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EFFECT OF SMALL HUB RADIUS CHANGE ON FREQUENCIES OF COUPLED VIBRATIONS OF A BEAM OF LINEARLY VARYING CROSS SECTION IN A CENTRIFUGAL FORCE FIELD

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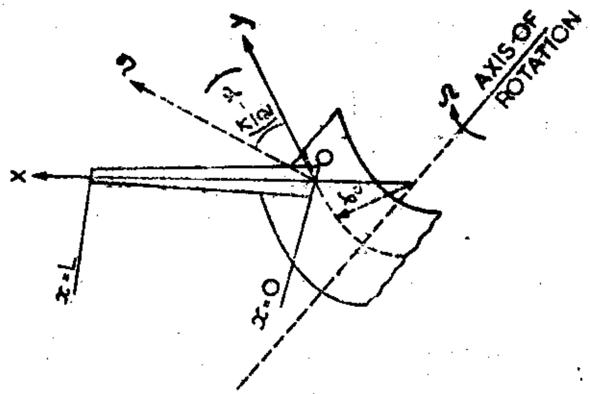
Introduction

The analysis presented in this paper considers vibration of a beam of linearly varying cross-section that could represent a turbine blade of simple geometry The shear centre of each cross-section does not coincide with the centre of gravity, consequently the torsional and bending oscillations are 'coupled'. The beam is attached to a hub of radius r, rotating at a constant angular velocity Ω, as shown in Fig. 1. The beam is allowed to vibrate in a plane making an angle $(\pi/2 - \psi)$ with the plane of rotation.

The frequencies of the coupled vibrations and the modal shapes of bending and torsional vibrations can be determined from the solutions of the following differential equations Tomar and Dhole (1975) with proper conditions :



oxy : Plane of Rotation oxv : Plane of Vibration

Fig. 1 The system under consideration

$$E \frac{\partial^2}{\partial x^2}\left(I_z \frac{\partial^2 v}{\partial x^2}\right) - \rho s_o \Omega^2 \left[\frac{d}{dx}\left(N \frac{\partial v}{\partial x}\right) + \sin^2 \psi \left(1 - \wedge \frac{x}{L}\right) v\right] + \rho s_x \frac{\partial^2}{\partial t^2}(v + x_\theta \theta) = 0$$

$$G \frac{\partial}{\partial x}\left(J_x \frac{\partial \theta}{\partial x}\right) - \frac{\partial}{\partial x}\left(C_x \frac{\partial^3 \theta}{\partial x^3}\right) - \rho s_x \, x_\theta \frac{\partial^2}{\partial t^2}(v + x_\theta \theta) - I\theta_x \frac{\partial^2 \theta}{\partial t^2} = 0 \quad (1)$$

$$N = \int_x^L (r+x)\left(1 - \frac{x}{L}\right)dx$$

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where EI_x, GJ_x and C_x are flexural, torsional and warping rigidities, respectively at the point x; ρ, s_x, $I_{\theta x}$ are density, area of cross section and mass moment of inertia about shear centre axis per unit length at the point x; x_θ is the distance between the shear centre axis and the centroidal axis at the point x.

For linearly varying cross section

$$I_x = I_0\left(1-\wedge\frac{x}{L}\right)^3, J_x = J_0\left(1-\wedge\frac{x}{L}\right), C_x = C_0\left(1-\wedge\frac{x}{L}\right)^5$$

$$S_x = S_0\left(1-\wedge\frac{x}{L}\right), I_{\theta x} = I_{\theta_0}\left(1-\wedge\frac{x}{L}\right)^3 x_\theta = x_\theta\left(1-\wedge\frac{x}{L}\right) \wedge = \left(1-\frac{S_L}{S_0}\right)$$

