Paper No. 222, Bull. ISET Vol. No. 19, 3 September 1982 pp. 91–101

DYNAMIC ANALYSIS OF PLANAR STRUCTURES USING DIRECT STEP-BY-STEP EXPLICIT INTEGRATION

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INTRODUCTION

The determination of dynamic response of structures subjected to dynamic loads is an important step in the design of structures. A large number of structures can be idealized as an assemblage of discrete beam elements or beam and truss elements in a plane or space. The combination of such elements in a plane form planar structures like multistorey frames, trusses, bridges, arches and dams etc. The idealization of structure as an assemblage of beam/truss elements with the masses concentrated at the connecting nodes is commonly termed as lumped mass model or mathematical model. The formulation of a mathematical model of a structure is an important step in the dynamic analysis and requires some judgement by the analyst.

Another important aspect of dynamic analysis is the method of analysis. The commonly used methods of dynamic analysis are (i) mode superposition method, (ii) direct integration method and (iii) frequency domain method. Which method should be used for dynamic analysis is dependent upon the type of the problem, objective of the analysis and sometimes on the preference of the analyst. The mode superposition method using response spectrum is the most common approach for determining earthquake response of structural problems. The method is applicable to linear problems only and is suitable for structures which respond in lower frequencies of vibration. The direct step-by-step integration methods are suitable when large number of modes participate in the response and in the high frequency dominant problems. The frequency domain or complex response method of dynamic analysis is convenient for problems requiring substructuring. Any of the methods outlined above could be employed for linear problems. However direct integration methods are sometimes preferred because of its broader application and the possibility of their extension to nonlinear problems.

This paper presents the application of explicit integration method in the dynamic analysis of planar structures subjected to earthquake

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motion. The main objectives of the study are (i) implementation of central difference scheme in plane frame pragram (ii) testing suitability of explicit integration in earthquake response determination problems-to determine whether any spurious response would occur and (iii) application to different planar problems like multistoreyed frame, cable stayed bridge, arches and dams.

DIRECT STEP-BY-STEP INTEGRATION

There are two types of alogrithms[1,2,7] that can be basically identified in direct integration methods (i) explicit integration method-requiring no solution of simultaneous equations, and (ii) implicit integration method requiring solution of simultaneous equations. The comparative merits and demerits of the two integration methods are given in Table 1. The earthquake response problems are generally low frequency dominant problems and are usually solved by implicit integration methods because of reasons (5), (6) and (7) given under implicit integration in the Table 1. Not many attempts have been made in the application of explicit integration methods for the solution of earthquake response problems. The purpose of this study was to employ explicit integration method-central difference scheme in the earthquake response problems of structures and to examine its adequacy. The explicit integration methods have some advantages over the implicit integration such as outlined in item 8 and 9 in Table 1. Particularly the explicit methods need very simple computer programming and require very small computer storage. Very large size problems can be easily handled with this integration. There is always a fear of undersirable high frequency response to be picked up when explicit integration methods are employed. This aspect can be studied by solving the different problems and examining dominating frequencies in time history response. The earthquake response problem by explicit integration is shown by a block diagram in Fig. 1.

