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DYNAMIC ANALYSIS OF ASYMMETRIC MULTI-STOREYED WITH SHEAR WALLS

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INTRODUCTION

Increasing heights of multi-storeyed buildings, a better understanding of the wind, blast and seismic forces as well as the need for achieving maximum possible economy coupled with stringent safety requirements is leading to a greater, attention to the dynamic behaviour of buildings.

The behaviour of plane frames with shear walls under dynamic loads has been investigated in detail (1, 2). It is possible to predict their mode shapes and frequencies with reasonable accuracy. An actual building, however, is a three dimensional structure consisting of rigid floors supported by vertical members.

These floors behave like rigid bodies under lateral loads i.e., they have practically no inplane deformations, but rotate as rigid bodies influencing the stress distribution in the vertical members significantly.

A method for the static analysis of multi-storeyed buildings with rigid floors under lateral loads has been reported by the authors elsewhere (3). The present paper extends this analysis to the dynamic behaviour of such buildings.

An experimental investigation has also been carried out to determine the reliability of the theoretical approach.

ASSUMPTIONS

The proposed analysis is based on the following assumptions:

The meterial is isotropic and elastic.

Translation and rotation of the frames are small.

In case of shear walls the beam between the centre and the edge of wall is infinitely rigid.

Distributed mass of vertical members is replaced by lumps at floor levels.

Damping effect is neglected.

ANALYSIS

A typical building scheme consisting of $n \times l$ columns and m storeys is shown in fig. 1. It is treated as two systems of plane frames, lying in the x and y planes respectively, orthogonal to each other and connected by rigid floors. Under the action of the lateral loads the floors undergo a rigid body movement as shown in fig. 2. Since the slabs are rigid in their own planes it is assumed that the lateral loads on each of the frame systems act at the outside joint on the side of the origin only.

Equations of equilibrium of each of the floors in the x, y and θ (rotation) direction

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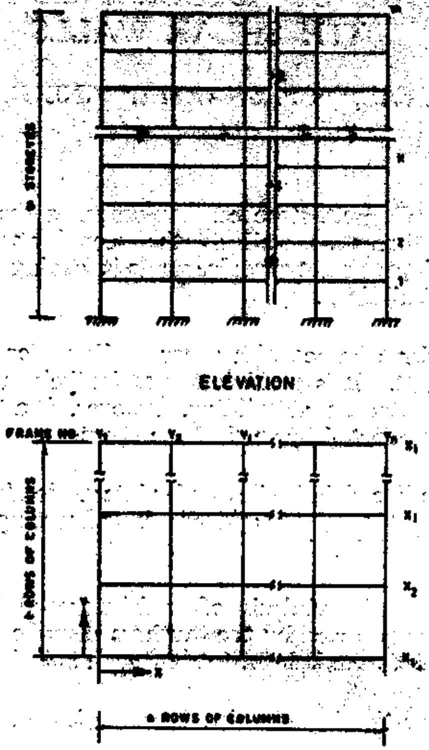

Fig. 1. A Typical Building Scheme

can be derived as shown in Appendix I assuming that the rotation of each floor is so small that sin θ can be replaced by θ. Finally, the force deformation relations for all the floors can be expressed as shown below:

$$M\ddot{\delta} + K\delta = 0$$

where M is the mass matrix

K is the stiffness matrix

It may now be assumed that

$$\delta = \bar{\delta} \sin pt$$

where $\delta = \{U\ V\ \theta\}_{3m \times 1}$ and $\bar{\delta}$ represents the amplitudes of $\{U\ V\ \theta\}$. Natural frequencies and mode shapes can now be found by computing eigen values and eigen vectors of the matrix.

$$[M^{1/2}\ K\ M^{-1/2}]$$

Components of $[M]$ corresponding to U and V are the total mass on each floor (same for U and V) and an equivalent mass for the rotation θ is calculated as shown below.

