GROUND DISPLACEMENT AND ACCELERATION CAUSED BY EARTHQUAKES

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SYNOPSIS

This paper is an outcome of an enquiry into the processes which lead to the attenuation of seismic waves in the earth. A knowledge of the variation of their amplitude with distance, and with depth, in the case of surface waves, would be of considerable interest to Seismology in general, and to Earthquake Engineering in particular.

The factors governing the amplitude of the ground displacement at a given epicentral distance have been examined in detail. However, as a progressive wave suffers from the cumulative effects of all these causes, the individual effects can be inferred only indirectly. Typical values representing the dissipation of seismic energy are also discussed.

INTRODUCTION

Most of the studies on the amplitude of seismic waves have been made with a view to understanding the structure of the earth and the nature of earthquakes. To an Earthquake Engineer the problem is an inverse one i.e., to estimate the amplitude of the seismic waves or the displacements caused by them when they emerge at the surface. Energy travels outwards from a focus, usually regarded as being small enough to approximate to a point, in the form of waves. These include body waves which may penetrate to all parts of the Earth's interior, as wall as surface waves which only travel along surfaces of discontinuity, especially over the free surface of the earth. An advancing wave is normally attenuated as its energy spreads out over the enlarging wavefonrt, and also by the partitioning of energy at the elastic discontinuities. Other factors contributing to this include diffraction, scattering, dispersion and deviation from perfect elasticity.

1. Geometrical Spreading of the Wavefront

In the simplest case of spherical symmetry about a focus, the solution of the wave equation, for waves advancing outward from the focus, has the form $A = \frac{1}{r} F(r - vt)$, where F is a function representing initial conditions. The solution implies that A decreases inversely as the distance. This accounts for most of the decrease in amplitude at distances not too close to the focus, where more complicated phenomena predominate.

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The expression for the displacement of the outer surface arising from a wave travelling towards it from a focus without change of type can be written as (see Appendix),

$$y = K\,T.\,Z.\sqrt{E.\,\frac{\cos i}{\sin e.\,\sin\Delta}\left(\frac{di}{dt}\right)} \qquad (1)$$

where K = a constant depending upon the fraction of the energy E passing into the wave

$Z = \dfrac{\text{ground displacement (y)}}{\text{amplitude of the incident wave}}$

Δ = epicentral distance

i = angle which the ray leaving F makes with the level surface through it

e = angle between the emergent ray and the surface.

Part of the energy released by an Earthquake travels along the free surface of the Earth and along other surfaces of discontinuity in the Earth (Jeffreys, 1963 pp. 56–57). The amplitudes of surface waves arising from a shock of a given size decrease with focal depth according to factor e^{-Gh} where h is the depth of focus and G a function of wave velocities (Jeffreys 1959 page 58). Thus when the focus is not too deep, surface waves will spread outward over the other surface falling in amplitude according to the inverse square root of the distance. However, surface waves exhibit considerable dispersion.

2. Partitioning of Energy at the Elastic Discontinuities

At each discontinuity encountered, where either the velocity changes discontinuously or its gradient is abnormally high, a wave is partly reflected and partly refracted into the new medium causing a division of its energy. In general an incident wave results in four transformed waves, the reflected and the refracted P and S waves except in certain special cases when the nature of the incident vibration may preclude some of these components (Jeffreys, 1963, pp.29–30). In order to calculate amplitude of any of these derived waves, it is only necessary to apply the boundary conditions corresponding to the continuity of stress and displacement across the entire boundary at all instants of time (Jeffreys,1963 pp.31–32). For the limiting case when the differences in the physical properties of the region vary but slightly, the analysis shows that loss of energy by transmission amounts to a second order quantity only, and if the medium can be considered homogeneous enough within a wavelength, the energy is transmitted virtually undamped along rays.

The ratio of the refracted wave amplitude to the incident wave amplitude can be represented by an appropriate transmission factor

$$F = \left[1 - \left(\frac{A}{A_0}\right)^2\right]$$

In general an emergent wave may have changed type at the various discontinuities. The foregoing argument, however, still holds but the appropriate transmission factor has to be calculated for every encounter and introduced in equation (1) to allow for the losses discussed here. The resulting expression for the displacement would now be given by,

$$y = KTZ\sqrt{(F_1.F_2. \ldots F_n).E.\frac{\text{Cos } i}{\text{Sin } e \,.\, \text{Sin } \triangle}\left(\frac{di}{d\triangle}\right)} \quad (2)$$

3. (a) Diffraction

However, if the discontinuity encountered involves a curvature large compared with that of the incident wavefront, the reflected and the refracted wavefronts will be sharply curved. This is the case of diffraction. It assumes great significance in the vicinity of the source of disturbances where the conditions of the ray theory do not obtain. The mathematical theory required to explain diffraction effects tends to be complicated owing to a wide variety of problems involved. No experimental work seems to have been done to elucidate these effects.

