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DYNAMIC ANALYSIS OF CONTINUOUS SPAN HIGHWAY BRIDGE

N. Munirudrappa and H.N. Dhruvaraja Iyengar
U.V.C.E. Bangalore University, Jnana Bharathi, Bangalore - 560 056

ABSTRACT

Dynamic analysis of highway bridges for the moving vehicles is often made using number of simplifications. Most of the bridges are either of uniform or non-uniform cross sections and are modeled as straight beams with elastic or rigid supports and their mass is either elastically attached or distributed throughout. Obtaining the exact solutions even after this simplification is tedious and it becomes still more complicated when an account has to be made for various other parameters on which the dynamic behaviour of the bridges and vehicles depend. These parameters include the surface roughness, stiffness, mass and damping characteristics of the bridges; speed, spring stiffness, mass, number of axles and damping characteristics of the vehicle, etc. Hence, approximate iterative procedures are adopted for evaluating the response.

This paper presents the development of a computer program catering for the computer-aided analysis of dynamic response of continuous span highway bridges. The program developed is capable of calculating the dynamic bending moments, dynamic shear force and dynamic deflections with corresponding dynamic amplification factors (DAF). The results for some typical bridges are presented in the form of tables and graphs. Further, the existence of a shallow area of settlement in approach road may lead to impact factors appreciably greater than those usually considered in bridge design, especially for sprung loads. It is observed from the results that the dynamic amplification factors increase with vehicle speed, span, length and road surface roughness.

KEYWORDS: Dynamic Response, Continuous Beams, Moving Load, Dynamic Amplification Factors

INTRODUCTION

The various parameters on which the dynamic response of an highway bridge depends, are classified into the following three categories:

1. Vehicle parameters
2. Bridge parameters
3. Road surface roughness

From the review of literature, it can be concluded that the problems related to dynamic amplification factors (DAF) were recognised in the nineteenth century (Cantieni, R.K., Fleming, F. 1960). Further, many investigators worked on this more realistically in the 20th century, wherein a major effort has been directed towards the problems of highway bridge dynamic amplification after taking into account the parameters cited above (Fryba, L. 1973, Inbanathan and Wiebew 1987, Tidemann, J.L. 1993, Zuraski, P.D. 1991).

The dynamic effects of moving loads on bridges are conveniently expressed in terms of an impact factor. This factor is usually defined as the ratio of maximum dynamic deflection produced in a bridge during the passage of the load to the maximum deflection that would occur if the load were acting statically. Impact factor is an important parameter used in the design of bridges, and design formulas usually indicate it as a percentage by which the static design loads should be increased to compensate for the dynamic effects of the moving load. The impact factor in most of the bridge codes is related to the natural frequency of the bridge. But this still does not account for parameters like speed, weight and dynamic characteristics, road surface irregulatiry, expansion joints at bridges supports, soil-structure interaction, etc. The complete dynamic analysis of the bridge-vehicle interaction problems requires a thorough knowledge of many parameters mentioned above. In addition to this, a study has to be carried out for a large number of vehicles with different dynamic vehicle characteristics.

