SEISMIC RESPONSE OF BRIDGES WITH SLIDING ISOLATION DEVICES

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ABSTRACT

Seismic response of bridges with sliding isolation system between superstructure and substructure is investigated. Frictional force of the isolation system is assumed to have ideal Coulomb-friction characteristics. In addition, a linear restoring force is also provided by the isolation system. The governing equations of motion of the isolated bridge system are derived. Seismic response of the isolated bridge system in both longitudinal and transverse directions is obtained by solving the non-linear equations of motion (non-linearity due to sliding system) in the incremental form using Newmark's method. The system is subjected to real earthquake ground motion (i.e. El-Centro 1940) in both horizontal directions. In order to study the effectiveness of the isolation devices, the seismic response of the isolated bridge is compared with the corresponding response of non-isolated bridge (i.e. bridge without isolation devices). In addition, a parametric study is conducted to investigate the effectiveness of sliding system for aseismic design of the bridges with different combinations of the superstructures and the piers. It is shown that the sliding system is quite effective in reducing the seismic response of bridges.

KEY WORDS

Sliding system, earthquake, bridge, effectiveness, isolation and system parameters.

INTRODUCTION

Seismic design of bridges draws great significance since bridges come under the category of *lifeline structures*. Disruption in the transportation network due to partial or total collapse of bridges after a major earthquake would seriously hamper the relief and rehabilitation operations. The traditional method of providing earthquake resistance in a structure is by increasing the strength as well as energy absorbing capacity (ductility) of the structural members. The development of ductility implies some damage; therefore, the current design method can be called fail-safe approach, which would prevent collapse but not cause damage. An alternative method is to isolate the structure

by the use of isolators. Seismic isolation is a strategy that attempts to reduce the seismic forces to or near the elastic capacity of a member, thereby, eliminating or reducing the inelastic deformations. The main concept in isolation is to reduce the fundamental frequency of structural vibration to a value lower than the predominant energy-containing frequencies of earthquakes. The other purpose of an isolation system is to provide means of energy dissipation, which dissipates the seismic energy transmitted to the system. Accordingly, by using an isolation device the structural system is essentially uncoupled from the earthquake ground motions.

A variety of isolation devices including laminated rubber bearings (LRB), frictional or sliding bearings and roller bearings have been developed. Among the various isolation systems, the sliding isolation systems are more common. Some of the commonly proposed sliding isolation systems include: the pure-friction (P-F) devices (Mostaghel and Tanbakuchi 1983), the resilient-friction base isolator (R-FBI) system (Mostaghel and Khodaverdian 1987), Alexisismon isolation system (Ikonomou 1984), the friction pendulum system (Zayas et al. 1990), and elliptical rolling rods (Jangid and Londhe 1998). The sliding systems perform very well under a variety of severe earthquake loading and are very effective in reducing the large levels of the superstructure's acceleration. These isolators are characterised by insensitivity to frequency content of earthquake excitation. This is due to tendency of sliding system to reduce and spread the earthquake energy over a wide range of frequencies. The above systems are extensively used for the seismic isolation of buildings (Buckle and Mayes 1990; Jangid and Datta 1995). There are few applications of these isolation devices for aseismic design of bridges as well (Bessason 1993; Jangid and Banerji 1995; Kartoum et al. 1992, Tandon 1996; Tsopelas et al. 1996; Wei et al. 1992). Bessason (1993) and Wei et al. (1992) also investigated effectiveness of lead rubber bearings for aseismic design of bridges. Jangid and Banerji (1995) found that pure-friction devices installed between superstructure and substructure of a bridge could effectively control the earthquake response of the system. Kartoum et al. (1992) and Tsopelas et al. (1996) conducted experimental studies on seismic response of bridges using sliding system and found that such devices are quite effective for seismic isolation of bridges. Although the above studies confirm the suitability of the isolation system for the bridges, there is still a need for understanding the dynamic behaviour of the seismically isolated bridge system under parametric variations.

In this paper, earthquake response of bridges isolated by the sliding system in longitudinal and transverse directions is investigated. In order to investigate the effectiveness of sliding system, the seismic response of the isolated bridge system is compared with corresponding bridge without an isolation system. In addition, effects of type of the superstructure and the pier of the bridge system on the effectiveness of the sliding system are also investigated.