When the equations are put in dimensionless variable $\xi = x/L$ and the substitutions

$$\beta^2 = \frac{EI}{\rho s_0 L^4}, \alpha^2 = \frac{GJ_0}{\rho s_0 L^2}, C_1 = \frac{C_0}{\rho s_0 L^4}, I_\theta' = \frac{I_{\theta_0}}{\rho s_0}, \bar{r} = \frac{r}{L}$$

are used, the equations become

$$\beta^2\left[(1-\wedge\xi)^3\frac{\partial^4 v}{\partial\xi^4} - 6\wedge(1-\wedge\xi)^2\frac{\partial^3 v}{\partial\xi^3}\right] + \left[6\wedge^2\beta^2(1-\wedge\xi)\frac{\partial^2 v}{\partial\xi^2} - \Omega^2\right.$$

$$\left.\left\{\frac{d}{d\xi}\left(\bar{N}\frac{\partial v}{\partial\xi}\right)\right\} - \Omega^2(1-\wedge\xi)\sin^2\psi\, v + (1-\wedge\xi)\frac{\partial^2}{\partial t^2}(v + x_\theta\theta)\right] = 0$$

$$\alpha^2\left[(1-\wedge\xi)\frac{\partial^2\theta}{\partial\xi^2} - \wedge\frac{\partial\theta}{\partial\xi}\right] - C_1\left[(1-\wedge\xi)^5\frac{\partial^4 v}{\partial\xi^4} - 5\wedge(1-\wedge\xi)^4\frac{\partial^3\theta}{\partial\xi^3}\right]$$

$$-(1-\wedge\xi)x_\theta\frac{\partial^2}{\partial t^2}(v + x_\theta\theta) - I_\theta'(1-\wedge\xi)^3\frac{\partial^2\theta}{\partial t^2} = 0 \qquad (2)$$

$$\bar{N} = \int_\xi^1 (\bar{r} + \xi)(1-\wedge\xi)\, d\xi$$

The solution of the equations are of the form

$$v = A f(\xi) e^{i\omega t}$$

and

$$\theta = B \phi(\xi) e^{i\omega t} \qquad (3)$$

Where A and B are constants which are not independent; and $f(\xi)$ and $\phi(\xi)$ are functions of ξ only which are also respectively the mode shapes of bending and torsional vibrations.

$$\beta^2\left[(1-\wedge\zeta)^3\frac{d^4 f}{d\xi^4} - 6\wedge(1-\wedge\xi)^2\frac{d^3 f}{d\xi^3}\right] + [6\wedge^2\beta^2(1-\wedge\xi)]\frac{d^2 f}{d\xi^2} - \Omega^2\frac{d}{d\xi}$$

$$\left\{\bar{N}\frac{df}{d\xi}\right\} - \gamma^2(1-\wedge\xi)f - x_0(1-\wedge\xi)^2\omega^2\phi\frac{B}{A} = 0$$

$$\alpha^2\left[(1-\wedge\xi)\frac{d^2\phi}{d\xi^2} - \wedge\frac{d\phi}{d\xi}\right] - C_1\left[(1-\wedge\xi)^5\frac{d^4\phi}{d\xi^4} - 5\wedge(1-\wedge\xi)^4\frac{d^3\phi}{d\xi^3}\right] + (I_\theta' + x_0^2)$$

$$(1-\wedge\xi)^3\omega^2 \times \phi + x_0(1-\wedge\xi)^2\omega^2 f\frac{A}{B} = 0$$

$$\bar{N} = \int_{\xi}^{1} (\bar{r} + \xi)\,(1-\Lambda\xi)\,d\xi, \qquad \gamma^2 = (\omega^2 + \Omega^2 \sin^2 \psi) \tag{4}$$

For the beam clamped at the root the boundary conditions can be written as

$$f = \frac{df}{d\xi} = \phi = \frac{d^2\phi}{d\xi^2} = 0 \qquad \text{at} \quad \xi = 0$$

$$\frac{d^2f}{d\xi^2} = \frac{d^3f}{d\xi^3} = \frac{d\phi}{d\xi} = \frac{d^3\phi}{d\xi^3} = 0 \qquad \text{at} \quad \xi = 1 \tag{5}$$