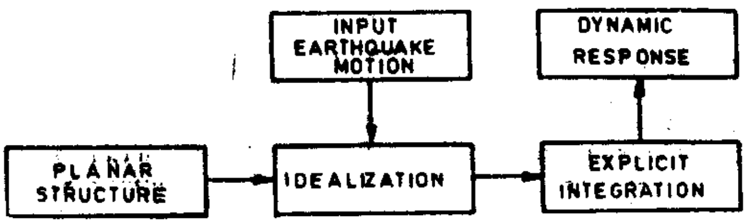

Fig. 1. Earthquake Response Problem by Direct Bxplicit Integration

EXPLICIT INTEGRATION

The equations of motion at time t can be written as,

$$M\ddot{u}_t + C\dot{u}_t + Ku_t = R_t \quad (1)$$

TABLE 1—Comparative Merits and Demerits of Implicit and explicit Integration

No.	Implicit Integration	Explicit Integration
1.	For the evaluation of dynamic response at time $t+\Delta t$, the equilibrium equation is satisfied at time t	For the evaluation of dynamic response at time $t+\Delta t$, the equilibrium equation is satisfied at time $t+\Delta t$
2.	Assembly of stiffness matrix is required	Assembly of stiffness matrix is not required
3.	Solution of simultaneous equations are necessary	Simultaneous equations are not required to be solved
4.	Mass and damping matrix could be diagonal or consistent	Mass and damping matrix must be diagonal
5.	Time step size is large. Integration method could be unconditionally stable, also time step is independent of the size of elements.	Time step size is very small. Integration method is conditionally stable. Δt should be less than critical time step Δt_{cr}. Time step is dependent on the size of the elements.
6.	Numerical damping is controllable. High frequency response can be eliminated by numerical damping.	Numerical damping is usually small and cannot be controlled. High frequency response cannot be eliminated by numerical damping.
7.	Computationally economical for structural problems subjected to low frequency loading such as earthquake.	Computationally economical for structural problems with high frequency response dominant and wave propagation problems.
8.	Computer storage requirements are larger	Computer storage requirements are very small.
9.	Applicable to both linear and nonlinear problems	Applicable to both linear and nonlinear problems. Economical for strongly nonlinear problems such as involving both material and geometric nonlinearity.
10.	Computer time for linear structural problems is much smaller than explicit method.	Computer time for linear structural problems is usually larger than implicit method.

where, M and C are mass and damping matrices, K is the stiffness matrix of plane beam element. R_t is a vector of externally applied dynamic forces which for ground motion is—$M\ddot{y}$.u_t, $\dot{u}_t$, $\ddot{u}_t$ are displacement, velocity and acceleration vectors. The acceleration $\ddot{u}_t$ and velocity $\dot{u}_t$ in equation (1) can be replaced by the following central difference formulae,

$$\ddot{u}_t = \frac{1}{\Delta t^2}(u_{t-\Delta t} - 2u_t + u_{t+\Delta t} \tag{2}$$

$$\dot{u}_t = \frac{1}{2\Delta t}(-u_{t-\Delta t} + u_{t+\Delta t}) \tag{3}$$

Substituting (2) and (3) in equation (1), we obtain the following equation.

$$M^* u_{t+\Delta t} = R^* \tag{4}$$

where

$$M^* = \frac{M}{\Delta t^2} + \frac{C}{2\Delta t}$$

$$R^* = R_t - P_t - \frac{2M}{\Delta t^2} u_t + \left(\frac{C}{2\Delta t} - \frac{M}{\Delta t^2}\right) u_{t-\Delta t} \tag{5}$$

Equation (4) can be solved for $u_{t+\Delta t}$ directly by knowing R_t, $P_t = Ku_t$, and displacement u_t and time t and $t - \Delta t$. The scheme requires a special starting procedure at $t = 0$, because right hand side, R^* in equation (5) requires $u_{t-\Delta t}$.

It should be noted that explicit calculation of $u_{t+\Delta t}$ is possible from equation (4) without requiring solution of simultaneous equations. The assembly of stiffness matrix is not required at any stage of time stepping. The main calculation at every time step consists in working out R_t and P_t and assembly of these nodal forces vector. The mass and damping matrices should be obviously diagonal for explicit solution of equation (4) to be possible. The diagonal damping matrix is possible if mass proportional damping is adopted,

$$C = \alpha M \tag{6}$$

The value of α can be obtained by prescribing damping in first mode of vibration.

STABILITY OF EXPLICIT INTEGRATION

The explicit integration is a conditionally stable scheme. The time step of integration should be smaller than the critical time step Δt_{cr}. The critical time step can be approximately obtained from the transit time of longitudinal wave between the two nearest nodes,

$$\Delta t_{cr} = \frac{L}{C_p} \tag{7}$$

where, L = shortest distance between the two nodes and C_p = velocity of propagation of longitudinal wave.

