

## LATERAL DYNAMIC RESPONSE OF PILE IN NON LINEAR SOIL MEDIUM

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### ABSTRACT

Soil pile interaction problem, under dynamic loading condition, has drawn the attention since 1934. In this paper a time domain approach of the soil pile interaction problem, under lateral dynamic loading, has been attempted. The material property of the vertical pile is linearly elastic homogeneous and isotropic. The motion of the pile is limited in the vertical plane. The time domain has been discretised into small intervals. The non linearity of soil property has been taken into account by using equivalent linear (E.L.M) soil properties. The need for non linear analysis becomes particularly important when soil system experience a large non recoverable strain. The inadequacy of linear models in describing the hysteretic energy dissipation mechanism demands the necessity of incorporating a truly non linear formulation of the problem. Suitable software has been developed which keeps track of the stress path and load reversal points of each element and the computer program takes decision about the appropriate instantaneous soil properties. In the experimental part of the work authors have carried out a small scale test, in the laboratory. Model pile embedded in dry dense sand in a tank was excited under two different dynamic loading conditions. In the first study the base of the tank was excited by a large motor and oscillator assembly and the pile top was free. In the second case the pile top was excited by a small motor and oscillator assembly and the base was at rest. The acceleration response of the pile was recorded with proper data acquisition system for both the cases. Comparison of the experimental and numerical studies have revealed that a proper judgement on the mathematical modelling of soil behaviour is very essential for any reliable analytical prediction.

### INTRODUCTION

Rapid growth of various complexes on poor soil demands in depth study of dynamic response analysis of Soil-Pile-Structure system under dynamic loading. Source of dynamic loading on pile may be : (i) Direct or Active, e.g. machine foundation, where pile transmits the excitation to soil, (ii) Indirect or Passive, e.g. earthquake or blast, where soil excites the pile. A rational idealization of the earth media underlying and surrounding the pile is the most important aspect in the aseismic analysis of pile. There is a need to establish the soil-pile interface behaviour under earthquake type loading and to identify a suitable numerical scheme for the dynamic design of pile.

### STATE OF THE ART

This 'State of the Art' presents methods for the analysis of soil pile system in (i) active, (ii) passive dynamic loading case, (iii) the role of soil constitutive model and (iv) experimental study. The aim of this SOA is to highlight the present methods with their limitations.

Soil pile interaction problem under dynamic loading condition has drawn the attention since 1934. Barkan (1962) has presented a brief account of early days development. Simplified design methodology has been presented by Richter et al. (1970), Srinivasulu (1976) and Major (1962). Prakash (1981), Prakash and Puri (1988) have briefly reviewed the dynamic analysis and design procedures.

### Active Dynamic Loading

The methodologies adopted in the dynamic analysis of pile foundation may be classified as : (i) The finite element method (Kuhlemeyer, 1979), (ii) The beam model in elastic or visco-elastic media (Tucker, 1964; Chandrasekaran, 1974; Matlock, 1973), (iii) The lumped mass model (Prakash and Chandrasekaran, 1973). Time domain & frequency domain approach are equally popular. Each approach has the advantage and disadvantage of its own.

Hayashi (1973), Prakash and Sharma (1969) and Prakash and Gupta (1970) determined the natural frequencies of the pile soil system by considering the pile as an equivalent cantilever. Penzien et al. (1964, 1970) have used the lumped mass model where the Mindlin's static solution for point load in an elastic isotropic half space was used to evaluate the spring stiffness, representing the soil reaction. The pile and soil, both, were idealized as an assemblage of lumped masses (Prakash and Chandrasekaran, 1973, 1977). This method does not take into account of radiation damping which was duly considered by Novak (1974), Novak and Nogami (1978), and others, Baranov's plane strain solution was the Greens function for soil reaction in these methods. Novak and Aboul-Ella (1978) extended this method, by standard matrix formulation, for layered soil media. In these methods the stiffness and energy dissipation characteristic of the overall pile soil system, or of a small element of this, has been expressed by impedance functions which depend on (i) the frequency, (ii) length and radius of the pile, (iii) stiffness and hysteretic characteristic of pile and soil media, (v) mode of vibration. These functions assume the stiffness of pile soil system as elastic which deviates substantially at high intensity of dynamic loading. These particular functions assume uniform soil but in reality soils are rarely so. These methods assume perfect bonding between soil and pile. Under high intensity of dynamic loading gapping or slippage may take place at pile soil contact, resulting a reduction in stiffness and damping properties. Prakash (1986), Prakash and Puri (1988) have discussed the limitation of this approach.

