

Static deformation of differently coupled two orthotropic elastic half-spaces due to a very long vertical strike-slip dislocation

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Abstract

The closed-form analytic expressions for the displacement and stresses at any point of either of two differently coupled orthotropic elastic half-spaces due to a very long vertical strike-slip dislocation situated in the upper elastic half-space have been obtained. The interface between two semi-infinite media is assumed to be horizontal and parallel to one plane of elastic symmetry of the orthotropic elastic medium. The coupling between two semi-infinite media is assumed to be either 'welded' or 'smooth-rigid' or 'rough-rigid'. The variations of the displacement and stresses with the horizontal distance from the fault are compared to study the effect of the different types of the coupling. It is found that the horizontal displacement for 'welded' coupling lies between the corresponding displacements for the 'rough-rigid' and 'smooth-rigid' couplings for different positions of the observer.

Key words: Static deformation; orthotropic medium; vertical strike-slip fault; half-spaces; welded contact.

## 1. Introduction

The upper part of the Earth is anisotropic (Dziewonski and Anderson 1981). Most anisotropic media of interest in seismology have, at least approximately, a horizontal plane of symmetry. A plane of symmetry is a plane in which the elastic properties have reflection symmetry. A medium with three mutually orthogonal planes of symmetry is known as orthorhombic. When one of the planes of symmetry in an orthorhombic symmetry is horizontal, the symmetry is termed as orthotropic symmetry (Crampin 1989). The orthotropy symmetry is also exhibited by olivine and orthopyroxenes, the principal rock-forming minerals of deep crust and upper mantle.

The sites of most earthquakes are along geological faults which are surfaces of material discontinuity in the Earth. The critical region where faulting often takes place is near or at the interface boundary. This region may be visualized as a two-phase medium of infinite extent and such a model has been used in both fundamental and applied research. Although, at present, the half-space model is considered to be adequate for most applications, the two-phase model is useful in considering the effect of internal structural discontinuities in the Earth by ignoring the effect of the free surface of the Earth.

In case of long faults, one is justified in using the two-dimensional approximation which simplifies the algebra to a great extent and one gets a closed-form analytical solution. Sharma et al. (1991) obtained the closed-form analytic expressions for the displacement and stresses at any point of either of two homogeneous isotropic elastic half-spaces in welded contact due to very long strike-slip dislocations. Singh et al. (1992) obtained closed-form analytic expressions for the Airy stress function, displacements and stresses at any point of either of two homogeneous, isotropic perfectly elastic half-spaces in welded contact due to various two-dimensional sources.

Garg et al.(1996) provided an integral representation of two-dimensional seismic sources causing antiplane strain deformation in an infinite orthotropic elastic medium.They used this representation to obtain the static deformation of an orthotropic semi-infinite elastic medium,consisting of a horizontal layer in welded contact with an orthotropic elastic half-space, due to an inclined two-dimensional strike-slip fault situated either in the layer or in the half-space.

In the present paper,we have obtained the closed-form analytic expressions for the displacement and stresses at any point of either of two differently coupled homogeneous orthotropic elastic half-spaces due to a very long vertical strike-slip dislocation situated in the upper elastic half-space. The interface between two semi-infinite elastic media is a horizontal plane which is also the plane of orthotropic symmetry.We assume that the upper semi-infinite orthotropic elastic medium is either in 'rough' or in 'smooth' or in 'welded'contact with the lower elastic semi-infinite orthotropic medium.Different deformations of the upper orthotropic elastic medium corresponding to various types of its contact with the lower orthotropic elastic medium have been obtained analytically. The lower semi-infinite medium(consisting of solid mantle) can be approximated to be rigid (Bott and Dean 1973)and it coincides with our problem that the upper semi-infinite elastic medium is in 'rough-contact' with lower semi-infinite rigid medium.The rigid semi-infinite medium upon which the upper elastic semi-infinite medium lies is not absolutely rigid but may possess a certain elasticity. It may be approximated to be rigid for some situations such as, even under extremely high temperature towards the central part of the Earth, the liquid nature of its core has acquired the properties of solid and rigid because of tremendous overlying pressure. Physically,the 'smooth-rigid' boundary condition is applicable where, in the lower half-space, the petroleum materials

and natural gas are present. It may also be used to study the effect of horizontal boundary which is lubricated.

The occurrence of intermediate earthquakes of focal depths between 30 to 300 Km is well known in regions of Europe such as the Calabrian. Intermediate depth shocks, range from 60 to 150 Km, with greater concentration has been located between Granada and Malaga, Spain (Buforn et al 1991). For intermediate earthquakes, the model of the Earth consisting of a lithosphere lying over an asthenosphere may be used. The welded case of the present problem together with the correspondence principal of linear viscoelasticity (Biot 1954) can be used to study intermediate earthquakes.