When there is a small change in the hub radius from r to $r_1 = r + \Delta r$, while all other conditions remain the same equations (4) become

$$\beta^2 \left[(1-\Lambda\xi)^3 \frac{d^4 f_1}{d\xi^4} - 6\,\Lambda\,(1-\Lambda\xi)^2 \frac{d^3 f}{d\xi^3} \right] + 6\,\Lambda^2 \beta^2 (1-\Lambda\xi) \frac{d^2 f}{d\xi^2} - \Omega^2 \frac{d}{d\xi}$$

$$\left\{ \bar{N}_1 \frac{df_1}{d\xi} \right\} - \gamma_1^2 (1-\Lambda\xi)\, f_1 - x_0\,(1-\Lambda\xi)^2\,\omega_1^2\,\phi_1 \frac{B}{A} = 0$$

$$\alpha^2 \left[(1-\Lambda\xi) \frac{d^2\phi_1}{d\xi^2} - \Lambda \frac{d\phi_1}{d\xi} \right] - C_1 \left[(1-\Lambda\xi)^5 \frac{d^4\phi_1}{d\xi^4} - 5\,\Lambda\,(1-\Lambda\xi)^4 \frac{d^3\phi_1}{d\xi^3} \right]$$

$$+ (I_\theta' + x_0^2)\,(1-\Lambda\xi)^3\,\omega_1^2\,\phi_1 + x_0\,(1-\Lambda\xi)^2\,\omega_1^2\, f_1 \frac{A}{B} = 0 \tag{6}$$

$$\bar{N} = \int_{\xi}^{1} (\bar{r}_1 + \xi)\,(1-\Lambda\xi)\,d\xi, \quad \bar{r}_1 = \bar{r} + \frac{\Delta r}{L} = \bar{r} + \delta$$

$$\gamma_1^2 = (\omega_1^2 + \Omega^2 \sin^2 \psi)$$

Here ω_1 and f_1, ϕ_1 are the new frequency and mode shapes corresponding to new hub radius r_1 The boundary conditions are not affected by the change of hub radius. They remain the same as given by (5) except f and ϕ replaced by f_1 and ϕ_1.

When first and second equations of (4) are multiplied recseptively by f_1 and ϕ_1 and integrated from 0 to 1, one gets

$$\beta^2 \int_0^1 (1-\Lambda\xi)^3 \frac{d^4 f}{d\xi^4} f_1\, d\xi - 6\,\Lambda\,\beta^2 \int_0^1 (1-\Lambda\beta)^2 \frac{d^3 f}{d\xi^3} f_1\, d\xi + 6\,\Lambda^2 \beta^2 \int_0^1 (1-\Lambda\xi)$$

$$\frac{d^2 f}{d\xi^2} f_1\, d\xi + \Omega^2 \int_0^1 \bar{N} \frac{df}{d\xi} \frac{df_1}{d\xi} d\xi - \gamma^2 \int_0^1 (1-\Lambda\xi)\, f\, f_1\, d\xi - \omega^2 x_0$$

$$\frac{B}{A} \int_0^1 (1-\Lambda\xi)^2\, \phi\, f_1\, d\xi = 0$$

$$\alpha^2 \int_0^1 (1-\Lambda\xi) \frac{d^2\phi}{d\xi^2} \phi_1\, d\xi - \Lambda\,\alpha^2 \int_0^1 \frac{d\phi}{d\xi} \phi_1\, d\xi - C_1 \int_0^1 (1-\Lambda\xi)^5 \frac{d^4\phi}{d\xi^4} \phi_1\, d\xi$$

$$+ 5 \wedge C_1 \int_0^1 (1-\wedge\xi)^4 \frac{d^3\phi}{d\xi^3} \phi_1 \, d\xi + \omega^2 \left\{ (x_0^2+I_s') \int_0^1 (1-\wedge\xi)^3 \phi\phi_1 \, d\xi \right.$$

$$\left. + x_0 \frac{A}{B} \int_0^1 (1-\wedge\xi)^3 f \phi_1 \, d\xi \right\} = 0 \tag{7}$$

where
$$\int_0^1 \frac{d}{d\xi}\left(\bar{N}\frac{df}{d\xi}\right) f_1 \, d\xi = -\int_0^1 \bar{N}\frac{df}{d\xi}\frac{df_1}{d\xi}\, d\xi$$