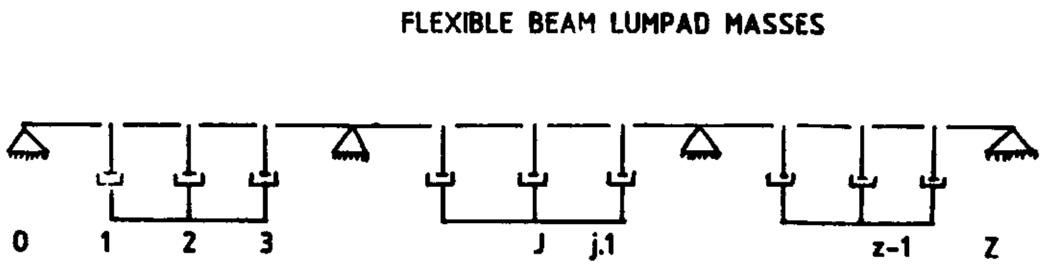

Fig. 1 Model of the bridge

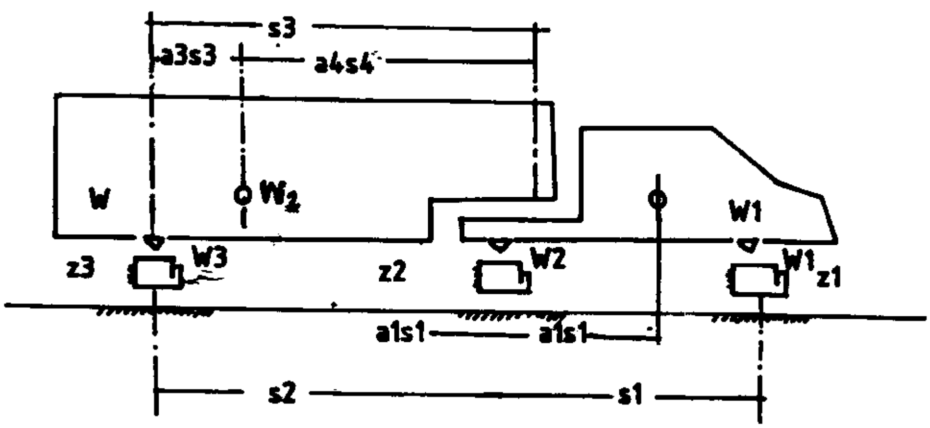

Fig. 2 Model of the vehicle

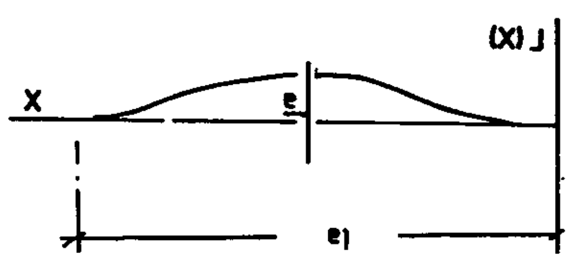

Fig. 3 Model of road surface

In this paper, a theoretical analysis of the DAF (dynamic amplification factor) or impact factor for continuous span highway bridges is carried out by taking some of the above mentioned parameters. The continuous span bridge is idealized as a continuous beam with discrete masses being lumped at the nodal points. The beam can be either of uniform cross section or of variable cross-section. The road surface irregularity is modelled by using a sine function. The damping in the bridge is represented by a series of dashpots connected to each node. Boundary conditions at supports are assumed to be simply supported, and all the deformations are assumed to be elastic. The vehicle is modeled as a planar rigid body with distributed mass; and tyres are modeled by concentrated point masses attached to the suspension system. The tyre-suspension system for each axle is represented by two linearly elastic springs in series.

The related bridge model, the vehicle model and the road surface model are shown in Figures 1, 2 and 3, respectively.

ASSUMPTIONS

Following are some of the assumptions made in this analysis:

(i) The tyres of the vehicles are assumed to remain in contact with the road surface at all times.

(ii) The rotations of the tractor and trailer are small, such that sines of the angles of the rotations can be taken to be the angles themselves and cosines of the angles may be taken as unity.

(iii) The dynamic deflection curve of the neutral axis of the beam is of the same shape as would occur if the weights of the moving loads on the beam and weight of the beam itself were statically applied to the beam.

DISPLACEMENT CO-ORDINATES

The mass concentration points and support points are referred to as nodes and are numbered in an increasing order starting with zero at the left end and ending with z at the right end (see Figure 1). The j th panel between nodes $j-1$, and j the number of concentrated masses attached to the deflecting nodes is denoted by N. The vehicle is assumed to move from left to right, and its axles are numbered from 1 to 3 starting with front axle.

The configuration of the beam at its position of static equilibrium, i.e., when acted upon only by its self-weight is specified by a function d(x). This function represent the deviation of the beam surface from a horizontal line passing through a point directly over the left support (Figure 4) and may include the effects of dead load deflection, initial camber, grade, roadway roughness or any combination of these factors.

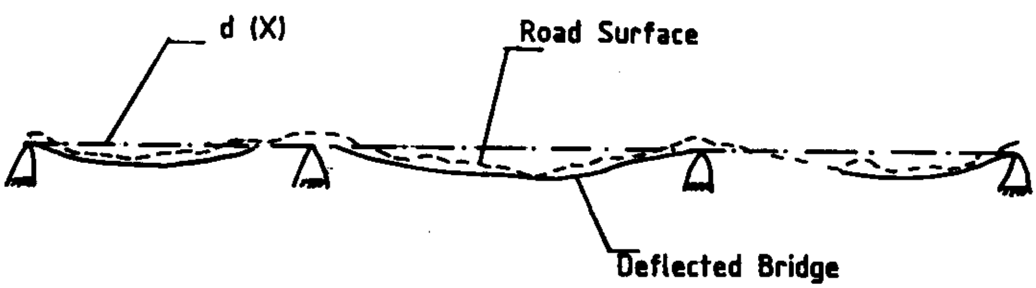

Fig. 4 Function d(x)

The deflection of the jth node, measured from the position of static equilibrium is denoted by y_j. The configuration of the vehicle is specified by the vertical displacements, z_i, of the points of support of the tractor and trailer masses as shown in Figure 2. These displacements are measured from the position of static equilibrium of the vehicle, when the front axle is over the left support.

EQUATION OF MOTION FOR BRIDGE

The equation of motion for the rth mass, M_r, is written as,

$$M_r \ddot{y}_r + C_r y_r = \sum_{i=1}^{3} Q_{ri} P_i \varphi_i + \sum_{j=1}^{n} R_{rj} y_j \tag{1}$$

in which,

C_r = Damping coefficient for the damper at the rth node;

P_i = The interacting force between the ith axle of the load unit and the beam;

φ_i = The position function, a function of speed of vehicle and time, which determines the fractional part of the load to be considered acting on mass M_r. The φ_i functions used are shown below and are given by the equation,

$$\varphi_i = A \sin \pi (x_i - ct/l) + B \cos \pi (x_i - ct/l) + c \tag{2}$$

Q_{ri} and R_{rj} = influence coefficients as defined below:

Q_{ri} represents the reaction induced at the rth node by a unit concentrated force at the point of application of the ith interacting force when all the nodes are fixed against deflection as shown in Figure 5. This quality is referred to as "reaction load co-efficient".