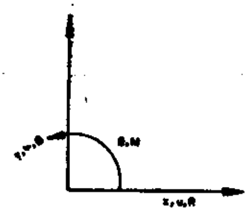

FIG. 2a POSITIVE DIRECTIONS OF COORDINATES DISPLACEMENT AND FORCES

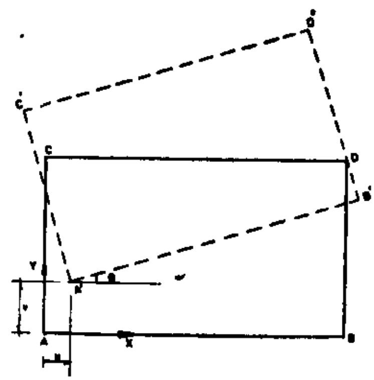

Fig. 2b. Rigid Body Movement of a Floor

Figure 3 shows the plan of a typical floor having a distributed mass *ABCD*. The moment of an element about the origin

$$= \rho r^2 \ddot{\theta}\, dx\, dy$$

where r is the radius vector and ρ the intensity of mass (per unit area). Also

$$r^2 = x^2 + y^2$$

Since θ and $\ddot{\theta}$ are constant, total moment

$$= \ddot{\theta} \int_{q}^{q+b} \int_{p}^{p+a} \rho\,(x^2 + y^2)\, dx\, dy$$

If ρ is assumed to be constant, total moment

$$= \ddot{\theta}\left[\frac{\rho ab}{3}(a^2 + 3ap + 3p^2 + b^2 + 3bq + 3q^2)\right]$$

