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STATIC AND QUASI-STATIC DEFORMATTION OF THE EARTH

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ABSTRACT

Strains and stresses within the earth constitute important precursors of earthquake. Therefore, the determination of the static deformation of the earth around surface faults is important for any scheme for the prediction of earthquake. Ben-Menahem and Singh (1976a) derived explicit expressions for the deformation of a homogeneous sphere due to an internal dislocation of arbitrary orientation and depth. Singh (1970) formulated the problem of the static deformation of a multilayered model of the earth in terms of layer matrices. This formulation is now being widely used to compute the residual strain, stress and tilt fields in half-space earth models. Singh and Garg (1985) applied the method of layer matrices to solve the two-dimensional problem of a long displacement dislocation in a multilayered half space.

It is reasonable to consider the postseismic crustal movements to be controlled by a quasi-static process of the relaxation of stress changes produced by the sudden appearance of a fault in a viscoelastic medium. The correspondence principle of linear viscoelasticity has been used to obtain the quasi-static solutions from the solutions of the corresponding elastic problems (Singh and Rosenman 1974, Singh and Singh 1989a).

STATIC DEFORMATION OF A SINGLE-LAYERED HALF-SPACE

The residual displacement, strain and tilt fields associated with an earthquake are diagnostic of the source mechanism. Geodetic observations indicate that strain accumulation is a continuing phenomenon in seismic belts. It is natural, therefore, to associate earthquakes with a sudden release of strain energy accompanying an abrupt change in strength over a surface.

Steketee (1958) represented a fault as a displacement dislocation surface in an elastic half-space. The basic assumption in this procedure is that the displacement following an earthquake can be modelled by the corresponding fields of a dislocation sheet with displacement discontinuity matching the observed slip. Chinnery (1961, 1963) used Steketee's results in calculating the displacement and stress field in the vicinity of a vertical rectangular strike-slip fault. Maruyama (1964) calculated all the sets of Green's functions necessary for calculating the displacement and stress field around faults in a semi-infinite medium. Following Steketee (1958) and Maruyama (1964), the elasticity theory of dislocations has been widely used to calculate the static deformation of a uniform elastic half-space (See e.g., Savage (1980), Rybicki (1986)).

Observations of far-field residual strains caused by the Alaskan earthquake of March 28, 1964 gave a fillip to theoretical studies related to the static deformation of the earth. Although seismologists had been familiar for many years with the phenomenon of permanent deformation in the vicinity of earthquake faults, the existence of such strains at teleseismic distance was quite doubtful prior to the above date. Press (1965) computed and contoured the residual displacement, strain and tilt fields for vertical, rectangular strike-slip and dip-slip faults. Using these results, Press showed for the first time, that the distance fields from major earthquake are large enough to be detected by modern instruments. In particular, he accounted for the order of magnitude ($\sim 10^{-8}$) of strain steps recorded from the great Alaskan earthquake at teleseismic distances.

Ben-Menahem and Singh (1968b) obtained a formal solution for the static deformation field due to an arbitrary multipolar source in a uniform half-space. Since any extended source can be decomposed into a series of multipolar components, the solution found by Ben-Menahem and Singh (1968b) can be used to calculate, in principle, the displacement field associated with an arbitrary finite source. This solution is expressed in terms of source coefficients i_m, j_m and k_m, which have been explicity given for a double couple source. Explicit expressions for the surface displacements in a layer over half-space configuration have been obtained by Ben-Menahem and Singh (1968b) for arbitrary source location in the layer of the substratum.