3. (b) Scattering

Scattering arises when the irregularities, very much smaller than the predominating wavelengths, are encountered. In such a case the incident wave will be irregularly scattered and partially degenerate into heat. Jeffreys (1963 p. 41) has developed a quantitative treatment of this effect with analogy to kinetic theory of gases and presents a quantity,

$$\tau \simeq (l/3\, a\, \varepsilon') \quad (3)$$

corresponding to the kinematic viscosity in gases, to be incorporated in a firmoviscous law; where l is the average grain diameter, a the longitudinal wave velocity, and it is assumed that the wave velocities vary from grain to grain by a factor of ε'. The firmoviscous law in its simplest form may be written as

$$Pij = 2\mu\, Eij + 2\nu \frac{dEij}{dt} \quad (4)$$

and corresponds to the Kelvin model,

where, Pij = deviatoric stress
Eij = deviatoric strain
μ = rigidity
ν = viscosity

The perfect elasticity stress–strain relations and the elastic afterworking equations are respectively,

$$Pij = 2\mu\, Eij \quad (5)$$

and, $$Pij + \tau \frac{d\,Pij}{dt} = 2\mu\, Eij + 2\nu \frac{d\,Eij}{dt} \quad (6)$$

It can be shown that the replacement of (5) by (4) introduces a damping factor of the order of exp $(-\nu\, \gamma^2 x/2\mu\, v)$ in the amplitudes of waves of period $2\pi/\gamma$ and speed v over a distance x. For waves in the crust of the earth, a comparison with observations gives an effective value of $\nu/\mu = 0.003$ secs. For deeper portions of the earth the value seems to be still less. For surface waves with speeds of the order of 3 km/sec., the damping factor would be (1/e) over distances of 50 and 5000 km. in waves of periods 1 and 10 secs., respectively. For body waves penetrating deeply into the earth, damping is still less.

Equation (6) similarly involves a damping factor, but in this case contrary to observation, short waves would not be more severely affected than longer ones, as in the case of firmoviscous relation (4). From this result Jeffreys concludes that scattering rather than the type of imperfections observed in laboratory experiments is the main source of departure from perfect elasticity theory in elastic wave propagation. He estimated the linear diameter of crustal irregularities responsible for scattering to be of the order of 5 meters.

Jeffreys further indicates that the wavefronts leaving the focus would be slightly blunted by scattering because of the associated small spread introduced into the values of the wave velocity. Using the firmoviscous law equation (4), he has shown that at a given point, the most rapid change of displacement in the onset would take place at the instant given by the perfect elasticity theory based on equation (5) but the blunting would be spread about this instant over a time-interval of the order of $\sqrt{(2\nu t/\mu)}$, where $\left(\frac{\nu}{\mu}\right)$ may be taken to be a measure of the average scattering during a time of transit t. For the longitudinal waves travelling through the earth's centre to the anticentre, $t \simeq 20$ min., and if ν/μ has everywhere the suggested surface value of 0.003 secs., the blunting would be spread over about 2 secs., which does not fit in with the observations of seismograms. It is, therefore, concluded that scattering is largely confined to the outermost 40 km. of the earth.

Rayleigh scattering will predominate if the grain size is smaller than the wavelength λ of the seismic wave concerned. Scattering in such a case will be proportional to $(1/\lambda^4)$ being greater for shorter wavelengths and smaller for longer ones.

4. Internal Friction

Besides these effects both laboratory experiments on rock samples as well as field measurements on rocks in situ, point to the elastic absorption in rocks i.e., in a steady state vibration, the amplitude is found to decrease with time. The corresponding loss of energy appears as heat and the processes causing this are collectively known as internal friction.

Internal friction may be expressed in terms of a function Q analogous to that of an electrical system and can be similarly obtained from the shape of the resonance curve. Alternatively, the relation between Q and one of the following quantities may be used:

1. The logarithmic decrement ε

2. The fractional loss of energy per unit cycle $\left(\frac{\Delta E}{E}\right)$

3. The phase difference between the applied stress and the resulting displacement δ. Accordingly,

$$\text{Internal friction} \cong \frac{\Delta E}{E} = 2\varepsilon = \frac{2\pi}{Q} = 2\pi \tan \delta \quad (7)$$

One may also define internal friction in terms of the attenuation of a plane wave. The amplitude A_x of a damped wave at a distance x from a reference position x_o, including the effect of divergence of the wave front, is normally expressed by the following relation.

$$A_x = \left(\frac{A_{xo}}{x^m}\right) e^{-kx} \quad (8)$$

where k is the absorption coefficient and m the appropriate geometrical factor for the wave. If k varies in the body, it has to be replaced by the corresponding integral $\int k(x)\,dx$.