Blaney et al. (1976) and Kuhlemeyer (1979) have successfully attempted the dynamic soil pile interaction problem by finite element method. Roesset (1980) has presented a detailed comparison of different approaches.

Pise (1985) presented the theoretical results of fixed head piles. He used the concept of elastic continuum reaction to study the effect of a layered soil system on the behaviour of pile subjected to lateral loads. Dobry (1982) put forward a beam on elastic foundation model in a finite element code (Blany, 1976) to evaluate the equivalent horizontal spring and dynamic coefficients at the top of an end bearing pile embedded in a uniform, linear-hysteretic soil layer. Soil surrounding pile was discretized by means of toroidal elements, the effect of foundation in the far field was reproduced through a consistent boundary matrix. Takamiya et al. (1981) presented the study of dynamic soil-pile-structure interaction, subjected to harmonic excitation at the base rock level, as a combination of two phenomena : (i) the free field soil layer vibration due to incoming incident wave without piles, (iii) the pile vibration subjected to the loading at its head because of the super-structure and to the soil reaction along its length because of the scattering waves from the piles. Hayashi et al. (1985) used experimental results of static and the alternating loading tests to investigate the dynamic behaviour of piles with simplified equivalent cantilever approach. Singh et al. (1977) presented a single degree of freedom mass spring dashpot model with the concept of an equivalent cantilever

technique. Tazimi (1969) presented a theoretical analysis on a basis of the three dimensional wave propagation theory in an elastic stratum. Kagwa et al. (1980) presented a finite element method to evaluate dynamic p-y relationship of pile. Angelides et al. (1981) attempted to explore the effect of nonlinear soil behavior on the dynamic response of piles subjected to lateral loads, using a finite element model for the soil region adjoining the pile, a consistent boundary matrix at some distance to produce radiation effects and an iterative equivalent linerization technique to estimate the variation of soil properties with level of strain. The Hardin and Drenevich soil model has been used to predict the relation between shear modulus and damping ratio. Gazetes et al. (1983) presented the results of a systematic parametric investigations of the static and dynamic response of single free head and bearing pile embedded in soil stratum, the modulus of which increase linearly with depth. The study was conducted with a dynamic finite element formulation (Balaney, 1976). Particular emphasis was given to assess the effects of soil inhomogeneity on pile deflection at and above the resonant frequencies of the system. Saha et al. (1986) presented an analysis to study the dynamic response of pile embedded in a homogeneous, elastic soil half space subjected to coupled horizontal and rotational mode of vibration. Finite difference technique was used in frequency domain to solve the differential equation of damped horizontal vibration of pile. The soil considered was linearly elastic. The material damping of the soil medium was neglected.

Davies (1985) presented a boundary element formulation, in frequency domain, for the dynamic analysis of axially and laterally loaded single pile and pile groups. The piles are represented by compressible beam column element and the soil as a hysteretic half space. The fundamental solutions for a periodic point force in a half space are coupled with a numerical solution for the motion of the soil domain by satisfying equilibrium and compatibility at the pile soil interface.

### Passive Dynamic Loading

Earthquake introduce lateral force on piles. The energy supplied to a structure may be absorbed in the elasto-plastic deformation of the superstructure, the superstructure and the soil. Vertical loads, transmitted from the superstructure, in association with seismically induced lateral load may cause buckling of the pile (Prakash, 1985, 1987). Fukuoda (1966) reported classical damage to Showa bridge, supported on pile, in the Nigita earthquake of 1964. Prakash (1981) has presented the concept of spectral response method for the design and analysis of pile against earthquake.

The problem of aseismic response of pile is somewhat different from response of pile under forced vibration (Arnold et al., 1977; Matlock, 1978; Prakash and Sharma, 1969). In the former case the foundation input motion vector, which corresponds to the response of the foundation to seismic excitation in the form of elastic waves, entails the solution of scattering mixed boundary value problem (Bycroft, 1980). The stress transfer, dissipation of energy, from pile to its adjacent soil in the active dynamic case and the transfer of stress, energy input, from neighbouring soil to pile in the passive dynamic case has been treated in the same way. The passive dynamic loading case is the interaction of three phenomena : (i) free field incoming wave, (ii) stress transfer from soil to pile and (iii) vibration of pile because of the soil reaction and superstructure. In the earthquake type loading there is a need to differentiate between the near and far field soil behaviour. The soil surrounding the pile experiences a highly in-elastic interaction between pile and soil, Bear et al. (1979, 1980) has modeled the soil by two sets of inelastic springs and dashpots. Near field soils are taken to be those whose properties and response are directly influenced by the motion of the pile, whereas, the far field

soil are influenced to a small degree by the motion of the pile. The time domain analysis, based on this model, has been successfully attempted by many authors. The real nature of this problem has been pointed out in a better way by Luco et al. (1986). They have evaluated the foundation input motion from the far field ground motion within the region to be occupied by the foundation.