The present problem is an improvement of the earlier paper discussed by Sharma et al. (1991) for isotropic medium in two ways- the boundary conditions at the interface between the two elastic half-spaces may be different and the medium may be orthotropic instead of isotropic. The results obtained by Sharma et al. (1991) can be recovered from our results as a particular case.

## 2. Formulation of the problem and Basic equations:

We consider an anisotropic elastic infinite medium comprising two elastic half-spaces. The upper half-space  $z < 0$  (termed as medium I) and the lower half-space  $z > 0$  (termed as medium II) with  $z$ -axis vertically downwards. The origin of a cartesian coordinate system  $Oxyz$  is placed on the interface  $z=0$ .

We further assume that both elastic mediums are homogeneous and orthotropic. The upper orthotropic semi-infinite elastic medium may either be in 'welded' contact with another lower orthotropic semi-infinite elastic medium or in 'rough' or 'smooth' contact with a rigid medium.

For an orthotropic elastic medium, with co-ordinate planes coinciding with the planes of symmetry and one plane of symmetry being horizontal, the stress-strain relation in matrix form is (Chung 1996)

$$\begin{bmatrix} \tau_{11} \\ \tau_{22} \\ \tau_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{22} \\ e_{33} \\ 2e_{23} \\ 2e_{13} \\ 2e_{12} \end{bmatrix} \quad (1)$$

where the two-suffix quantities  $c_{ij}$  are elastic constants of the medium and  $\tau_{ij}$  and  $e_{ij}$  are, respectively, stress and strain tensors.

For an isotropic elastic medium

$$c_{11} = c_{22} = c_{33} = \lambda + 2\mu, \quad c_{12} = c_{13} = c_{23} = \lambda, \quad c_{44} = c_{55} = c_{66} = \mu \quad (2)$$

where  $\lambda$  and  $\mu$  are the Lamé constants.

Let  $(u, v, w)$  be the components of the displacement vector. Let elastic medium under consideration be under the conditions of antiplane strain deformation in the  $yz$ -plane due to a very long vertical strike-slip dislocation parallel to the  $x$ -axis. In this case, the displacement vector is parallel to the direction of the fault strike and depends upon  $y$  and  $z$  coordinates only. Thus, under the state of antiplane strain deformation,  $v = w = 0$  and  $u = u(y, z)$ . The equilibrium equation in terms of non-zero displacement component  $u$  is (Garg et al. 1996, Eq.(10))

$$\frac{\partial^2 u}{\partial y^2} + \frac{1}{\alpha^2} \frac{\partial^2 u}{\partial z^2} = 0 \quad (3)$$

for zero body forces. The non-zero stresses are

$$\tau_{12} = \alpha^2 \frac{\partial u}{\partial y}, \quad \tau_{13} = c \frac{\partial u}{\partial z} \quad (4)$$

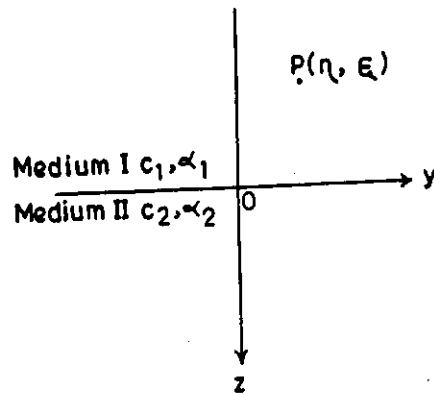


Fig.1. Section  $x=0$  of the model consisting of two orthotropic elastic half-spaces with a line source in the upper half-space.

where

$$c=c_{55}, \alpha = (c_{66}/c_{55})^{1/2} \quad (5)$$

The values of elastic constants  $\alpha$  and  $c$  depend upon the values of  $c_{66}$  and  $c_{55}$ . We assume that the values of  $\alpha$  and  $c$  are positive and real. In case of an isotropic elastic medium,  $c=\mu$  and  $\alpha=1$ .

### 3. Line source in an elastic orthotropic infinite medium

We assume that a line-source, parallel to x-axis, intersects the yz-plane of elastic symmetry at the point  $P(\eta, \xi)$  in the upper orthotropic elastic half-space (Fig.1). At the point  $P(\eta, \xi)$  either a single couple (12) or a single couple (13) acts. The single couple (12) is a couple in the xy-plane with equal and opposite forces in the x-direction with its arm in the y-direction (Ben-Menahem and Singh 1981). Similarly, (13) is a single couple in the xz-plane with forces in the x-direction and arm in the z-direction (Fig 2).