STARTING PROCEDURE

With u_0, $\dot{u}_0$ and $\ddot{u}_0$ as known, $u_{-\Delta t}$ can be obtained from the equations (2) and (3) as,

$$u_{-\Delta t} = u_0 - \Delta_t \dot{u}_0 + \frac{\Delta_t^2}{2} \ddot{u}_0 \tag{8}$$

$\ddot{u}_0$ can be obtained from equation (1) at $t = 0$ as,

$$\ddot{u}_0 = M^{-1} (R_0 - Ku_0 - C\dot{u}_0) \tag{9}$$

Substituting $\ddot{u}_0$ from equation (9) in (8) we obtain,

$$u_{-\Delta t} = u_0 - \Delta t \, \dot{u}_0 + \frac{\Delta_t^2}{2} M^{-1} (R_0 - Ku_0 - C \, \dot{u}_0) \tag{10}$$

IMPLEMENTATION OF EXPLICIT SCHEME IN PLANE FRAME PROGRAM

The following steps of calculation will be required in the implementation of central difference scheme in plane frame program,

(i) Initialize u_0, $\dot{u}_0$, $\ddot{u}_0$

(ii) Form stiffness matrix K, for each element, store in unassembled form

(iii) Form mass matrix M, for each element and assemble, store it in assembled form

(iv) Work out $u_{-\Delta t}$, at $t = 0$ from equation (10)

(v) For every time step, work out $R_t = - M\ddot{y}$, for ground motion problem

(vi) For every time step, work out, $P_t = Ku_t$,

(vii) Assemble P_t forces, in all the elements

(viii) Form right hand side R^* in equation (4)

(ix) Solve for displacements from equation (4)

(x) Repeat steps (v) to (ix), for every time step, for complete time history.

A plane frame computer program with these steps is written for earthquake response of planar structures.

NUMERICAL EXAMPLES

The numerical examples of a two storey frame, circular arch, gravity dam and cable stayed bridge are chosen to demonstrate the application of explicit integration in determination of earthquake response. The structures are assumed to be assemblage of beam elements[4,6]. The beam element, Fig. 2, is assumed to have three degree of freedom at every node. The axial, shearing and bending deforma-

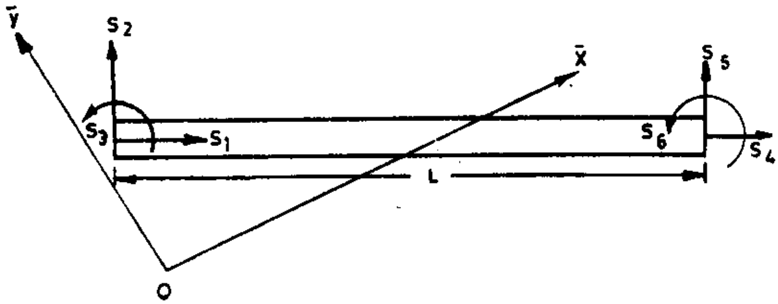

Fig. 2. Beam element with axial, shear forces and bending moment for two dimensional Structures

tions are included in 6×6 stiffness matrix of beam element[6]. The lumped masses of diagonal mass matrix are obtained on the basis of physical lumping of mass half and half of element at each node. The rotary inertia of the element ρIh ($\rho =$ mass density, $I =$ moment of inertia, $h =$ length of the element) is also lumped half and half at the nodes. The critical time step and the time step adopted in different problems are given in Table 2. The $N - S$ component of May 18, 1940 Elcentro earthquake is applied at the base of structure in every case. Time history response of displacement, bending moment, axial force and shear force are obtained at different sections of the structure.

RESULTS OF DYNAMIC ANALYSIS

Two storey frame: Figure 3 shows the time history response of a two storeyed building frame. Fig. 3b shows deflected shape at a specific time. The displacement, shear force and bending moment time histories indicate the dominance of fundamental mode. There are no high frequencies seen in the response.

Circular arch: The earthquake response of a 100 m fixed circular arch is shown in Fig. 4. The deflected shape at a specific time, displacement, axial force, shear force and bending moment response are shown in Fig. 4 b, c, d, e and f. The participation of higher modes is clearly visible in axial force, shear force and bending moment response. But no spurious frequencies (very high frequencies) are present in the

TABLE 2

Critical Time Step and Actual Time Step in Different problems.

Structure	Critical time step $\Delta t = \frac{L}{C_p}$ (Second)	Δt adopted (Second)
Two bay frame Fig. 3	0.0015	0.001
Circular Arch Fig. 4	0.0018	0.001
Gravity dam Fig. 5	0.0014	0.001
Cable stayed Bridge Fig. 6	0.00151	0.001

response. This shows that earthquake motion excites those frequencies in the structure which lie in the frequency range of accelerogram only.

