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LOVE WAVE PROPAGATION IN AN INHOMOGENEOUS ANISOTROPIC MEDIUM WITH A COLUMN OF DIFFERENT ELASTIC PROPERTIES[1]

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Introduction

Inhomogeneities in the earth's crust and upper mantle have significant effects on surface wave characteristics such as phase and group velocities and amplitudes. In the last decade geophysical studies have shown that the earth's crust is not only vertically but also laterally inhomogeneous.

Many authors have already studied vertically inhomogeneous media by taking different variations. Some of the investigators have tried to discuss lateral inhomogeneity by taking vertical discontinuities in the earth. Alsop (1966) developed an approximate method for calculating reflection and transmission coefficients for Love waves incident on a vertical discontinuity. The concept of coupling coefficients between modes on either side of the boundary is introduced. Sinha (1964) discussed transmission of SH-waves in a homogeneous vertical layer sandwiched in a homogeneous medium. He obtained reflection and transmission coefficients for SH-waves incident normally in the planes of discontinuity. Singh (1974) studied Love wave dispersion in a transversely isotropic and latterally inhomogeneous crustal layer.

In the present study we have discussed Love wave propagation in an inhomogeneous and transversely medium with a column of different elastic properties.

Formulation of the problem

The geometry of the problem is shown in Fig. 1. The system is referred to a rectangular co-ordinate system with z-axis directed vertically downwards and the origin at the free surface. The column with thickness 2a is inhomogeneous and transversely isotropic.

Let the directional rigidity and density for the coloumn be N_1, L_1 and ρ_1 respectively and for the halfspace be N, L and ρ respectively.

The equation of SH-type motion is given by,

$$\frac{\partial}{\partial x}(\tau_{xy}) + \frac{\partial}{\partial z}(\tau_{yz}) = \rho \frac{\partial^2 v}{\partial t^2} \tag{1}$$

The displacements independent of the y-coordinate, therefore

$$\frac{\partial}{\partial y} \equiv 0 \tag{2}$$

The stresses are given by

$$\tau_{xy} = N \frac{\partial v}{\partial x}$$

and

$$\tau_{yz} = L \frac{\partial v}{\partial z} \tag{3}$$

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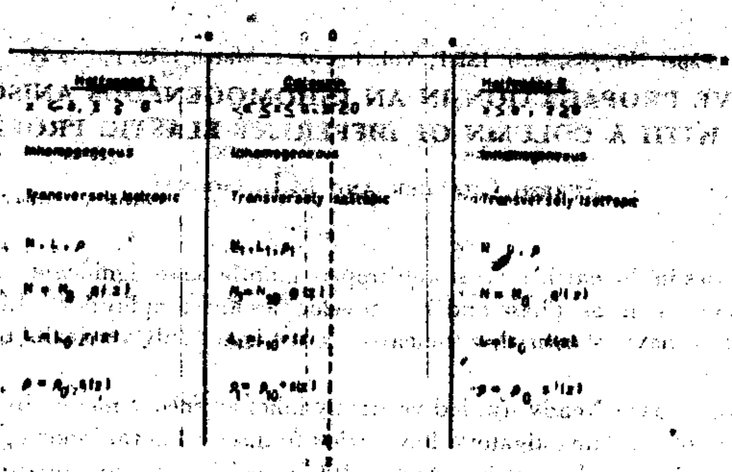

Fig. 1

Putting (3) in (1), we get

$$N\frac{\partial^2 v}{\partial x^2}+\frac{\partial N}{\partial x}\frac{\partial v}{\partial x}+L\frac{\partial^2 v}{\partial z^2}+\frac{\partial L}{\partial z}\frac{\partial v}{\partial z}=\rho\frac{\partial^2 v}{\partial t^2} \qquad (4)$$

Assuming that the disturbance in the medium is simple harmonic w.r.t. time with angular frequency w, i.e., $v\,e^{-iwt}$, the equation (4) becomes,

$$N\frac{\partial^2 v}{\partial x^2}+\frac{\partial N}{\partial x}\frac{\partial v}{\partial x}+L\frac{\partial^2 v}{\partial z^2}+\frac{\partial L}{\partial z}\frac{\partial v}{\partial z}+w^2\rho v=0 \qquad (5)$$

where v is now a function of x and z only.

We assume $N=N_0\,q(z)$, $L=L_0\,r(z)$ and $\rho=\rho_0\,s(z)$ for the halfspace I and II, and for the coloumn $N_1=N_{10}\,q(z)$, $L_1=L_{10}\,r(z)$ and $\rho_1=\rho_{10}\,s(z)$, where N_0, L_0, ρ_0, N_{10}, L_{10} and ρ_{10} are constants.