In a recent paper (Garg et al. 1996), henceforth referred to as Paper I, the displacement  $u_0$  (say), parallel to the x-axis, due to either a line source or single couple (12) or couple (13) in an unbounded orthotropic elastic medium has been obtained in the following integral form

$$u_0 = \int_0^{\infty} [A_0 \sin k(y-\eta) + B_0 \cos k(y-\eta)] e^{-k\alpha|z-\xi|} dk \quad (6)$$

Here,  $\alpha$  is the elastic constant of the orthotropic unbounded elastic medium, defined in equation (5) and the source coefficients  $A_0$  and  $B_0$  for various single couples are given in Table 1. These co-efficients are independent of the variable of integration  $k$ .

Table 1

Source co-efficients for various seismic sources.

$\text{Sign}(z-\xi)$  is the sign function which has the value of  $-1$  for  $z < \xi$  and  $+1$  for  $z > \xi$ .  $F_{12}$  and  $F_{13}$  are, respectively, the moments of couples (12) and (13).

Source	$A_0$	$B_0$
Single couple (12)	$\frac{F_{12}}{2\pi c}$	0
Single couple (13)	0	$\text{sign}(z-\xi) \frac{F_{13}}{2\pi c}$

#### 4. Solution of the formulated problem

For a line source parallel to the x-axis acting at the point  $P(\eta, \xi)$  of the upper half-space, suitable expressions for the displacement in the two-phase medium are of the form

$$u^{(1)} = u_0 + \int_0^{\infty} \left[ A_1 \sin k(y-\eta) + B_1 \cos k(y-\eta) \right] e^{-\alpha_1 kz} dk \quad (7)$$

$$u^{(2)} = \int_0^{\infty} \left[ A_2 \sin k(y-\eta) + B_2 \cos k(y-\eta) \right] e^{-\alpha_2 kz} dk \quad (8)$$

The superscript(1) is used to indicate medium I ( $z < 0$ ) and the superscript (2) for the medium II ( $z > 0$ ). Both the displacements  $u^{(1)}$  and  $u^{(2)}$  satisfy the equilibrium equation (3). The coefficients  $A_1, B_1, A_2, B_2$  are to be determined from the boundary condition at the interface  $z=0$  of the two-phase medium.



The corresponding stresses  $\tau_{13}^{(1)}$  and  $\tau_{13}^{(2)}$  in the two-phase medium can be obtained from the equations (4),(7) and (8). We find

$$\tau_{13}^{(1)} = c_1 \alpha_1 \int_0^{\infty} \left[ -\text{sign}(z-\xi) \left\{ A_0 \sin k(y-\eta) + B_0 \cos k(y-\eta) \right\} e^{-\alpha_1 k |z-\xi|} + \left\{ A_1 \sin k(y-\eta) + B_1 \cos k(y-\eta) \right\} e^{\alpha_1 kz} \right] k dk \quad (9)$$

$$\tau_{13}^{(2)} = -c_2 \alpha_2 \int_0^{\infty} \left[ A_2 \sin k(y-\eta) + B_2 \cos k(y-\eta) \right] e^{-\alpha_2 kz} k dk \quad (10)$$

#### 4.1 Welded Contact

When the coupling between two orthotropic elastic half-spaces at the interface  $z=0$  is welded, displacement  $u$  and stress  $\tau_{13}$  across the horizontal plane  $z=0$  are continuous (Singh 1970). That is,

$$u^{(1)} = u^{(2)}, \quad \tau_{13}^{(1)} = \tau_{13}^{(2)} \quad \text{at } z=0 \quad (11)$$

Equations (7)-(11) determine the coefficients  $A_1, B_1, A_2, B_2$  which are appearing in equations (7)-(8). Since the horizontal line source parallel to  $x$ -axis passes through the point  $P(\eta, \xi)$  and this point lies in the upper half-space ( $z \leq 0$ ), therefore  $\xi < 0$  and  $|\xi| = -\xi$ . We finally find the values of coefficients for welded contact as given below:

$$A_1^{(W)} = \left[ \frac{1-m}{1+m} \right] A_0 e^{\alpha_1 k \xi}, \quad A_2^{(W)} = \left[ \frac{2}{1+m} \right] A_0 e^{\alpha_1 k \xi} \quad (12)$$

$$B_1^{(W)} = - \left[ \frac{1-m}{1+m} \right] B_0 e^{\alpha_1 k \xi}, \quad B_2^{(W)} = \left[ \frac{-2}{1+m} \right] B_0 e^{\alpha_1 k \xi}$$