R_{rj} represents the reaction induced at the rth node by a unit deflection of the jth node when all other nodes are supported against deflection, as shown in Figure 5, and is referred to as the "reaction deflection co-efficient".

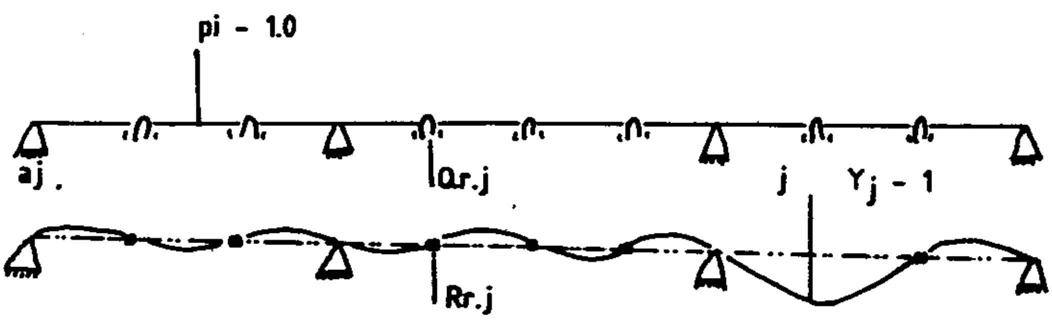

Fig. 5 Influence coefficient

The reaction deflection coefficient R_{rj} depends only on the properties of the beam and is evaluated only once for each problem. Whereas, the co-efficient Q_{ri} is time-dependent and a function of both the characteristics of the beam and positions of interacting forces which may vary as the vehicle moves along the span.

By the application of Equation (1) to each concentrated mass of the beam, one can obtain as many equations as there are degrees of freedom for the beam. The interacting forces depend on the motions of both the vehicle and the bridge. Their variations with respect to time are not known at the outset, and it is necessary to also consider, the equations of motion for the vehicle. It is this interdependence of the motions of the vehicle and the bridge that complicates the analysis.

EQUATIONS OF MOTION FOR THE VEHICLE

Let P_{sti}, denote the force exerted by the ith axle load, when the vehicle is in its position of static equilibrium. The equation of motion of the vehicle can be expressed in terms of the interacting forces $(P_i - P_{st,i})$ as shown below :

$$M[a]\{\ddot{z}_i\} = \{P_i - P_{st,i}\}$$
$$\{\ddot{z}_i\} = g/W \times [a]^{-1}\{P_i - P_{st,i}\} \quad (3)$$

where,

g = acceleration due to gravity,
W = total weight of the vehicle,
M = total mass of the vehicle, and
$[a]^{-1}$ = the inverse of a 3×3 matrix $[a]$ whose elements are given below:

$$a_{11} = (a_1^2 + a_1a_2\rho_1)W_1/W + a_5(a_2^3 + a_3a_4\rho_2)W_2/W + W_1/W$$
$$a_{12} = a_{21} = a_1a_2(1-\rho_2)W_1/W + a_5(1-a_5)(a_3^2 + a_3a_4\rho_2)W_2/W$$
$$a_{13} = a_{31} = a_3a_4a_5(1-\rho_2)W_2/W$$
$$a_{22} = (a_2^2 + a_1a_2\rho_1)W_1/W + (1-a_5)^2(a_3^2 + a_3a_4\rho_2)W_2/W + W_2/W$$
$$a_{23} = a_{32} = a_3a_4(1-a_5)(1-\rho_2)W_2/W$$
$$a_{33} = (a_4^2 + a_3a_4\rho_2)W_2/W + W_3/W$$

The terms ρ_1 and ρ_2 are dimensionless measures of the rotary moments of inertia of the tractor and trailer masses, defined by:

$$\rho_1 = 1/a_1a_2 \times [r_1/s_1]^2 \text{ and } \rho_2 = 1/a_3a_4 \times [r_2/s_2]^2$$

in which s_1 and s_2 are axle spacings as shown in Figure 2,

r_1 = radius of gyration of the tractor about an axis which passes through the center of gravity and is normal to the plane of the paper, and

r_2 = the corresponding quantity for the trailer mass.

NUMERICAL COMPUTATIONS

The theoretical procedure described above is applied to study the dynamic responses of some standard 3-span continuous highway bridges. The differential equation of motion has been integrated numerically using an iterative procedure over finite interval within each integration step. Computer programs are developed to compute the dynamic amplification factors and to obtain the influence line diagrams for the dynamic bending moments, shear force and deflections of the bridges under the passage of moving vehicle. The time required to run the entire program depends on the numbest of mass concentration points used in each span of the bridge, the number of spans, number of axles of the vehicle and the time interval of integration. The program can be used for single span or multispan bridges with either uniform or variable cross sections.

The objectives of the aplication of the method are to study the variation of impact factors for the continuous bridges with respect to the speed of the vehicle, span of the bridge, surface roughness, natural frequency, weight of the vehicle and weight of the bridge.

Following are the typical three-span highway bridges used for the application of the method discussed above.

