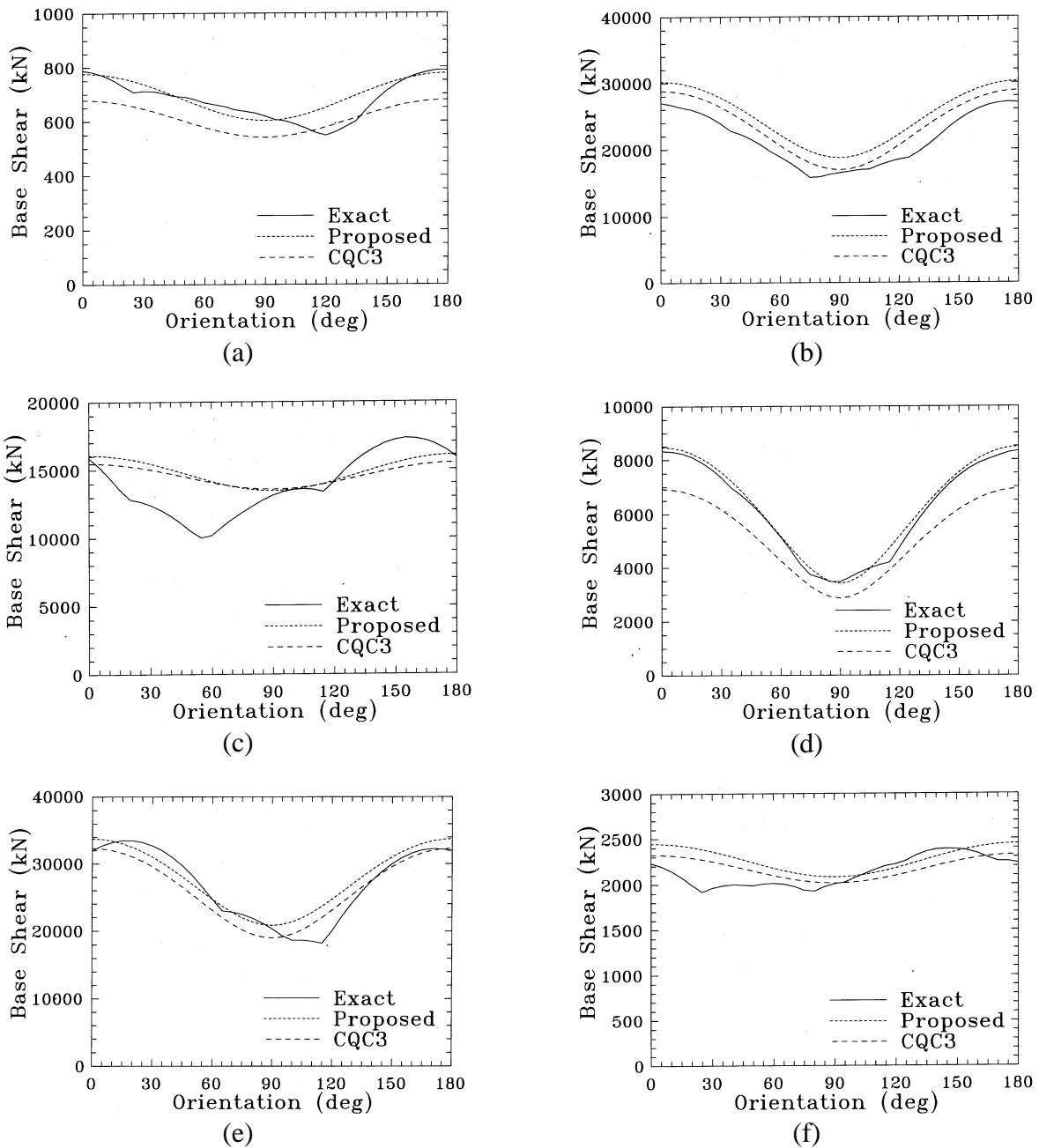


Fig. 4 Variation of largest base shear in the X_s -direction with orientation θ for (exact) time-history analysis, proposed rule, and CQC3 rule in the case of (a) Borrego Mountain, (b) Imperial Valley, (c) Kern County, (d) Michoacan, (e) Parkfield, and (f) San Fernando earthquake motions

