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LATERAL PRESSURE IN BINS DUE TO EARTHQUAKE TYPE HORIZONTAL LOADS

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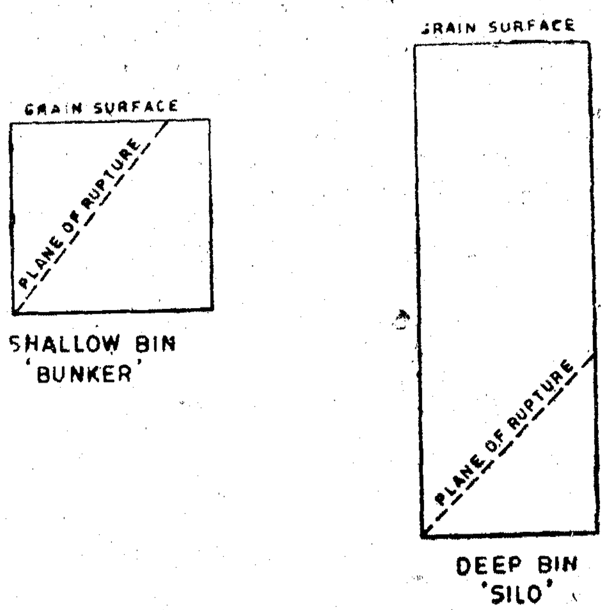
Synopsis

Methods are not available, so far, to determine lateral pressures in bins under dynamic loading conditions. This paper describes an approximate method of determining lateral pressures in bins due to earthquake type horizontal loads. In this method, Airy's theory[1]†, which is for lateral pressures in bins under static conditions, has been modified. Expressions for pressures in shallow bins as well as deep bins have been obtained. Pressures have been determined for various height to diameter ratio of bin and lateral seismic coefficients.

Introduction

Large size containers which are used to store fills like grains, cement, coal etc., are termed as silos and bunkers. Shallow containers, in which the plane of rupture meets the grain surface before it strikes the opposite side of the container, are called bunkers while deep bins are called silos. (Figure 1).



SILOS AND BUNKERS
FIG. 1

Lateral pressures in bins are somewhat akin to those due to fill behind retaining walls but because of the limited extent of the fill in bins, the pressures are actually different. Methods are available for determining the increment in pressures on retaining walls due to earthquake type horizontal loads acting on fills. One such method is that due to Mononobe[2] who had modified Coulomb's wedge theory of earth pressure to consider the effect of earthquake type horizontal load. For design purposes, it is usual to assume the earthquake effects to be represented by an equivalent horizontal static load having a magnitude equal to seismic coefficient times the total weight. This load has been termed here as earthquake type horizontal load. Airy's theory for determination of lateral pressure in bins under static conditions is based on Coulomb's wedge theory. In the method proposed here, Airy's theory has been modified to determine lateral pressure in bins due to eartquake type horizontal load.

Modified Theory

In Airy's theory[1] it is assumed that the pressures on wall are due to a wedge of

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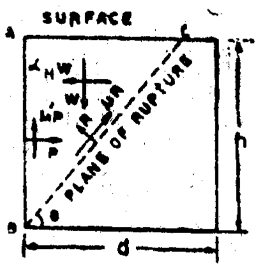
† Refers to serial numbers of references listed at the end of the paper.

material between the wall and the plane of rupture. (Thus, they are due to mass ABC in case of shallow bin and mass ABCD in case of deep bin (Figure 2), where BC is plane of rupture).

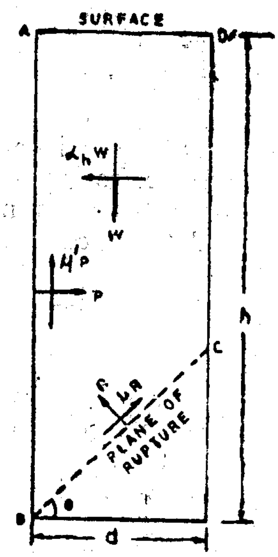
In addition to the above, in the modified theory it is assumed that the earthquake type load acts at the point of concurrence of forces.