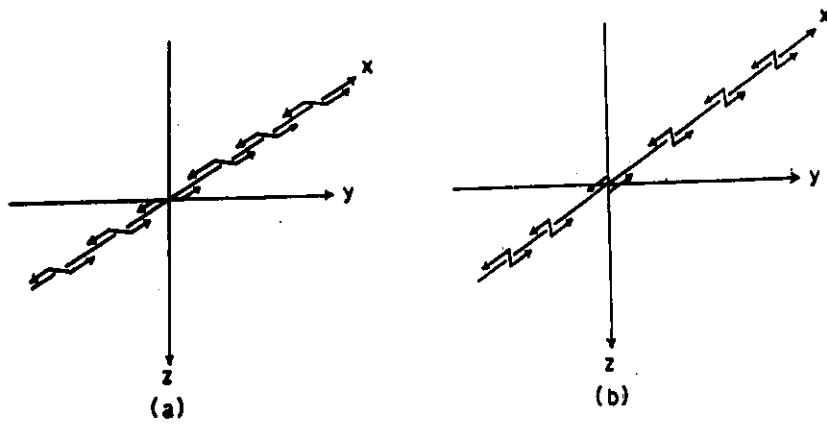


Fig.2 Summation of point couples.(a) a line couple(12) and (b) a line couple (13).

where

$$m = (c_2^{\alpha_2} / c_1^{\alpha_1}) \quad (13)$$

The values of constants  $A_0$  and  $B_0$  are to be taken from Table 1. In

the case of two welded isotropic half-spaces,  $m$  becomes the ratio of their rigidities. In Table 1, we see that the value of the source coefficient  $B_0$  changes for  $z > \xi$ . In equation (12),  $B_0^1$  is the value of  $B_0$  when  $z < \xi$  and  $B_0 = -B_0^1$  for  $z > \xi$ . We shall now obtain explicit closed-form expressions for the displacement at any point of the two-phase medium as a result of either a line couple (12) or a line couple (13) acting at every point on the source line (which is parallel to x-axis) lying in the medium I (Fig. 2)

#### 4.1.1 Displacement due to a line couple (12)

Substituting the values of  $A_1, B_1, A_2, B_2$  from equation (12) and the values of  $A_0, B_0$  from the Table 1 in equations (7) and (8), the integral expressions for the displacements are obtained. After evaluating these integrals analytically with the help of the standard transform integrals (Erdélyi 1954), we obtain the following closed-form expressions for the displacement at any point of two welded orthotropic elastic half-spaces due to a line couple (12) acting at every point of the line source passing through the point  $P(\eta, \xi)$  in the medium I:

$$u_{(12)}^{(1)} = \frac{F_{12}}{2\pi c_1^{\alpha_1}} \left[ \frac{(y-\eta)}{(y-\eta)^2 + \alpha_1^2 (z-\xi)^2} + \frac{(c_1^{\alpha_1} - c_2^{\alpha_2})}{(c_1^{\alpha_1} + c_2^{\alpha_2})} \frac{(y-\eta)}{(y-\eta)^2 + \alpha_1^2 (z+\xi)^2} \right] \quad (14)$$

$$u_{(12)}^{(2)} = \frac{F_{12}}{(c_1^{\alpha_1} + c_2^{\alpha_2})\pi} \left[ \frac{(y-\eta)}{(y-\eta)^2 + (\alpha_2 z - \alpha_1 \xi)^2} \right] \quad (15)$$

The shear stresses  $\tau_{12}^{(1)}, \tau_{13}^{(1)}$  and  $\tau_{12}^{(2)}, \tau_{13}^{(2)}$  can be easily obtained from the above displacements and the stress-displacement

relations given in (4). The displacements for the entirely isotropic elastic two-phase medium with welded contact can be derived from above result as a special case (on taking  $\alpha_1 = \alpha_2 = 1$ ,  $c_1 = \mu_1$  and  $c_2 = \mu_2$ ).

#### 4.1.2 Displacement due to a line couple (13)

Similar to the above process, we obtain the following closed-form expressions for the displacements at any point of two welded orthotropic elastic half-spaces due to a line couple (13) acting at every point on the line source passing through the point  $P(\eta, \xi)$  in the medium I:

$$u_{(13)}^{(1)} = \frac{F_{13}}{2\pi c_1} \left[ \frac{\alpha_1(z-\xi)}{(y-\eta)^2 + \alpha_1^2(z-\xi)^2} + \left( \frac{c_1\alpha_1 - c_2\alpha_2}{c_1\alpha_1 + c_2\alpha_2} \right) \frac{\alpha_1(z+\xi)}{(y-\eta)^2 + \alpha_1^2(z+\xi)^2} \right] \quad (16)$$

$$u_{(13)}^{(2)} = \frac{-\alpha_1 F_{13}}{(c_1\alpha_1 + c_2\alpha_2)\pi} \frac{(\alpha_2 z - \alpha_1 \xi)}{(y-\eta)^2 + (\alpha_2 z - \alpha_1 \xi)^2} \quad (17)$$

#### 4.2 Rough-Rigid contact

When the upper orthotropic elastic half-space is in rough contact with a lower semi-infinite rigid base at the interface  $z=0$ , the boundary condition (Dundurs and Hetenyi 1965 & Small and Booker 1984) is

$$u^{(1)} = 0 \text{ at } z=0 \quad (18)$$

The coefficients  $A_1^{(R)}$ ,  $B_1^{(R)}$  for a line source in the upper orthotropic elastic half-space in rough contact with a rigid medium are to be found from equations (7) and (18). We find

$$A_1^{(R)} = -A_0 e^{\alpha_1 k x}, \quad B_1^{(R)} = B_0^1 e^{\alpha_1 k x} \quad (19)$$