If we put

$$v=v_1\,L^{-1/2} \qquad (6)$$

in order to simplfy equation (5), we get the equations for the halfspace and for the coloumn as

$$\frac{\partial^2 v_1}{\partial x^2}+\frac{L_0}{N_0}\frac{r(z)}{q(z)}\frac{\partial^2 v_1}{\partial z^2}+\left[\frac{L_0}{N_0}\frac{r(z)}{q(z)}\left\{\frac{1}{4}(r_z/r)^2-\frac{1}{2}r_{zz}/r\right\}+\frac{\rho_0}{N_0}\frac{s(z)}{q(z)}w^2\right]v_1=0$$

and

$$\frac{\partial^2 v_1}{\partial x^2}+\frac{L_{10}}{N_{10}}\frac{r(z)}{q(z)}\frac{\partial^2 v_1}{\partial z^2}+\left[\frac{L_{10}}{N_{10}}\frac{r(z)}{q(z)}\left\{\frac{1}{4}(r_z/r)^2-\frac{1}{2}r_{zz}/r\right\}+\frac{\rho_{10}}{N_{10}}\frac{s(z)}{q(z)}w^2\right]v_1=0 \qquad (7)$$

respectively.

Where $r_z=\dfrac{\partial r}{\partial z}$ and $r_{zz}=\dfrac{\partial^2 r}{\partial z^2}$

We assume the solution of equations in (7) in the from

$$V_1(x,z)=\begin{cases} Z(z)\,A\,e^{ikx}+B\,e^{-ikx} & \text{for } x<-a \\ Z(z)\,C\,e^{ikx}+D\,e^{-ikx} & \text{for } -a\leqslant x\leqslant a \\ Z(z)\,F\,e^{ikx} & \text{for } x>a \end{cases} \qquad (8)$$

where A, B, C, D and F are constants, w is the angular frequency, k and k_1 are the wave numbers for the region $x < -a$, $x > a$ and $-a \leqslant x \leqslant a$, respectively. We see that Z(z) satisfies the equations,

$$\frac{d^2Z}{dz^2} + \left\{\frac{w^2}{\beta_0^2}\frac{s(z)}{r(z)} - \frac{N_0}{L_0}\frac{q(z)}{r(z)}k^2 + \frac{1}{4}(r_z/r)^2 - \frac{1}{2}(r_{zz}/r)\right\} Z = 0 \tag{9}$$

in the halfspaces, and

$$\frac{d^2Z}{dz^2} + \left\{\frac{w^2}{\beta_{10}^2}\frac{s(z)}{r(z)}q - \frac{N_{10}}{L_{10}}\frac{q(z)}{r(z)}k_1^2 + \frac{1}{4}(r_z/r)^2 - \frac{1}{2}(r_{zz}/r)\right\} Z = 0 \tag{10}$$

in the coloumn, where

$$\beta_0^2 = \frac{L_0}{\rho_0} \qquad \text{and} \qquad \beta_{10}^2 = \frac{L_{10}}{\rho_{10}}$$

The boundary conditions are that the displacements and the stresses should be continous across the surface of discontinuity, i.e.,

$$\begin{aligned} [\tau_{xy}] &= 0 \qquad \text{at } x = \pm a \qquad \text{for all } z, \\ [\,v\,] &= 0 \qquad \text{at } x = \pm a \qquad \text{for all } z \end{aligned} \tag{11}$$

The square bracket in (11) denotes the change in the value of a quantity across a surface of discontinuity. The continuity of stress and displacement for all z requires that the function Z(z) must be the same for the regions $x < -a$, $-a \leqslant x \leqslant a$ and $x > a$. In otherwards, the differential equations (9) and (10) for Z(z) must be the same. This gives

$$\frac{w^2}{\beta_0^2}\frac{s(z)}{r(z)} - \frac{N_0}{L_0}\frac{q(z)}{r(z)}k^2 = \frac{w^2}{\beta_{10}^2}\frac{s(z)}{r(z)} - \frac{N_{10}}{L_{10}}\frac{q(z)}{r(z)}k_1^2 \tag{12}$$

since $w = kc = k_1 c_1$, where c and c_1 are the phase velocities for regions, $x > a$ and $x < -a$, $-a \leqslant x \leqslant a$, respectively. Equation (12) can be written as

$$k_1^2 = \frac{N_{10}}{L_{10}}k^2\frac{N_0}{L_0} + \frac{s(z)}{q(z)}\left(\frac{c^2}{\beta_{10}^2} - \frac{c^2}{\beta_0^2}\right) \tag{13}$$