STRUCTURAL MODEL

Fig. 1 shows the model of a three-span continuous bridge seismically isolated by providing the sliding isolation system between the superstructure and pier. The sliding system considered is similar to the R-FBI system (Mostaghel and Khodaverdian 1987) in which the isolation effects are provided by parallel action of friction and restoring force. Friction in the isolation system ensures limited transmission of earthquake forces from piers to the superstructure and dissipates seismic energy over the wide frequency range. On the other hand, the restoring force in the isolation system reduces the isolator displacement and brings back the structure to its original position. Various assumptions made for the structural system under consideration are as follows :

1. Bridge superstructure and piers are assumed to remain in the elastic state during the earthquake excitation. This is a reasonable assumption as the isolation attempts to reduce the earthquake response in such a way that the structure remains within the elastic range.



Fig. 1. (a) Three-span continuous deck isolated bridge



Fig. 1. (b) Mathematical idealization



Fig. 1. (c) Schematic diagram of the sliding system.

- 2. The bridge superstructure is resting over the sliding systems provided both at the pier and abutment locations. The abutments are modeled as a rigid body fixed to the ground (Kartoum et al. 1992).
- 3. Stiffness contribution of non-structural elements (i.e. kerbs, parapet walls and wearing coat) is neglected. However, their mass, producing inertial forces, is considered in the analysis.
- 4. The sliding isolation system is isotropic, implying that it has the same properties in two orthogonal directions.
- 5. Coefficient of friction of the sliding system remains constant throughout the motion of the structure. Also, coefficient of dynamic friction is taken to be same as coefficient of static friction.
- 6. The restoring force provided by the sliding system is linear (i.e. proportional to relative displacement). In addition, sliding isolation system also provides a viscous damping.
- 7. The interaction between frictional forces of the sliding system in two orthogonal directions is ignored.

Based on the above assumptions, the sliding isolation system is characterized by the parameters, namely: the lateral stiffness (k_b) , the damping constant (c_b) and coefficient of friction (μ) as shown in Fig. 1(c). The viscous damping constant of the sliding system is expressed in terms of the damping ratio as

$$c_{\rm b} = 2\xi_{\rm b}m_{\rm b}\omega_{\rm b} \tag{1}$$

where $\xi_{\rm b}$ is the damping ratio of the sliding system; $m_{\rm b}$ is the corresponding mass of the bridge superstructure transmitted to the sliding system; $\omega_{\rm b} = 2\pi/T_{\rm b}$ is the isolation frequency; and $T_{\rm b}$ is the period of isolation defined as

$$T_{b} = 2\pi \sqrt{\frac{m_{b}}{k_{b}}}$$
(2)

Note that T_b can be taken as the fundamental time period of the bridge for rigid superstructure and pier conditions. However, the flexibility of superstructure and pier may slight change the fundamental time period of the bridge beyond T_b .

The limiting value of frictional force F_L , of the sliding system at the abutment or pier is expressed by

$$F_{\rm L} = \mu m_{\rm b} g \tag{3}$$

where g is the acceleration due to gravity.

The system is subjected to real earthquake ground motion in two horizontal directions. Both superstructure and substructure are modelled as a lumped mass system and assume to have been divided into number of small discrete elements. Adjacent elements are connected by the node. The mass of each element is assumed to be distributed between the two adjacent nodes in the form of point masses. At each node, two dynamic degrees-of-freedom (DOF) in the horizontal directions are considered as shown in Fig. 1(b).

GOVERNING EQUATIONS OF MOTION

The equations of motion for the bridge system with lumped mass model (refer Fig. 1(b)) are expressed by the following matrix form (Chopra 1996) as

$$[M]{\ddot{z}} + [K]{z} + {F} = -[MIr]{\ddot{z}g}$$
(4)

$$\{z\} = \{x_1, x_2, x_3, \dots, x_n, y_1, y_2, y_3, \dots, y_n\}^{T}$$
(5)

$$\left\{\ddot{\mathbf{z}}_{g}\right\} = \left\{\begin{matrix} \ddot{\mathbf{x}}_{g} \\ \ddot{\mathbf{y}}_{g} \end{matrix}\right\}$$
(6)

where [M], [K] and [C] represents the mass, stiffness and damping matrix, respectively of the bridge structure; $\{ \ddot{z} \}$, $\{ \dot{z} \}$ and $\{ z \}$ represent structural acceleration, structural velocity and structural displacement vectors, respectively; $\{ F \}$ is the frictional force vector; [r] is the matrix of earthquake influence coefficients; $\{ \ddot{z}_g \}$ is the

(3)

earthquake ground acceleration vector; \ddot{x}_g and \ddot{y}_g represents the earthquake ground acceleration in longitudinal and transverse directions of the bridge, respectively; and x_i and y_i are the displacements of the ith node of the bridge system in longitudinal and transverse directions, respectively as shown in Fig. 1(b).