STATIC DEFORMATION OF A MULTILAYERED HALF-SPACE

Singh (1970, 1971) applied the method of layer matrices to solve the problem of the static deformation of a multilayered elastic half-space by buried sources. The point source is represented as a discontinuity, at the source level, in the z-dependent coefficients of the displacement and the stress integrands. It has been shown that the elements of these source matrices are :

Spheriodal Field

$$(D_m)_1 = -[1-(-1)^{m+1}](i_m + j_m),$$

$$(D_m)_2 = [1-(-1)^{m}](i_m - j_m),$$

$$(D_m)_3 = \mu[1-(-1)^{m+1}](i_m + \delta j_m),$$

$$(D_m)_4 = -\mu[1-(-1)^{m+1}](i_m - \delta j_m),$$

where μ is the rigidity, k the bulk modulus and

$$\delta = \frac{3k+\mu}{3k+7\mu}.$$

Toroidal Field

$$(D_m)_1 = [1-(-1)^{m}]k_m,$$

$$(D_m)_2 = -\mu[1-(-1)^{m+1}]k_m.$$

The coefficients i_m, j_m and k_m characterize the source. The values of these source coefficients for the six elementary sources of Steketee (1958) have been given by Ben-Menahem and Singh (1968b).

Two representations of two-dimensional seismic sources causing plane strain deformation are given by Singh and Garg (1986). In the first representation, the Airy stress function corresponding to various sources in an isotropic homogeneous infinite medium is obtained. Knowing the stress function, the field due to the source in a layered medium can be obtained by solving the corresponding boundary-value problem. In the second approach, the source are represented in terms of the jumps at the source level in the Fourier integral representation of the displacements and stresses due to the source in an isotropic homogeneous infinite medium.

The problem of the static deformation of a multilayered half-space by two-dimensional source has been formulated by Singh (1985), Singh and Garg (1985) and Garg (1986). In this two-dimensional case, the plane strain problem and the antiplane strain problem are decoupled and, therefore, can be treated separately.

Singh (1986) studied the problem of the axially-symmetric deformation of a transversely isotropic multilayered half-space by surface loads in terms of layer matrices. It has been shown that the general problem splits into two independent problems, called the torsional problem and the plane problem. The torsional problem is solved by integrating the equation of equilibrium directly. The plane problem is solved by expressing the stresses in terms of the generalized Love's strain potential. The particular cases of a torque and a vertical force are considered in detail.

Garg and Singh (1985) studied the two-dimensional problem of the static deformation of a multilayered isotropic half-space by surface loads in detail, the Thomson-Haskell matrix method is used to obtain the stress field at any point of the medium. The partiular cases of a normal line load and a shear line load are considered. Explicit expressions for stresses caused by these loads on a uniform half-space are derived. The corresponding problem of the static deformation of a transversely isotropic multilayered half-space has been discussed by Garg and Singh (1987).

STATIC DEFORMATION OF A SPHERICAL EARTH MODEL

The infinite space static Green's function, also known as the somigliana tensor, is given by

$$\vec{G} = \frac{1}{4\pi\mu}\left[\frac{\vec{I}}{R} - \frac{1}{4(1-\sigma)}\left(\frac{\vec{I}}{R} - \frac{\vec{R}\,\vec{R}}{R^3}\right)\right],$$

where μ is the rigidity, σ the Poisson ratio and $\vec{I}$ the unit tensor. Ben-Menahem, and Singh (1968a) obtained the eigenvector expansion of the Somigliana tensor in cylindrical and spherical coordinates. The authors also obtained the Green's tensor for a uniform sphere with zero surface tractions. This tensor was then used, in conjunction with the Volterra relation, to evaluate the static field generated in a uniform elastic sphere by a shear dislocation. It was found that the static field in a sphere associated with the Legendre polynomial of the first degree poses some problems. In this case, one has to invoke the principles of conservation of the angular momentum and the centre of mass of the sphere, to evaluate the field. These conservation principles imply

$$\int_V \vec{r} \times \vec{u}\, dv = 0, \qquad \int_V \vec{u}\, dv = 0.$$

It was pointed out by the authors that the field due to an arbitrary shear dislocation can be expressed as a linear combination of the fields due to a vertical strike-slip fault, a vertical dip-slip fault and a 45° dip-slip fault.