The above identical relation can be verified by integrating by parts and making use of the boundary conditions. Similarly, when first and second equations of (6) are multiplied respectively by f and ϕ, and integrating from 0 to 1, one obtains

$$\beta^2 \int_0^1 (1-\wedge\xi)^3 \frac{d^4 f_1}{d\xi^4} f \, d\xi - 6\wedge\beta^2 \int_0^1 (1-\wedge\xi)^2 \frac{d^3 f_1}{d\xi^3} f \, d\xi + 6\wedge^2\beta^2 \int_0^1 (1-\wedge\xi) \frac{d^2 f_1}{d\xi^2} f \, d\xi$$

$$+ \Omega^2 \int_0^1 \bar{N_1} \frac{df}{d\xi}\frac{df_1}{d\xi} d\xi - \gamma_1^2 \int_0^1 (1-\wedge\xi) f f_1 \, d\xi - \omega_1^2 x_0 \frac{B}{A} \int_0^1 (1-\wedge\xi)^3 \phi_1 f \times d\xi = 0$$

$$\alpha^2 \int_0^1 (1-\wedge\xi) \frac{d^2\phi_1}{d\xi^2} \phi \, d\xi - \wedge \alpha^2 \int_0^1 \frac{d\phi_1}{d\xi} \phi \, d\xi - C_1 \int_0^1 (1-\wedge\xi)^5 \frac{d^4\phi}{d\xi^4} \phi \, d\xi$$

$$+ 5\wedge C_1 \int_0^1 (1-\wedge\xi)^4 \frac{d^3\phi_1}{d\xi^3} \phi \, d\xi + \omega_1^2 \left\{ (x_0^2+I_s') \int_0^1 (1-\wedge\xi)^3 \phi_1 \phi \, d\xi \right.$$

$$\left. + x_0 \frac{A}{B} \int_0^1 (1-\wedge\xi)^3 f_1 \phi \, d\xi \right\} = 0 \tag{8}$$

Subtracting equation (7) from the respective equation (8), we get

$$\beta^2 \int_0^1 (1-\wedge\xi)^3 \left[\frac{d^4 f_1}{d\xi^4} f - \frac{d^4 f}{d\xi^4} f_1\right] d\xi - 6\wedge\beta^2 \int_0^1 (1-\wedge\xi)^2 \left[\frac{d^3 f_1}{d\xi^3} f - \frac{d^3 f}{d\xi^3} f_1\right] d\xi$$

$$+ 6\wedge^2\beta^2 \int_0^1 (1-\wedge\xi) \left[\frac{d^2 f_1}{d\xi^2} f - \frac{d^2 f}{d\xi^2} f_1\right] d\xi + \Omega^2 \int_0^1 (\bar{N}_1 - \bar{N}) \frac{df}{d\xi}\frac{df_1}{d\xi} d\xi - (\gamma_1^2 - \gamma^2)$$

$$\int_0^1 (1-\Lambda\xi)\, ff_1\, d\xi - x_o \frac{B}{A}\left\{\omega_1^2 \int_0^1 (1-\Lambda\xi)^2 \phi_1 f\, d\xi - \omega^2 \int_0^1 (1-\Lambda\xi)^2 \phi f_1\, d\xi\right\} = 0$$

$$a^2 \int_0^1 (1-\Lambda\xi)\left[\frac{d^2\phi_1}{d\xi^2}\phi - \frac{d^2\phi}{d\xi^2}\phi_1\right] d\xi - \Lambda a^2 \int_0^1 \left[\frac{d\phi_1}{d\xi}\phi - \frac{d\phi}{d\xi}\phi_1\right] d\xi$$