The mass matrix is a diagonal matrix having corresponding lumped masses at various nodes. Stiffness matrix of the bridge superstructure and piers is constructed separately and static condensation is carried out to eliminate the rotational degreesof-freedom. Damping matrix of the bridge system is not explicitly known and it is constructed from assumed modal damping in each mode of vibration using its modeshapes and frequencies.

INCREMENTAL SOLUTION PROCEDURE

The seismic response of the bridge system is obtained by solving the equations of motions in the incremental form. For the present study, the solution of equations of motion is obtained by Newmark's method assuming linear variation of acceleration over a very small time interval, Δt . In the incremental form, the governing equations (i.e. Eq. (4)) can be written as :

$$[M] \{\Delta \ddot{z}\}^{t+\Delta t} + [C] \{\Delta \dot{z}\}^{t+\Delta t} + [K] \{\Delta z\}^{t+\Delta t} + \{\Delta F\}^{t+\Delta t} = -\{\Delta P\}^{t+\Delta t}$$
(7)

where $\{\Delta z\}^{t+\Delta t}$ is the incremental displacement vector from time t to $t + \Delta t$, $\{\Delta P\}^{t+\Delta t}$, is the incremental inertial load induced by the ground motion; and $\{\Delta F\}^{t+\Delta t}$ is the vector of incremental frictional forces generated at the isolator interface.

Based on the assumption of linear variation of acceleration over small time interval (t, the incremental acceleration and velocity vectors are expressed as:

$$\{\Delta \ddot{z}\}^{t+\Delta t} = a_0 \{\Delta z\} + a_1 \{\dot{z}\}^t + a_2 \{\ddot{z}\}^t$$
(8)

$$\left\{\Delta \dot{z}\right\}^{t+\Delta t} = b_0 \left\{\Delta z\right\} + b_1 \left\{\dot{z}\right\}^t + b_2 \left\{\ddot{z}\right\}^t$$
(9)

where $a_0 = 6/\Delta t^2$, $a_1 = -6/\Delta t$, $a_2 = -3$, $b_0 = 3/\Delta t$, $b_1 = -3$ and $b_2 = -\Delta t/2$.

Substituting Eqs. (8) and (9) in Eq. (7), it can be expressed as

$$\left[\hat{K}\right]\left\{\Delta z\right\}^{t+\Delta t} = \left\{\hat{P}\right\}^{t+\Delta t}$$
(10)

and

$$[\hat{K}] = a_0[M] + b_0[C] + [K]$$
(11)

$$\left[\hat{P}\right]^{t+\Delta t} = \left[\Delta P\right]^{t+\Delta t} - \left\{\Delta F\right\}^{t+\Delta t} - \left[M\right]\left(a_{1}\left\{\dot{z}\right\}^{t} + a_{2}\left\{\ddot{z}\right\}^{t}\right) - \left[C\right]\left(b_{1}\left\{\dot{z}\right\}^{t} + b_{2}\left\{\ddot{z}\right\}^{t}\right)$$
(12)

After solving for incremental displacement vector from Eq. (10), the velocity vector at time $t+\Delta t$ is obtained from Eq. (9). The final acceleration vector is obtained from the direct equilibrium of Eq. (4) instead of using the Eq. (8) to avoid the unbalanced forces generated in the numerical integration scheme.

The DOF of superstructure above the sliding system is only considered when the isolation system is in the sliding phase. The sliding in the isolation system takes place as soon as the frictional force exceeds the limiting value expressed by Eq. (3). Transition from sliding to the non-sliding phase takes place whenever the relative velocity of the sliding system becomes zero. The response of the bridge system is found to be sensitive to the starting and ending times of the sliding and non-sliding phases implying that the digitised time interval, Δt should be very small. The $\Delta t = 0.02/1000$ is employed for both sliding and non-sliding phases. Still smaller $\Delta t = 0.02/2000$ has been used in the neighbourhood of transition of phases. Further, the sliding velocity less than 1×10^{-8} m/sec is assumed to be zero for checking the transition from sliding to non-sliding phase.