Equation (13) gives the change in the wave number, as the wave is transmitted from the halfspace I into the coloumn and from the coloumn to the half space II. From boundary conditions (11) and equation (8), we get

At $x = -a$

$$\left.\begin{aligned} &k L_0^{1/2}\{A e^{-ika} - B e^{ika}\} = k_1 L_{10}^{1/2}\{C e^{-ik_1a} - D e^{ik_1a}\} && \text{(i)} \\ &L_0^{1/2}\{A e^{-ika} + B e^{ika}\} = L_{10}^{-1/2}\{C e^{-ik_1a} + D e^{ik_1a}\} && \text{(ii)} \\ &\text{and at } x = +a \\ &k_1 L_{10}^{1/2}\{C e^{ik_1a} - D e^{-ik_1a}\} = k L_0^{1/2} F e^{ika} && \text{(iii)} \\ &L_{10}^{-1/2}\{C e^{ik_1a} + D e^{-ik_1a}\} = L_0^{1/2} F e^{ika} && \text{(iv)} \end{aligned}\right\} \tag{14}$$

From equations (iii) and (iv) of (14), we get

$$C = e^{-2ik_1a}\frac{(L_{10}k_1 + L_0 k)}{(L_{10}k_1 - L_0 k)} D$$

$$D = \left(\frac{L_{10}}{L_0}\right)^{1/2}\left(\frac{L_{10}k_1 - L_0 k}{2L_{10}k_1}\right) e^{i(k+k_1)a}\, F$$

Therefore we obtain,

$$C = PF \quad \text{and} \quad D = QF \tag{15}$$

where

$$P = \left(\frac{L_{10}}{L_0}\right)^{1/2} \left(\frac{L_{10}\,k_1 + L_0\,k}{2L_{10}\,k_1}\right) e^{i(k-k_1)a}$$

and

$$Q = \left(\frac{L_{10}}{L_0}\right)^{1/2} \left(\frac{L_{10}\,k_1 - L_0\,k}{2L_{10}\,k_1}\right) e^{i(k+k_1)a}$$

From equations of (i) and (ii) of (14), we get

$$B = A\left(\frac{1-G}{1+G}\right) e^{-2ika} \tag{16}$$

$$F = A\left(\frac{L_{10}}{L_0}\right)^{1/2} \frac{2}{1+G}\,\frac{1}{P\,e^{-ik_1a} + Q\,e^{ik_1a}}\, e^{-ika} \tag{17}$$

where

$$G = \frac{k_1}{k}\;\frac{L_{10}}{L_0}\;\frac{(P\,e^{-ik_1a} - Q\,e^{ik_1a})}{(P\,e^{-ik_1a} + Q\,e^{ik_1a})} \tag{18}$$

From equations (15) to (18), we get

$$\frac{B}{A} = \left[\frac{(kL_0/k_1L_{10})\,\{(L_{10}\,k_1 + L_0 k)\,e^{i(k-2k_1)a} + (L_{10}\,k_1 - L_0\,k)\,e^{i(k+2k_1)a}\}}{\Delta_1} - \frac{\{(L_{10}\,k_1 + L_0 k)\,e^{i(k-2k_1)a} - (L_{10}\,k_1 - L_0 k)\,e^{i(k+2k_1)a}\}}{\Delta_1}\right] e^{-2ika} \tag{19}$$

$$C/A = \frac{2k/k_1\,(L_{10}\,k_1 + L_0 k)\,e^{-ik_1a}}{\Delta_1} \tag{20}$$

$$D/A = \frac{2k/k_1\,(L_{10}\,k_1 - L_0 k)\,e^{-ik_1a}}{\Delta_1} \tag{21}$$

$$F/A = \frac{4\,(L_{10}/L_0)\,k\,L_0\,e^{-ik_1a}}{\Delta_1} \tag{22}$$

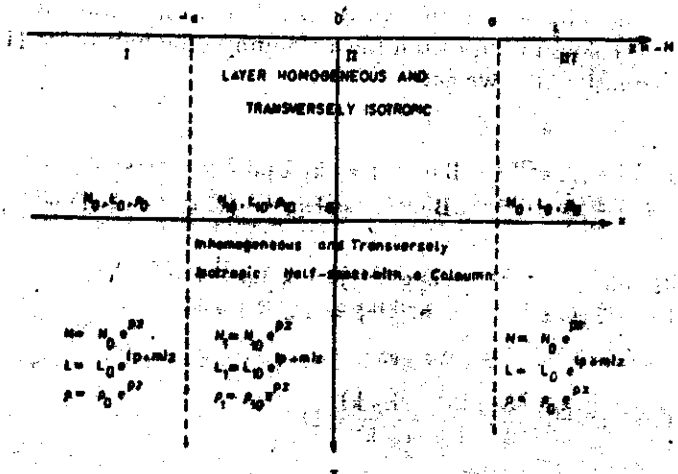

Fig. 2

where $$\Delta = \frac{k}{k_1}\left(\frac{L_0}{L_{10}}\right)^{\frac{1}{2}} (L_{10}\, k_1 + L_0 k)\; e^{i(k-2k_1)a}$$