A relation between $\frac{\Delta E}{E}$ obtained from a rock bar sample and k obtained from the damping of a wave in the field follows from a suggestion by Born (1941) which states that

$$\frac{\Delta E}{E}(\text{bar}) = (2.1)\frac{\Delta E}{E}(\text{bulk}), \quad (9)$$

so that

$$\frac{\Delta E}{E}(\text{bar}) = \frac{2kv}{f} \quad (10)$$

$$\therefore k = \frac{f}{2v}\frac{\Delta E}{E}(\text{bar}) = \frac{\pi}{TvQ}(\text{bar}) \quad (11)$$

No general theory is available to describe the actual physical mechanisms responsible for causing dissipation in a medium, for obvious mathematical difficulties. However, the simplest way to introduce dissipative effects in the equation of motion is to represent them either as a function of velocity or of the absolute value of all acting forces, i.e. COULOMB FRICTION (Förtsch, 0., 1956).

Following this simple models have been proposed by various authors which suggest the elastic behaviour of solids for the particular mechanisms, some of these important models are briefly summarized in Table 1. The mathematical form wherever possible, has been translated in mechanical terms as a combination of springs (elastic elements), and dashpots (viscous elements).

Models 1 to 5 produce a frequency dependent variation in the internal friction. Model 4 proposed by Boltzmann implies that the behaviour of a solid under stress is a function of its entire previous history. The solid may be visualized as a combination of springs and dashpots, but the resulting equations, in general are not soluble. This difficulty has been partially overcome by Sokoloff and Scriabin who assumed, on experimental evidence, that the function ψ (see Table 1) may be expressed as a negative exponential involving constants of the rocks, $\psi(T-t) = B\,e^{-b(T-t)}$ (12)

Models 5 and 6 were specifically designed to render Q independent of the frequency in order to satisfy the results obtained from some of the laboratory experiments such as those by Kimball and Lowell (1927), Ide (1937, 8,000 cps.), Birch and Bancroft (1938, 120-4, 500 cps.).

Knopoff and Mac Donald (1958) have shown that the dissipative characteristics of many solids cannot be accounted for by any linear mechanism of attenuation. A particular model explored by them involves a nonlinear hysterisis loop resulting from nonrecoverable deformation at small stresses. The model, as they conclude, is by no means unique and other models involving some frictional dissipation could also account for the observation. Fortsch (1956) attributes the Coulomb Friction to be a mechanism for attenuation.

Lomnitz (1962) observed, "there is no evidence that the deformation of solids is governed by linear mechanism at the intercrystalline or molecular level, therefore a linear empirical relation for the creep function in such as Boltzmann's equation,

$$\varnothing(t) = q \log_e (1+at) \tag{13}$$

can be expected to fit only a restricted region of the strain-time continuum". He further emphasized the need for more detailed data and that it would seem premature to reach any conclusions or to introduce further refinements into the mathematical treatment.

However, in most measurements of this type the errors are relatively large i.e.. of the order of 20 percent. Bruckshaw and Mahanta in 1954, working in the frequency range of 40-120 cps., noted a small increase in Q with decreasing frequency, the gradient being greater at the lower frequencies. Usher in 1962 (2-40 cps.) noted a similar frequency dependence of Q by an improved apparatus reducing the error to within 10 percent. The variation of Q amounted to as much as 100 percent but most of it occurred between 2 and 10 cps., thus explaining the failure of other workers, using higher frequencies, to observe it. A brief review of laboratory measurements of internal friction in different rock types over different frequency ranges by various investigators is given in Table 2. Usher (1962) studied the effect of oil and water also on internal friction (Table 3).

The variation of Q in rocks in situ can be studied by two different means,

(i) by measuring the amplitude as a function of the distance, and

(ii) by examining the frequency spectrum of the wave at various distances.

The first type of experiments were performed by Evison (1951), Collins and Lee (1956) and Mac Donald (1959), and point to the frequency independence of Q. But these results are largely inconclusive.

Gutenberg (1935), Kendal (1941), and Ricker (1953), have followed the second line of approach using frequencies below 50 cps. and have found a linear relationship between Q and the time period. Born (1941), using freqencies in the range (200-4000 cps.) also concluded that Q was independent of frequency but varied when a certain amount of moisture was present. Internal friction would thus appear to have two components, one suggesting a solid friction type of mechanism and the other a viscous type. Usher's (1962) results are more relevent to the seismic wave propagation as they pertain to low frequencies encountered in seismic work. They suggest a solid viscous type of mechanism operating at low frequencies which are probably associated with the grain boundaries. However, more experimental work in the low frequency region under varying pressure would permit more realistic correlation between laboratory rock samples and the rocks in situ.