NUMERICAL STUDY

In the present study, nine different types of three-span continuous deck bridges are considered from *Bridges in Maharashtra* (1997). Dynamic properties of these bridges are given in Table 1. These bridges are derived by the combination of different types of superstructures and piers. The properties of superstructure and piers of the bridge system are given in Table 2 and 3, respectively. A comparison of fundamental time period of isolated (with $T_b = 2.5$ sec) and non-isolated bridges is also shown in Table 1. It is seen from the table that the fundamental time period of isolated bridges is elongated in comparison to the non-isolated bridges. Further, the time period of the isolated bridge is mainly governed by the isolation period of the sliding system and it is very close to it for all bridges. The system is subjected to El-Centro 1940 earthquake ground motion in two horizontal directions. NOOE component of above earthquake excitation is applied in the longitudinal direction of the bridge. The other orthogonal component is subjected in the transverse direction. The comparison of the seismic response of above bridges will provide the effects the flexibility of superstructure and piers on the effectiveness of the sliding system. The responses are obtained under different coefficient of friction (in the range of 0.03 to 0.1) of the sliding system. However, the other parameters of the sliding isolation system (i.e. period of isolation and damping ratio) are held constant throughout and the values taken for these are $T_b = 2.5$ sec and $\xi_b = 0.1$. The damping in the non-isolated bridge is assumed as 5% of the critical damping in all modes of vibration.

Bridge No.	Fundamental of non-isolated	time period l bridge (sec)	Fundamental time period of isolated bridge (sec)		Type of	Type of Super-
	Longitudinal	Transverse	Longitudinal	Transverse	Pier	structure
1	0.442	0.801	2.502	2.503		S-1
2	0.643	1.221	2.505	2.507	P-1	S-2
3	0.779	1.452	2.507	2.508		S-3
4	0.452	0.366	2.503	2.503		S-1
5	0.523	0.638	2.507	2.508	P-2	S-2
6	0.713	0.698	2.510	2.512		S-3
7	0.585	0.991	2.532	2.533		S-1
8	0.816	1.470	2.568	2.570	P-3	S-2
9	0.977	1.754	2.599	2.601		S- 3

Table 1 : Details of the bridges considered in the analysis $(T_{b} = 2.5 \text{ sec}).$

(For details of the superstructure and pier refer Table 2 and 3, respectively)

Type of superstructureS-1S-2S-3Span length (m)2537.550Area of cross-section (m²)4.306.216.88Moment of inertia in transverse16.8125.61687.24			
Type of superstructure	S-1	S-2	S-3
Span length (m)	25	37.5	50
Area of cross-section (m ²)	4.30	6.21	6.88
Moment of inertia in transverse direction (m ⁴)	16.81	25.616	87.24
			↓

Table 2 : Geometric properties of variousbridge superstructures

Table	3	:	Geometric	properties	various	bridge	piers.
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Particulars	P-1	P-2	P-3
Pier height (m)	10	20	30
Shape of the pier	Solid circular	Oblong	Hollow circular
Cross-sectional area of pier (m ²)	1.767	10.645	3.461
Moment of inertia in longitudinal direction (m ⁴)	0.2485	2.701	4.524
Moment of inertia in transverse direction (m ⁴)	0.2485	14.184	4.524

The response quantities of interest are the base shear in the piers and the relative displacement of the isolation devices. The base shear in the pier indicates the forces exerted in the bridge due to earthquake ground motion. On the other hand, the relative displacement of the sliding system is crucial from the design of the isolation system and expansion joints in superstructure. The response of the isolated bridge system is compared to the corresponding response of the bridge without isolation devices (referred as non-isolated bridges). The piers in non-isolated bridges are connected to the superstructure above by a fixed bearing, which allows rotation but no displacement (Kartoum et al. 1992). In order to study the effectiveness of the sliding system, it is convenient to express the response in terms of the response ratio R defined as

$$R = \frac{\text{Peak base shear in the pier of the bridge with sliding system}}{\text{Peak base shear in the pier of the bridge without sliding system}}$$
(13)

The ratio R is a measure of effectiveness of the sliding system and value being less than unity implies that the sliding system is effective in reducing the earthquake response of bridges.