The forces acting on the wedge are as shown in figures 2A and 2B. W is the weight of the wedge of material, α_H is the seismic coefficient, α_H W is the equivalent horizontal load, P is the total pressure on one wall and R is the reaction of wedge mass on the plane of separation, h is the height of bin, d is the lateral dimension of the bin, μ is the coefficient of friction of fill on fill, μ' is the coefficient of friction of fill on wall, w is the density of the fill and p denotes pressure at any depth from top.

Force P and R are expressed in terms of W and θ and then θ is found for maximum value of P.



SHALLOW BIN
FIG. 2A



DEEP BIN
FIG. 2B

FORCES FOR ANALYSIS

Shallow Bin (Figure 2A)

Weight of wedge ABC = W = $\frac{1}{2}$ wh² Cot θ

Resolving forces horizontally and vertically and equating them

$$P - R\sin\theta + \mu R\cos\theta - \alpha_H W = 0 \qquad (1)$$

and

$$\mu' P - W + R\cos\theta + \mu R\sin\theta = 0 \qquad (2)$$

Eliminating R between these equations

$$P = \frac{1}{2} wh^2 \frac{\{(\alpha_H - \mu) + \tan\theta(1 + \alpha_H\mu)\}}{\tan\theta(1 - \mu\mu') + \tan^2\theta(\mu + \mu')} \qquad (3)$$

For maximum value of P, $\frac{dP}{d\theta} = 0$

which gives

$$\tan\theta = \frac{\mu - \alpha_H}{1 + \mu\alpha_H} + \frac{\{(\mu + \mu')(\mu - \alpha_H)(1 - \mu'\alpha_H)(1 + \mu^2)\}^{\frac{1}{2}}}{(1 + \mu\alpha_H)(\mu + \mu')} \qquad (4)$$

The maximum value of P for shallow bin is obtained by substituting (4) in (3),

$$P_{sb} = \frac{\frac{1}{2} wh^2 (1 + \mu\alpha_H)^2}{[\{(\mu - \alpha_H)(\mu + \mu')\}^{\frac{1}{2}} + \{(1 + \mu^2)(1 - \mu'\alpha_H)\}^{\frac{1}{2}}]^2} \qquad (5)$$

The pressure p at any depth from top

$$p_{sb} = \frac{dP}{dh} = \frac{wh\,(1+\mu\alpha_H)^2}{[\{(\mu-\alpha_H)(\mu+\mu')\}^{\frac{1}{2}}+\{(1+\mu^2)(1-\mu'\alpha_H)\}^{\frac{1}{2}}]^2} \tag{6}$$

If $\alpha_H = 0$, the pressure obtained is that due to static case, namely,

$$p_{static} = \frac{wh}{[\{\mu(\mu+\mu')\}^{\frac{1}{2}} + (1+\mu^2)^{\frac{1}{2}}]^2} \tag{7}$$

The pressure 'p' varies linearly with height h. The depth upto which the container acts as a shallow bin is equal to d tan θ. Thus, equation 6 is applicable for a height, h_{sb}, equal to

$$h_{sb} = d\left[\frac{\mu-\mu_H}{1+\mu\alpha_H} + \frac{\sqrt{(\mu+\mu')(\mu-\alpha_H)(1+\mu^2)(1-\mu'\alpha_H)}}{(1+\mu\alpha_H)(\mu+\mu')}\right] \tag{8}$$

Deep Bins (Figure 2 B)

Proceeding similarly to the above case,

$$W = \frac{wd}{2}(2h - d\tan\theta) \tag{9}$$

Equating forces and eleminating R,

$$P = \frac{wd}{2}(2h - d\tan\theta)\frac{\{(\alpha_H-\mu) + \tan\theta\,(1+\alpha_H\mu)\}}{(1-\mu\mu') + \tan\theta\,(\mu+\mu')} \tag{10}$$

For maximum value of P,

$$\tan\theta = [-(1-\mu\mu')(1+\mu\alpha_H) + \sqrt{\{(1-\mu\mu')(1+\mu\alpha_H)(1+\mu^2)(1-\mu'\alpha_H)+2h/d\,(1+\mu^2)(1-\mu'\alpha_H)(\mu+\mu')(1+\mu\alpha_H)\}}] \div (\mu+\mu')(1+\mu\alpha_H) \tag{11}$$