Table 1 : Standard 3-Span Continuous Bridges

Sl. No.	Span (m)	Weight (kN/m)	I m^{4}	E kN/m^2	Fundamental Freq. Hz	Type of Bridge
1	12-15-12	48.38	0.038	2 x 10^7	2.99	I Beam
2	16-20-16	50.20	0.007	"	2.12	"
3	20-24-20	52.21	0.095	"	1.17	"
4	24-30-24	54.84	0.038	"	1.49	"

Each span of the bridge is divided into four equal panels using three lumped masses in each span. The material of the bridge is assumed to be concrete and the coefficient of damping is assumed as 1% of critical damping. Each node is assigned a single-degree-of-freedom corresponding to vertical translatory motion. The boundary conditions at supports are assumed to be simply supported and continuous. The bridge is assumed to be of uniform mass distribution and flexural rigidity. The vehicle model used is as shown in Figure 2.

DISCUSSION

The results of dynamic analysis of highway bridges are shown in Table 1 and have been presented in the form of graphs in Figures 6 to 12.

The history curves for dynamic analysis are obtained for various velocities ranging from 45 kmph to 120 kmph. Typical history curves in Figures 6 to 9 are drawn for a velocity of 120 kmph. The abscissa in these figures represent the distance between the left support and the position of the front axle of vehicle on the bridge.

The maximum value of dynamic increment obtained when the load is near the position producing the maximum value of the particular effect under consideration will be identified as the "Critical Dynamic Increment". For deflection at the center of the span, the critical dynamic increment is the peak ordinate obtained when the load is at or close to node 7 (Figures 8 and 9). From these curves, it can be seen that the dominant period of the waves is that of the fundamental natural mode, indicating that this mode is the principal contributor to these responses. Also, the largest dynamic increments for deflection occur at node 6, indicating that for the type of the structure and loading under consideration, the critical section for deflection is at node 6.

1. Effect of Initial Vehicle Oscillation and Interleaf Friction

Time histories of dynamic deflection of Bridge 4 for an initial oscillating unit load are shown in Figure 4. Initial value of the interacting force is assumed to be 30% greater than the static value, and the initial value of the bouncing velocity of the sprung mass is assumed to be zero. The solid curve corresponds to the suspension spring, whereas the dotted curve refers to the suspension system in its normal operating condition. The limiting interleaf frictional force in the latter case is considered to be 15% of the static axle load and the initial value of frictional force is assumed to be zero. From these figures, it can be seen that, in the absence of energy dissipation in the vehicle suspension system, the dynamic effects produced by an initially oscillating vehicle may be significantly greater than those induced by the same load moving smoothly across the span. Further, the interleaf friction in the vehicle suspension dynamically changes the feature of the response curves and significantly reduces the magnitudes of the peak effects.

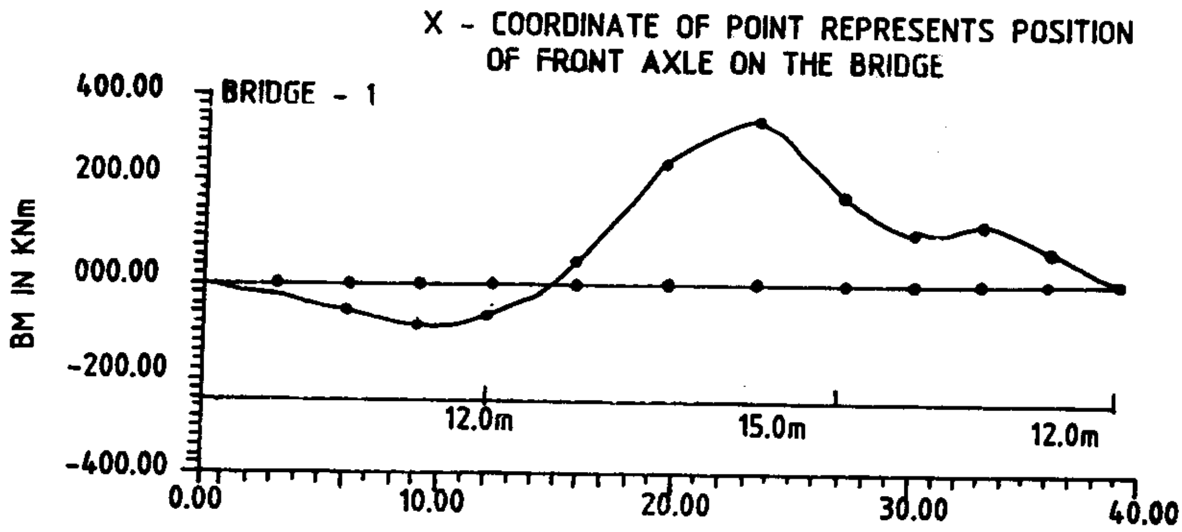

Fig. 6 Influence line diagram for B.M.