$$-C_1 \int_0^1 (1-\Lambda\xi)^5 \left[\frac{d^4\phi_1}{d\xi^4}\phi - \frac{d^4\phi}{d\xi^4}\phi_1\right] \times d\xi + 5\Lambda C_1 \int_0^1 (1-\Lambda\xi)^4 \left[\frac{d^3\phi}{d\xi^3}\phi - \frac{d^3\phi}{d\xi^3}\phi_1\right] d\xi$$

$$+ (\omega_1^2-\omega^2)(x_0^2+I_e') \int_0^1 (1-\Lambda\xi)^3 \phi\,\phi_1\, d\xi + x_o \frac{A}{B}\left\{\omega_1^2 \int_0^1 (1-\Lambda\xi)^2 f_1 \phi\, d\xi - \omega^2 \int_0^1 (1-\Lambda\xi)^2 f \phi_1\, d\xi\right\} = 0 \qquad (9)$$

Assuming that for small changes of hub radius, the change of modal shape is small and can be neglected. Thus $f = f_1$ and $\phi = \phi_1$ which after substituting in eqn. (9) gives

$$\Omega^2 \delta \int_0^1 \left\{ \int_\xi^1 (1-\Lambda\xi)\, d\xi \right\}\left(\frac{df}{d\xi}\right)^2 d\xi - (\gamma_1^2-\gamma^2)\int_0^1 (1-\Lambda\xi)\, f^2\, d\xi$$

$$- (\omega_1^2-\omega^2)\frac{B}{A} x_o \int_0^1 (1-\Lambda\xi)^2 f\phi\, d\xi = 0$$

$$(\omega_1^2-\omega^2)\left\{ (x_0^2+I_e')\int_0^1 (1-\Lambda\xi)^3 \phi^2\, d\xi + x_o \frac{A}{B}\int_0^1 (1-\Lambda\xi)^2 f\phi\, d\xi\right\} = 0 \qquad (10)$$

Second of eqn. (10) gives

$$\frac{B}{A} = -\frac{x_o \int_0^1 (1-\Lambda\xi)^2 f\phi\, d\xi}{(x_0^2+I_e')\int_0^1 (1-\Lambda\xi)^3 \phi^2\, d\xi}$$

Substituting in the first of equ. (10)

$$\Omega^2 \delta \int_0^1 \left(\int_\xi^1 (1-\Lambda\xi)\, d\xi \right)\left(\frac{df}{d\xi}\right)^2 d\xi - (\gamma_1^2-\gamma^2)\int_0^1 (1-\Lambda\xi)\, f^2\, d\xi$$

$$+ (\omega_1^2-\omega^2)\, x_0^2 \frac{\left[\int_0^1 (1-\Lambda\xi)^2 f\phi\, d\xi\right]^2}{(x_0^2+I_e')\int_0^1 (1-\Lambda\xi)^3 \phi^2\, d\xi} = 0$$

$$\Omega^2 \delta \int_0^1 \left(\int_\xi^1 (1-\Lambda\xi)\, d\xi \right) \left(\frac{df}{d\xi} \right)^2 d\xi - (\gamma_1^2-\gamma^2) \left\{ \int_0^1 (1-\Lambda\xi)\, f^2\, d\xi \right.$$

$$\left. - \frac{x_0^2}{(x_0^2+I_0')} \frac{\left[\int_0^1 (1-\Lambda\xi)^2 f \phi\, d\xi \right]^2}{\int_0^1 (1-\Lambda\xi)^3 \phi^2\, d\xi} \right\} = 0$$

Since $\qquad \gamma_1^2 - \gamma^2 = \omega_1^2 - \omega^2$

or

$$\Omega^2 \delta\, k_1 - (\gamma_1^2-\gamma^2)\,(k_2-bk_3) = 0 \qquad (11)$$

where

$$k_1 = \int_0^1 \left(\int_\xi^1 (1-\Lambda\xi)\, d\xi \right) \left(\frac{df}{d\xi} \right)^2 d\xi, \qquad k_2 = \int_0^1 (1-\Lambda\xi)\, f^2\, d\xi \qquad (12)$$

$$k_3 = \frac{\left[\int_0^1 (1-\Lambda\xi)^2 f \phi\, d\xi \right]^2}{\int_0^1 (1-\Lambda\xi)^3 \phi^2\, d\xi} \text{ and } b = \frac{x_0^2}{x_0^2 + I_0'}$$

Equation (11) is the governing equation for the application of the 'perturbation method'.