Table 4, contains a valuable set of field results giving absorption coefficient, k, and the corresponding internal friction (1/Q) for different waves with different periods. The data show that the absorption in the case of surface waves, varies considerably with period, being smaller for longer periods and vice versa. Karnik (1956) combined data for Rayleigh waves with periods of between 0.001 sec. (k=200 per kilometer) and 200 secs. (k=0.00002 per kilometer) and found that all data can be represented fairly well by the following expression,

$$k = 0.017\,(T)^{-1.42} \tag{14}$$

Furthermore, the value of the absorption coefficient depends on the path which the corresponding wave has travelled. Gutenberg (1945a) obtained the following value of k corresponding to 20 secs. period for different paths.

Continental path,	$k = 0.00016$
Around the earth or across the Pacific,	$k = 0.00030$
Along the boundary of the Pacific,	$k = 0.00050$

He attributes the loss of energy along different paths, just mentioned, to reflection and diffraction of waves along the part of the path which crosses and recrosses repeatedly the discontinuity between the Pacific and the continental structure. The G waves with wave lengths of several hundred kilometers, much in excess of the probable maximum depth at which there is a distinct difference in elastic contants and density between the material below the Pacific Basin and the surrounding continents, do not seem to show any similar loss of energy.

However, the available data seem to be very scanty and more data for the value of absorption coefficients corresponding to different periods and paths, would be required to extend the methods of magnitude determinations of Gutenberg and Richter for maximum amplitudes of surface waves corresponding to 20 secs., to that corresponding to other periods.

On the other hand the data for body waves show that $k = 0.00006$ is fairly constant over waves of different periods.

Equation (2) could now be further modified as follows in order to include the dissipative effects discussed above. Thus,

$$y = \text{KTZ} \sqrt{(F_1 \ldots F_n)\, E. \frac{\text{Cos i}}{\text{Sin e . Sin} \triangle}\left(\frac{di}{d\triangle}\right) . e^{-2k\triangle}} \qquad (15)$$

The distance along the wave path has been replaced by the epicentral distance $\triangle$, as this involves only a small error within tolerable limits, for most seismic waves which do not encounter the core mantle boundary.

The various quantities involved in equation (15) can be computed for various conditions. Thus the term $\left(\frac{di}{d\triangle}\right)$ is a function of velocity along the path, Z is a function of the angle of emergence and the Poisson's ratio just below the surface, and the F_n depends upon the angle of incidence at the discontinuity, the wave type, and the value of density and wave velocity on either side of the discontinuity. It is, therefore, possible to prepare tables of Z and F_n for given types of waves and discontinuities.

5. Dispersion

The change of wave velocity with period is known as dispersion. This phenomenon in fact does not lead to any dissipation of energy, but to a lengthening of the pulse. The life-time of a pulse undergoing dispersion is thus increased at the expense of its amplitude. We may consider the dispersion of surface waves and body waves separately.

SURFACE WAVE DISPERSION

Whenever the velocity of seismic waves changes with depth h in the earth, the surface waves undergo dispersion, as the velocity of surface waves depends on λ/h where λ is the wave length.

The equation for the velocity of Rayleigh waves, propagating along the free surface, of the earth regarded as plane, is given by,

$$\frac{C^6}{\beta^6} - 8\frac{C^4}{\beta^4} + C^2\left(\frac{24}{\beta^2} - \frac{16}{\alpha^2}\right) - 16\left(1 - \frac{\beta^2}{\alpha^2}\right) = 0 \qquad (16)$$

where α, β, and C are respectively the longitudinal, transverse and the Rayleigh wave velocities in the medium.

The expression for C does not show any dispersion effect, but the earth is not homogeneous as is assumed here. The theory of Rayleigh waves in the presence of two and three layers have been worked out by Stoneley and Tillotson, and the equations for velocity show that the waves are dispersed.

The equation for the velocity C' of Love waves in the presence of a surface layer with transverse wave velocity β' overlying a homogeneous medium with transverse wave velocity β, so that $\beta' < C' < \beta$, is given as:

$$\mu\left(1 - \frac{C'^2}{\beta^2}\right)^{\frac{1}{2}} - \mu'\left(\frac{C'^2}{\beta'^2} - 1\right)^{\frac{1}{2}} . \tan\left[KH'\left(\frac{C'^2}{\beta'^2}\right)^{\frac{1}{2}}\right] = 0 \qquad (17)$$

where $K = \frac{2\pi}{\lambda}$, λ being the wave length.

Equation (17) shows that the velocity depends on K i.e., the wave length and hence on the period, causing dispersion of Love waves.