For a more direct comparison of the performances of the proposed and CQC3 rules in the case of the base shear in the X_s -direction, absolute error is averaged over the entire range of θ between 0° – 180° and plotted with respect to $T_{g,\text{mean}}/T_{n,\text{mean}}$ for each of the six ground motions. Here, $T_{g,\text{mean}}$ is the average of the dominant periods of the major and intermediate principal components of the ground motion, and $T_{n,\text{mean}}$ is the average of the periods of the example building in the X_s - and Y_s -directions. Since there is not much difference in the periods in the X_s - and Y_s -directions, it is assumed that $T_{g,\text{mean}}/T_{n,\text{mean}}$ would be a good estimate for the mean of the ratios of dominant period to natural period in the two principal directions and thus this parameter would properly describe the extent to which the building is stiff to the ground motion. Higher the ratio, stiffer would be the building relative to the ground motion. Figures 5(a)–5(f) show the plots of absolute error with $T_{g,\text{mean}}/T_{n,\text{mean}}$ for the proposed and CQC3 rules in the cases of the Borrego Mountain, Imperial Valley, Kern County, Michoacan, Parkfield, and San Fernando motions, respectively. It may be observed that the performance of the proposed rule is quite good with the average error remaining within 16% in all the cases considered here. CQC3 rule is associated with greater errors in most of the cases, even though these errors do not exceed 22%. The performance of the proposed rule is significantly better than that of the CQC3 rule, particularly when the building is stiffer relative to the ground motion ($T_{g,\text{mean}}/T_{n,\text{mean}} \gg 1$). In the case of a narrow-band motion like Michoacan motion, this happens for $T_{g,\text{mean}}/T_{n,\text{mean}} > 1$. On the other hand, for a broad-band motion like Kern County motion, there is no clear value of $T_{g,\text{mean}}/T_{n,\text{mean}}$ (in the range of building periods considered) above which the proposed rule performs significantly better than the CQC3 rule. It may also be observed that when the building periods fall within the band-width of the ground motion and $\delta_{0,jq}^k = C_{jq}$, both rules lead to similar errors which are due to the approximations made for the nonstationarity factors.

The performance of the proposed rule is evaluated next in the estimation of the second largest, third largest, ... response peaks. Figure 6 shows the higher-order peak base shear along the X_s -direction, after normalization with respect to the largest value, in the case of the Borrego Mountain motion, with the example building assumed to have mass and stiffness properties for Case VI (with natural period equal to 1.1 s in the X_s -direction) and for $\theta = 75^\circ$. First 20 peaks are considered for this plot and results obtained from the proposed rule are compared with those from the time-history analysis. These results indicate that the ratio of the largest peak to a higher-order peak, as in Equation (19), works well in the example considered. Similar trends have been observed with the other ground motions as well. For a more comprehensive evaluation, absolute error values of the estimated first 20 peaks (from the proposed rule with respect to the time-history results) are averaged over θ (varying between 0° and 180°), and those are compared for the example ground motions in Figure 7 (with mass and stiffness properties remaining same as for Case VI). It is clear from this figure that for the first 10 peaks, the proposed formulation leads to very good estimates, with the absolute average error remaining close to 10%. The error increases for the next 10 peaks due to the effects of nonstationarity being dependent on the order of peak, as discussed earlier. Even for these peaks, the absolute average error remains within 30%, except in the case of Imperial Valley and Parkfield motions. It may be noted that the example building considered for these results is stiff with respect to the Borrego Mountain, Michoacan, and San Fernando motions, and is flexible with respect to the Imperial Valley, Kern County, and Parkfield motions. Thus, the results presented in Figure 7 cover a wide range of relative flexibility of building systems with respect to the ground motions.