We note that the values of elastic constants of medium II do not affect the values of coefficients in equation(19). Comparing equations (12) and (19), it is observed that the values of

coefficients  $A_1^{(R)}, B_1^{(R)}$  for a line source in elastic half-space in rough contact with the rigid medium can be obtained from the corresponding values of the coefficients  $A_1^{(W)}, B_1^{(W)}$  due to welded coupling by taking  $m \rightarrow \infty$  in equation (12).

#### 4.3. Smooth-Rigid Contact

When the upper orthotropic elastic half-space is in smooth contact with a lower semi-infinite rigid medium at the interface  $z=0$ , the boundary condition (Small and Booker 1984) is

$$\tau_{13}^{(1)} = 0 \quad \text{at } z=0 \quad (20)$$

The coefficients  $A_1^{(S)}, B_1^{(S)}$  for a line source in the upper orthotropic elastic half-space in smooth contact with a rigid medium are to be found from equations(9) and (20). We find

$$A_1^{(S)} = A_0 e^{\alpha_1 k x}, \quad B_1^{(S)} = -B_0^1 e^{\alpha_1 k x} \quad (21)$$

We note that the values of elastic constants of medium II do not affect the values of coefficients in equation(21). Also it is observed that the values of coefficients  $A_1^{(S)}, B_1^{(S)}$  for a line source in elastic half-space in smooth contact with a rigid medium can be obtained from that of welded coupling by putting  $m=0$  in equation (12). We further note that the boundary condition (20) in the present problem is the same as for the stress-free condition of the half-space ( $z \leq 0$ ).

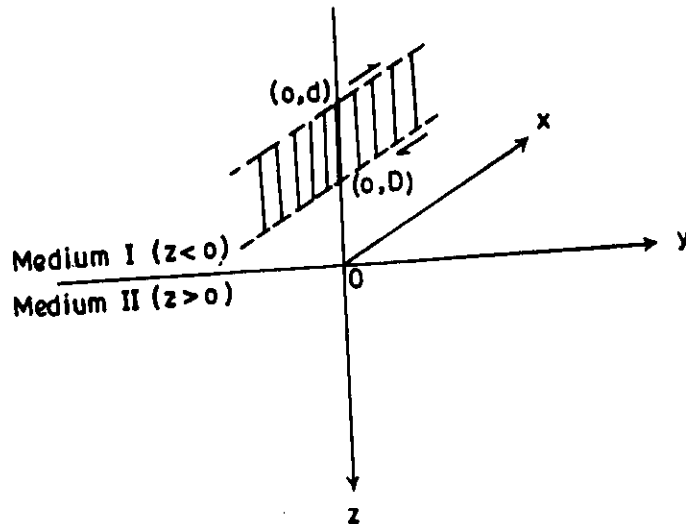


Fig.3. Geometry of a long vertical strike-slip fault occupying the region  $y=0$ ,  $d \leq z \leq D \leq 0$  in the medium I.

## 5. Very long Vertical strike-slip dislocation in orthotropic elastic medium I

In paper I, it has been observed that a very long vertical strike-slip dislocation may be represented by the single couple (12) when the couple moment  $F_{12}$  is taken as

$$F_{12} = -c_1 \alpha_1^2 b d \xi \quad (22)$$

where  $b$  is the slip,  $d\xi$  is the infinitesimal width of the line dislocation (parallel to  $x$ -axis) and  $c_1, \alpha_1$  are elastic constants of the orthotropic elastic medium I in which the line dislocation lies.

### 5.1 Welded contact

Equations (14)-(15) and (22) determine the deformation at any point of the medium of two welded orthotropic half-spaces due to a very long vertical strike-slip dislocation situated in the medium I.

The deformation field at any point of the welded medium due to a very long vertical strike slip fault with finite vertical extent is then obtained by integrating with respect to  $\xi$  from  $d$  to  $D$ , where  $L=D-d$  is the vertical width of the fault. For the sake of simplicity, we assume that the vertical strike-slip fault passes through the  $z$ -axis, i.e.  $\eta=0$  (Fig.3.). The displacements and stresses due to this fault at any point of the two-phase orthotropic elastic medium with welded contact at the interface are found to be

$$u^{(1)} = \frac{-b}{2\pi} \left[ \tan^{-1} \frac{\alpha_1 (\xi - z)}{y} + \left( \frac{1-m}{1+m} \right) \tan^{-1} \frac{\alpha_1 (\xi + z)}{y} \right] \Bigg|_d^D \quad (23)$$