The equivalent mass to be used in the $[M]$ matrix, therefore, is

$$\frac{\rho ab}{3}(a^2 + 3ap + 3p^2 + b^2 + 3bq + 3q^2)$$

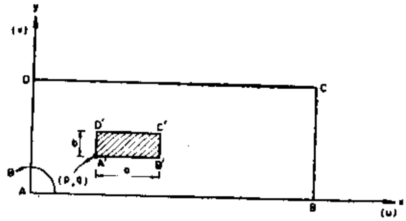

Fig. 3. Typical Floor Plan with Distributed Mass ABCD

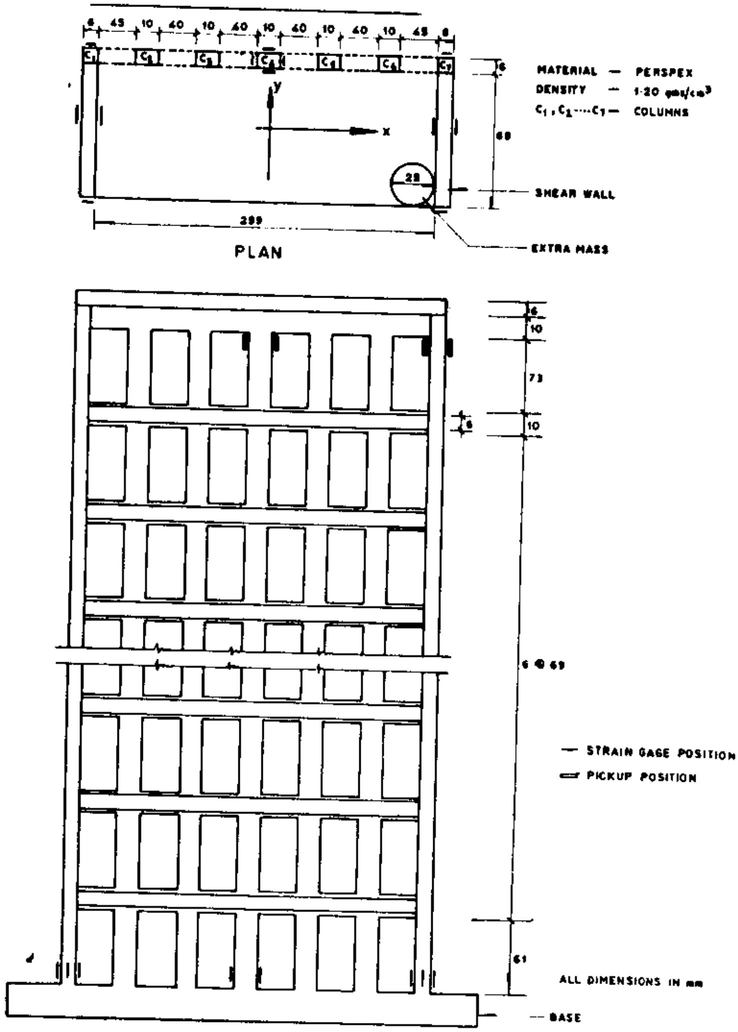

Fig. 4. Model Dimension

For the concentrated mass W having a radius vector R, the equivalent mass is

$$W R^2$$

The mass matrix $[M]_{3m \times 3m}$ is thus a diagonal matrix whose elements are

$$\xi_1 \ldots \xi_m, \xi_1 \ldots \xi_m, \eta_1 \ldots \eta_m$$

where

ξ_i = lumped mass on the floor i, and

η_i = total 'equivalent mass' corresponding to rotation θ at the floor i.

In the foregoing derivation it is assumed that the other two inertia forces corresponding to the accelerations $\ddot{U}$ and $\ddot{V}$ do not contribute to the moment. This is possible only if the origin is chosen to coincide with the centre of mass. If this is not done, the mass matrix has two sets of off-diagonal elements and the problem becomes highly complex in as much that the eigen values of a large unsymmetric matrix have to be computed.

While the formulation of a mass matrix for an arbitrary origin is straight forward the complexity pointed out above has restricted the scope of present investigation to such problems where the centres of mass on all the floors fall on a single vertical line.

ILLUSTRATIVE EXAMPLE

The method has been used to analyse the building shown in fig. 4. The choice of the building was dictated by the availability of this model in the laboratory.

Frequencies and mode shapes were found for the following three cases:

Case I Self-weight only.

Case II Two masses (corresponding to the mass of the two acceleration pickups used in the experimental investigation) shared between the two top floors.

Case III Cylindrical mass on all floors except the top floor as shown in fig. 4.

EXPERIMENTAL INVESTIGATION

An experimental investigation was carried out by S. K. Agarwal to ascertain the lowest three resonant frequencies of this model under steady state vibration at the Structural Dynamics Laboratory of the School of Research and Training in Earthquake Engineering, University of Roorkee (4). The model was mounted on a plane vibration table fitted with a mechanical oscillator and a speed controlled motor fig. 5. Amplitude measurements were recorded using.

(i) acceleration pickups mounted on the vertical members as near to the top floor as possible (fig. 4), and,

(ii) strain gages pasted on columns and shear walls near the base (fig. 4). The frequency-amplitude plots for the three cases referred were obtained and are given in fig. 6, 7, 8 and 9.