The disturbance, in surface waves, propagates in groups of waves which tend to become sinusoidal as the travel time T and the epicentral distance $\triangle$ increase, the periods being approximately the same at different stations for groups whose group velocity $C = \frac{\triangle}{T}$ is the same. The group velocity C in terms of phase velocity C′ is given as

$$C = C' - \lambda \left(\frac{dC'}{d\lambda} \right) \tag{18}$$

The surface wave equations as cited above and the more generalized ones may be used to fit the broad features of the observed dispersion of earlier arriving surface waves. The equation (17) for the velocity of Love waves entails a sinusoidal dispersion of an initially confined disturbance.

Jeffreys (1959, page 39), has given an approximate expression for the displacement caused by a surface wave undergoing dispersion. The main features of the expression include a factor common to all wave groups which will cause the ratios of the amplitudes of waves of given periods to be constant on the Earth's outer surface. However, the amplitude would vary according to the wave group, the nature of the initial disturbance and that of dispersion.

BODY WAVE DISPERSION

The cause of irregular dispersion, of body waves or more correctly of their oscillatory motion, is not yet clearly understood. Jeffreys has considered the effects of scattering, the complex initial conditions at the focus, fluctuations in the local gravity value during the passage of a disturbance, imperfections in elasticity and the departures from homogeneity within the earth. If the original disturbance at the focus is assumed to be oscillatory, it would explain oscillations in P and S, but in such a case the duration of the oscillations should be the same for all distances whereas it is actually observed to increase with distance. The cause of bodily wave dispersion must therefore be sought in terms of heterogeneities inside the earth, most probably in the outermost 40 km., but the precise way in which it occurs must await more detailed knowledge of the crustal structure than is yet available. The following suggestions seem to hold promise:

(i) diffuse refraction at irregular interfaces in the crustal layers,

(ii) subsidiary waves arising from the fact that the travel times of reflected waves, though stationary are not true minima,

(iii) The longitudinal and transeverse wave velocities have been derived on the assumption that the density ρ, compressibility k' and rigidity μ are constant.

But these conditions do not hold rigorously as they are functions of initial strain, temperature and chemical composition. To a first approximation we may take ρ, k' and μ as slowly varying functions of depth between a limited number of surfaces of discontinuity within the earth. Since in the derivation of the equation of motion, viz.,

$$\rho f_i = \left(k' + \frac{\mu}{3}\right) \frac{\partial \theta}{\partial x_i} + \mu \nabla^2 u_i \tag{19}$$

terms involving $\frac{\partial k'}{\partial x_i}$ and $\frac{\partial \mu}{\partial x_i}$ are assumed to be zero, which no longer holds if k' and μ are taken as variables, additional terms would appear in the wave equations,

$$\frac{\partial^2 \theta}{\partial t^2} = \left(\frac{k + \frac{4}{3}\mu}{\rho}\right) \nabla^2 \theta \tag{20}$$

$$\frac{\partial^2 \xi}{\partial t^2} = \left(\frac{\mu}{\rho}\right) \nabla^2 \xi \tag{21}$$

The result would be a dispersion of body waves analogous to the dispersion of surface waves. Such an effect would be pronounced if longitudinal and transverse wave velocities vary significantly over distances comparable with the predominating wavelengths of the disturbance, and such changes in both horizontal and vertical components are most likely to occur in the crust of the earth (outermost 40 km) and probably contribute to the observed dispersion of body waves. The details are not yet clearly understood. In the mantle, except perhaps in the neighbourhood of a limited number of discontinuities ρ, k' and μ change sufficiently slowly and the dispersion effects may be neglected.

CONCLUSION

Damage to structures is mainly caused by the horizontal component of the ground acceleration. The longitudinal waves are, therefore, relatively harmless to structures as the resulting acceleration is mainly in the vertical direction and hardly ever exceeds that due to gravity, which the structures are normally able to withstand. We are thus, chiefly interested in the various attenuating effects on shear waves. These will include the bodily transverse waves and the surface waves. The ground acceleration depends both upon the frequency of vibration and its amplitude. At greater distances from the epicentre the surface waves inspite of their longer periods have greater effect as they lose their energy less rapidly by divergence. It is also commonly assumed that both the effects of scattering and of internal friction are more significant for shear waves than for the compressional waves as the stresses involved are deviatoric rather than symmetrical. However, it is extremely difficult to give a quantitative definition of these effects particularly of scattering in the crust which widely varies in character.

Furthermore, the amplitudes of ground motion arising from nearby shocks are of great interest to Earthquake Engineers. But no general theory can be produced for these owing to, extremely complicated conditions obtaining near the source. The amplitudes of near earthquake phases, however, seem to fall something like the inverse of distance, with an irregular variation superposed on it. An estimation of this latter effect seems to be fraught with difficulties at this stage owing to our lack of clear understanding of the actual mechanisms of the Earthquakes involved.