$$\tau_{12}^{(1)} = \frac{c_1 \alpha_1^3 b}{2\pi} \left[ \frac{(\xi - z)}{y^2 + \alpha_1^2 (\xi - z)^2} + \left( \frac{1-m}{1+m} \right) \frac{(\xi + z)}{y^2 + \alpha_1^2 (\xi + z)^2} \right] \Bigg|_d^D \quad (24)$$

$$\tau_{13}^{(1)} = \frac{bc_1 \alpha_1}{2\pi} \left[ \frac{y}{y^2 + \alpha_1^2 (\xi - z)^2} - \left( \frac{1-m}{1+m} \right) \frac{y}{y^2 + \alpha_1^2 (\xi + z)^2} \right] \Bigg|_d^D \quad (25)$$

for  $z \leq 0$  (medium I) and

$$u^{(2)} = \frac{-b}{(1+m)\pi} \left[ \tan^{-1} \frac{\alpha_1 \xi - \alpha_2 z}{y} \right] \Bigg|_d^D \quad (26)$$

$$\tau_{12}^{(2)} = \frac{bc_2 \alpha_2^2}{(1+m)\pi} \left[ \frac{(\alpha_1 \xi - \alpha_2 z)}{y^2 + (\alpha_1 \xi - \alpha_2 z)^2} \right] \Bigg|_d^D \quad (27)$$

$$\tau_{13}^{(2)} = \frac{bc_2 \alpha_2}{(1+m)\pi} \left[ \frac{y}{y^2 + (\alpha_1 \xi - \alpha_2 z)^2} \right] \Bigg|_d^D \quad (28)$$

for  $z \geq 0$  (medium II). Here

$$f(\xi) \Bigg|_d^D = f(D) - f(d) \quad (29)$$

We have verified that the boundary conditions given in (11) in terms of the continuity of the displacement and stress across  $z=0$  are satisfied by (23), (26) and (25) and (28).



### 5.2 Rough-Rigid contact:

The expressions for the displacement and stresses due to a very long vertical strike-slip fault of finite width  $L$  in the upper orthotropic elastic semi-infinite medium I in rough contact with a rigid medium can be found similarly as in the welded case. We note that this deformation field is the same as obtained from the deformation field for the welded contact on letting  $m \rightarrow \infty$ .

### 5.3 Smooth-Rigid contact:

The expressions for the displacement and stresses due to a very long vertical strike-slip fault of finite width  $L$  in the upper orthotropic elastic semi-infinite medium I in smooth contact with a rigid medium can be found similarly as in the welded case. We note that this deformation field is the same as obtained from deformation field for the welded contact on taking  $m = 0$ .

## 6. Numerical Results

In this section, we wish to examine the effect of the different types of coupling between two orthotropic elastic semi-infinite media upon the variations of the horizontal displacement  $u$  and the shear stresses  $\tau_{12}$  and  $\tau_{13}$  with the horizontal distance  $y$  from the very long vertical strike slip fault of finite width  $L$ . We consider the case when the fault touches the interface, i.e.,  $D=0$  and  $d=-L$ .

For numerical computation, we use the values of orthotropic elastic constants for Olivine materials given by Verma (1960) for the medium I. These constants are  $\alpha_1 = .9894$ ,  $c_1 = 8.10 \times 10^{11}$  dynes/cm<sup>2</sup>. For medium II, we use the values of orthotropic elastic constants given by Love (1944) for Baryte type materials which are  $\alpha_2 = .9824$ ,  $c_2 = 2.87 \times 10^{11}$  dynes/cm<sup>2</sup>.

Figures 4-5 show the variation of the horizontal displacement  $u$

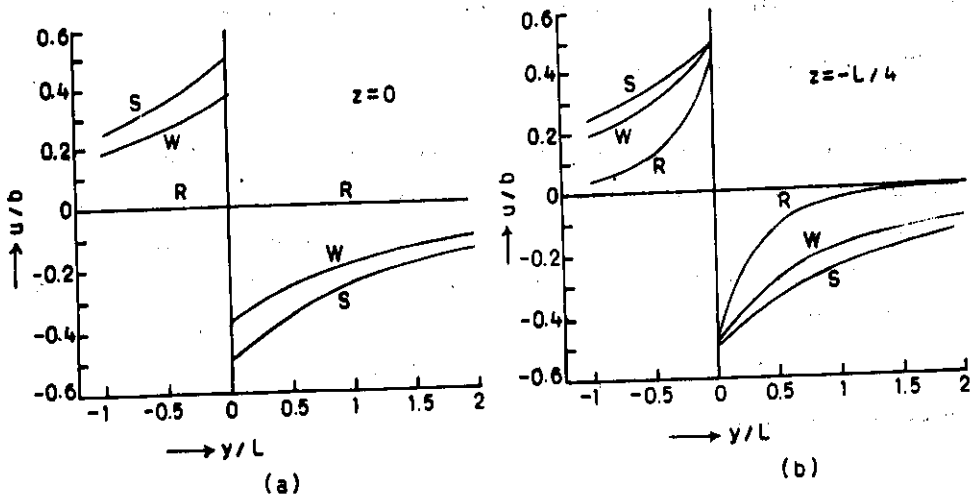


Fig.4. Variation of the dimensionless displacement  $u/b$  with the dimensionless horizontal distance  $y/L$  from a vertical strike-slip fault. W denotes that the curve is for 'welded' coupling, S is for 'smooth-rigid' coupling and R denotes the curve for 'rough-rigid' coupling. (a) for  $z=0$  and (b) for  $z=-L/4$