Substituting 10 in 11, the maximvm value of P for deep bin is obtained

$$P_{db} = \frac{1}{2}\frac{wd^2}{(\mu+\mu')^2}\left[\left\{\frac{2h}{d}(\mu+\mu') + (1-\mu\mu')(1+\mu\alpha_H)\right\}^{\frac{1}{2}} - \left\{(1+\mu^2)(1-\mu'\alpha_H)\right\}^{\frac{1}{2}}\right]^2 \tag{12}$$

The pressure p at any depth h below top is $p = \frac{dP}{dh}$

$$= \frac{wd(1+\mu\alpha_H)}{(\mu+\mu')}\left[1 - \frac{\{(1+\mu^2)(1-\mu'\alpha_H)\}^{\frac{1}{2}}}{[\{2h/d\,(\mu+\mu') + 1-\mu\mu'\}(1+\mu\alpha_H)]^{\frac{1}{2}}}\right] \tag{13}$$

If $\alpha_H = 0$, the pressure obtained is that due to static case, namely-

$$p_{static} = \frac{wd}{(\mu+\mu')}\left[1 - \frac{(1+\mu^2)^{\frac{1}{2}}}{\{2h/d\,(\mu+\mu') + 1-\mu\mu'\}^{\frac{1}{2}}}\right] \tag{14}$$

Discussion of Results and Conclusions

Expressions obtained by this theory have been used to determine pressures for various height to diameter ratio of bins, seismic coefficients and fills. The height to diameter ratio had various values upto 5·0 so that both shallow and deep bins cases are extensively covered. The depth upto which a bin can be considered as shallow depends upon μ, μ' and α_H and, therefore, on type of fill, the surface of side wall of the bin and seismic coefficient. In general, for a particular μ and μ', as α_H increases the depth as for shallow bin decreases.

Three values of seismic coefficient, α_H, namely, 0.05, 0.10 and 0.20 have been tried. Five sets of values of μ and μ' have been used and they are, (i) 0.466 and 0.444, corresponding to wheat in a concrete bin, (ii) 0.70 and 0.70, corresponding to coal in a concrete bin, (iii) 0.316 and 0.554, corresponding to cement in a concrete bin, (iv) 0.472 and 0.263, corresponding to peas in a steel bin and (v) 0.30 and 0.30, corresponding to an arbitrary condition. (For cases (i) to (iv), values have been taken from references 1 and 3).

Tables 1 to 5 give static pressure as well as ratio of dynamic (seismic) to static pressure for the various conditions. The dotted line indicates the demarcation between shallow and deep bins. For one particular μ and μ', namely for value of 0.466 and 0.444, graphs (figures 3, 4 and 5) have been plotted to indicate variation of pressures with respect to h/d ratios.