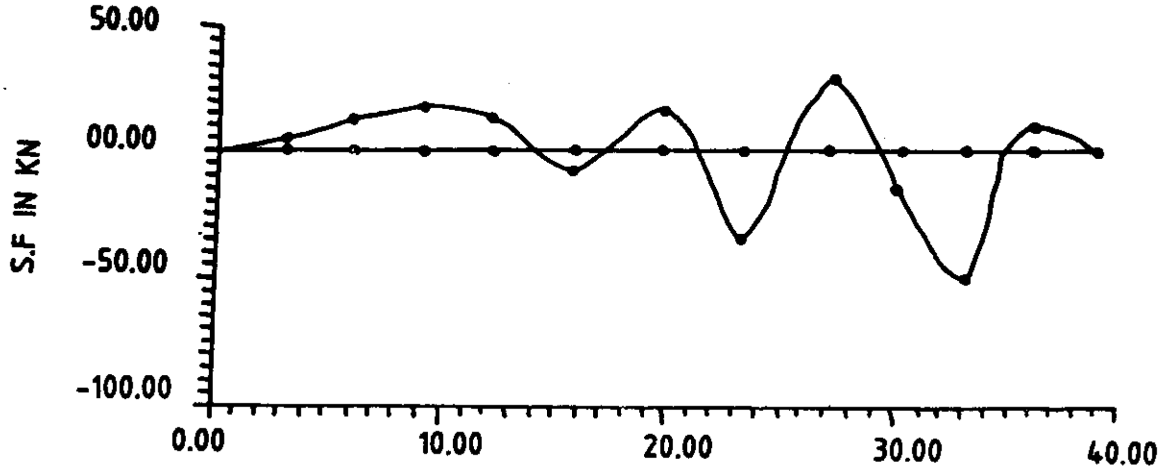

Fig. 7 Influence line diagram for S.F.