Kagwa et al. (1981) describes a nonlinear numerical method, based on beam on Winkler's foundation approach for soil pile structure interaction in saturated sand. The method consists of two steps : (i) nonlinear free field analysis and (ii) nonlinear soil pile structure analysis. Nonlinear stress strain relations were approximated by the hyperbolic model. With the computed free field soil motion, a nonlinear soil pile structure analysis was carried out in time domain.

### Experimental Study

Gaul (1958) reported for the first time the test on a dimensionally scaled model of a vertical pile under dynamic loads. Hayashi and Nobou (1962) conducted tests on H-piles embedded vertically in sand and subjected to forced vibration by vibrating generator installed at the pile head. Apart from the general findings they observed that the :

1. Natural frequency and resonance curve of single vertical piles could be calculated by conceiving a simplified system of vibration.
2. Damping coefficient measured in the free vibration tests depend on the relative density of the subgrade and the length of free part of piles.

Chandrasekaran et al. (1973) presented full scale field tests results of experiments conducted on (Franki) load bearing pile and (Simplex) friction piles. Agarwal (1973) presented experimental tests results conducted on a small dimension instrumented pile as a cantilever beam. Discrete element method was used to solve the fourth order differential equation in a finite difference form to compare with experimental results. Novak et al. (1976) conducted some dynamic experiments with small pile foundations in the field. These experimental results were compared with theoretical predictions made on the basis of a theory described by Novak (1974).

Scott (1982), Alpan (1973), Gle (1984), Woods (1984) have reported experimental studies. Finn and Gohl (1987) has discussed the study in Centrifuge.

### Role of Geomechanics Modeling

The dynamic response of the pile soil system is a function of the shape, periodicity, frequency of the loading and a function of the mass, damping and stiffness (Veletos, 1977) of the system. Soil, through its stiffness, strength and energy dissipation characteristics highly influences the dynamic response. Rate of loading, number of cycles of loading, initial stress and stress reversal effects, strain rate are among the most important parameters which governs the soil stiffness and its strength (Sangrey et al. 1974). Laboratory controlled tests on clays (Hardin, 1978) have clearly indicated that stress history is also another equally important parameter. If the cyclic shear strain level is less than its endurance strength the soil will try to achieve a elastic non failure equilibrium condition. But, if it exceeds the endurance strength, the soil stiffness deteriorates. The frequency domain approach can not take into account of these non linear phenomena. The essential parameters, which represent these non linearities of soil at the pile soil interface, could be evaluated only from experimental data, either in the

laboratory or in the field. The properties of the earth media determine to a large extent the response of seismically forced pile structure. Unfortunately, the current state of the art does not allow for an accurate representation of soil behaviour during seismic events. The present state of the development in experiments is such that all soil samples are disturbed. A proper judgement is very essential before incorporating the necessary soil data in the numerical analysis. Drumm et al. (1986) have presented a methodology to represent the cyclic shear stress deformation response of dry sand concrete interface by using a modified Ramberg Osgood model which permits representation of stiffening as well as degradation behaviour. Interpretation and evaluation of parameters from the cyclic response of laboratory controlled test data have been presented in detail. Very little research has been performed on the effect of constitutive models on the predicted dynamic response of embedded structures.

Underestimation or overestimation of soil modulus, both, may turn grave implications with regard to cost and safety of the pile supported structures.

Summarizing this SOA it may be concluded that in depth study is needed to merge dynamic response theory with dynamic lateral load test results on pile. This liason could be brought by developing efficient numerical schemes, proper geodynamic modelling of soil and sophisticated lateral dynamic test on pile. These much needed developments will offer confidence to the engineer to use analytical models employing nonlinear hysteretic soil reaction characteristics.

## EXPERIMENTAL STUDY

A general view of the experimental set-up is shown in Fig. 1.