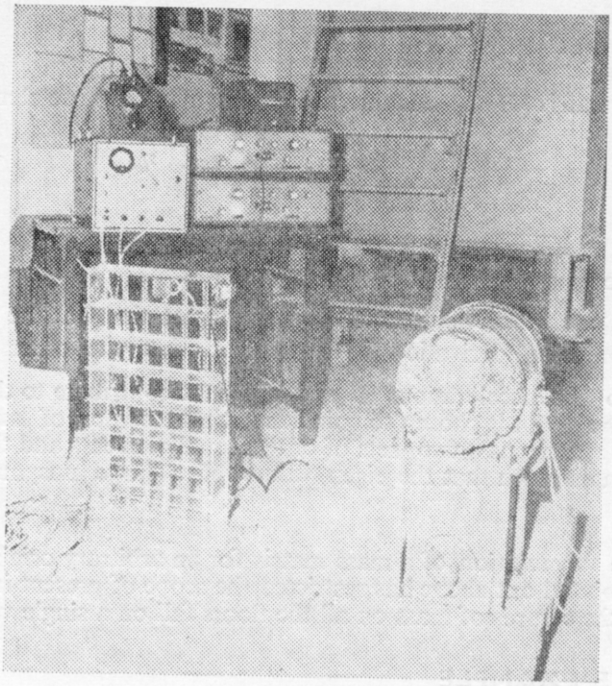

Fig. 5

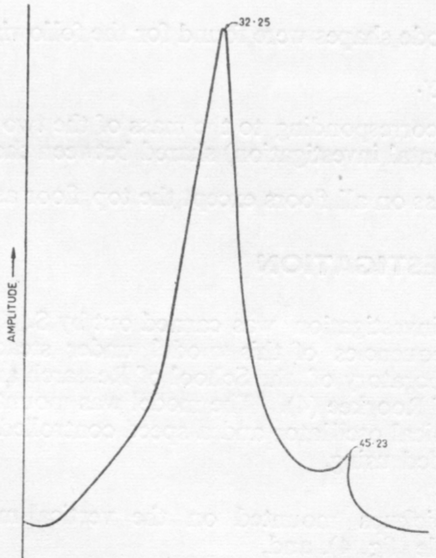

Fig. 6. Amplitude-Frequency Plot for Case I X-Direction

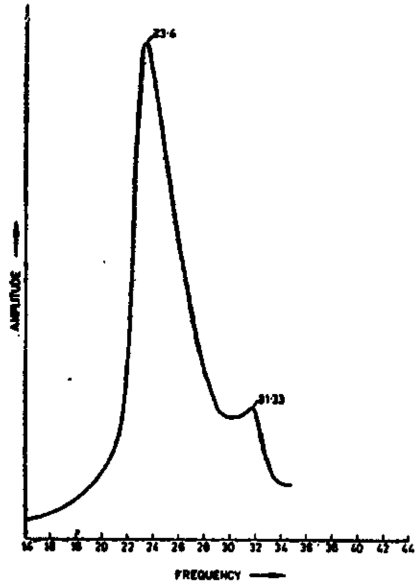

Fig. 7. Amplitude-Frequency Plot for Case II X-Direction

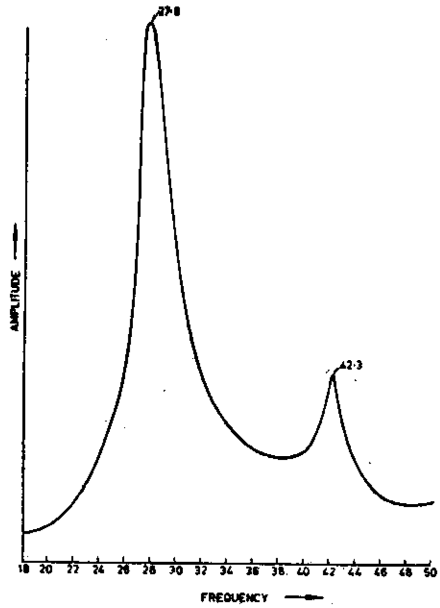

Fig. 8. Amplitude-Frequency Plot for Y-Direction Case II

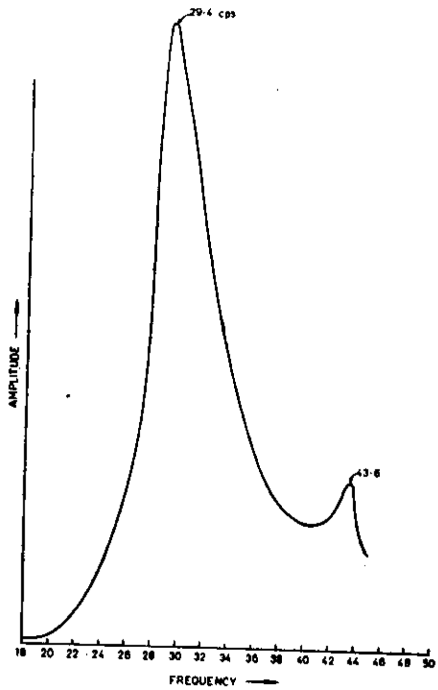

Fig. 9. Amplitude-Frequency Plot for Y-Direction Case III

TABLE I

Case	Periods (Secs)				
	Theoretical			Experimental	
	Mode 1	Mode 2	Mode 3	Mode 1 & 2	Mode 3
I	0.033	0.032	0.022	0.03	0.023
II	0.038	0.035	0.027	0.036	0.024
III	0.050	0.043	0.027	0.042	0.031