It will be useful to also judge the performance of the proposed rule on the basis of algebraic percentage error and to consider this over all 20 peaks in an average sense. With this purpose, the averaging is done now over both θ (varying between 0° and 180°) and the order of peak (for the first 20 peaks), and the average error values are given in Table 5 for all 42 combinations of building periods and example motions. Negative values in this table indicate that the estimates from the proposed rule on average are greater than the time-history results. There is no specific trend available from these results. However, the proposed formulation seems to overestimate the first 20 response peaks much more often than underestimating, and the extent of error typically ranges from 10% to 20%. There are cases like Parkfield motion in which same nonstationarity factor cannot be assumed irrespective of the order of

peak, and therefore there is a need for further improvement in this direction. Nevertheless, it is clear that besides being more accurate than the CQC3 rule (for the largest response peak) in specific situations, the proposed rule provides reasonably accurate estimates for the higher-order response peaks in a simple manner.

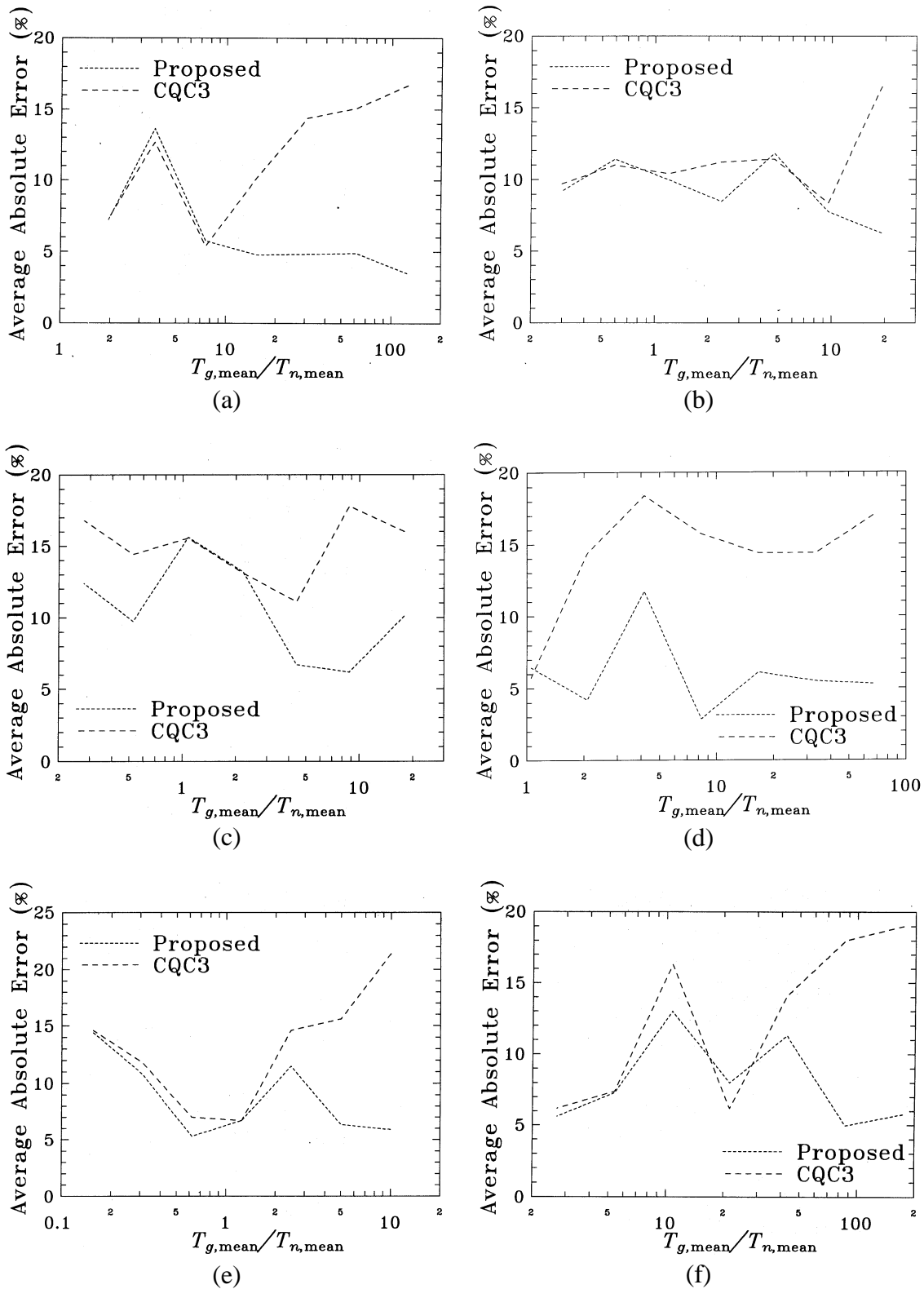


Fig. 5 Variation of absolute error averaged over θ with $T_{g,mean}/T_{n,mean}$ for the proposed and CQC3 rules in the case of (a) Borrego Mountain, (b) Imperial Valley, (c) Kern County, (d) Michoacan, (e) Parkfield, and (f) San Fernando earthquake motions

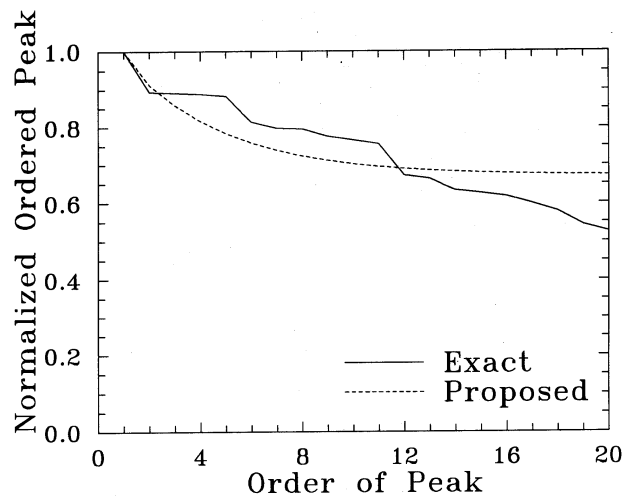


Fig. 6 Comparison of normalized ordered peak base shear in Case VI for $\theta = 75^\circ$ as obtained from the exact (time-history) analysis and the proposed rule in the case of Borrego Mountain motion

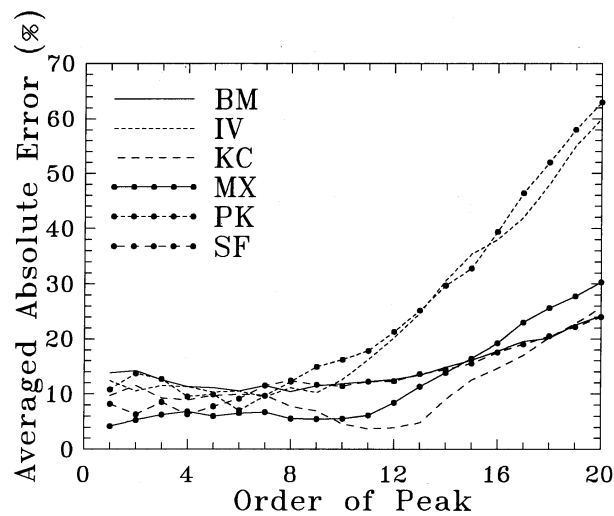


Fig. 7 Variation of the averaged (over θ) absolute error in the ordered peak amplitude with the order of peak for the Borrego Mountain (BM), Imperial Valley (IV), Kern County (KC), Michoacan (MX), Parkfield (PK), and San Fernando (SF) motions

Table 5: Percentage Error with the Proposed Rule as Averaged over Orientation and Order of Peak for Different Cases of Mass and Stiffness Properties and Ground Motions