So far the general processes which contribute to the loss of energy in a progressive wave have been outlined. However, for particular cases additional terms would force consideration. Thus, while discussing the divergence effect, it was assumed that the energy is transmitted equally in all directions. Besides the anisotropy of the region surrounding the focus, the nature of the source and the manner of energy release may prevent the equipartition of energy by all azimuths.

Furthermore, it has been observed that the amplitudes of waves (Kazim Ergin, 1953) reflected at the mantle core boundary show anomalous behaviour i.e., they have larger amplitudes than could be predicted theoretically. This may be caused by a particular type of anisotropy which, however, lacks a general theory.

Similarly, while computing the amplitudes of surface waves it would be essential to consider additional factors which may arise from circumstances such as the linear extension of the focus, and from the occurrence of disturbances under varying depths of the oceans.

APPENDIX

An expression for the ground displacement at an epicentral distance Δ due to energy propagated in body waves can be obtained under simplified conditions. These are,

(i) that the Earth is spherically symmetrical about its centre,

(ii) that the conditions of ray theory obtain, and

(iii) that the surfaces of discontinuity are distant enough from the source to permit the application of plane wave theory of reflection and refraction.

Let us consider a group of waves which travel outwards from F in all directions between the angles i and (i+di) forming a conical shell (see figure 1) which will subtend a solid angle equal to $\frac{2\pi r^2 \text{Cos } i \, di}{r^2}$

If E be the energy passing through unit solid angle, the total energy in the shell will be equal to $2\pi E \text{ Cos } i \, di$.

If the earth is treated as being spherically symmetrical and FO, a polar axis (figure 2) these group of waves will emerge on the surface of the earth bounded by the colatitudes Δ and $(\Delta+d\Delta)$. Using spherical coordinates, the area of this region can be readily seen to be

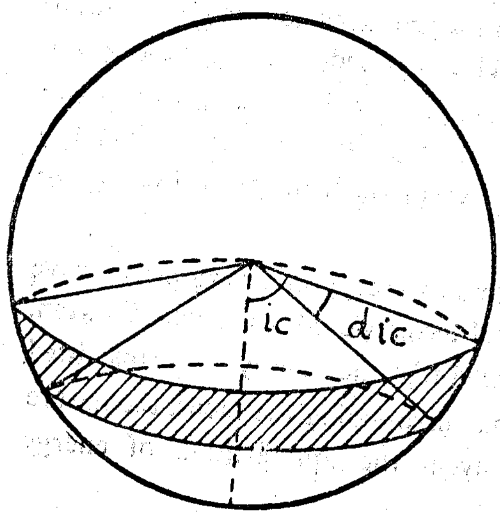

Figure 1

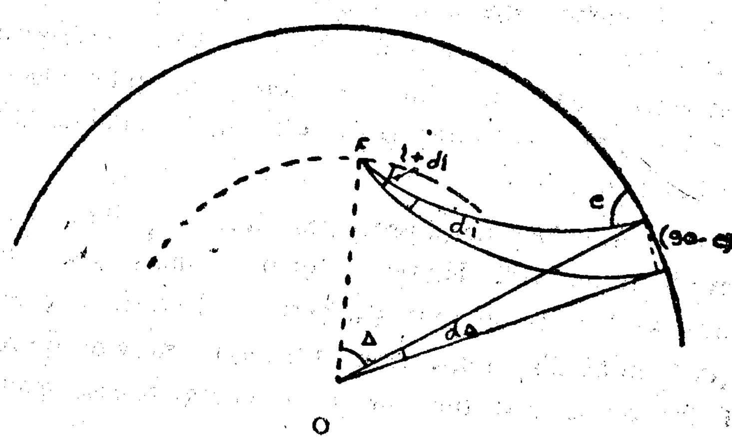

Figure 2

equal to $2\pi R^2 \text{ Sin } \triangle d\triangle$ and that of the emergent wavefront to $2\pi R^2 \text{ Sin } \triangle d\triangle . \text{ Sin e}$. Therefore the energy per unit area of the wavefront emergent at E can be written as

$$\frac{E \text{ Cos i di}}{R^2 . \text{ Sin e Sin } \triangle \text{ d}\triangle} \qquad \text{(A—1)}$$

It can be further shown that the mean energy per unit wavelength of a train of waves is directly proportional to the square of its amplitude and inversely proportional to the amplitude of its period and therefore,

$$A = \frac{T}{C} \sqrt{\frac{E \text{ Cos i}}{R^2 \text{ Sin } \triangle \text{ Sin e}} \left(\frac{di}{d\triangle} \right)} \qquad \text{(A—3)}$$

where, C is a constant.