$$+ (L_{10}\, k_1 - L_0\, k)\, e^{i(k+2k_1)a} + \left(\frac{L_{10}}{L_0}\right)^{\frac{1}{2}} (L_{10}\, k_1 + L_0 k).$$

$$.\; e^{i(k-2k_1)a} - (L_{10}\, k_1 - L_0\, k)\, e^{i(k+2k_1)a} \tag{23}$$

Transposing in T-ratios, (19) to (23) we get

$$\Delta_1 = \frac{4\, kL_0\, k_1\, L_{10} \operatorname{Cos} 2\, k\, k_1\, a - 2i\, (k^2\, L_0^2 + k_1^2\, L_{10}^2) \sin 2k_1 a}{k_1\, L_{10}}\; e^{ika}$$

and

$$\frac{B}{A} = \frac{2i\, (L_{10}^2\, k_1^2 - L_0^2\, k^2)\, \sin\, (2k_1\, a)\, e^{-2ika}}{\Delta} \tag{24}$$

$$\frac{D}{A} = \frac{2\, (L_{10}\, L_0)^{1/2}\, (L_{10}\, k_1 - L_{0k})\;\; e^{-i(k-k_1)a}}{\Delta} \tag{25}$$

$$\frac{F}{A} = \frac{(4k\, k_1\, L_{10}\, L_0)\, e^{-2ika}}{\Delta} \tag{26}$$

where $$\Delta = 4k\, k_1\, L_0\, L_{10} \cos 2k_1\, a - 2i\, (L_0^2\, k^2 + L_{10}^2\, k_1^2) \sin 2\, k_1\, a$$

Writing $$\frac{B}{A} = R\;\; e^{i\theta} \qquad \text{and} \qquad \frac{F}{A} = R_1\, e^{i\phi}$$

We have

$$R^2 = \frac{(L_{10}^2\, k_1^2 - L_0^2\, k^2) \sin^2 2\, k_1\, a}{4k^2\, k_1^2\, L_0^2\, L_{10}^2 \cos^2 2k_1\, a + (L_0^2\, k^2 + L_{10}^2\, k_1^2) \sin^2 2k_1\, a} \tag{27}$$

$$R_1^2 = \frac{4\, k^2\, k_1^2\, L_0^2\, L_{10}^2}{4k^2\, k_1^2\, L_0^2\, L_{10}^2 \cos^2 2k_1\, a + (L_0^2\, k^2 + L_{10}^2\, k_1^2) \sin^2 2k_1\, a} \tag{28}$$

$$\theta = \tan^{-1}\left[\frac{2k\, k_1\, L_0\, L_{10} \cos 2k_1\, a \cos 2k\, a + (L_{10}^2\, k_1^2 + L_0^2\, k^2) \sin 2k_1\, a \sin 2k\, a}{2k\, k_1\, L_0\, L_{10} \sin 2k\, a \cos 2k_1\, a - (L_{10}^2\, k_1^2 + L_0^2\, k^2) \sin 2k_1\, a \cos 2k\, a}\right] \tag{29}$$

$$\phi = \tan^{-1}\left[\frac{(L_0^2\, k^2 + L_{10}^2\, k_1^2) \cos 2k\, a \sin 2k_1\, a - 2k\, k_1\, L_0\, L_{10} \sin 2ka \cos 2k_1 a}{(L_0^2\, k^2 + L_{10}^2\, k_1^2) \sin 2k_1\, a \sin 2ka + 2k\, k_1\, L_0\, L_{10} \cos 2ka \cos 2k_1 a}\right] \tag{30}$$

since there is no accumulation of energy, the relation

$R^2 + R_1^2 = 1$, is satisfied.

Equation (27) shows that perfect transmission is possible when $\sin 2k_1\, a = 0$ i.e. $k_1\, a = (n\pi/2)$, $n = 0, 1, 2, \ldots\ldots$ using the relation $w = k_1\, c_1$, we find that prefect transmission is possible at the frequency,

$$w = (n\pi/2a)\; c_1$$

In such case there is no reflected wave and the amp'tude of the transmitted wave is maximum being equal to the amplitude of the incident wave.

