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FUNDAMENTAL TIME PERIOD OF FRAMED BUILDINGS

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ABSTRACT

A method suitable for hand calculations or for use with desk top computers is presented for the determination of fundamental time period of framed buildings. Stiffness matrix and mass matrix are formed with respect to deflections at a fewer level than the number of storeys. Only one level is shown to yield good results. The computation work is practically independent of the number of storeys.

Key words : Framed Building, Fundamental Time Period, Shear Beam, Base Shear.

INTRODUCTION

IS 1893 Criteria for Earthquake Resistant Design of Structures permits (reference 4) the use of seismic coefficient method for the evaluation of earthquake loading for buildings which are less than 40 m in height in all the zones and for those which are greater than 40 m and less than 90 m in height in zones I to III. One of the parameters that governs the magnitude of the base shear is the coefficient C which defines the flexibility of the structure and is dependent on the fundamental time period, T, of the structure. A rational method is recommended for the estimation of T, in the absence of which an empirical formula ($T = 0.1\,n$, n = number of storeys) is given in reference 4. This formula does not take into account the distribution of stiffness and mass properties along the height in a rational way.

An accurate method (Clough, King and Wilson, 1964) for determining the dynamic characteristics takes into account translational, rotational and vertical degrees of freedom at joints of a frame. The rotational and vertical degrees of freedom may be condensed out to yield the lateral stiffness matrix of each frame and by stiffness transformation the lateral

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stiffness of the structure is obtained. Assuming the masses to be lumped at floor levels an eigenvalue problem of size equal to the number of storeys is formed. For an approximate analysis a frame may be idealised as a shear beam (Basu and Nagpal, 1980) and Holzer's method (Clough and Penzien, 1975) may be used to obtain the dynamic characteristics. The computational work in Holzer's method is proportional to the number of storeys.

In this paper a method suitable for hand calculations or desk top computers is presented. The frame is idealised as a shear beam. By forming the stiffness matrix and the mass matrix with respect to lateral displacements at a fewer number of levels along the height, the size of the problem is kept small.

METHOD

Under lateral loading a frame exhibits shear mode behaviour in which a floor may be visualized as sliding over the floor below it. Fig. 1 shows this behaviour in which the jth floor has a relative displacement d_j with respect to the (j—1)th floor, when the jth storey has a storey shear V. The storey shear required to cause a unit sway angle is defined as the shearing rigidity of the jth storey. A procedure to obtain the storey shearing rigidity is available (Basu and Nagpal, 1980).

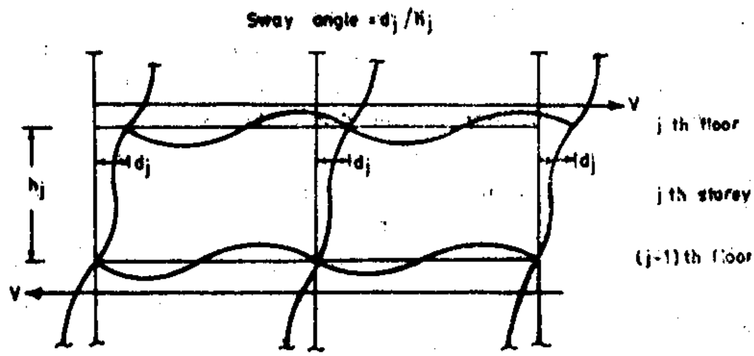

Fig. 1. Shear Mode Behaviour of Frames

All the frames in a building are replaced by respective shear beams. The shearing rigidities of respective storeys of these shear beams are added to form an equivalent shear beam. A single equivalent shear beam thus replaces the entire building.

To reduce the computation work the stiffness and mass matrix of the equivalent shear beam are formed with respect to lateral deflections at a selected number of reference levels, the number of such reference levels are fewer than the number of floors in a building.