The apparatus consists of steel tank (2 x 1 x 0.15 m) resting on four frictionless steel wheels on smooth steel plates. Subrounded uniform Solani sand got deposited, in layers, inside the tank by slow rain method, with a free fall of 30 cm. As the tank got filled up with sand, to the desired height, Aluminium rod (Table 1) was placed vertically. Filling was then continued till the mid depth of the embedment of the rod. Two acceleration pickups, one for sensing vertical vibration another for horizontal, were placed closed to the rod. Rest of the filling was then completed. This stepwise filling was undertaken to avoid driving of the pile and to ensure the pile tip not in touch with the inside base of the tank, thus ascertaining a free tip condition. Two pickups, similar combination as at bottom, were attached at the pile top. These four pickups were connected to pen recorder through amplifier for data acquisition. The top surface of the sand bed was levelled. The tank was then subjected to vibration with the Motor Oscillator assembly, attached at the bottom outside of the base, at the resonant frequency of the sand bed for sufficient duration to achieve maximum density (Table 1) of the deposit. A small Motor Oscillator assembly, hung from a tripod at the level of the pile top, was connected with it.

Passive periodic loading case was simulated by exciting the base of the tank with large oscillator and keeping the top oscillator at rest. For active periodic case the top small oscillator was excited when the bottom large oscillator was kept free (Fig. 2). For creating passive impact loading case the side of the tank was struck by a steel pendulum hammer on wooden plank fixed on the outside of its vertical wall and for active dynamic loading, only the pile top was struck by a small weight wooden pendulum bob, hung from the tripod. The energy input, for both these impact cases, were controlled by the swing of the pendulum.

**TABLE 1**  
**MATERIAL PROPERTIES**

Soil	Pile
<u>Type : Poorly graded sand</u>	<u>Material : Aluminium</u>
Effective size D <sub>10</sub> = .15 mm	Diameter = 1.90 cm
Uniformity coefficient = 1.67	Weight = .927 kg
Specific gravity = 2.6	Length = 119 cm
Max. density = 1.81 kg/cm <sup>3</sup>	Modulus of elasticity = $7 \times 10^{10}$ N/m <sup>2</sup>
Min. density = 1.38 kg/cm <sup>3</sup>	Weight density = 2.81 t/m <sup>3</sup>
Relative density = 0.51	Wt. of oscillator + Motor = 9.3 kg
	Wt. of pickup and accessories = 285 gm

Chosen parameters for this experimental studies were :

- (i) Eccentricity of the mass of the oscillator
- (ii) Rotational speed of the oscillator
- (iii) Rotational amplitude of the pendulum
- (iv) Depth of embedment of the model pile

As the total length of the pile kept same, increase in the depth of embedment reduced the free length of the pile above sand bed.

The aim of this experimental investigation was to compare the active and passive dynamic case on :

- (i) Location of the resonant frequency of pile soil system.
- (ii) Soil properties evaluated through a numerical scheme.

#### Passive Dynamic Loading Case

In this case excitation to the tank was given by bottom motor oscillator assembly. In the Fig. 3, amplitude of vibration for curve 1 ( $e = 36$ ) increased from 0.15 mm at 140 rpm (2.33 cps) to 0.56 mm at 180 rpm (3.0 cps) giving the resonant peak. Then with the increase in operating frequency the amplitude decreases to .06 mm at 340 rpm (5.67 cps). For curve 2 ( $e = 32$ ) the amplitude increased from 0.08 mm at 136 rpm (2.26 cps) to 0.49 mm at the resonant frequency of 176 rpm (2.93 cps).

Fig. 4 shows the same trend of frequency response curve for the 40 cm depth of embedment of pile. For smaller depth of embedment resonant frequency occurred at 150 rpm with amplitude of vibration .54 mm for  $e = 32$  and for  $e = 36$  the resonant frequency and amplitude of vibration were 160 rpm and 0.59 mm respectively. Comparing Fig. 3 and Fig. 4, it is clear that with the decrease in depth of embedment, the amplitude at resonant frequency increased. Also the resonant frequency had a clear demarcation in two different cases of depth of embedments. The decrease in depth of embedment showed a decrease in resonant frequency at same level of

of force of excitation. The resonant frequencies for  $e = 36$  were 180 rpm and 160 rpm for depth of embedment 51 cm and 40 cm respectively. Similarly for  $e = 32$  the resonant frequencies were 176 rpm and 150 rpm respectively.

#### Active Dynamic Loading Case

The response curve for this case are plotted in Fig. 5. In this case the excitation was given by small motor oscillator assembly attached to the top of pile. This figure shows a set of response curves for the depth of embedment 40 cm and 51 cm. For the depth of embedment of 51 cm the amplitude of vibration increased from 2.17 mm at 2.14 cps to 11.89 mm at 2.27 cps where it found its resonant frequency. With further increase in frequency, amplitude decreased. Similar trend was observed for the depth of embedment 40 cm and it located its resonant frequency at 2.2 cps. Thus with increase in depth of embedment the resonant frequency and amplitude of vibration decreased.