If K denotes another constant determining the fraction of the energy, E, actually passing into the wave under consideration (P, SV, or SH) and Z the ratio of ground displacement to the wave amplitude, we can write the resultant ground displacement y as,

$$y = K \, T \, Z \sqrt{\frac{E \text{ Cos i}}{R^2 \text{ Sin } \triangle \text{ Sin e}} \left(\frac{di}{d\triangle} \right)} \qquad \text{(A—3)}$$

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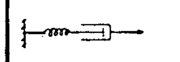
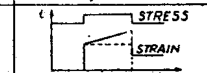
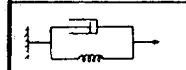
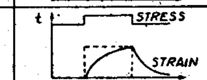
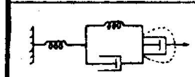
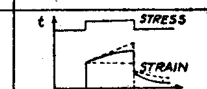
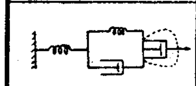
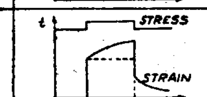
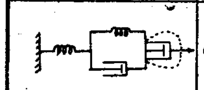
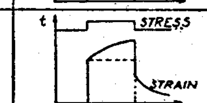
TABLE 1

Model	Strain-Stress relation	MECHANISM	EFFECT UPON $\frac{\Delta E}{E} \propto \epsilon$	STRAIN TIME GRAPH
1. Maxwell : (elasticoviscosity)	$2\mu\ \dot{e} = \dot{p} + p/\tau$		ϵ = CONSTANT AT L F $\epsilon \propto \frac{1}{f}$ AT H F	
2. Voigt : (firmoviscosity)	$e + \tau \dot{e} = p/\mu$		$\epsilon \propto f$ AT L F ϵ = CONSTANT AT H F	
3. Solido–fluid :	—	VISCOUS COMBINATION OF SPRING AND DASHPOTS OF VARIABLE RELATIVE SIZE	$\epsilon \propto f$ AT L F $\epsilon \to 0$ (LARGE τ)	$\tau \to \infty$ (IDEAL SOLID) $\tau \to 0$ (IDEAL FLUID)
4. Boltzmann : (creep function, ϕ and memory function, ψ)	$e(\tau) = \frac{1}{\mu}\left[p(\tau) + \int_{-\infty}^{t} p(t)\,\dot{\phi}(\tau - t)\,dt\right]$, $p(\tau) = \mu\left[e(\tau) - \int_{-\infty}^{t} e(t)\psi(\tau - t)dt\right]$	A COMBINATION OF SPRINGS AND DASHPOTS	NOT GENERALLY SOLUBLE THE GENERAL EFFECT IS TO PRODUCE A MAXIMUM AT SOME FREQUENCY	
5. Sokoloff and Scriabin : Simplification of Boltzmann's memory function.	$\psi = B\,e^{-b(\tau - t)}$ in Boltzmann's relations, where B, b are constants.		$\epsilon \to 0$ AT V. L. F. $\epsilon \to 0$ AT V.H.F.	
6. Lomnitz : Simplification of Boltzmann's creep function.	$\phi = q \log (1 + a\,t)$ in Boltzmann's relations, q, a are constants.		ϵ = CONSTANT	
7. Jeffreys : Modification of Lomnitz's law of creep in Rocks.	$\phi(t) = \frac{q}{\alpha}\left[\left\{1 + a\,t\right\}^{\alpha} - 1\right]$		ϵ = CONSTANT	
8. Knopoff : Solid Friction.	Dissipative forces proportional to applied forces.		ϵ = CONSTANT	
9. Thermal Mechanisms :	—	THERMAL ISOTHERMAL AND ADIABATIC	MAX ϵ AT V.L.F.	

Not:—The functions ϕ and ψ in models 4, 5, 6 and 7 are related as follows:

$$L(\psi) = L(\dot{\phi}) / [1 + L(\dot{\phi})],$$

where $L(\psi)$ is the Laplace Transform of ψ.

TABLE 2

Internal Friction for Different Kinds of Rocks in Different Frequency Ranges.

Rock	Investigator	Frequency Range (Cycles per sec.)	Internal Friction $\frac{1}{Q} \times 10^{-3}$	$\frac{\Delta E}{E}$
Sylvan Shale	Born	3000–12,000	14.3	0.09
Hunton Limestone	„	2000–10,000	14.3	0.09
Amherst Sandstone	„	900–4000	17.5	0.11
Cockfield Yequa	„	3000–11,000	14.3	0.09
Solenhofen Limestone	Birch and Bancroft	about 10,000	6.7	0.042
Slate (Pa)	„	„ „	3.8	0.024
Quincy Granite	„	„ „	10.0	0.063
Rockport Granite	„	„ „	7.7	0.048
Diabase	„	„ „	1.7	0.011
Quincy Granite	„	140 – 1600	5.0–10.0	0.03–0.06
(Very soft) Sandstone	Bruckshaw and Mahanta	40 – 120	47.8	0.3
Oolite	„	„ „	23.9	0.15
Shelly Limestone	„	„ „	15.9	0.10
Granite	„	„ „	15.9	0.10
Dolerite	„	„ „	9.6	0.06
Diorite	„	„ „	7.2	0.045
Chalk	Evison (seismic)	600	12.7	0.08
Various Sedimentary Rocks	Born „	Seismic	12.7–19.1	0.08–0.12
„ „	Howell „	„	—	—
Pierre Shale	MacDonal „	500	7.7	0.048

(Contd.)