Love Waves in Layer laying over a half-space

The geometry of the problem is shown in the Fig. 2. The column with thickness 2a and the half-space is inhomogeneous and transversely isotropic. The layer of thickness h is homogeneous and transversely isotropic. The vertical variation in the column and the half-space has been taken the same. The density and directional rigidities in the half-space and for the column vary exponentially.

The equation of motion for the homogeneous layer can be obtained from equation (4) as

$$N_0 \frac{d^2 v'}{dx^2} + L_0 \frac{d^2 v'}{dz^2} = \rho_0 \frac{d^2 v'}{dt^2} \tag{31}$$

The solution of (31) can be written as

$$v' = \begin{cases} Z(z)\,[A\, e^{ikx} + B\, e^{-ikx}] & \text{for } x < -a \\ Z(z)\,[C\, e^{ik_1 x} + D\, e^{-ik_1 x}] & \text{for } -a < x < a \\ Z(z)\, F\, e^{ikx} & \text{for } x > a \end{cases} \tag{32}$$

We see that Z(z) satisfies the equation

$$\frac{d^2 Z}{dz^2} + \left[\frac{\rho_0 k^2 c^2 - N_0 k^2}{L_0}\right] Z = 0 \tag{33}$$

for the half-space and for the column

$$\frac{d^2 Z}{dz^2} + \left[\frac{\rho_{10} k_1^2 c^2 - N_{10} k_1^2}{L_{10}}\right] Z = 0 \tag{34}$$

The solution of (33) can be written as

$$Z = A_0 e^{ia_2 z} + B_0 e^{-ia_2 z} \tag{35}$$

where $a_2^2 = k^2 \left[\frac{\rho_0 c^2 - N_0}{L_0}\right]$

$$v'(x, z) = \begin{cases} (A_0 e^{ia_2 z} + B_0 e^{-ia_2 z})\,(A\, e^{ikx} + B\, e^{-ikx}) & \text{for } x < -a \\ (A_0 e^{ia'_2 z} + B_0 e^{-ia'_2 z})\,(C\, e^{ikx} + D\, e^{-ikx}) & \text{for } -a < x < a \\ (A_0 e^{ia_2 z} + B_0 e^{-ia_2 z})\, F\, e^{ikx} & \text{for } x > a \end{cases} \tag{36}$$

Where $(a'_2)^2 = k_1^2 \left[\frac{\rho_{10} c^2 - N_{10}}{L_{10}}\right]$ (37)

Now the equation of motion for the half-space can be written from equation (7) by putting

$$q(z) = e^{pz}, \quad r(z) = e^{(p+m)z}, \quad s(z) = e^{pz} \tag{38}$$

we get the equations for the halfspace and for the column as

$$\frac{d^2 v_1}{dx^2} + \frac{L_0}{N_0}\frac{d^2 v_1}{dz^2} e^{mz} + \left[\frac{\rho_0}{N_0} w^2 - \tfrac{1}{4}(p+m)^2 \frac{L_0}{N_0} e^{mz}\right] v_1 = 0$$

and (39)

$$\frac{d^2 v_1}{dx^2} + \frac{L_{10}}{N_{10}}\frac{d^2 v_1}{dz^2} e^{mz} + \left[\frac{\rho_{10}}{N_{10}} w^2 - \tfrac{1}{4}(p+m)^2 \frac{L_{10}}{N_{10}} e^{mz}\right] v_1 = 0$$

respectively.

To simplify equation (39), we use the method of separation of variables. Putting

$$v_1(x, z) = X(x)\, Z(z) \tag{40}$$

we get

$$\frac{1}{X(x)}\frac{d^2X}{dx^2} + \frac{L_0}{N_0} e^{mz} \frac{1}{Z(z)}\frac{d^2Z}{dz^2} + \left[\frac{\rho_0}{N_0} w^2 - \tfrac{1}{4}(p+m)^2 \frac{L_0}{N_0} e^{mz}\right] = 0 \tag{41}$$

Then X(x) and Z(z) satisfies the following equations,

$$\frac{d^2X}{dx^2} + k^2 X = 0, \tag{42}$$

$$\frac{d^2 Z}{dz^2} + \left[\left(\frac{w^2}{\beta_0^2} - \frac{N_0}{L_0} k^2\right) e^{-mz} - \tfrac{1}{4}(p+m)^2\right] Z = 0 \tag{43}$$

for the halfspace and for the column

$$\frac{d^2 X}{dx^2} + k_1^2 X = 0, \tag{44}$$

$$\frac{d^2 Z}{dz^2} + \left[\left(\frac{w^2}{\beta_{10}^2} - \frac{N_{10}}{L_{10}} k_1^2\right) e^{-mz} - \tfrac{1}{4}(p+m)^2\right] Z = 0 \tag{45}$$