Gravity dam: Gravity dam idealized as an assemblage of beam element is shown in Fig. 5. The deflected shape, shear force and bending moment at a specific time are shown in Fig. 5 c, d and e. The time history of displacement, shear force and bending moment at a selected node/member are shown in Fig. 5f, g and h. The higher mode frequencies are visible in shear force and bending moment response. There are no spurious frequencies seen in the response.

Cable Stayed Bridge: Figure 6a shows the mathematical model of a cable stayed bridge consisting of assemblage of beam and truss elements. Fig. 6b shows deflected shape at a specific time which is of antisymmetric type. Fig. 6 c, d, e show time history response of displacements, shear force and bending moment. The high frequencies are seen in shear force and bending moment which is quite common in earthquake response of structures.

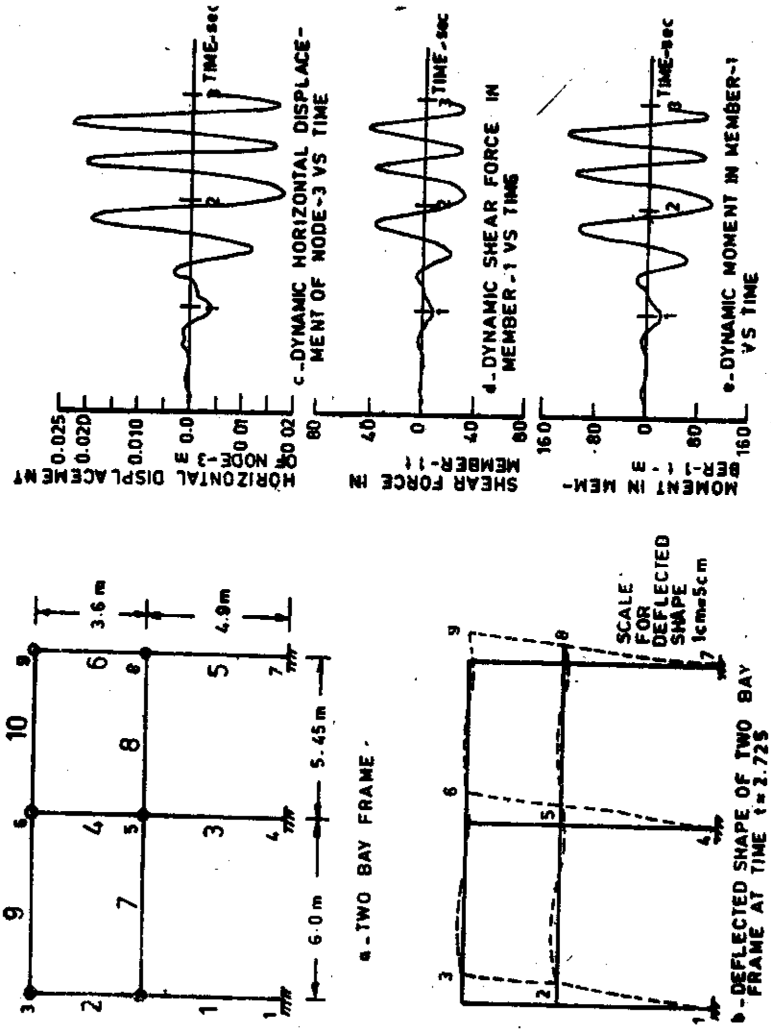

Fig 3. Dynamic Response of Two Bay Frame due to Elcentro Earthquake May 18, 1940 N-S Component

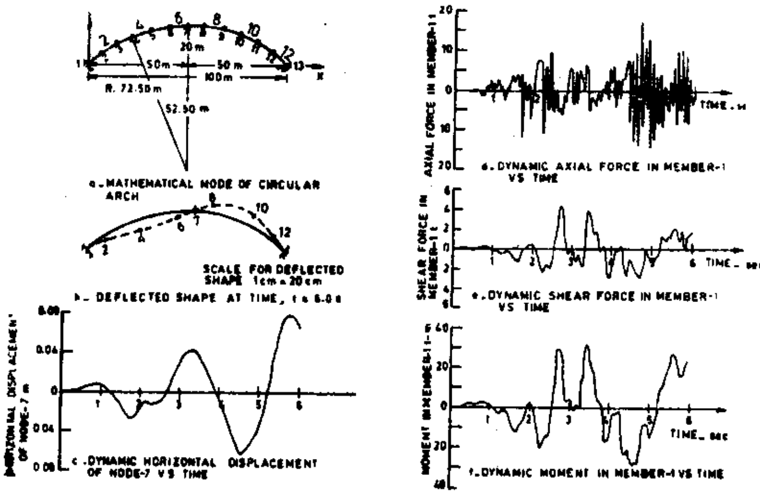

Fig. 4. Dynamic Response of Circular Arch due to Elcentro Earthquake May 18, 1940, N-S Component.