Perturbation Method

The method of perturbation as used by Hsu Lo (1960) has been extended to this coupled problem. For a small change of hub-radius, Δr the $\delta \ll 1$, and the new frequency parameter can be expressed in terms of a power series of δ. Thus

$$\gamma_1 = \gamma \sum_{n=0}^{\infty} a_n\, \delta_n \qquad (13)$$

Where the co-efficients a_n are to be determined by the method of perturbation. Substitution of eqn. (13) into eqn. (11) gives

$$\Omega^2\, \delta\, k_1 - \gamma^2 \left[\left(\sum_{n=0}^{\infty} a_n\, \delta^n \right)^2 - 1 \right] (k_2-bk_3) = 0$$

Expanding this equation into a series with ascending powers of δ, we get

$$\delta^0 \{-\gamma^2 (a_0^2-1)\}\,(k_2-bk_3) + \delta\, \{\Omega\ \ k_1-\gamma^2\,(2a_0\, a_1)\ (k_2-bk_3)\}$$
$$+ \delta^2 \{-\gamma^2 (2a_0\, a_2+a_1^2)\}\, (k_2-bk_3) + \delta^3 \{-\gamma^2 (2a_0\, a_3+2a_1\, a_2\} \, (k_2-bk_3)$$
$$+ \delta^4 \{-\gamma^2 (2a_0\, a_4 + 2a_1\, a_3 + a_2^2)\}\, (k_2-bk_3) + \ldots\ldots = 0 \qquad (14)$$

Equating the co-efficient of the δ^0 term equal to zero, we get

$$a_0 = \pm 1 \qquad (15)$$

The positive sign should be used so that the condition that at $\delta = 0$, $\gamma_1 = \gamma$ is satisfied. Equating the coefficient of δ — term to zero, we get

$$a_1 = \Omega^2 k_1 \mid 2 \gamma^2 (k_2 - bk_3) \qquad (16)$$

Neglecting the second and higher order terms of δ, the following first order solution is obtained

$$\gamma_1 = \gamma (1 + a_1 \delta) \qquad (17)$$

This gives a linear relation between the frequency parameter γ_1 and δ, with a_1 as the slope of this linear relation.

Substituting the expressions for γ_1 and γ in eqn. (17) and solving for ω_1 one obtains the relation between the new frequency ω_1 and the parameter δ as given by

$$\omega_1 = [(\omega^2 + \Omega^2 \text{Sin}^2 \psi) (1 + a_1 \delta)^2 - \Omega^2 \text{Sin}^2 \psi]^{1/2} \qquad (18)$$

This relation can be put in a linear form by expanding into a series and discarding terms of higher powers of δ.

$$\omega_1 = \omega \left[1 + a_1 \frac{(\omega^2 + \Omega^2 \text{Sin}^2 \psi)}{\omega^2} \delta \right] \qquad (19)$$

For the particular case $\psi = 0$, this reduces to

$$\omega_1 = \omega (1 + a_1 \delta) \qquad (20)$$

The results (20) and (19) confirm the results previously by Boyce (1956) and Hsu La (1960) if in addition $x = 0$ and $S_l/S_o = 1$.