Stiffness Matrix:-Let the portion of the shear beam between $(i-1)$th and ith reference levels be designated as the ith segment. A segment spans across a number of storeys (Fig. 2). Fig. 3 shows an isolated segment (ith)

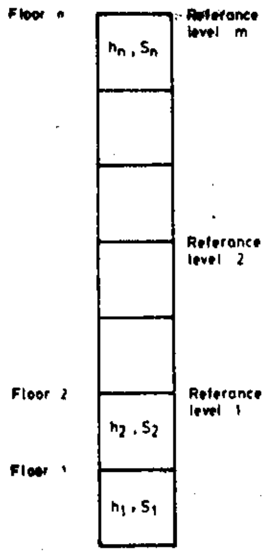

Fig. 2 Shear Beam Model

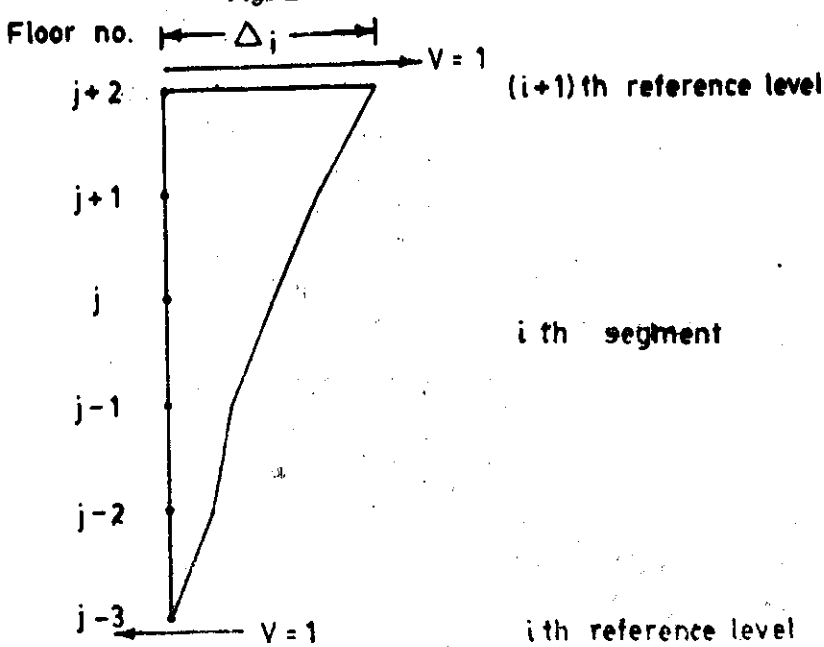

Fig. 3. Force-Displacement Relationship for a Segment of the Shear Beam

with a shear V equal to unity in it. The relative lateral displacement between the (i—1)th and ith reference level is given by

$$\Delta_i = \Sigma h_j/s_j \qquad (1a)$$

in which h_j, s_j are height and shearing rigidity respectively of the jth storey and the summation extends over the number of storeys in the segment.

The force k^i required to cause unit relative displacement between the (i—1) th and the ith reference level is, therefore, given by

$$k^i = 1/(\Sigma h_j/s_j) \qquad (1b)$$

The lateral stiffness matrix of the shear beam with respect to displacements at reference levels, is obtained by giving a unit displacement at each levels successively with displacements at other levels restrained and calculating the lateral forces, at three levels, namely, the level at which unit displacement is given and the levels above and below it

When the ith reference level is given such a unit displacement (Fig. 4) the resulting forces at the (i—1) th, ith and (i+1) th reference levels are equal to $-k^i$, $k^i + k^{i+1}$ and $-k^{i+1}$ respectively with the forces at all the other levels equal to zero. The stiffness matrix [K] of the shear beam is, therefore, tridiagonal the elements of which are

$$K(i, i) = k^i + k^{i+1} \qquad (2a)$$
$$(\text{for } i = 1 \text{ to } m)$$

$$K(i, i+1) = -k^{i+1} \qquad (2b)$$
$$(\text{for } i = 1 \text{ to } m-1)$$

$$K(i+1, i) = K(i, i+1) \qquad (2c)$$
$$(\text{for } i = 1 \text{ to } m-1)$$

in which m = number of reference levels. The second term in Eq (2a) vanishes for i = m.