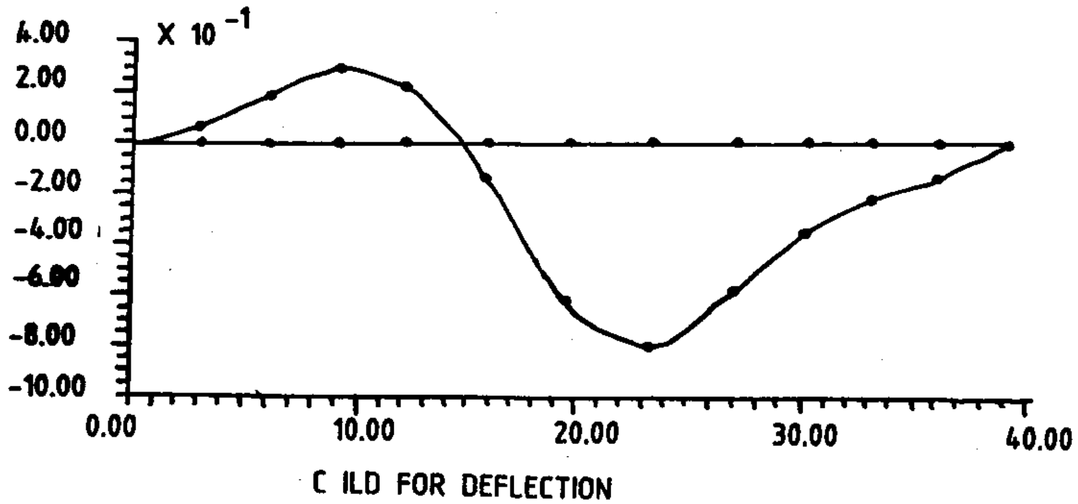

Fig. 8 Static influence diagrams for Bridge 1 for deflection at node 6

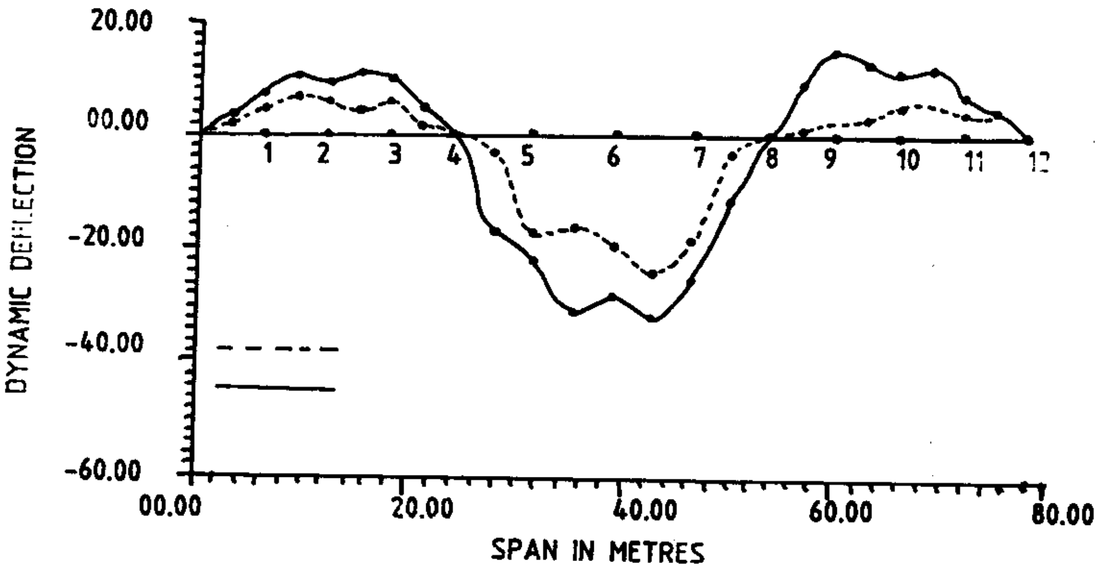

Fig. 9 Effect of initial vehicle oscillation and interleaf friction

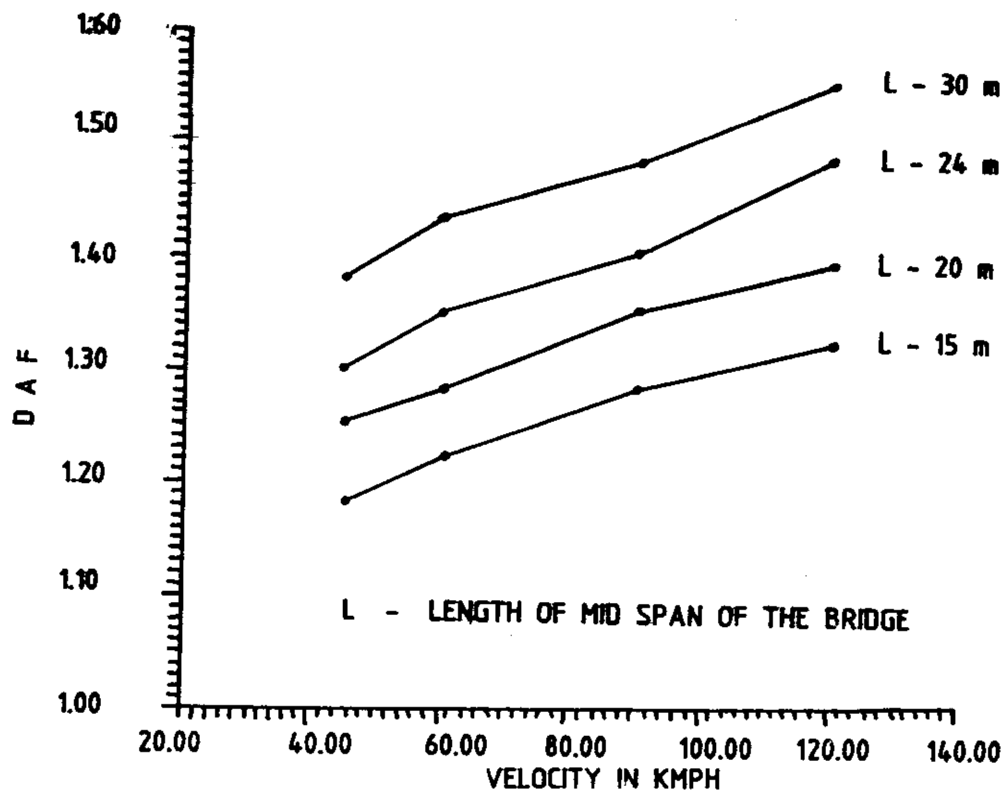

Fig. 10a Variation of DAF with respect to velocity of vehicle

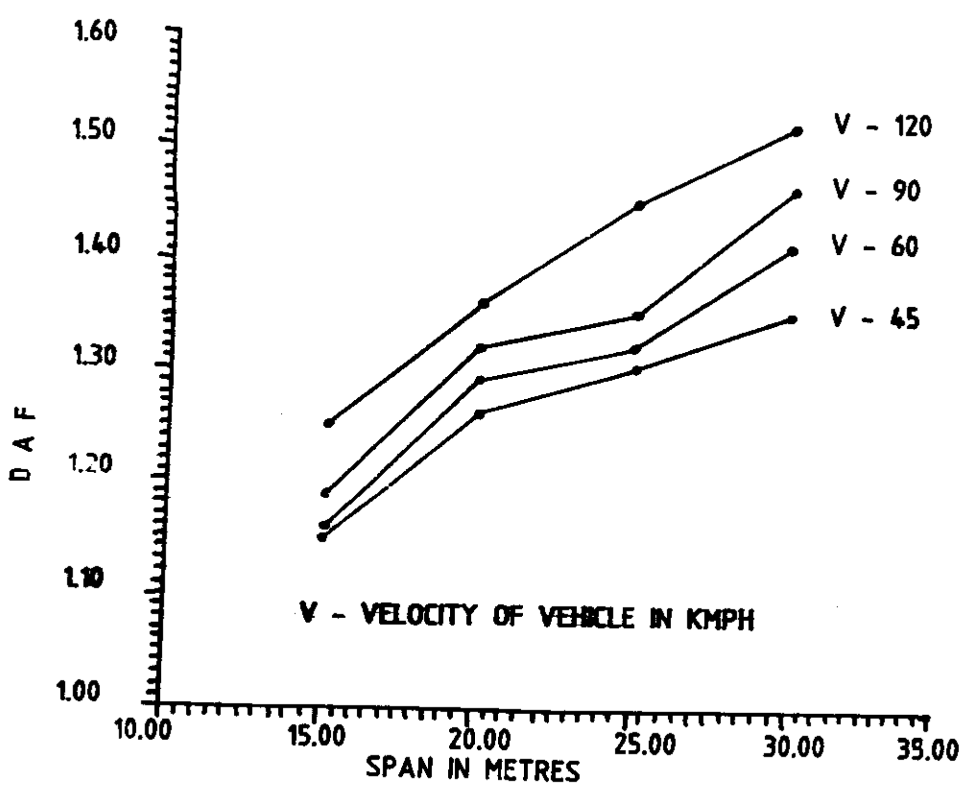

Fig. 10b Variation of DAF with respect to span of the bridge

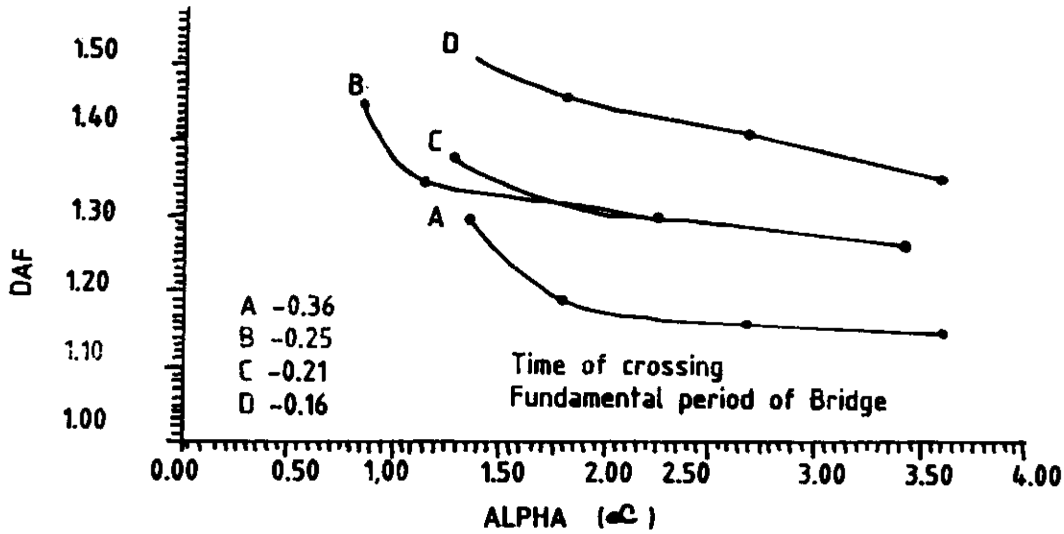

Fig. 11 Effect of fundamental frequency and mass ratio on DAF

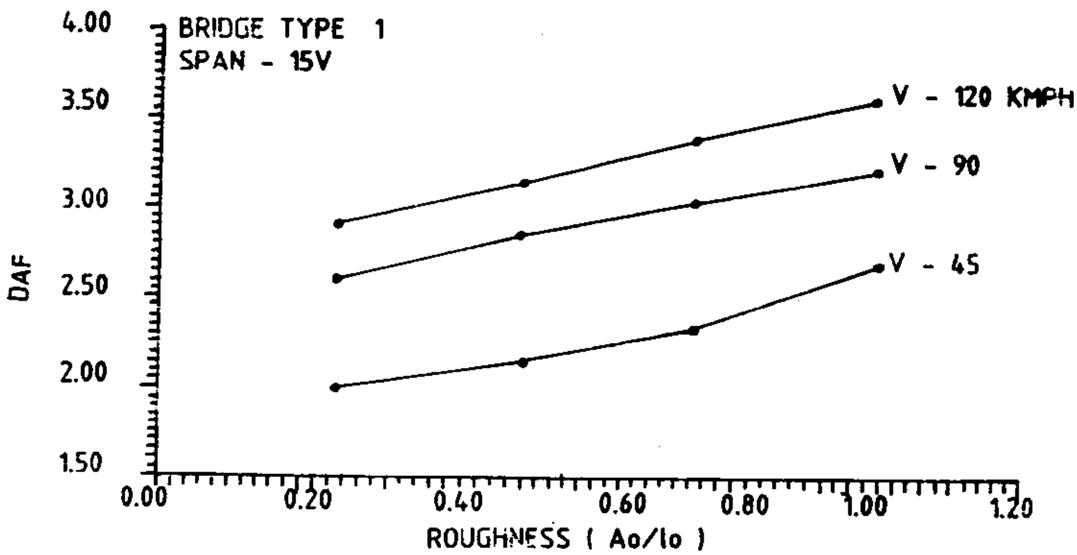

Fig. 12a Effect of surface roughness on DAF

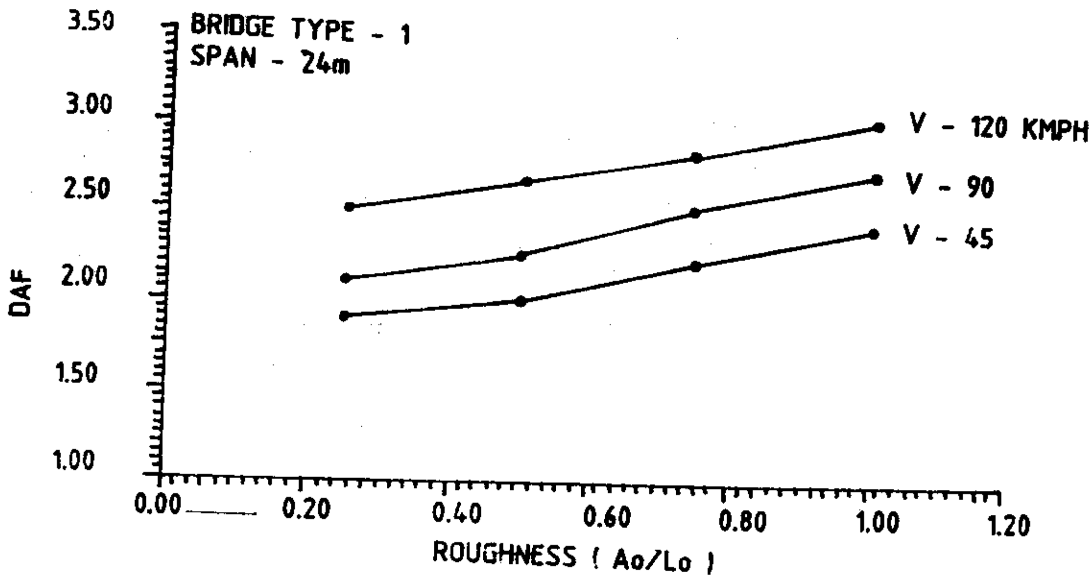

Fig. 12b Effect of surface roughness on DAF

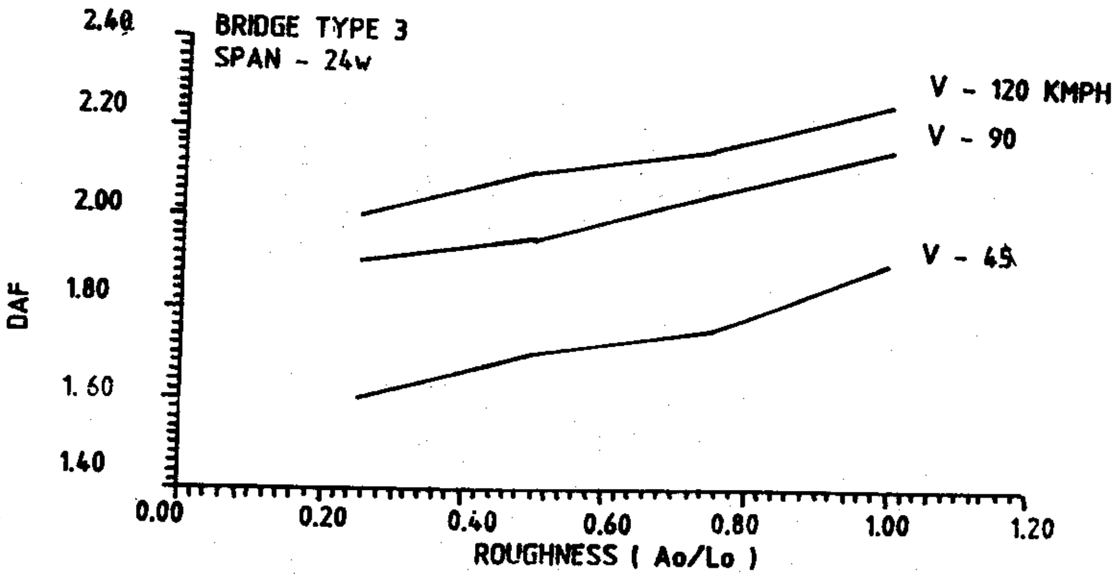

Fig. 12c Effect of surface roughness on DAF

2. Effect of Speed of Vehicle