Example Motion	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Borrego Mountain	-11.6	-8.84	-3.36	-2.58	-14.7	-11.5	1.58
Imperial Valley	-6.7	-7.23	-18.4	-21.9	-13.1	-18.3	-11.6
Kern County	-17.1	-20	-6.1	-9.3	-19.2	-2.83	12.6
Michoacan	21	12.54	8.8	5.1	-24	-9.41	-14.5
Parkfield	-38.8	-36.5	-24.1	-27.6	-25.4	-19.8	-15.6
San Fernando	-1.42	0.26	-7.74	-3.53	-10.9	-10.6	-3.7

The proposed rule requires SV ordinates of the principal components of the input ground motion for the estimation of $\delta_{0,iq}^k$ (see Equation (18)). This may limit the usefulness of the proposed rule because the

characterization of seismic hazard at the site under consideration is not always available in terms of the SV curves. For such situations, we propose to approximate SV ordinates in terms of the pseudo-spectral acceleration (PSA) curves and mean period T_c of the ground motion as (Gupta, 2008)

$$SV_j^k = \begin{cases} \frac{1}{\omega_j} \sqrt{(PSA_j^k)^2 - (PGA^k)^2}; & \omega_j > \frac{2\pi}{T_c^k} \\ \frac{1}{\omega_j} PSA_j^k & ; \omega_j \leq \frac{2\pi}{T_c^k} \end{cases} \quad (22)$$

where $PSA_j^k (= \omega_j^2 SD_j^k)$ is the largest peak amplitude of the pseudo-acceleration response (of the SDOF oscillator with ω_j frequency and ζ_j damping ratio), PGA^k is the peak ground acceleration, and T_c^k is the period corresponding to the center of gravity of the undamped PSV curve, for the k th principal component of ground acceleration. It may be observed that for SV_j^k equal to PSA_j^k/ω_j for all natural frequencies of the system, the proposed rule in the case of the largest response peak will become same as the CQC3 rule. It has been observed that by approximating SV ordinates as in Equation (22), the numerical results undergo only minor variations and therefore same observations can be made as before (for example, see Table 6 for the recomputed results of Table 5 on using the SV approximation of Equation (22)).

Table 6: Percentage Error with the Proposed Rule, as Averaged over Orientation and Order of Peak for Different Cases of Mass and Stiffness Properties and Ground Motions, on Using the SV Approximation

Example Motion	Case I	Case II	Case III	Case IV	Case V	Case VI	Case VII
Borrego Mountain	-11.2	-7.5	-2.4	-1.98	-12.1	0.12	3.11
Imperial Valley	-8.2	-7.1	-16.6	-19.1	-17.3	1.3	-13.3
Kern County	-28.1	-21.2	-8.4	-12.7	-33.8	-3.06	-7.0
Michoacan	16.2	12.4	9.63	4.9	-19.2	1.04	-9.4
Parkfield	-32.3	-41.5	-30.3	-29.2	-36.2	-10.2	-39.6
San Fernando	-7.6	0.07	7.3	-3.54	-9.45	-0.04	-31.2

CONCLUSIONS

A new modal combination rule has been formulated for the ordered peak response of a MDOF system subjected to multi-component ground motion. Both, the excitation and the response, have been assumed to be stationary, and the effect of nonstationarity has been included with the help of the response spectrum characterization of the ground motion. Following assumptions have been made in order to arrive at a simple form of the rule. First, the peak factors have been assumed to be same for (i) the largest modal displacement and largest modal velocity responses, and (ii) the system response and modal displacement, for any order of peak. Secondly, the effects of nonstationarity have been assumed to be same for (i) the largest modal displacement and largest modal velocity responses, and (ii) the largest modal displacement and the system response, for any order of peak. It has been also assumed that the ratio of the peak factor for a higher-order peak to that for the largest peak in the modal displacement response is dependent only on the order of the peak, irrespective of the mode. The proposed rule requires (as input) characterization of the seismic hazard in form of the SD and SV spectra for the principal components, and the orientation of the major principal axis of the ground motion with respect to the building. No assumptions have been made regarding the cross-correlation between different modes and regarding the nature of the input excitation.