Equation (43) can be written as,

$$\frac{d^2 Z}{dz^2} - [a_1^2 - a_2^2 e^{-mz}]\, Z = 0 \tag{46}$$

where $$a_1^2 = \tfrac{1}{4}(p+m)^2 \quad \text{and} \quad a_2^2 = \left(\frac{w^2}{\beta_0^2} - \frac{N_0}{L_0} k^2\right) \tag{47}$$

Putting

$$Z(z) = U(x), \qquad x = \frac{2}{m} e^{-mz/2}.\ a_2 \tag{48}$$

Equation (46) reduces to

$$x^2 \frac{d^2U}{dx^2} + x \frac{dU}{dx} - (a_3^2 - x^2)\, U = 0 \tag{49}$$

is a Bessel's differential equation. Where

$$a_3 = \frac{2a_1}{m} \tag{50}$$

The solution of equation (49) is given by,

$$U = A_1 J_{a_3}(x) + B J_{-a_3}(x) \tag{51}$$

The equations (48) and (51) gives

$$Z(z) = A_1 J_{2a_1/m}\left(\frac{2}{m} a_2 e^{-mz/2}\right) + B_1 J_{2-a_1/m}\left(\frac{2}{m} a_2 e^{-mz/2}\right) \tag{52}$$

The solutions of equations (42) and (45) can be written as

$$X(x) = \begin{cases} (A e^{ikx} + B e^{-ikx}) & \text{for } x < -a \\ (C e^{ik_1x} + D e^{-ik_1x}) & \text{for } -a < x < a \\ F e^{ikx} & \text{for } x > a \end{cases} \tag{53}$$

For equations (6), (40), (52) and (53), we get

$$V(x, z) = L_0^{-1/2} e^{-(p+m)z/2} \left\{ A_1 J_{2a_1/m}\left(\frac{2}{m} a_2 e^{-mz/2}\right) + B_1 J_{-2a_1/m}\left(\frac{2}{m} a_2 e^{-mz/2}\right)\right\} . \left(A\, e^{ikx} + B\, e^{-ikx}\right) \tag{54}$$

for $x < -a$

and for the medium $-a \leqslant x \leqslant a$,

$$V(x, z) = L_{10}^{-1/2} e^{-(p+m)z/2} \left\{ A_1 J_{2a_1/m}\left(\frac{2}{m} a_4 e^{-mz/2}\right) + B_1 J_{-2a_1/m}\left(\frac{2}{m} a_4 e^{-mz/2}\right)\right\} . \left(C\, e^{ik_1x} + D\, e^{-ik_1x}\right) \tag{55}$$

where $$a_4^2 = \left(\frac{w^2}{\beta_{10}^2} - \frac{N_{10}}{L_{10}} k_1^2\right) \tag{56}$$

and for the medium $x > a$

$$v(x, z) = L_0^{-1/2} e^{-(p+m)z/2} \left\{ A_1 J_{2a_1/m}\left(\frac{2}{m} a_2 e^{-mz/2}\right) + B_1 J_{-2a_1/m}\left(\frac{2}{m} a_2 e^{-mz/2}\right)\right\} . F\, e^{ikx} \tag{57}$$

since displacement vanishes at infinity, we get

$B_1 = 0$ and hence (54) reduces to

$$v = L_0^{-1/2} e^{-(p+m)z/2} A_1 J_{2a_1/m}\left(\frac{2}{m} e^{-mz/2}\right) \left(A\, e^{ikx} + B\, e^{-ikx}\right) \tag{58}$$

The boundary conditions are :

(i) The displacement should be continuous i.e.,

$v' = v$ at $z = 0$, and

(ii) the tangential stress should be continuous i.e.

$\tau_{yz} = \tau'_{yz}$ at $z = 0$

(iii) the surface is stress free i.e.