Fig. 6 compares the pile response under active and passive dynamic loading cases. A typical example for depth of embedment 51 cm has been compared here. In the passive dynamic case the resonant frequency was at 3 cps and in active dynamic case, it decreased to 2.27 cps. Similar trend has been observed for lesser depth of embedment, i.e. 40 cm, it was 150 rpm and 132 rpm for passive and active dynamic cases respectively. In these two cases all other variables e.g. pile material, pile diameter and soil were same, only the mode of force of excitation was different. In the active dynamic case the excitation was applied at the pile top and wave, travelling downward, excited the soil grain but in the passive dynamic case the vibration of the tank excited the soil grain and the soil grain, in turn, transferred the excitation to pile. Thus the stress transfer mechanism are different for these two cases. In the passive case soil grains excite the pile, but in active case the pile excites the soil grains. The observation establishes that the stiffness modulus of soil in passive dynamic case is more than that in active dynamic case.

TABLE 2

	Embedment = 51cm		Embedment = 40 cm	
	Active	Passive	Active	Passive
Natural Freq. (CPS)	2.55	2.51	2.27	2.25
Max. Pile Acc. mm/sec <sup>2</sup>				
Top	594.5	495.4	1090	508.6
Soil	363.3	18.17	754.6	23.6
Ratio	1.64	27.26	1.44	21.55

In the Fig. 7, comparison between active and passive dynamic loading cases for depth of embedment 51 cm and 40 cm is presented. In impact case natural frequencies obtained from passive case (2.55 cps) is higher than the obtained in active case (2.51 cps). Natural frequency is higher for higher depth of embedment. The ratio of acceleration on pile top to the soil (at mid depth of embedment) are 1.6 and 27.26 for passive and active dynamic cases respectively. This may be concluded from this experimental investigation that behaviour of soil under active and passive dynamic loading are different and there is a need to evaluate the dynamic shear modulus of soil for these two cases separately.

## NUMERICAL MODELLING

Design practice, in the past, has frequently made use of empirical information for response analysis of pile under lateral dynamic loading. In recent years two theoretical approaches have been advanced :

1. Elastic approach where soil is considered to be a linear, isotropic, elastic continuum.
2. Subgrade reaction approach where the soil is characterised as series of discrete unconnected spring which ignores the continuous nature of the soil.

The use of elastic continuum approach model, for analysis, is more satisfactory but complicated. It takes into account of radiation damping of waves from pile to soil and it considers the interaction of a point with any other point in the pile soil medium. These are not considered in the subgrade reaction approach. Despite several limitations of the subgrade reaction model its simplicity and its merit to consider the nonlinearity of soil behavior in an easy and in a tractable way have made this Winkler's approach popular.

Winkler's soil model considers that the soil deformation  $u$  (cm) and its resistance  $p$  ( $\text{kg}/\text{cm}^2$ ) can be related as :

$$P = k_h \cdot u \quad (1)$$

where  $k_h$  = modulus of subgrade reaction ( $\text{kg}/\text{cm}^3$ )

Considering the pile as a thin strip, the governing beam equation of small elemental length  $dz$  at a depth  $z$  from the top can be written as :

$$E_p I_p \frac{d^4 u}{dz^4} = -p \cdot d \quad (2)$$

where

$$\begin{aligned} E_p &= \text{modulus of elasticity of pile } (\text{kg}/\text{cm}^2) \\ I_p &= \text{moment of inertia of pile section } (\text{cm}^4) \\ d &= \text{diameter or width, perpendicular to bending plane of pile } (\text{cm}). \end{aligned}$$

Considering the effect of axial load equation 2 modifies to

$$E_p I_p \frac{d^4 u}{dz^4} + \frac{d}{dz} (P(z) \frac{du}{dz}) + k_h \cdot du = 0 \quad (3)$$

where  $P(z)$  = axial load (kg) on elemental length  $dz$ .

Equation (3) implicitly assumes that in spite of the deformation of pile under vertical and lateral load at any depth  $z$  the contribution of axial load  $P(z)$  may be considered vertical on any elemental length  $dz$ .