The proposed combination rule has been illustrated with the help of a 5-story building having seven different sets of floor mass and story stiffness properties and by using six recorded ground motions with

significantly different frequency characteristics. Results show in the case of the largest base shear response that the estimates by the proposed rule follow the (exact) time-history estimates reasonably well for the entire range of the orientation of the major principal axis, and that those are more accurate compared to the estimates from the CQC3 method, particularly when the building is much stiffer to the ground motion. The maximum absolute error averaged over different orientations is about 16% in the case of the proposed rule. Unlike the CQC3 rule, the proposed rule also estimates the higher-order peak amplitudes and in a very simple way. The estimates from the proposed rule for the largest 20 peaks are found to be often larger than the time-history estimates, with the extent of error typically ranging between 10–20%. For the largest 10 peaks, however, the average absolute error remains close to 10%. Considering that only first few orders of peaks are important for the nonlinear response, larger errors for the lower orders of peaks is not a serious limitation. The proposed rule requires additional input data in form of the SV spectra of the principal components, but this requirement can be easily addressed by using the PSA (or PSV) spectra of these components.

REFERENCES

1. Amini, A. and Trifunac, M.D. (1985). "Statistical Extension of Response Spectrum Superposition", *International Journal of Soil Dynamics and Earthquake Engineering*, Vol. 4, No. 2, pp. 54–63.
2. Anagnostopoulos, S.A. (1981). "Response Spectrum Technique for Three-Component Earthquake Design", *Earthquake Engineering & Structural Dynamics*, Vol. 9, No. 5, pp. 459–476.
3. ATC (1978). "Tentative Provisions for the Development of Seismic Regulations for Buildings", Report ATC-3-06, Applied Technology Council, Redwood City, U.S.A.
4. Christian, J.T. (1989). "Generating Seismic Design Power Spectral Density Functions", *Earthquake Spectra*, Vol. 5, No. 2, pp. 351–368.
5. Chu, S.L., Amin, M. and Singh, S. (1972). "Spectral Treatment of Actions of Three Earthquake Components on Structures", *Nuclear Engineering and Design*, Vol. 21, No. 1, pp. 126–136.
6. Der Kiureghian, A. (1981). "A Response Spectrum Method for Random Vibration Analysis of MDOF Systems", *Earthquake Engineering & Structural Dynamics*, Vol. 9, No. 5, pp. 419–435.
7. Der Kiureghian, A. and Nakamura, Y. (1993). "CQC Modal Combination Rule for High-Frequency Modes", *Earthquake Engineering & Structural Dynamics*, Vol. 22, No. 11, pp. 943–956.
8. Goodman, L.E., Rosenblueth, E. and Newmark, N.M. (1953). "Aseismic Design of Firmly Founded Elastic Structures", *ASCE Transactions*, Vol. 120, pp. 782–802.
9. Gupta, V.K. (1994). "Higher Order Peaks in the Seismic Response of Multistoried Buildings", Report 94-03, Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur.
10. Gupta, V.K. (2002). "Developments in Response Spectrum-Based Stochastic Response of Structural Systems", *ISET Journal of Earthquake Technology*, Vol. 39, No. 4, pp. 347–365.
11. Gupta, V.K. (2008). "Short Communication: A New Approximation for Spectral Velocity Ordinates at Short Periods", *Earthquake Engineering & Structural Dynamics* (in press, DOI: 10.1002/eqe.877).
12. Gupta, I.D. and Trifunac, M.D. (1987a). "Order Statistics of Peaks in Earthquake Response of Multi-Degree-of-Freedom Systems", *Earthquake Engineering and Engineering Vibration*, Vol. 7, No. 4, pp. 15–50.
13. Gupta, I.D. and Trifunac, M.D. (1987b). "Order Statistics of Peaks of the Response to Multi-Component Seismic Excitation", *Bulletin of Indian Society of Earthquake Technology*, Vol. 24, No. 3-4, pp. 135–159.
14. Gupta, V.K. and Trifunac, M.D. (1989). "Investigation of Building Response to Translational and Rotational Earthquake Excitations", Report CE 89-02, University of Southern California, Los Angeles, U.S.A.
15. Gupta, V.K. and Trifunac, M.D. (1990). "Response of Multistoried Buildings to Ground Translation and Rocking during Earthquakes", *Probabilistic Engineering Mechanics*, Vol. 5, No. 3, pp. 138–145.
16. Hernandez, J.J. and Lopez, O.A. (2002). "Response to Three-Component Seismic Motion of Arbitrary Direction", *Earthquake Engineering & Structural Dynamics*, Vol. 31, No. 1, pp. 55–77.