Figs. 2 and 3 show time histories of the isolator displacement and base shear in the pier of the Bridge-2 (i.e. with superstructure S-2 and pier P-1) in the longitudinal and transverse directions, respectively. The responses are plotted for two values of friction coefficient of the sliding system i.e. $\mu = 0.03$ and 0.05. The peak values of corresponding response quantities are expressed in Table 4. It is observed that the base shear of the isolated bridge system in two directions is quite less in comparison to the non-isolated bridge system. There is 49.61 and 29.57 % reduction in the base



Fig. 2. Time histories of isolator displacements and pier base shear in the longitudinal direction of Bridge-2 (i.e. bridge with P-1 and S-2).



Fig. 3. Time histories of isolator displacements and pier base shear in the transverse direction of Bridge-2 (i.e. bridge with P-1 and S-2).

shear of the bridge in longitudinal and transverse direction, respectively for $\mu = 0.03$ indicating the effectiveness of sliding system. Thus, the sliding system can effectively reduce the seismic response of the bridge system.

Direction of bridge \rightarrow	Longitudinal		Transverse	
Response quantity \downarrow	$\mu = 0.03$	μ = 0.05	$\mu = 0.03$	$\mu = 0.05$
Peak isolator displacement (mm)	125.04	85.33	106.98	81.46
Peak base shear of isolated bridge (10 ⁶ N)	2.24	2.25	1.63	1.79
Peak base shear of non-isolated bridge (10 ⁶ N)	4.	46	2	.31
Reduction in the base shear (%)	49.61	49.55	29.57	22.37





Fig. 4. Effects of friction coefficient of the sliding system on variation of the peak isolator displacement and base shear (longitudinal direction).



Fig. 5. Effects of friction coefficient of the sliding system on variation of the peak isolator displacement and base shear (transverse direction).

Fig. 4 shows variation of the response ratio R and peak isolator displacement against the friction coefficient of the sliding system. The responses are shown in the longitudinal direction for different types of bridges with varying superstructures. As the coefficient of friction increases, the response ratio R increases rendering to less effectiveness of the sliding system. Thus, the effectiveness of the sliding system is reduced for higher values of coefficient of friction. On the other hand, the relative sliding displacement is significantly reduced for higher values of coefficient of friction. Thus, the earthquake forces transmitted to the bridge system can be reduced at the expense of increasing relative sliding displacement of the isolation system. However, the sliding displacements have a practical limitation. Therefore, in designing the sliding system a compromise is made between transmitted earthquake forces and the relative displacement at the pier level.

It is also observed from the Fig. 4 that the response ratio R is relatively higher for the bridges having higher non-isolated fundamental time period (refer Table 1) indicating that the sliding system is more effective for the stiff bridges in comparison to the flexible bridges. However, there is not much significant variation in the sliding displacement of bridges. Similar trends of the effectiveness of the sliding system are depicted in Fig. 5 for the corresponding response in the transverse direction. However, it is seen from the figure that for bridges with superstructure S-3 and pier P-1 and P-3, the response ratio R is close to unity for higher values of μ implying no reduction in the earthquake response of the bridge system. This happens because of the fact that these bridges are quite flexible (fundamental time period is of the order of 1.452 and 1.754 sec as shown in Table 1) in the transverse directions. As a result, further elongation of time period of the bridge by the isolation system does not reduce the response of the system. Thus, the sliding system is found to be more effective for stiff bridges in comparison to flexible bridges.

Figs. 6 to 8 show the comparison of the performance of various types of superstructures on the seismic response of the isolated bridge system for the pier type P-1, P-2 and P-3, respectively. The response ratio R in the longitudinal direction is less than unity for all types of superstructure combinations and the friction coefficient of sliding system. There is more reduction in the earthquake response of the pier with superstructure S-1 in comparison to S-2 and S-3. This is again due to fact that this bridge system is relatively stiff as compared to the other bridges. Further, the response ratio R in the transverse direction of all the bridges is relatively higher and at the same time the sliding displacements are low. This is expected due to the fact that all bridges are relatively flexible in the transverse direction in comparison to longitudinal direction (refer Table 1) and as observed the isolation is more effective for the stiff bridges. The other reason may be because of the fact that the N00E component (applied in the longitudinal direction) of the El-Centro earthquake motion is relatively strong as compared to the other orthogonal component. As a result, in the longitudinal direction, the isolator will be more in the sliding phases resulting in more dissipation of seismic energy through friction and thus, increasing the effectiveness of isolation.