Table 1 shows the theoretical and experimental values of the first three frequencies for the three cases. In this particular case the first two resonance frequencies were too near to be distinguished experimentally.

It is seen that the agreement between the theoretical and experimental values is excellent. The maximum deviation is only 16%. The maximum rotation is induced in the structure in the third mode. Theoretical U, V and θ plots are given in fig. 10, 11 & 12.

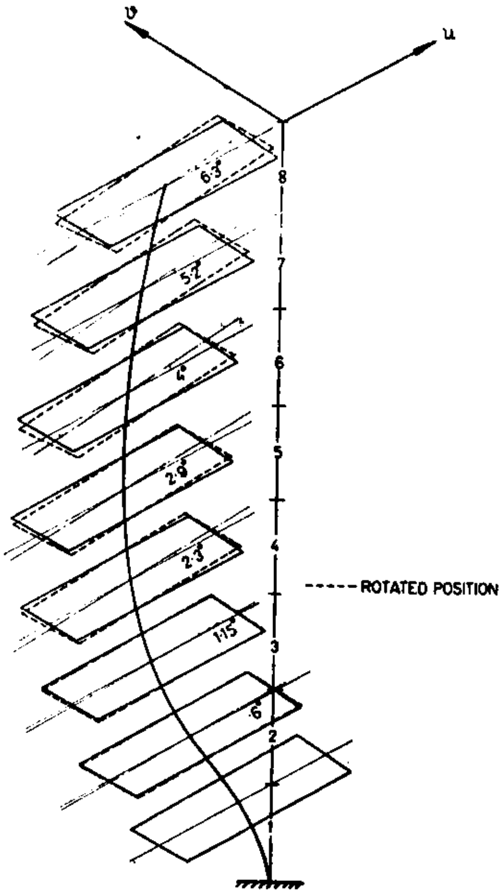

Fig. 10. u, v, θ Plot for Mode 3 Case I

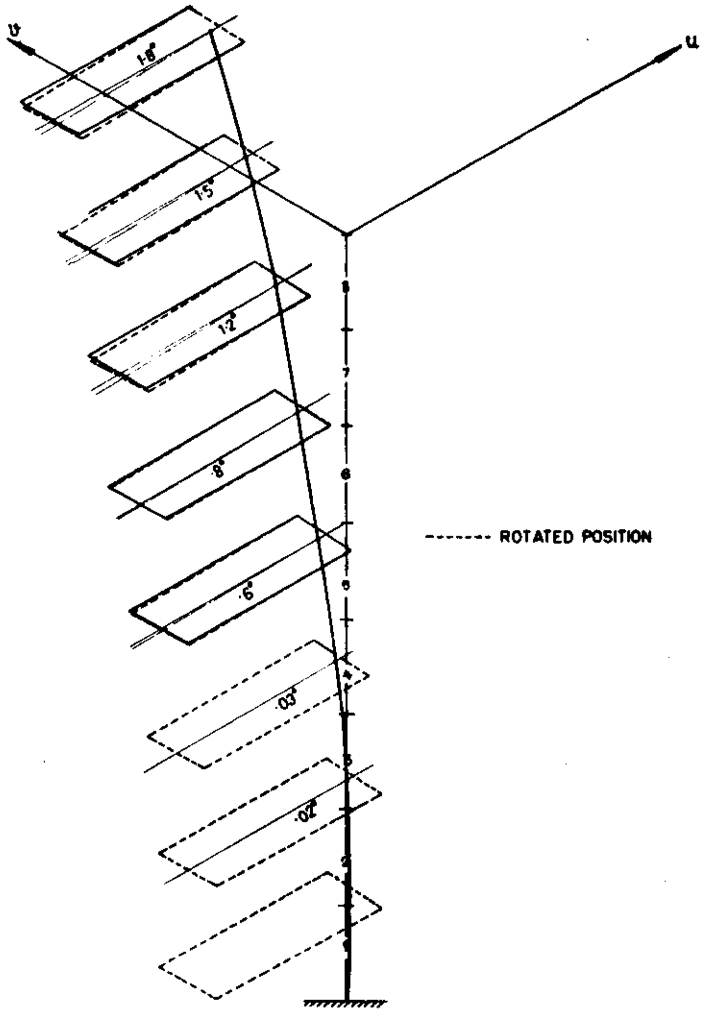

Fig. 11. u, v, θ Plot for Mode 1, Case III

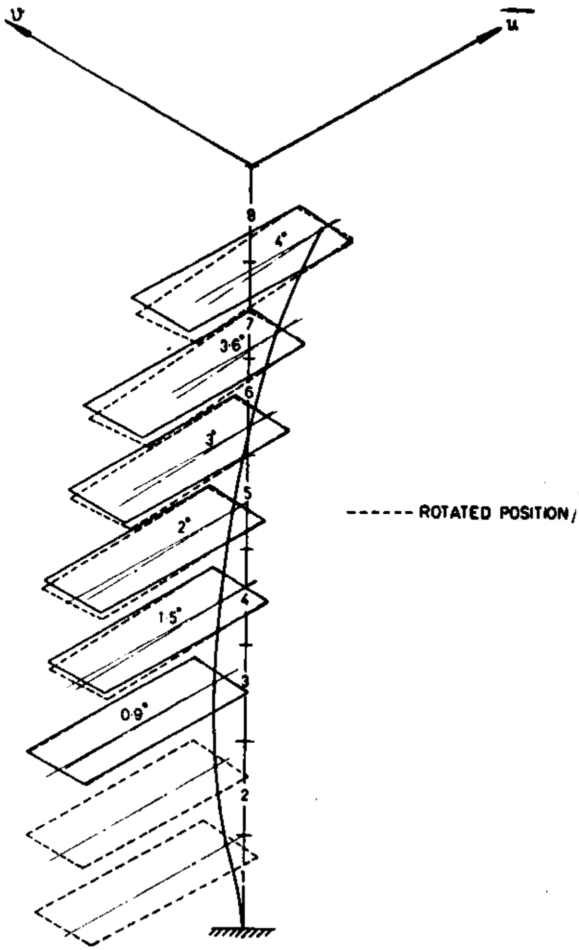

Fig. 12. u, v, θ Plot for Mode 3, Case 3

MODEL ANALYSIS

Knowing the response U_r, V_r and θ_r (of centre of mass) for the rth floor, the model displacements at rth floor for the jth frame were computed using equations (6) and (7). Assuming 6% damping in all modes and using Housner's average spectra curves the most probable values of frame/shear wall displacements were evaluated in accordance with IS: 1893-1974 for Case I and III and are plotted in figs. 13 (a & b).

In case I the most probable response of frame/shear walls was calculated considering these as two dimensional plane frames and is shown in fig. 13(a). The effect of torsion was superimposed over these values as per IS: 1893–1974. Table 2 gives the comparison of forces on frame and shear walls when considered as independent plane elements, considering torsion as per code provisions and the reported approach.