Mass Matrix :— The inertia force, Q^j of a floor mass in the ith segment

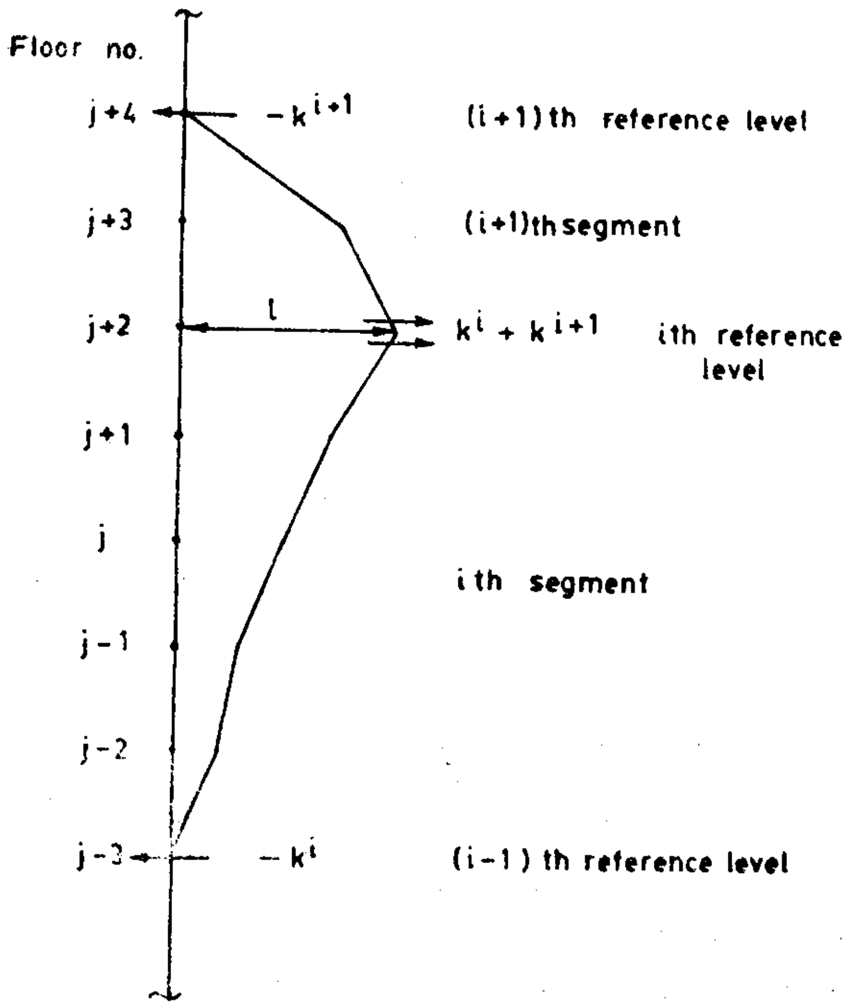

Fig. 4. Forces Due to a Unit Displacement at the ith Reference Level

(Fig. 5) is given by

$$Q^j = -\omega^2 y_j m_j \tag{3a}$$

in which y_j, m_j are lateral deflection and mass at the jth floor respectively and ω is the natural frequency.

This inertia force is assumed to be replaced by the inertia forces Q^j_{i-1} and Q^j_i at the adjacent reference levels as the reactions of a simply supported beam of span equal to the length of the segment :