The explicit expressions for the displacement field generated in a uniform sphere by a shear dislocation obtained by Ben-Menahem and Singh (1968a) form the basis for the numerical results reported by Singh and Ben-Menahem (1969), BenMenahem et al. (1969), Ben-Menahem and Singh (1970) and Ben-Menahem et al. (1970). Singh and Ben-Menahem (1969) computed the residual displacement and strain fields at the surface of a non-gravitating, homogeneous, isotropic, elastic sphere resulting from faulting inside the sphere. The field was exhibited through the use of equi-displacement and equi-strain lines. It was found that for nearly horizontal and nearly vertical faults, the field due to a dip-slip source shows a strong dependence on the dip angle. Ben-Mehahem et al. (1970) numerically integrated the results for a point source over the fault area to obtain the field due to a finite source in a sphere. The computation of the double integrals was done by a successive use of the Simpson's rule, first in one coordinate, then in the second.

Wason and Singh (1972) used the Thomson-Haskell matrix device to solve the problem of the static deformation of a multistoreyed spherical earth model by buried sources. The point source is represented as a discontinuity in the motion-stress vector across the spherical surface passing through the source (Wason and Singh, 1971).

Israel et al. (1973) gave a scheme for calculating the residual field in a gravitating, radially inhomogeneous model of the earth caused by a buried dislocation source. The equations of equilibrium can be obtained from the equations of the free oscillations of the earth by setting the frequency equal to zero. The problem of evaluating the residual displacement field splits into two independent problems. The first problem deals with the spheroidal field and is represented by six radial functions. The second problem deals with the toroidal field and is represented by two radial functions.

QUASI-STATIC DEFORMATION OF A VISCO-ELASTIC HALF-SPACE

One approach to improve admittedly inaccurate models for the behaviour of the earth following a seismic event is to try to incorporate the effects of inelastic processes. If the effects of viscosity are taken into account, then energy supplied to a system via a sudden slip along an earthquake fault can be partially stored within the system and released at subsequent times, giving rise to such relatively slowly varying phenomena as creep and relaxation.

The quasi-static approximation to the behaviour of the system is a useful preliminary step in the attempt to improve the model, so long as one is not trying to calculate short-lived phenomena such as seismic waves. In quasi-static processes, the stress equilibrium exists at every point at each instant of time. Therefore, the inertial term in the equation of motion can be disregarded. The quasi-static behaviour of the system is thus determined by the equation of equilibrium and the equations relating stress, strain and displacement, subject to boundary or initial conditions. A standard technique in the solution of such a problem is the use of the Laplace transform to eliminate any explicit time dependence. Upon comparing the resulting set of equations for an isotropic, linearly viscoelastic medium with the corresponding set for an isotropic, elastic medium, one sees that they are formally identical with the exception that the bulk modulus k and the shear modulus μ characterizing the elastic case are replace by a "transform bulk modulus" k^* and a "transform shear modulus" μ^* in the linearly viscoelastic case. The corresponding principle is the resulting conclusion that we can obtainthe Laplace transformed viscoelastic solution from the corresponding transformed elastic solution on replacing the elastic modulii k and μ with the transform modulii k^* and μ^*, respectively.

Singh and Rosenman (1974) obtained explicit expressions for the surface displacements induced by a point source and a vertical rectangular finite source. It is shown that, for a vertical dip-slip source, surface displacements for viscoelastic case are identical with the corresponding elastic results. In the case of a vertical strike-slip fault, detailed numerical results are obtained for both a point source and a finite rectangular source. It is found that the results for the viscoelastic models differ significantly from the corresponding elastic results.