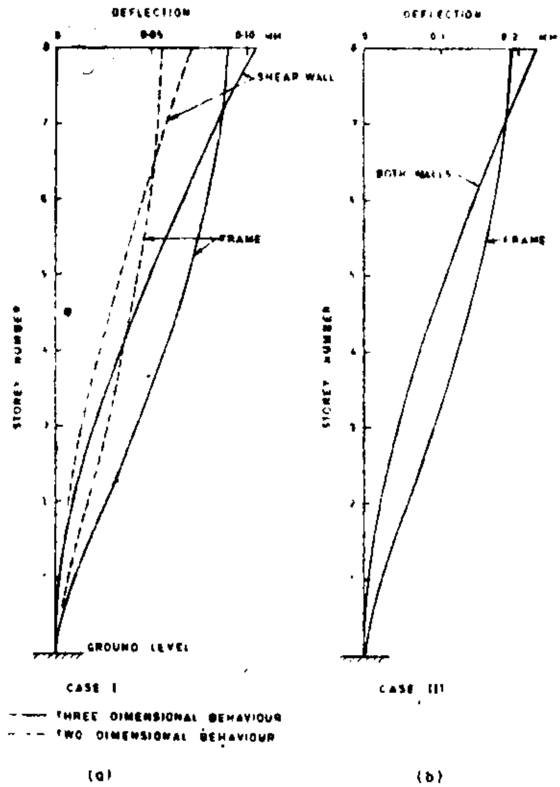

Fig. 13. Most Probable Response

During the investigation it was observed that in case III the response of two shear walls was approximately equal though the mass was placed eccentrically. Another extreme case was considered in which the superimposed mass was increased by a factor of 10 and placed at one of the shear walls. Even in this case the response of the two shear walls was indentical.

NUMERICAL PROBLEMS

It is seen from Table 2 that the storey shears for the two shear walls are different while the symmetry of the structure demands the two to be identical. It was also noticed that the deformations (most probable response) tallied to three figures. The difference is due to numerical problems associated with computing shears from given deformations —moments will be less susceptible. However, the values obtained are good enough for a general comparison since the difference between the three approaches is clearly brought out.

COMPARISON OF THE THREE APPROACHES

It is observed that, for the frame, the 2D and the IS code approaches yield nearly same storey shears. This shows that eccentricity affecting the frame is small. The 3D

TABLE 2

(Storey Shears in Gms.)

Storey	Frame			Shear walls			
	Considering 2D action	As per IS 1893	Reported approach	Considering 2D action	As per IS 1893	Reported approach	
						Left	Right
8	57.2	57.3	93.0	34.0	62.0	56.0	25.0
7	119.8	120.1	196.0	64.0	123.0	86.0	102.0
6	175.6	175.3	289.0	87.0	174.0	114.0	116.0
5	224.5	225.4	370.0	104.0	216.0	141.0	151.0
4	265.5	266.5	438.0	117.0	249.0	167.0	162.0
3	297.1	298.2	489.0	125.0	273.0	192.0	175.0
2	318.5	319.7	523.0	130.0	289.0	213.0	191.0
1	327.1	328.3	536.0	133.0	296.0	239.0	201.0

values, on the other hand are significantly higher indicating a greater participation of the frame in the oscillations of the structure.

The shear wall values are nearly doubled when the eccentricity is considered in accordance with code provision while the corresponding values for the 3D approach though higher than the 2D values, are considerably lower.

It would appear that the proposed approach shows a more balanced participation of the two elements the frame and the shear walls. On this example 2D approach underestimates the forces in both the elements and the codal provision results in underestimation in one and over estimation in the other.

CONCLUSION

The agreement between experimental and predicted values shows that the proposed approach may be used with confidence for the dynamic analysis of asymmetric multistoreyed buildings with shear walls.

Comparison of design forces (most probably values) obtained by analysis of constituent plane elements, by following the codal provisions (IS 1893–1974) for eccentricity and those found using the proposed approach shows that in this example the former two lead to an underestimation. While no definite conclusion can be drawn from a single example it is obvious that the methods currently used may lead to underdesigning. Whether they consistently underestimate these forces merits further examination. The possibility of an underestimation has, however, been established.

ACKNOWLEDGEMENT

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APPENDIX I

Stiffness matrix for a framed building system with rigid floors.