$$Q^j_{i-1} = -\omega^2 m_j y_j (1 - x_j/L_i) \tag{3b}$$

$$Q^j_i = -\omega^2 m_j y_j x_j/L_i \tag{3c}$$

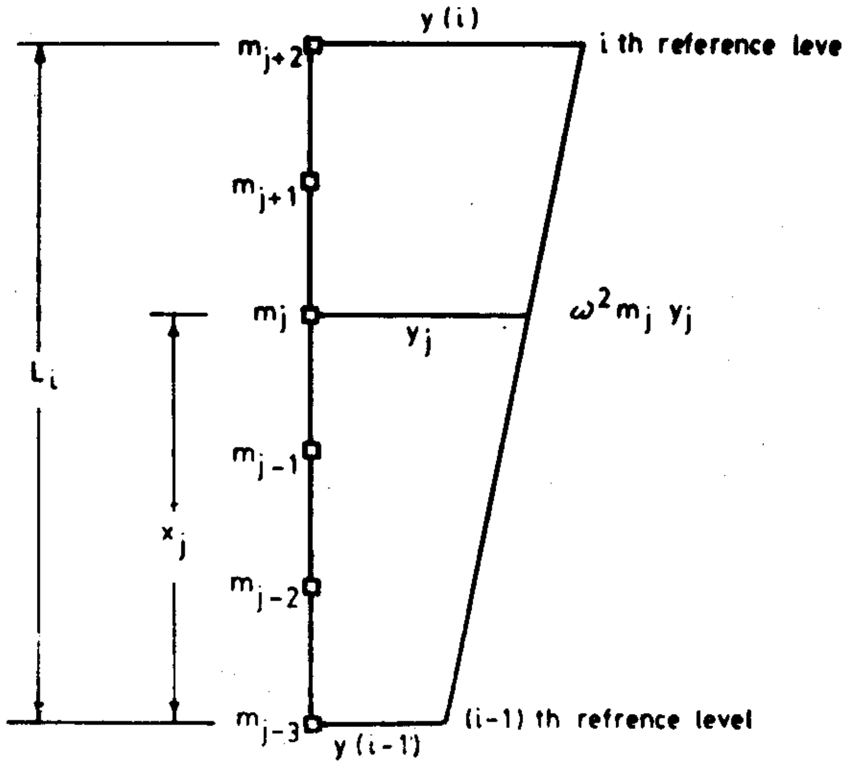

Fig. 5. A Segment With Floor Masses

in which x_j = distance of the jth mass from the (i—1) th reference level and L_i = length of the ith segment.

Assuming further, for free vibrations, the displacements at floor levels to be linear between the adjacent reference levels, the lateral displacement y_j is expressed in terms of the displacements at the reference levels as

$$y_j = y(i-1) + [y(i) - y(i-1)]\ x_j/L_i \tag{4}$$

in which y(i–1), y(i) are lateral displacements at the (i—1)th and ith reference levels respectively.

The total inertia forces Q_{i-1} and Q_i at the reference levels are obtained by summing the contribution of all the masses in a segment. Making use of Eq. (4) these may be written as

$$\begin{bmatrix} Q_{i-1} \\ Q_i \end{bmatrix} = -\omega^2 \begin{bmatrix} a^i & b^i \\ b^i & c^i \end{bmatrix} \begin{bmatrix} y(i-1) \\ y(i) \end{bmatrix} \tag{5}$$

in which

$a^i = \Sigma m_j (1 - x_j/L_i)^2$ (6a)

$b^i = \Sigma m_j x_j/L_i (1 - x_j/L_i)$ (6b)

$c^i = \Sigma m_j (x_j/L_i)^2$ (6c)

the summation extending over a number of masses in a segment. The superscript i for a,b and c refers to the ith segment.

The terms in the square bracket on the right hand side of Eq. (5) form the mass marix of the ith segment. The mass matrix [M] of the shear beam model is tridiagonal the elements of which are given by

$M(i, i) = c^i + a^{i+1}$ (7a)

(for $i=1$ to m)

$M(i, i+1) = b^{i+1}$ (7b)

(for $i=1$ to $m-1$)

$M(i+1, i) = M(i, i+1)$ (7c)

(for $i=1$ to $m-1$)

The second term in Eq. (7a) vanishes for $i=m$.