Using the results of Singh and Rosenman (1974), Rosenman and Singh (1973a, b) derived expressions for quasi-static surface strains, tilts and stresses resulting from a finite, rectangular, vertical, strike-slip fault in a Maxwellian viscoelastic half-space. The variation with time and epicentral distance is studied. Detailed numerical calculations reveal significant differences between the viscoelastic and the elastic results. It is found that all nonvanishing stress components at the free-surface die exponentially with time. This is in contrast to the behaviour of the displacements and strains, which, in general, do not vanish for large times.

Garg and Singh (1988) used the corresponding principle to obtain theoretical expressions for the quasi-elastic surface displacement and shear stress caused by a long strike-slip dislocation in an elastic layer overlying a Maxwell viscoelastic half-space. Variation of the surface displacement and shear stress with horizontal distance is studied for various times and vertical extents of the fault. It is seen that, for large vertical extents of the fault, the quasi-static response differs significantly from the corresponding elastic response. Garg and Singh (1989) obtained analytic expressions for the quasi-static stresses caused by a two-dimensaional shear line load acting on the boundary of a semi-infinite medium consisting of a homogeneous elastic layer lying over a Maxwell viscoelastic half-space.

Singh and Singh (1989a, b) obtained expressions for the quasi-static displacements, strains and stresses as convolution integrals for an arbitrary shear dislocation or an explosive source situated in an isotropic homogeneous half-space which is linear viscoelastic of the most general kind. In the expressions for the displacements, strains and stresses the elastic moduli μ and k occur in various combinations. In particular, in the expressions for strains, the following combinations occur .

$$Q_1 = \frac{1}{3k+4\mu}, \qquad Q_2 = \frac{2\mu}{3k+4\mu},$$

$$Q_3 = \frac{1}{3k+\mu}, \qquad Q_4 = \frac{2\mu}{3k+\mu}$$

Singh and Singh (1989b) have shown that the quasi-static viscoelastic strains can be obtained from the corresponding static elastic strains on replacing Q_i by the auxiliary functions $\bar{Q}_i(t)$. The values of the auxiliary functions $\bar{Q}_i(t)$, for the Kelvin material and the Maxwell material are given below. In deriving these values, it has been assumed that the source time function is $\delta(t)$.

KELVIN MODEL

The constitutive equation for a Kelvin material is -

$$\tau = q_o e + q_1 (\partial e / \partial t),$$

where τ is the stress, e is the strain and the parameters q_o and q_1 characterize the material. We assume that the material is elastic in dilatation and Kelvin viscoelastic in distortion. The auxiliary functions are found to be

$$Q_1(t) = \frac{1}{2q_1} \exp\left(- \frac{3k+2q_o}{2q_1} t\right),$$

$$Q_2(t) = \frac{1}{2} \left[\delta(t) - \frac{3k}{2q_1} \exp\left(- \frac{3k+2q_o}{2q_1} t\right)\right],$$

$$Q_3(t) = \frac{2}{q_1} \exp\left(- \frac{6k+q_o}{q_1} t\right),$$

$$Q_4(t) = 2 \left[\delta(t) - \frac{6k}{q_1} \exp\left(- \frac{6k+q_o}{q_1} t\right)\right].$$

MAXWELL MODEL

The constitute equation for a Maxwell material is -

$$\tau + p_1 (\partial \tau / \partial_t) = q_1 (\partial_e / \partial_t).$$

We assume that the material is elastic in dilatation and Maxwell viscoelastic in distortion. We find

$$Q_1(t) = \alpha [p_1 \delta(t) + 2q_1 \alpha \exp(-3k \alpha t)],$$

$$Q_2(t) = q_1 \alpha [\delta(t) - 3k \alpha \exp(-3k \alpha t)],$$

$$Q_3(t) = 2\beta [p_1 \delta(t) + q_1 \beta \exp(-6k \beta t)],$$

$$Q_4(t) = 2q_1 \beta [\delta(t) - 6k \beta \exp(-6k \beta t)],$$

where

$$\alpha = (3kp_1 + 2q_1)^{-1}, \qquad \beta = (6kp_1 + q_1)^{-1}.$$

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