Fig. 6. Variation of peak isolator displacement and base shear for bridges with pier type P-1 and combination of different superstructures.



Fig. 7. Variation of peak isolator displacement and base shear for bridges with pier type P-2 and combination of different superstructures.



Fig. 8. Variation of peak isolator displacement and base shear for bridges with pier type P-3 and combination of different superstructures.

CONCLUSIONS

Response of bridges isolated by the sliding system is investigated under real earthquake ground motion (i.e. El Centro 1940). In order to study the effectiveness of the sliding system for aseismic design of bridges, the response of the isolated bridge in both directions are compared with corresponding response of the non-isolated bridge. The effectiveness of the sliding system is investigated for bridges having various types of superstructures and piers under different values of friction coefficients of sliding system. From the trends of results of the present study, following conclusions may be drawn:

- 1. Sliding system can significantly reduce the earthquake forces in the piers of bridges and therefore, these isolators can be conveniently introduced for aseismic design of bridges.
- 2. Increase in the coefficient of friction of the sliding system decreases the relative displacements across isolators but it also increases the base shear in piers making isolation less effective.
- 3. Sliding system is found to be more effective for stiff bridges as compared to the flexible bridges.

REFERENCES

- 1. Buckle, I.G. and Mayes R.L. (1990). "Seismic isolation: history, application and performance- A world overview", Earthquake Spectra, Vol. 6, pp. 161-202.
- 2. Bessason, B. (1993). "Earthquake response of seismically isolated bridges", 2nd World Conf. on Structural Dynamics, Norway, Vol. 1, pp. 253-259.
- 3. "Bridges in Maharashtra" (1997). Publication by Public Works Department, Government of Maharashtra, India.
- 4. Chopra, A.K. (1996). "Dynamics of Structures", Prentice-Hall of India, New Delhi.
- 5. Ikonomou, A.S. (1984), "Alexisismon seismic isolation levels for translational and rotational seismic input", Proc. 8th World Conf. on Earthquake Engineering, Vol. 5, pp. 975-982.
- 6. Jangid, R.S. and Datta, T.K. (1995). "Seismic behavior of base isolated buildings: A state of the art review", Structures and Buildings, ICE, Vol. 110, pp. 186-203.

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- Jangid, R.S. and Banerji, P. (1995). "Seismic response of bridges with non-linear supports", 2nd Int. Conf. on Seismology and Earthquake Engineering, Vol. 1, pp. 556-568.
- 8. Jangid, R.S. and Londhe, Y.B. (1998). "Effectiveness of elliptical rolling rods for base isolation", Journal of Structural Engineering, ASCE, Vol. 124, pp. 469-472.
- 9. Kartoum, A., Constantinou, M.C., and Reinhorn, A.M. (1992). "Sliding system for bridges: Analytical study", Earthquake Spectra, Vol. 8, pp. 345-372.
- Mostaghel, N and Tanbakuchi, J. (1983). "Response of sliding structures to earthquake support motion", Earthquake Engineering and Structural Dynamics, Vol. 11, pp. 729-748.
- Mostaghel, N. and Khodaverdian, M. (1987). "Dynamics of resilient-friction base isolator (R-FBI)", Earthquake Engineering and Structural Dynamics, Vol. 15, pp. 379-390.
- 12. Tandon, M. (1996). "Earthquake resistant bridge construction", Symposium on Earthquake Effects on Structures, Plant and Machinery, Vol. 1, pp. 7.1-21.
- Tsopelas, P., Constantinou M.C., Okamota, S., Fujii, S. and Ozaki, D. (1996). "Experimental study of bridge seismic sliding systems", Engineering Structures, Vol. 18, pp. 301-310.
- Wei, Z., Kai, Q., Weihua, Z. and Zhengxin, F. (1992). "Test and analysis of bridge vibration isolation", 10th World Conference on Earthquake Engineering, Vol. 4, pp. 2237-2240.
- 15. Zayas, V.A., Low, S.S. and Mahin, S.A. (1990). "A simple pendulum technique for achieving seismic isolation", Earthquake Spectra, Vol. 6, pp. 317-333.