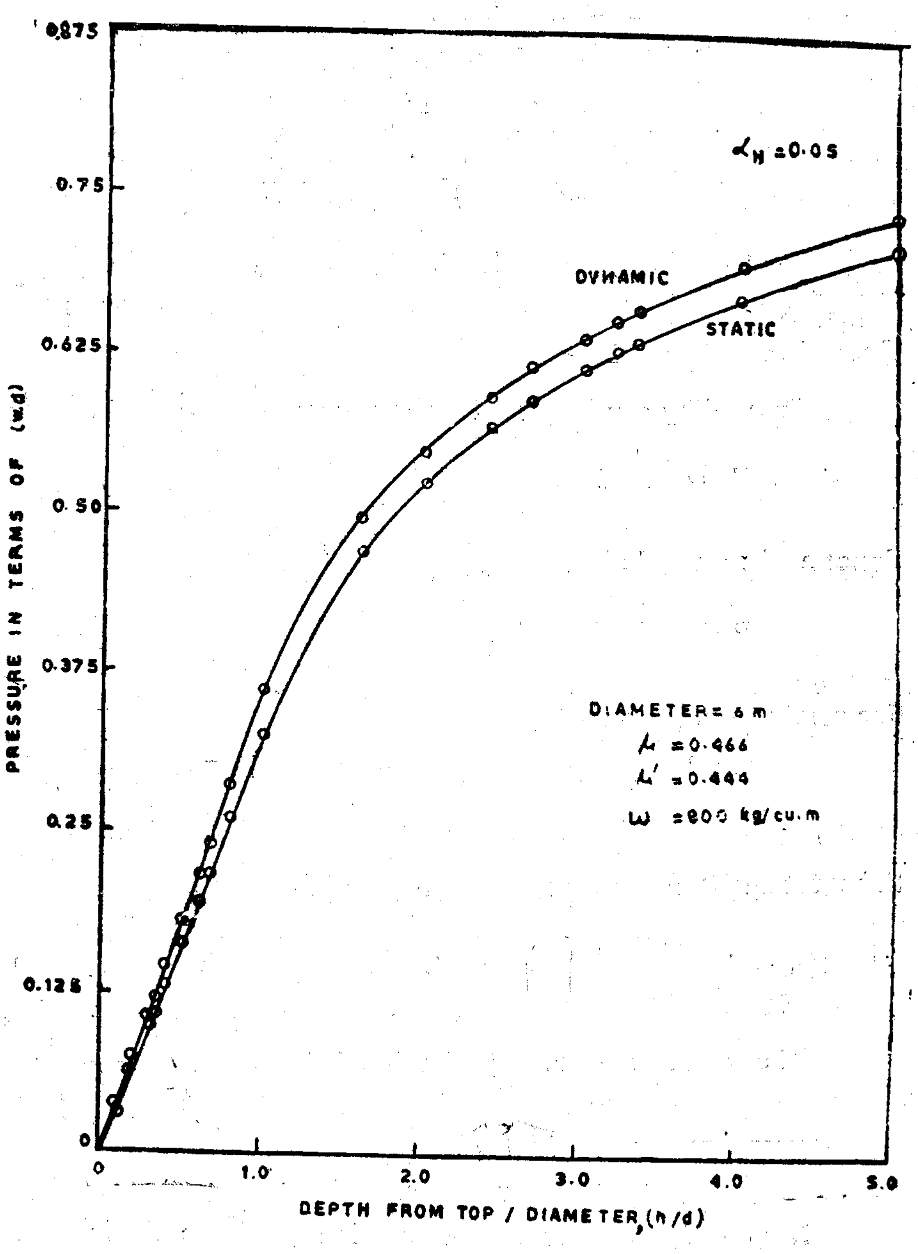


FIGURE 3

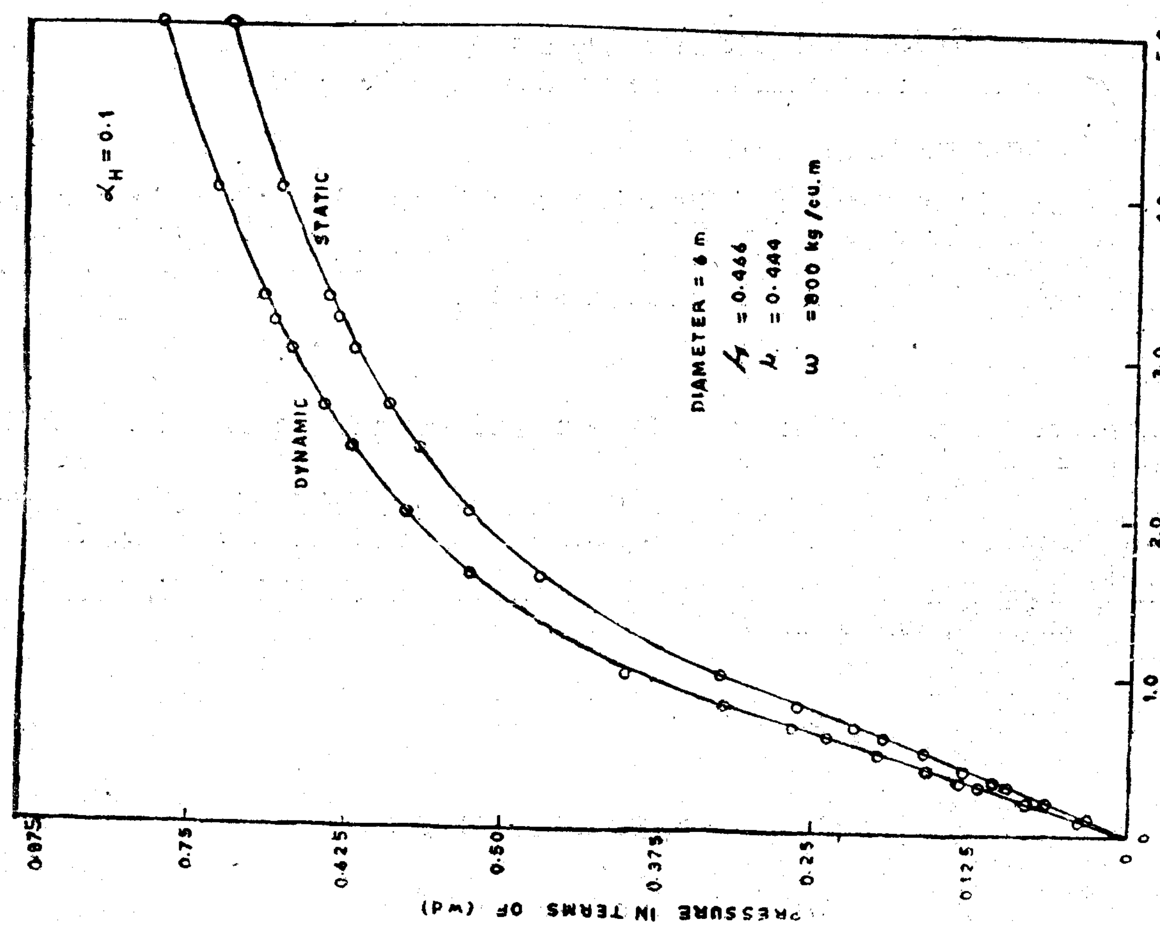


FIGURE 4

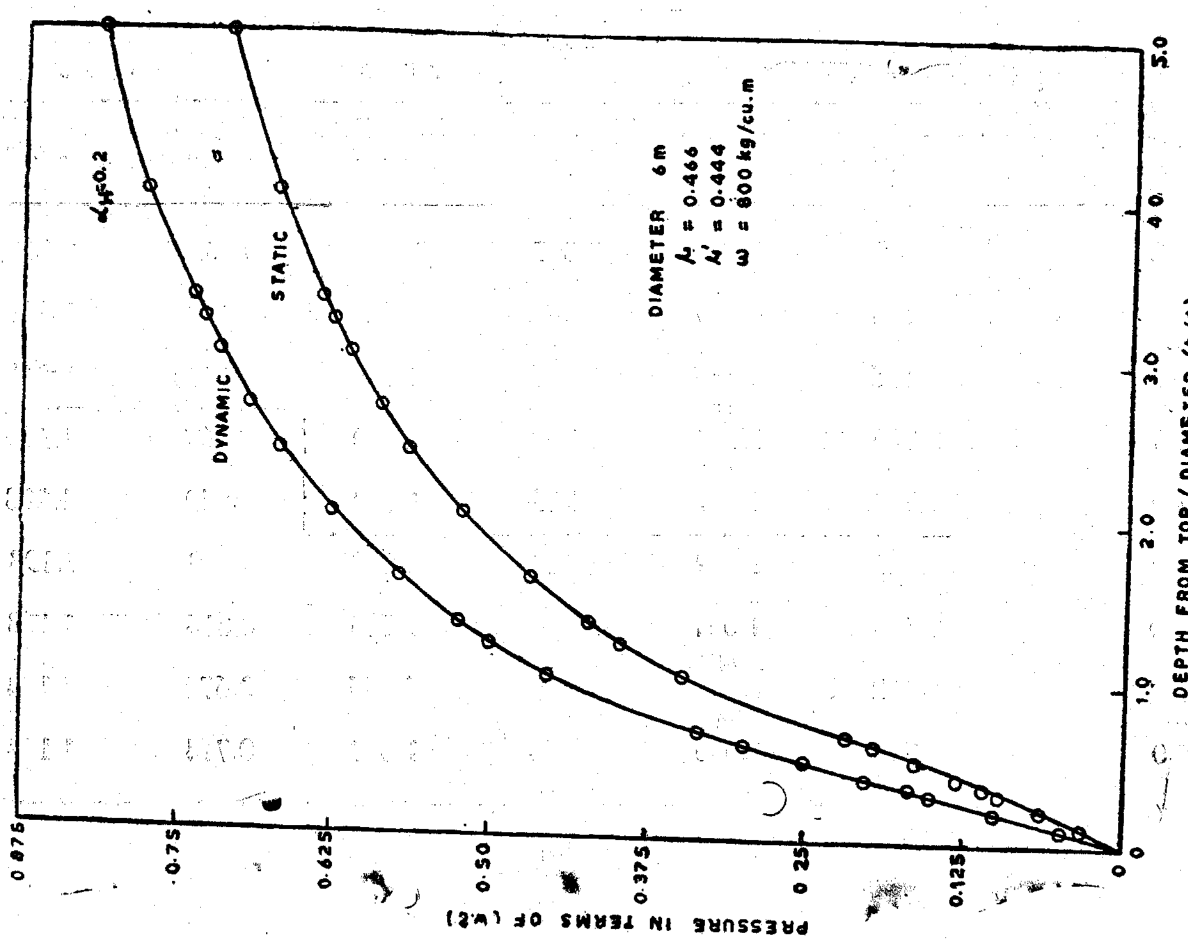


FIGURE 5

It is observed that percentage increase in pressure is more for shallow bins compared to deep bins. Also, for a particular fill in shallow bins, the ratio of dynamic to static pressure is constant. For a seismic coefficient of 0.10 and h/d ratio of 5.0 the maximum value of the ratio of dynamic to static pressure is about 10%.

References