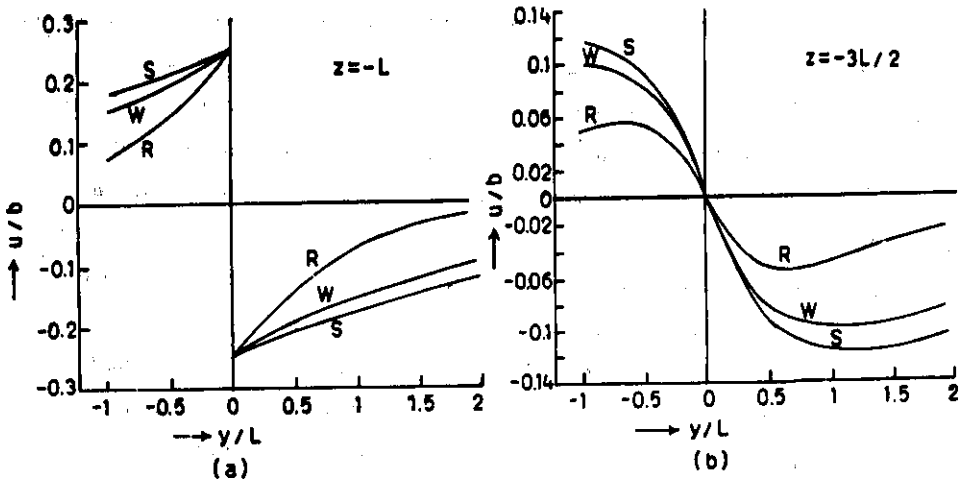


Fig.5 Variation of  $u/b$  with  $y/L$ . (a) for  $z=-L$  and (b) for  $z=-3L/2$ . Notation as in figure 4.

with the horizontal distance  $y$  from the very long vertical strike-slip fault for four different positions of the observer, namely,  $z=0, -L/4, -L$  and  $-3L/2$ . In each of these figures, three curves representing the horizontal displacement at points of the orthotropic elastic semi-infinite medium I, corresponding to different types of its coupling with a base are drawn. When  $z=0$ , the observer is at the interface and when  $z=-3L/2$ , the observer is above the strike-slip fault. These figures exhibit that the displacements due to 'welded' contact lies between the corresponding displacements due to 'rough-rigid' and 'smooth-rigid' contacts and thus the nature of coupling of the half-space affects significantly the displacement at every point of the medium. From these figures, we also note that the displacement is antisymmetric with respect to the distance from the fault.

Figure 4(a) shows that at the interface  $z=0$ , the amount of discontinuity in the horizontal displacement  $u$  at the point  $y=0$  is different for different types of coupling. It is also noted that the displacement  $u$  vanishes at the interface when the coupling is that of 'rough-rigid' type. This observation coincides with the boundary condition stated in equation (18) for the 'rough-rigid' contact. Figure 4(b) shows that there is a discontinuity of magnitude  $b$  in the horizontal displacement  $u$  at the point  $y=0$  for  $z=-L/4$  for all types of coupling of the medium I with the base. It follows that when  $-L < z < 0$ , the amount of discontinuity in the horizontal displacement  $u$  for all possible types of coupling is the same.

The subsurface  $z=-L$  is the upper horizontal boundary of the strike-slip fault. At this subsurface  $z=-L$ , there is a discontinuity of magnitude  $b/2$  in the horizontal displacement  $u$  at the point  $y=0$  for all types of coupling of the medium I with the base as shown in figure 5(a). The subsurface  $z=-3L/2$  is above the strike-slip fault and figure 5(b) shows that at  $z=-3L/2$  the displacement is continuous at  $y=0$  for each type of the coupling.