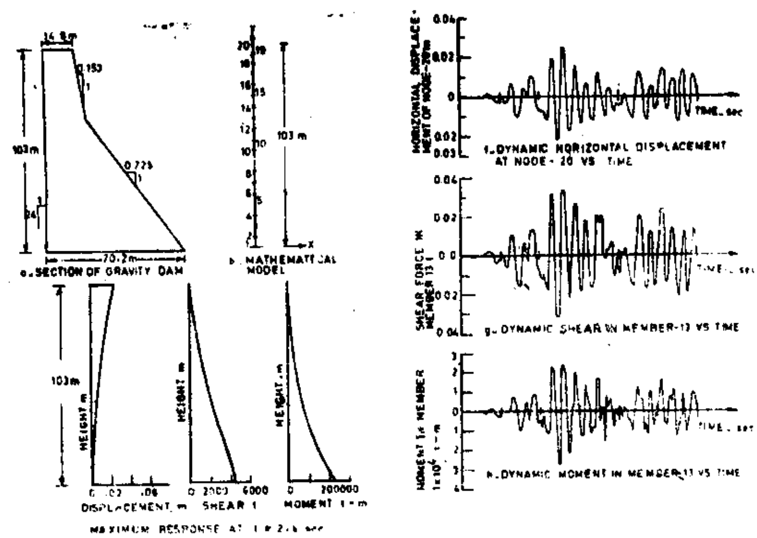

Fig. 5. Dynamic Response of Gravity dam due to Elcentro Earthquake May 18-1940, N-S component

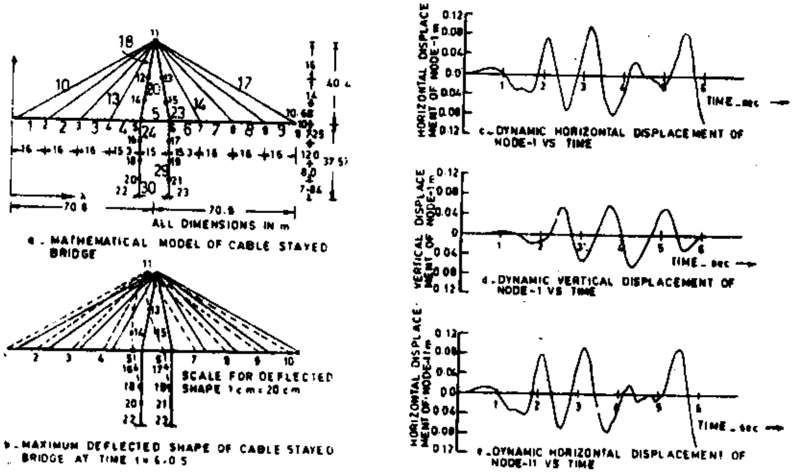

Fig. 6. Dynamic Response of Cable Stayed Bridge due to Elcentro Earthquake May 18, 1940 N-S Component

SUMMARY AND CONCLUSIONS

The application of explicit integration method—central difference scheme to planar structures is presented with special reference to earthquake problems. Steps to implement this method in plane frame program are described. The suitability of explicit integration to earthquake response problems of structures is demonstrated by solving several structural problems. The following main conclusions are derived from the above study.

1. Time history response of structures can be obtained reliably by direct step by step explicit integration method without giving rise to any spurious response.

2. The method is applicable to variety of planar structures. A single computer program can solve different types of structures built as an assemblage of beam elements.

3. The computer storage requirements are very small in explicit method. The method does not pose any problem in solving structures with large number of elements and nodes.

FURTHER DEVELOPMENTS

Application of explicit integration to two dimensional planar problems of earthquake response is established in this paper. There are further possibilities of extensions of explicit integration method to three dimensional space frame structures and nonlinear problems of earthquake response.

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