1. Ketchum, M S., "The Design of Walls, Bins and Grain Elevators". McGraw Hill Book Company INC. 1919.
2. Mononobe N, and Matuo H., "On the Determination of Earth-pressures During Earthquakes" World Engineering Congress, Tokyo, 1929, paper No. 388.
3. Jai Krishna and Jain, O.P., "Plain and Reinforced Concrete", Volume II, Nemchand and Brothers, Roorkee, (U.P.), 1966.

TABLE 1 PRESSURES IN BIN

$\mu = 0.466$; $\mu' = 0.444$

Wheat Filled in Concrete Bin

wd = 4800 Kg/m^2

{Depth from top / Diameter} h/d	$\alpha_H = 0.05$		$\alpha_H = 0.1$		$\alpha_H = 0.2$	
	Static pressure in terms of w.d.	Ratio Dyn/ Static	Static pressure in terms of w.d.	Ratio Dyn/ Static	Static pressure in terms of w.d.	Ratio Dyn/ Static
0.2	0.065	1.107	0 065	1.230	0.065	1.541
0.4	0.130	1.107	0.130	1.230	0.130	1.541
0.6	0 195	1.107	0.195	1.230	0.195	1.541
0.8	0.260	1.107	0.260	1.230	0.287	1.350
1.0	0.3 5	1.107	0.325	1.230	0.349	1.298
2.0	0.523	1.049	0.523	1.098	0.523	1.198
3.0	0.614	1.041	0.614	1.083	0.614	1.168
4.0	0.672	1.038	0.672	1.076	0.672	1.154
5.0	0.714	1.036	0.714	1.072	0.714	1.144