17. Kaul, M.K. (1978). "Stochastic Characterization of Earthquake through Their Response Spectrum", *Earthquake Engineering & Structural Dynamics*, Vol. 6, No. 5, pp. 497–509.
18. Menun, C. and Der Kiureghian, A. (1998). "A Replacement for the 30%, 40% and SRSS Rules for Multicomponent Seismic Analysis", *Earthquake Spectra*, Vol. 14, No. 1, pp. 153–163.
19. Newmark, N.M. (1975). "Seismic Design Criteria for Structures and Facilities—Trans-Alaska Pipeline System", *Proceedings of the U.S. National Conference on Earthquake Engineering*, Ann Arbor, U.S.A., pp. 94–103.
20. O'Hara, G.J and Cunnif, P.F. (1963). "Elements of Normal Mode Theory", Report 6002, Naval Research Laboratory, Washington, DC, U.S.A.
21. Penzien, J. and Watabe, M. (1975). "Characteristics of 3-Dimensional Earthquake Ground Motions", *Earthquake Engineering & Structural Dynamics*, Vol. 3, No. 4, pp. 365–373.
22. Rosenblueth, E. and Contreras, H. (1977). "Approximate Design for Multicomponent Earthquakes", *Journal of the Engineering Mechanics Division, Proceedings of ASCE*, Vol. 103, No. EM5, pp. 881–893.
23. Rosenblueth, E. and Elorduy, J. (1969). "Response of Linear Systems to Certain Transient Disturbances", *Proceedings of the Fourth World Conference on Earthquake Engineering*, Santiago, Chile, Vol. 1, Part A-1, pp. 185–196.
24. Sadhu, A. (2007). "Ordered Peak Response under Multi-Component Ground Motion via Modal Combination Rule and Its Correlation with Nonlinear Response", M.Tech. Thesis, Department of Civil Engineering, Indian Institute of Technology Kanpur, Kanpur.
25. Singh, M.P. and Mehta, K.B. (1983). "Seismic Design Response by an Alternative SRSS Rule", *Earthquake Engineering & Structural Dynamics*, Vol. 11, No. 6, pp. 771–783.
26. Smeby, W. and Der Kiureghian, A. (1985). "Modal Combination Rules for Multicomponent Earthquake Excitation", *Earthquake Engineering & Structural Dynamics*, Vol. 13, No. 1, pp. 1–12.
27. Trifunac, M.D. and Brady, A.G. (1975). "A Study on the Duration of Strong Earthquake Ground Motion", *Bulletin of the Seismological Society of America*, Vol. 65, No. 3, pp. 581–626.
28. Unruh, J.F. and Kana, D.D. (1981). "An Iterative Procedure for the Generation of Consistent Power/Response Spectrum", *Nuclear Engineering and Design*, Vol. 66, No. 3, pp. 427–435.
29. USNRC (1976). "Combining Modal Responses and Spatial Components in Seismic Response Analysis", *Regulatory Guide 1.92, Revision 1*, U.S. Nuclear Regulatory Commission, Washington, DC, U.S.A.
30. Wilson, E.L., Der Kiureghian, A. and Bayo, E.P. (1981). "A Replacement for the SRSS Method in Seismic Analysis", *Earthquake Engineering & Structural Dynamics*, Vol. 9, No. 2, pp. 187–194.