Numerical Examples

Example 1:—

A 10-storeyed, 3 bay frame (Fig. 6) is considered to illustrate the

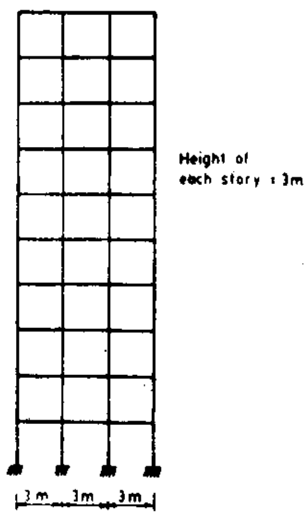

Fig. 6. Example Frame

procedure. All the columns in a storey and the beams at a floor level have the same flexural stiffnesses. The flexural rigidity of each column in a storey and each beam at floor levels as well as lumped masses and storey shearing rigidities are given in Table 1. Two cases are considered :

(1) two reference levels—at the 5th floor and the 10th floor (2) one reference level at the 10th floor level.

TABLE 1. Structural Properties of the Example Frame 1.

Floor/Storey No.	Column flexural rigidity in tonne m^2	Beam flexural rigidity in tonne m^2	Lumped mass in tonne sec^2/m	Shearing rigidity in tonne
1	3.88×10^4	1.82×10^4	3.40	1.15×10^5
2	1.75×10^4	1.21×10^4	5.74	3.04×10^4
3	1.75×10^4	1.21×10^4	3.40	3.10×10^4
4	1.75×10^4	1.21×10^4	3.40	3.10×10^4
5	1.75×10^4	1.21×10^4	3.40	3.36×10^4
6	9.71×10^3	6.00×10^3	3.40	1.80×10^4
7	9.71×10^3	6.00×10^3	3.40	1.60×10^4
8	9.71×10^3	6.00×10^3	3.40	1.60×10^4
9	9.71×10^3	6.00×10^3	3.40	1.60×10^4
10	9.71×10^3	6.00×10^3	1.70	2.01×10^4

For case (1) the stiffness matrix (in tonne/m) and the mass matrix (in tonne sec^2/m) are calculated to be

$$K = \begin{bmatrix} 3598 & -1131 \\ -1131 & 1131 \end{bmatrix}$$

$$M = \begin{bmatrix} 11.92 & 2.721 \\ 2.721 & 5.790 \end{bmatrix}$$

The fundamental frequency is obtained by solving the following eigen value problem.

$$\begin{bmatrix} 3598 & -1131 \\ -1131 & 1131 \end{bmatrix} \begin{Bmatrix} \phi(1) \\ \phi(2) \end{Bmatrix} = \omega^2 \begin{bmatrix} 11.92 & 2.721 \\ 2.721 & 5.790 \end{bmatrix} \begin{Bmatrix} \phi(1) \\ \phi(2) \end{Bmatrix} \quad (8)$$

in which ϕ (1) ϕ and (2) are the eigen vector components at levels 1 and 2 respectively.

Eq. (8) yields a fundamental frequency equal to 8 83 rad/sec (time period=0.71 secs).

For case (2) the stiffness and mass with respect to the 10th floor are 777 tonnes/m and 11.5 tonne sec^2/m respectively. A fundamental frequency equal to 8.22 rad/sec (time period = 0.764 secs) is obtained directly in this case as $\sqrt{K/M}$.

Table 2 gives the time period as obtained from the present method, exact value based on the frame program and the value using the empirical formula (ref : 4). The values of the coefficient C based on these time periods as read from the graph in ref. 4 are also included in the table. Since the base shear is directly proportional to C the errors in C and the base shear are the same. It is seen that the C value obtained from the present method for both the cases is in excellent agreement with that based on the frame program. The value based on the empirical formula is about 22 percent in error.

TABLE 2. Comparison of Time Period and Coefficient C for Example Frame 1.

Method	Time Period Seconds	C
Frame program	0.723	0.68
Present method (case 1)	0.710	0.68
Present method (case 2)	0.764	0.65
Ref. 4	1.0	0.53

Example 2 :—

A 8-storeyed 2-bay frame is shown in Fig. 7 wherein the flexural stiffnesses of members are also given. The frame has a lumped mass equal to 2.75 tonne sec^2/m each at floors 1 to 7 and 2 tonne sec^2/m at the 8th floor. Again two cases are considered : (1) two reference levels-at the 4th floor and the 8th floor (2) one reference level at the 8th floor.