TABLE 2 PRESSURES IN BIN

$\mu = 0.7$; $\mu' = 0.7$

Bituminous Coal Filled in Concrete Bin

wd = 4800 Kg/m^3

$\left\{\frac{\text{Depth from top}}{\text{Diameter}}\right\}$ h/d	$a_H = 0.05$		$a_H = 0.1$		$a_H = 0.2$	
	Static pressure in terms of w.d.	Ratio Dyn/ Static	Static pressure in terms of w.d.	Ratio Dyn/ Static	Static pressure in terms of w.d.	Ratio Dyn/ Static
0.2	0.041	1.129	0.041	1.276	0.041	1.639
0.4	0.082	1.129	0.082	1.276	0.082	1.639
0.6	0.123	1.129	0.123	1.276	0.123	1.639
0.8	0.164	1.129	0.164	1.276	0.164	1.639
1.0	0.205	1.129	0.204	1.276	0·204	1.639
2.0	0·351	1.069	0.351	1.141	0.351	1 286
3 0	0.422	1.059	0.422	1.120	0.422	1.244
4.0	0.460	1.055	0.460	1.110	0.460	1.223
5.0	0.485	1.052	0.485	1.104	0.485	1.211

TABLE 3 PRESSURES IN BIN

$\mu = 0.316$; $\mu' = 0.554$

Cement Filled in Concrete Bin

wd = 8650 Kg/m²

{Depth from top / Diameter} h/d	$a_H = 0.05$ Static pressure in terms of w.d.	$a_H = 0.05$ Ratio Dyn/ Static	$a_H = 0.1$ Static pressure in terms of w.d.	$a_H = 0.1$ Ratio Dyn/ Static	$a_H = 0.2$ Static pressure in terms of w.d.	$a_H = 0.2$ Ratio Dyn/ Static
0.2	0.081	1.112	0.081	1.248	0.081	1.638
0.4	0.162	1.112	0.162	1.248	0.162	1.638
0.6	0.263	1.112	0.263	1.248	0.268	1 363
0 8	0.324	1.112	0.340	1.212	0.340	1.280
1.0	0.397	1.057	0.397	1.116	0.397	1.236
2.0	0.569	1.038	0.569	1.077	0.569	1.156
3.0	0.660	1.032	0.660	1.065	0.660	1.131
4.0	0.718	1.029	0.718	1.058	0.718	1.118
5.0	0.760	1.027	0.760	1.054	0.760	1,110

TABLE 4 PRESSURES IN BIN

$\mu = 0.472$; $\mu' = 0.263$

Peas Filled in Steel Bin

wd = 4800 Kg/m²

{Depth from top / Diameter} h/d	$\alpha_H = 0.05$		$\alpha_H = 0.1$		$\alpha_H = 0.2$	
	Static pressure in terms of w.d.	Ratio Dyn/Static	Static pressure in terms of w.d.	Ratio Dyn/Static	Static pressure in terms of w.d.	Ratio Dyn/Static
0.2	0.070	1.098	0.070	1.209	0.070	1.482
0.4	0.139	1.098	0.139	1.209	0.139	1.482
0.6	0.208	1.098	0.208	1.209	0.208	1.482
0.8	0.278	1.098	0.278	1.209	0.278	1.482
1.0	0.348	1.098	0.348	1.209	0.378	1.292
2.0	0.590	1.048	0.590	1.096	0.590	1.194
3.0	0.706	1.041	0.706	1.082	0.706	1.165
4.0	0.781	1.037	0.781	1.075	0.781	1.151
5.0	0.836	1.035	0.836	1.071	0.836	1.142

TABLE 5 PRESSURES IN BIN

$\mu = 0.3$; $\mu' = 0.3$

General Case for Lower Value

Depth from top / Diameter h/d	$\alpha_H = 0.05$		$\alpha_H = 0.1$		$\alpha_H = 0.2$	
	Static pressure in terms of w.d.	Ratio Dyn/Static	Static pressure in terms of w.d.	Ratio Dyn/Static	Static pressure in terms of w.d.	Ratio Dyn/Static
0.2	0.093	1.096	0.093	1.210	0.093	1.532
0.4	0.186	1.096	0.186	1.210	0.186	1.532
0.6	0.278	1.096	0.278	1.210	0.304	1.337
0.8	0.371	1.096	0.394	1.142	0.394	1.259
1.0	0.469	1.050	0.469	1.108	0.469	1.218
2.0	0.710	1.035	0.710	1·071	0.710	1.143
3.0	0.849	1.030	0.849	1.059	0.849	1.120
4.0	0.940	1.027	0.940	1.054	0.940	1.108
5.0	1.005	1.025	1.005	1.050	1.005	1.101