Higher Order Solutions

Higher Order Solutions can be obtained by following the same procedure outlined. Setting the coefficient of δ^n—term $n = 0, 1, 2, 3$............of eqn. (14) to zero seperately, one obtains the following set of equations

0	Order	$a_0^2 - 1 = 0$	
1st	Order	$k_1 \Omega^2 - \gamma^2 (2a_0 a_1)(k_2 - bk_3) = 0$	
2nd	Order	$2a_0 a_2 + a_1^2 = 0$	(21)
3rd	Order	$2a_0 a_3 + 2a_1 a_2 = 0$	
4th	Order	$2a_0 a_4 + 2a_1 a_3 + a_2^2 = 0$	
n^{th}	Order	$\sum_{j=0}^{n} a_j a_{n-j} = 0$	

The parameters a_n can be calculated successively from eqn. (21). The first five parameters are

$$a_0 = 1 \quad , \quad a_1 = k_1 \Omega^2 \mid 2 \gamma^2 (k_2 - b\, k_3)$$

$$a_2 = - a_1^2/2, \quad a_3 = a_1^3/2, \quad a_4 = - 5\, a_1^4/8$$

It follows that the $\gamma_1 - \delta$ relation can be expressed as

1st order	$\gamma_1 = \gamma (1 + a_1 \delta)$	
2nd order	$\gamma_1 = \gamma (1 + a_1 \delta - a_1^2 \delta^2/2)$	
3rd order	$\gamma_1 = \gamma (1 + a_1 \delta - a_1^2 \delta^2/2 + a_1^3 \delta^3/2)$	(22)
4th order	$\gamma_1 = \gamma (1 + a_1 \delta - a_1^2 \delta^2/2 + a_1^3 \delta^3/2 - 5\, a_1^4 \delta^4/8)$	

Convergence

The convergence of the infinite series

$$\gamma_1 = \gamma \sum_{n=0}^{\infty} a_n \delta_n$$

and the determination of its radius of convergence can be treated as follows :

Let
$$g(\delta) = \sum_{n=0}^{\infty} a_n \delta_n$$

Eqn. (13) becomes

$$\gamma_1 = \gamma g(\delta) \tag{23}$$

Substituting this relation into eqn. (11) gives

$$\Omega^2 \delta k_1 - \gamma^2 [g^2(\delta) - 1](k_2 - b k_3) = 0$$

whereas
$$g(\delta) = \left(1 + \frac{\Omega^2 \delta k_1}{\gamma^2 (k_2 - b k_3)}\right)^{1/2}$$

When eqn. (16) is substituted it becomes

$$g(\delta) = (1 + 2 a_1 \delta)^{1/2} \tag{24}$$

Thus the infinite series

$$\sum_{n=0}^{\infty} a_n \delta_n$$

Converges to the sum given by eqn. (24). The radius of convergence is $| 2 a_1 \delta < 1 |$

or
$$| \delta | < \frac{1}{2 a_1}; \quad a_1 > 0 \tag{25}$$

It may be noted here that the process of proving the convergence of the series actually constitutes a direct solution of the problem without using the perturbation method. The presentation of the perturbation method is retained in this problem because of its logical use in obtaining the first order solution and because of the possibility of its extension to more complicated systems.

Conclusion

It has been shown that for coupled transverse and torsional vibrations of a rotating

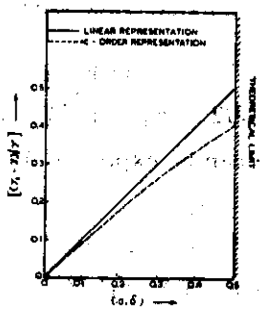

FIG. 2. COMPARISON BETWEEN LINEAR AND INFINITE ORDER REPRESENTATION

beam of linearly varying cross section, a small change in hub radius causes a corresponding change in the frequency parameter. The relation between them is approximately a linear one and the constant of proportionality can be determined from the known parameters.

Higher order representations of the $\gamma_1 - \delta$ relation are also obtained as given by equation (22). Because of the approximation used in equation (14) for calculation of a_1, the use of higher-order representation is deemed unnecessary. A chart is provided as shown in Fig. 2, to indicate the comparison between the linear and the infinite-order representation.

Moreover, $S_{(x)}$, $I_{(x)}$, $J_{(x)}$, $I_{p(x)}$, C_x X_g are functions of x in this case of linearly varying cross-section, the method will also be true for exponential beam and other type of beams of variable cross-section.

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