Figure 10(a) shows the speed of vehicle versus DAF and variation of DAF with respect to the non-dimensional speed parameter α is shown in Figure 11. As it is seen from these figures, DAF varies almost linearly with the speed parameters for the range of the speeds considered. Also it can be seen from Figures 10(a) and 13(b) that this increment in DAF due to increase in speed increases with the span length.

3. Effect of Span Length of Bridge

Figure 10(a) shows the variation of DAF with respect to the span of the bridge. The range of span considered in this study lies between 15 m to 30 m. It can be seen that DAF varies almost linearly with stiffness of the bridge for the range considered.

4. Effect of Fundamental Frequency and Mass Ratio on DAF

As it is seen from Figure 11, the value of DAF increases with decrease in ratio of masses of the load to mass of bridge. Further, it is seen from the same figure that DAF increases almost linearly upto about $\alpha = 1.5$ and then attains a maximum value.

5. Effect of Depth of Irregularity of Road Surface

As seen from Figures 12(a) to 12(c), the value of DAF increases with increase in the depth of irregularity for unsprung vehicle. In case of sprung vehicles, it is seen that its effect on DAF is almost zero.

CONCLUSIONS

1. Initial oscillations in sprung vehicles can significantly increase the value of DAF.
2. Vehicle damping reduces the increment in DAF due to initial oscillations of vehicle.
3. The dynamic amplification factor increases with the speed of the vehicle.
4. DAF increases with the stiffness of the bridge.
5. DAF increases with the depth of irregularity of the road surface in case of unsprung vehicles more then sprung vehicle.
6. DAF increases with the increase in the value of ratio of vehicle mass to the mass of the bridge.

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