From D'Alembert's principle the governing differential equation (3) for static case could be extended to dynamic case by including the following dynamic effects for any  $i$ th element (Poulos, 1980).



$$H_{ai} = \left(\frac{W_i}{g}\right) \left(\frac{d^2 u_i}{dt^2}\right) \quad (4)$$

$$H_{vi} = J_i \left(\frac{du_i}{dt}\right) \quad (5)$$

where

- $H_{ai}$  = inertial force  
 $H_{vi}$  = damping force  
 $W_i$  = weight of pile element  
 $J_i$  = damping factor at  $i$

Expressing the acceleration  $d^2u/dt^2$  and the velocity  $du/dt$  in terms of higher order backward differences, equations (4) and (5) for a time  $t$  and time increment  $\delta t$  becomes

$$H_{ai, t} = \frac{W_i}{g(\delta t)^2} (2u_{i,t} - 5u_{i,t-1} + 4u_{i,t-2} - u_{i,t-3}) \quad (6)$$

$$H_{vi, t} = \frac{J_i}{6(\delta t)} (11u_{i,t} - 18u_{i,t-1} + 9u_{i,t-2} - 2u_{i,t-3}) \quad (7)$$

It is considered that at any time  $t$ , the value of the dependent variable at previous time instants  $t-1$ ,  $t-2$  and  $t-3$  are known. The finite difference expression over space in equation(2) and over time in equation (6) and (7) leads to,

$$A_i u_{i-2,t} + B_i u_{i-1,t} + C_i u_{i,t} + D_i u_{i+1,t} + E_i u_{i+2,t} = G_i \quad (8)$$

where

$$A_i = F_{i-1} - \frac{\delta}{4} (R_{i-1,t} + P_{i-1,t})$$

$$B_i = -2(F_{i-1} + F_i)$$

$$C_i = F_{i-1} + 4F_i + F_{i+1} + \delta^3 S_{i,t} + \frac{\delta}{4} (R_{i-1,t} + \delta P_{i-1,t}) \\ + \frac{\delta}{4} (R_{i+1,t} + \delta P_{i+1,t}) + \delta^3 (2W_i/(\Delta t)^2 + \frac{J_i}{6\Delta t})$$

$$D_i = -2(F_i + F_{i+1})$$

$$E_i = F_{i+1} - \frac{\delta}{4} (R_{i+1,t} + \delta P_{i+1,t})$$

$$G_i = \delta^3 H_{i,t} + W_i (5 u_{i,t-1} - 4 u_{i,t-2} + u_{i,t-3}) \frac{\delta^3}{(\Delta t)^2}$$

$$+ \frac{J_i \delta^3}{6 \Delta t} (18 u_{i,t-1} - 9 u_{i,t-2} + 2 u_{i,t-3})$$

$$F_i = E_p I_p \text{ at } i$$

$$P_i = \text{axial load at } i$$

$$R_i = \text{Rotational restraint at } i$$

$$S_i = \text{Spring factor for lateral soil resistance at } i = k_{hi} \delta$$

$$H_i = \text{Applied lateral load at } i$$

$$\delta = \text{Distance between adjacent node points}$$

Equation (8) when applied to all interior node points of the pile along with top and tip boundary conditions offers an  $(n \times n)$  matrix (banded) of the form ;

$$A U = Q \quad (9)$$

which when solved for  $U$  offers the time response behavior of pile.

When the time domain of interest  $T$  is discretised into  $m$  small time interval  $t_1, t_2, \dots, t_m$  the above scheme (9) demands a matrix inversion  $A^{-1}$  at each time step. Though it solves the exact dynamic equation at each instant, repeated matrix inversion makes it extremely slow. A little sacrifice over the exact nature of the problem can make the scheme much faster. In this paper this compromise has been considered. Equation (3), (4) and (5) have been discretised into a set of a few first order partial differential equations :

$$\text{Slope } \phi = du/dz \quad (10)$$

$$\text{Moment, } M = EI d\phi/dz \quad (11)$$

$$\text{Shear force, } V = dM/dz \quad (12)$$

$$\text{Velocity, } v = du/dt \quad (13)$$

The shear force equilibrium equation :

$$\frac{dV}{dz} + R \cdot u + \frac{d}{dz} (P(z) \phi) + m \frac{dv}{dt} = 0 \quad (14)$$

where  $R$  = soil resistance, over the elemental length of pile;  $m$  = mass of pile element

The total space time frame has been discretised into  $n$  elements over space and  $m$  elements over time.