			Average 1/Q (2 c/S)	Average 1/Q (40 c/S)	Average △E/E (2 c/S)	Average △E/E (40 c/S)
Dolerite	Usher	2 – 40	2.2	5.4	0.014	0.034
Diorite	,,	,, ,,	2.4	5.1	0.015	0.032
Old Red Sandstone	,,	,, ,,	8.0	13.5	0.050	0.085
Micaceous Sandstone	,,	,, ,,	5.6	11.8	0.035	0.074
Oolite Limestone	,,	,, ,,	3.2	6.7	0.020	0.042
Slate	,,	,, ,,	1.9	4.1	0.012	0.026
Chalk	,,	,, ,,	5.6	9.1	0.035	0.057
Shelly Limestone	,,	,, ,,	1.6	3.8	0.010	0.024
Miles tonegrit	,,	,, ,,	5.91	23.9	0.10	0.15
Tufnol	,,	,, ,,	7.9	9.5	0.05	0.06
Wood	,,	,, ,,	6.4	7.0	0.040	0.044
Asbestos	,,	,, ,,	3.8	8.9	0.024	0.056

TABLE 3

Effect of Water and Oil on Internal Friction.

Rock	Saturation Moisture (by weight)	40 C/S, dry		40 C/S, Saturated	
		$\frac{\Delta E}{E}$	$\frac{1}{Q} \times 10^{-3}$	$\frac{\Delta E}{E}$	$\frac{1}{Q} \times 10^{-3}$
Dolerite	0.25%	not measurable			
Oolitic Limestone	5%	0.032	5.4	0.06	9.6
Millstone grit	9%	0.14	22.3	0.30	47.8
Micaceous Sandstone	1.0%	0.08	12.7	0.32	50.9
Old Red Sandstone	4.0%	0.07	11.1	0.24	38.2
Old Red Sandstone(OIL)	3.0%	0.07	11.1	0.41	65.3

TABLE 4

Absorption Coefficient (k) for Amplitudes, and the corresponding internal friction ($\frac{1}{Q}$) for different waves with different periods, obtained from the earthquake Records.

Reference	Wave Type	Period (Sec)	Absorption Coefficient K (km^{-1}) $\times10^{-6}$		Internal Friction $\frac{1}{Q}$ $\times10^{-3}$	
(a) Surface Waves						
Gutenberg (1924)	Love	100 ±	100		14.29	
			New Guina	Kamchatka	New Guina	Kamchatka
Sato (1958)	G	360	23	28	13.5	19.0
„ „	G	216	25	31	8.5	12.0
„ „	G	108	58	35	9.0	11.0
„ „	G	72	77	71	8.0	8.0
„ „	G	54	79	—	6.0	—
„ „	G	43	75	—	4.0	—
Press, Ben-Menaham and	G	400	—		10.95	
Toksöz (1961)	G	200	—		10.52	
„ „	G	100	—		8.19	
Bath and A.L. Arroyo (1962)	G	300	28.2		13.76	
„ „ „	G	200	25.6		7.92	
„ „ „	G	150	34.9		7.90	
„ „ „	G	120	53.0		9.47	
„ „ „	G	100	69.4		10.25	
„ „ „	G	86	82.3		10.35	
„ „ „	G	75	88.7		9.72	
Gutenberg (1945 a)	Rayleigh	20	200		5.0	
Ewing and Press (1954 a)	Rayleigh	215	22		6.65	
„ „ „	Rayleigh	140	36		6.73	
Ewing and Press (1964 b)	Rayleigh	250-350	8		4.10	
Bath (1958)	Rayleigh	120-260	—		4.80	
Press, Ben-Menahem, and Toksöz (1961)	Rayleigh	400	—		6.96	
„ „	Rayleigh	200	—		5.72	
„ „	Rayleigh	100	—		8.92	
(b) Body Waves						
Gutenberg (1945 b)	P,PKP	4 ±	60		.8	
Gutenberg (1958)	P,PP	2	60		.4	
„ „	P,PP	12	60		2.5	
„ „	S	12	60		1.4	
„ „	S	24	60		2.5	
Press (1956)	S	11	(90)		2.0	
Benioff Press and Smith(1961)	Free Oscillations	375–500	—		5.7	
			—		7.5	
„ „ „	„	666				
Jeffreys (1959)	Nutation	37×10^6	—		25.0	