$\tau'_{yz} = 0$ at $z = -H$

From boundary condition (ii) i.e.

$$\frac{\partial v}{\partial z} = \frac{\partial v'}{\partial z} \qquad \text{at } z = 0$$

we have

$$A_o - B_o = -\frac{L_o^{-1/2} A_1}{i\, a_2}\left[\frac{p+m}{2} + a_1 - \frac{a_2 J_{a_s+1}}{J_{a_s}} \frac{2}{m} a_2 \frac{2}{m} a_2\right] J_{a_s} \frac{2}{m} a_2^1 \tag{59}$$

From boundary condition (iii) i.e.

$$\frac{\partial v'}{\partial z} = 0 \qquad \text{at } z = -H,$$

we have

$$i(A_o + B_o)\tan a_2 H = A_o - B_o \tag{60}$$

From boundary condition (i), we have

$$A_o + B_o = L_o^{-1/2} A_1 J_{a_2}\left(\frac{2}{m} a_2\right) \tag{61}$$

From equations (59), (60) and (61), we get

$$\tan a_2 H = \frac{1}{a_2}\left[\frac{p+m}{2} + a_1 - \frac{a_2 J_{a_2+1}}{J_{a_2}}\left(\frac{2}{m} a_2\right)\right] \tag{62}$$

This is the frequency equation for the medium except the column.

Frequency equation for the column can be written as

$$\tan a_2' H = \frac{1}{a_2'}\left[\frac{p+m}{2} + a_1 - a_2' J_{a_2+1}\left(\frac{2}{m} a_2'\right) J_{a_2}\left(\frac{2}{m} a_2'\right)\right] \tag{63}$$

where $a_2' = a_4$

Numerical Calculations :

For numerical calculations frequency equations (62) and (63) can be written as

$$\tan Y\left(X^2 - \frac{N_o}{L_o}\right)^{\frac{1}{2}} = \frac{(p+m)H}{Y\left(X^2 - \frac{N_o}{L_o}\right)^{\frac{1}{2}}} - \frac{J_{(p+2m)/m}\left[\frac{2Y}{mH}\left(X^2 - \frac{N_o}{L_o}\right)^{\frac{1}{2}}\right]}{J_{(p+m)/m}\left[\frac{2Y}{mH}\left(X^2 - \frac{N_o}{L_o}\right)^{\frac{1}{2}}\right]} \tag{64}$$

and

$$\tan Y_1\left(X_1^2 - \frac{N_{10}}{L_{10}}\right)^{\frac{1}{2}} = \frac{(p+m)H}{Y_1\left(X_1^2 - \frac{N_{10}}{L_{10}}\right)^{\frac{1}{2}}} - \frac{J_{(p+2m)/m}\left[\frac{2Y_1}{mH}\left(H_1^2 - \frac{N_{10}}{L_{10}}\right)^{\frac{1}{2}}\right]}{J_{(p+m)/m}\left[\frac{2Y_1}{mH}\left(X_1^2 - \frac{N_{10}}{L_{10}}\right)^{\frac{1}{2}}\right]} \tag{65}$$

Where

$$X = C/\beta_0, \qquad Y = kH,$$
$$X_1 = C/\beta_{10}, \qquad Y_1 = k_1 H.$$

The values of constants are assumed to be

$$N_0 = 2.83 \times 10^{11} \text{ dynes/cm}^2,$$
$$L_0 = 3.25 \times 10^{11} \text{ dynes/cm}^2,$$
$$\rho_0 = 2.85 \text{ gm/cm}^3,$$
$$N_{10} = 3.45 \times 10^{11} \text{ dynes/cm}^2,$$
$$L_{10} = 4.02 \times 10^{11} \text{ dynes/cm}^2,$$
$$\rho_{10} = 3.05 \text{ gm/cm}^3$$
$$pH = 1.68,$$
$$mH = 0.42.$$

The computations were performed on the electronic computer TDC-316 of Kurukshetra University, Kurukshetra. The group velocity was obtained by the formula :

$$U = C + k\,\frac{dC}{dk}$$

i.e.
$$U/\beta_0 = X + Y\,\frac{dX}{dY}.$$

The group velocity was obtained by the above formula and calculated numerically. The group velocity and phase velocity for the other frequency equations have also been calculated. This dispersion curves have been exhibited in Fig. 3.

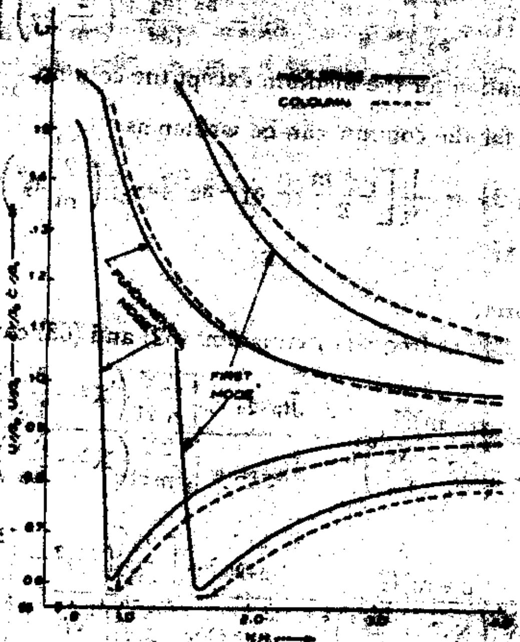

Fig. 3 Phase and group velocity curves for fundamental and first modes

Reflection and Transmission coefficients as a function of frequency have been calculated by using equations (27) and (28). These have been plotted as functions of frequency in Figs, 4 and 5, respectively.