Equation (10), (11) and (12) are integrated over space while equation (13) and (14) are integrated over time. The solution starts with initial at rest condition for all pile elements. The top and tip boundary conditions are satisfied at each step of marching over time. A close look at the set of equation (9) reveals that each pivotal point of the pile elements gets the contribution of response from two nodes above and two nodes below it. The nature of this band width is also maintained in the system of equation (10) through (12). While evaluating the acceleration  $dv/dt$  from equation (14) it assumes the soil resistance  $R_{i,t-1}$  remains constant during the period  $t-1$  to  $t$  and it satisfies the equilibrium equation at  $t$ th instant with  $R_{i,t-1}$  which was developed  $t-1$ . The scheme (9) does not propagate this inaccuracy. To reduce this inaccuracy authors have modified the scheme (14) by an iterative under relaxation scheme for each time jump. It has been found with under relaxation factor .5 the scheme offers convergence with 5% relative error within 3-4 cycles.

Soil resistance  $R$  plays the most crucial role in this non linear analysis. It has been assumed that

$$R = R_{EP} + R_V \quad (15)$$

where

$$\begin{aligned} R_{EP} &= \text{Elasto-plastic resistance of soil element} \\ R_V &= \text{Viscous part of the soil resistance} \\ R_{EP} &= f(u) \cdot K_H \\ R_V &= q S_j \cdot v \end{aligned}$$

where

$$\begin{aligned} f(u) &= u \neq u < q \\ f(u) &= q \neq u \geq q \end{aligned} \quad (16)$$

where

$$\begin{aligned} q &= \text{the maximum possible elastic deformation of soil beyond which soil element starts yielding plastically} \\ S_j &= \text{Soil damping constant of an element} \end{aligned}$$

Fig. 8 shows the loading path OAB and the unloading path BSDE as expressed by Eq. (15) and (16).

A computer programme was developed with the above mentioned formulation expressed by Eqn. (10) through (16).

#### Estimation of Error

To find the accuracy of the numerical method the response of a free free beam (length 100 cm, diameter = 2 cm, Unit weight = .0076 kg/cm<sup>3</sup> and Young modulus =  $2 \times 10^6$  kg/cm<sup>2</sup>) was found out through this scheme. Fig. 9 shows the time displacement behavior under impact loading and Fig. 10 shows frequency-amplitude response under forced vibration. Both of these cases estimate the natural frequency

of vibration as 40 rad/sec. The natural frequency of vibration of this problem from closed form solution, is 35 rad/sec which is very close to the numerical prediction.

### Analysis of the Experimental Results

The same numerical scheme was used for the estimation of dynamic stiffness modulus of the sand bed, by trial method. Fig. 11 shows the time displacement behavior under impact case and Fig. 12 shows the frequency Vs amplitude behavior of the pile top in the periodic case respectively where the stiffness modulus of the soil was selected as a triangular distribution with 0 at top and .50 kg/cm<sup>3</sup> at tip of the pile. The motion was imparted on the pile top and the prediction of the numerical scheme is closed to the observed behavior of pile under active dynamic loading. The same matching could be obtained for passive dynamic case only when the tip soil is .7 kg/cm<sup>3</sup>.

### CONCLUSION

In this paper authors have tried to establish the fact that response of pile under active and passive dynamic cases are different. Experimental study and its interpretation through a numerical scheme, developed in this paper, shows that in passive dynamic load soil stiffness modulus is higher than that of active dynamic loading case. Authors expect that full scale field experiment, where the depth of embedment is large, significant change in the observed behaviour may take place which may establish the fact more clearly.

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- 1 MODEL PILE
- 2 REACTION TO PILE RESPONSE
- 3 TOP PICKUPS
- 4 MOTOR
- 5 OSCILLATOR
- 6 SAND
- 7 TANK
- 8 SEALING AIR TANK
- 9 INSULATED PIT
- 10 MOTOR-OSCILLATOR