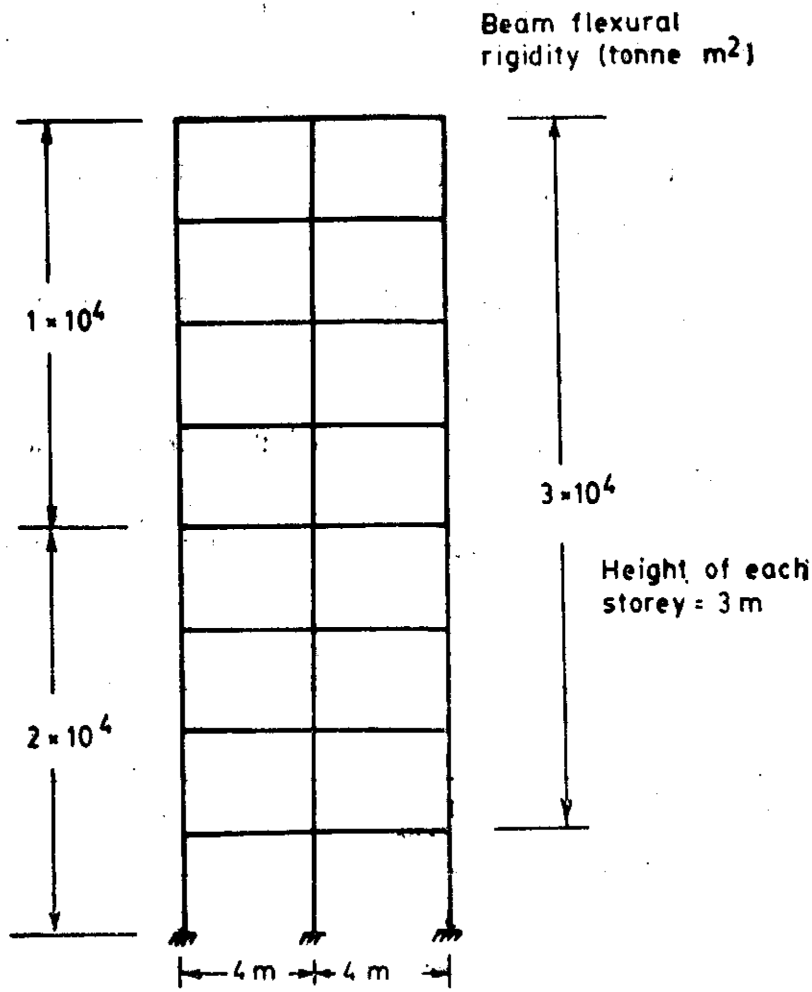

Fig. 7. Example Frame 2

The time periods and the C values are given in Table 3. For this

TABLE 3. Comparison of Time Period and Coefficient C for Example Frame 2.

Method	Time Period seconds	C
Frame program	0.52	0 80
Present method (case 1)	0.50	0.80
Present method (case 2)	0 51	0 80
Ref. 4	0.80	0.63

example also it is seen that the present method yields time periods and C values which are in excellent agreement with those obtained from the frame program. The C value based on the empirical formula is about 21 percent in error.

CONCLUSIONS

A method suitable for hand calculations/desk top computers has been presented for the evaluation of the fundamental time period of framed buildings. The method is practically independent of the number of storeys.

Notations

The following symbols are used in the paper:—

C = a coefficient defining the flexibility of the structure,

m = number of levels w.r.t. which mass and stiffness matrices are formed,

[K] = stiffness matrix,

Q^j = inertia force at the jth floor,

s_j = shearing rigidity of the jth storey,

T = time period,

x_j = distance of the jth floor mass from the (i—l)th connection level,

y_j = lateral displacement of the jth floor mass,

y (i) = lateral displacement at the ith reference level,

ω = natural frequency.

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