Consider a plane frame γ^i (as shown in fig. 1) loaded by lateral loads acting on joints 1, 2, m laying in the $x-z$ plane at $y=0$ only. Let $p^i\,(m \times 1)$ be this load vector, $\delta^i\,(m \times 1)$ the vector representing lateral movement (v) of the joints $1,..,m$, δ_1^i the vector representing all other deformations of all the joints arranged in any convenient order, then

$$\begin{bmatrix} K_1^i & K_2^i \\ K_3^i & K_4^i \end{bmatrix} \begin{bmatrix} \delta^i \\ \delta_1^i \end{bmatrix} = \begin{bmatrix} P^i \\ O \end{bmatrix} \tag{1}$$

The matrix at the extreme left is appropriately partitioned stiffness matrix of the frame Y^i.

Here

$$K_3^i = [K_2^i]^T$$

Equation (1) can be rearranged to get

$$K^i \delta^i = P^i \quad (i = 1, 2, \ldots n) \tag{2}$$

where

$$K^i = K_1^i - K_2^i\,[K_4^i]^{-1}\,K_3^i \tag{3}$$

Similarly for a frame x^j lying in the $y=0$ plane

$$K^j\,\delta^j = P^j \quad (j = 1, 2, \ldots l) \tag{4}$$

where

$$K^j = K_1^j - K_2^j\,[K_4^j]^{-1}\,K_3^j \tag{5}$$

Vectors δ^j and P^j represent the lateral movement (v) and lateral load on joints 1, 2...m of the frame X^j.

To relate δ^i and δ^j to the floor movements consider a typical floor shown in fig. 2 whose movement is defined by the displacements U_r and V_r in the x and y directions respectively and a rotation θ_r. Suffix r refers to the rth floor.

For the frame Y^i, lateral displacements $\delta_r{}^i$ ($r = 1, 2\ldots, m$) are given by

$$\delta_r{}^i = V_r + X_i \operatorname{Sin} \theta_r$$

where X_i is the ordinate of frame Y^i as shown in fig. 1, $\delta_r{}^i$ is the rth component of the vector δ^i, V_r that of the vector V and θ_r that of the vector θ.

Since θ_r is small

$\operatorname{Sin} \theta_r \approx \theta_r$

Thus

$$\delta^i = V + X_i\theta \tag{6}$$

Similarly

$$\delta^j = U - Y_j\theta \tag{7}$$

Vectors U, V and θ are the rigid body displacements of the floors.

Since the floors are in equilibrium under the action of loads P^i and P^j acting on the frame system Y^i and X^j, total forces in the X and Y directions are zero and the moment of the forces about, say, the origin is zero.

The external load vector in x-direction,

$$Q = \sum_{i=1,\ldots n} P^i \tag{8}$$

Total exteranal load vector in y-direction.

$$R = \sum_{j=1,\ldots l} P^j \tag{9}$$

Total moment about the origin.

$$M = \sum_{i=1,\ldots,n} X_i P^i - \sum_{j=1,\ldots,l} Y_j P^j \tag{10}$$

Using expression (2), (4), (6) and (7) equations (8), (9) and (10) may be expressed as

$$[K]\begin{bmatrix} U \\ V \\ \theta \end{bmatrix} = \begin{bmatrix} R \\ Q \\ M \end{bmatrix} \tag{11}$$

$[K]$, the stiffness matrix of the system is defined as

$$[K] = \begin{bmatrix} \alpha_1 & 0 & \alpha_2 \\ 0 & \alpha_3 & \alpha_4 \\ \alpha_2 & \alpha_4 & \alpha_5 \end{bmatrix}$$

$$\alpha_1 = \sum K^j \qquad \alpha_2 = -\sum Y_j K^j$$

$$\alpha_3 = \sum K^i \qquad \alpha_4 = \sum X_i K^i$$

$$\alpha_5 = \sum X_i^2 K^i + \sum Y_j^2 K^j$$

where

$$i = 1, 2, \ldots, n$$

$$j = 1, 2, \ldots, l$$