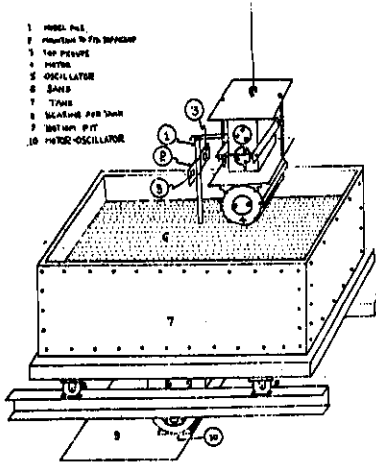


Fig. 1. A General View of the Experimental Apparatus

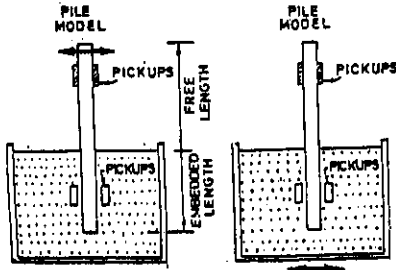


Fig. 2 Active Case Passive Case

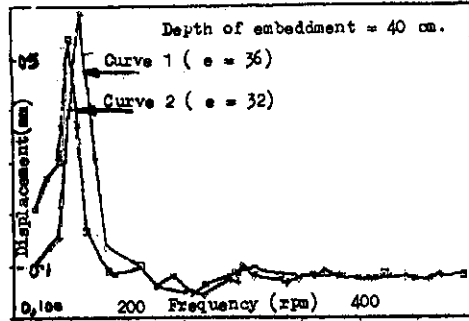


Fig. 4 Frequency Response Curve for Passive Loading

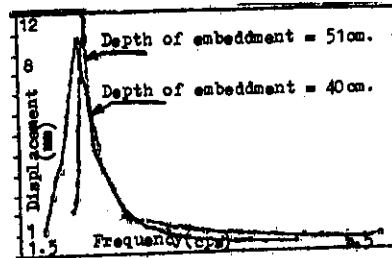


Fig. 5 Frequency Response Curve for Active Dynamic Loading

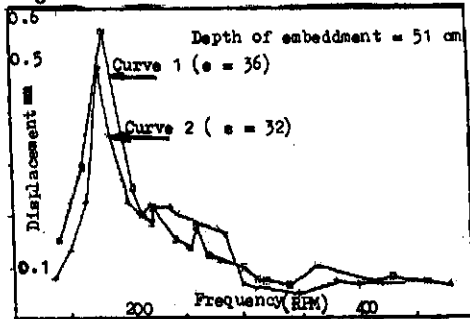


Fig. 3 Frequency Response Curve for Passive Dynamic Loading



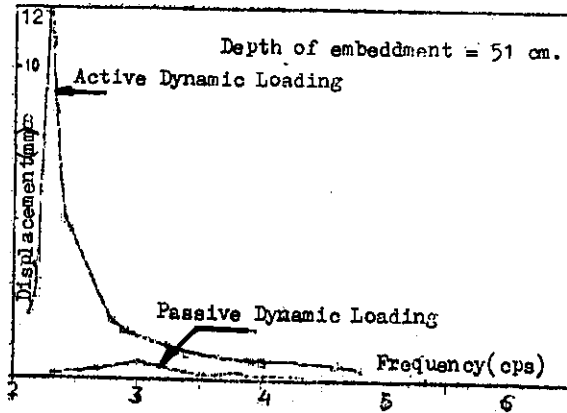


Fig. 6 Comparison of Active and Passive dynamic Loading Cases

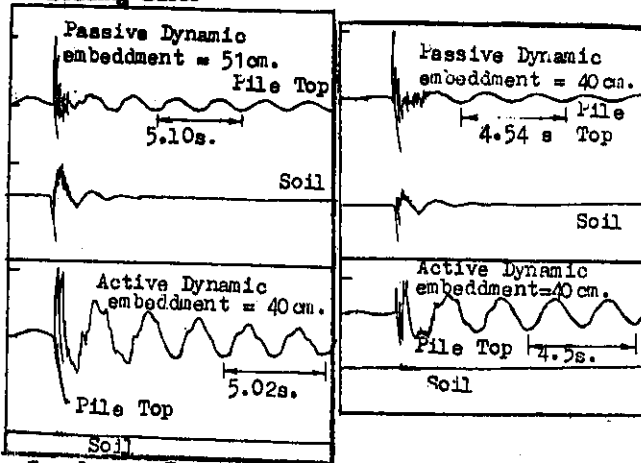


Fig. 7 Impact Cases

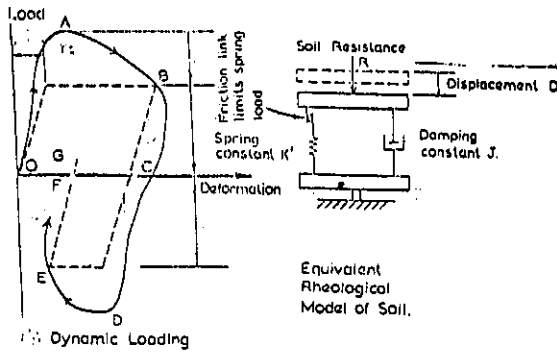


Fig.8 Soil Rheological Model