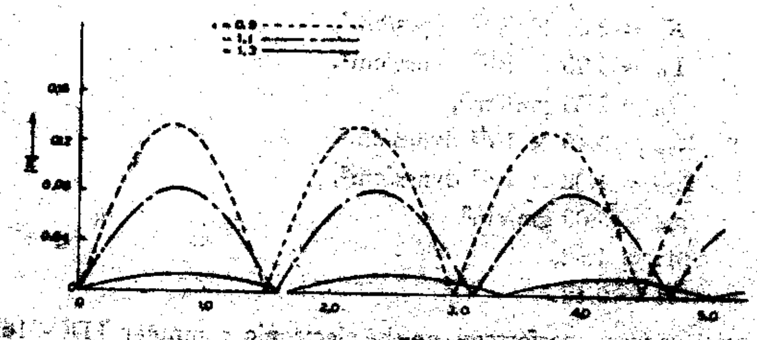

Fig. 4 The variation of the modulus of the reflection coefficient with the frequency

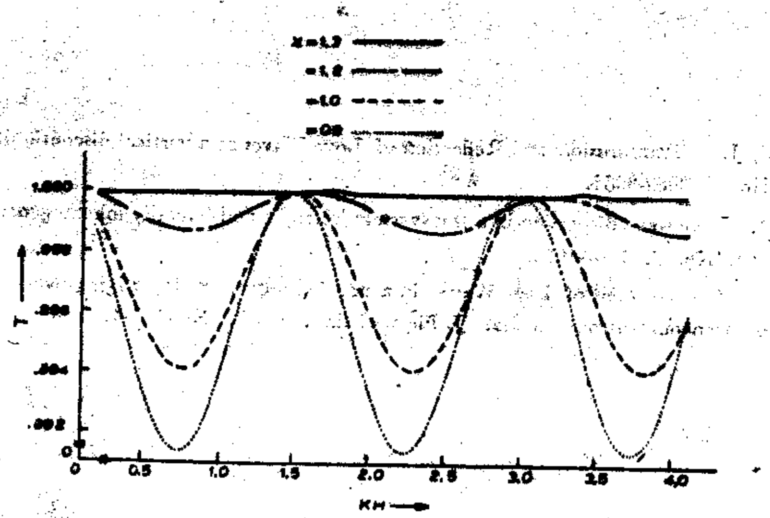

Fig. 5 The variation of the modulus of the transmission coefficient with the frequency

Conclusion :

The problem has been studied on the basis of mathematical analysis and numerical computations. The pnase and group velocity curves for the coloumn and the half-space have been drawn for the first two modes. For the fundamental mode, the phase velocity is less in the half-space than in the column upto the value of kH = 2.0, and thereafter the phase velocity in the half-space is more than the phase velocity in the coloumn. The phase velocity for the first mode in the half-space is always less than the phase velocity in the coloumn for all values of frequency. The deviation in the phase velocity at high and low frequency range is quite small and decreases with higher modes. For waves of small wave numbers, the group velocity for the fundamental mode decreases repidly with frequency and is minimum when kH = 0.9. As frequency increases beyond this value, the group velocity increases. For the first mode, the group velocity is minimum when kH = 1.6, beyond this value, if the frequency increases the value of the group velocity increases.

From Fig. 4, it is clear that the value of the modulus of the reflection coefficient, when plotted against frequency, undergoes a series of maxima and minima, the value of which decreases with the increasing value of X. Similar are the variations in the value of the transmission coefficient when plotted against frequency as can be seen from Fig. 5.

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References

Alsop, L.E., 1966, J. Transmission and Reflection of Love Waves at a vertical discontinuity. Geophysics, Res., 71, No. 16, 3969-3984.

Singh, V.P., 1974, Love wave dispersion in a transversely isotropic and laterally inhomogeneous crustal layer, B.S.S.A, 64, No. 6, 1967-1978.

Sinha, S.B., 1964, Transmission of Love Waves in a non-homogeneous layer lying over a vertically semi-infinite homogeneous isotropic media. J. Phys. of the Earth, 12, No. 1, 1-4.