The shear stress  $\tau_{13}$  at the interface  $z=0$  is zero everywhere

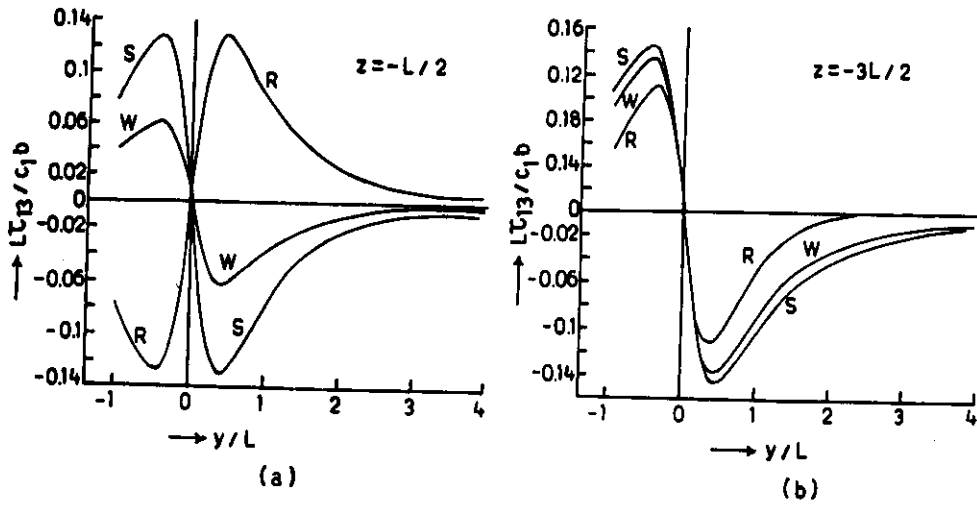


Fig.8. Variation of stress  $L\tau_{13}/c_1b$  with  $y/L$ . (a) for  $z=-L/2$  and (b) for  $z=-3L/2$ . Notation as in figure 4.

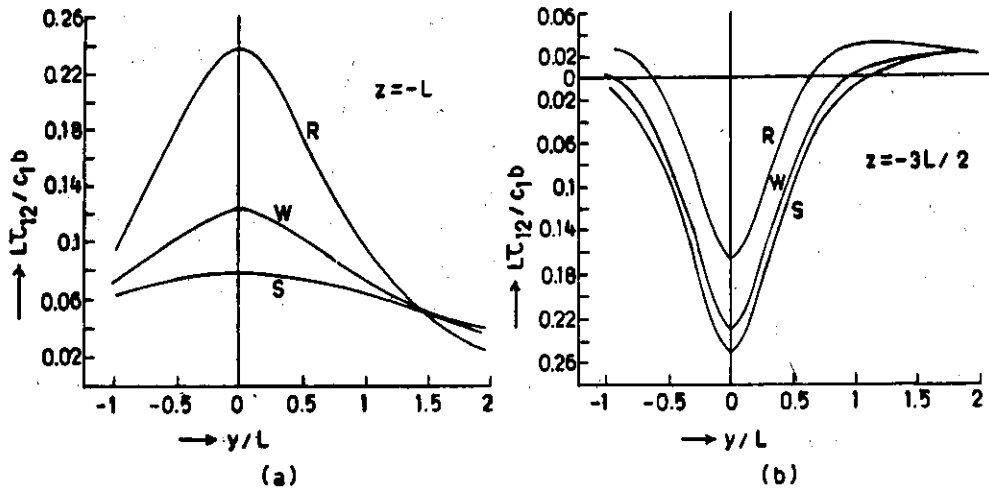


Fig. 7. Variation of stress  $LT_{12}/c_1 b$  with  $y/L$ . (a) for  $z = -L$  and (b) for  $z = -3L/2$ . Notation as in figure 4.

for the smooth-rigid type of coupling while  $\tau_{13}$  becomes approximately zero everywhere except at the point  $y=0$  where it becomes infinite for 'welded' as well as for 'rough-rigid' type of coupling of the medium I with the base. At the subsurface  $z=-L$ , the shear stress  $\tau_{13}$  is zero approximately everywhere except at the point  $y=0$  where it is infinite for each type of the coupling. The variation of shear stress  $\tau_{13}$  at the subsurface levels  $z=-L/2$  and  $z=-3L/2$  has been shown in figure 6(a,b). It is found that the shear stress  $\tau_{13}$  for the welded coupling lies between the corresponding stresses for the 'rough-rigid' and 'smooth-rigid' couplings. It is also noted that the shear stress  $\tau_{13}$  is antisymmetric with respect to  $y$ .

We have checked numerically that the shear stress  $\tau_{12}$  becomes zero at the interface  $z=0$  for the 'rough-rigid' type of coupling. We have noted that the shear stress  $\tau_{12}$  is symmetric with respect to  $y$ . We have also noted that the shear stress  $\tau_{12}$  is positive in the neighbourhood of the fault for  $-L \leq z \leq 0$  (Fig.7(a)) and above the fault the stress  $\tau_{12}$  becomes negative in the neighbourhood of  $y=0$  (Fig.7(b)). It is also observed that the stress  $\tau_{12}$  due to 'welded' case lies between the corresponding stresses for 'smooth-rigid' and 'rough-rigid' case.

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