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APPROXIMATE DYNAMIC ANALYSIS OF CABLE SYSTEMS

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INTRODUCTION

Cable systems have been used in recent years to support a number of long span roofs throughout the world. The increased popularity of these structures is a direct result of the wide variety of aesthetic shapes possible through their use, efficient use of material and their capability for covering large spans. However, because of their small flexural rigidity cable roofs may suffer large deformations unless adequately pretensioned, and in the process the roofing material may get damaged. In order to determine the dynamic response of cable suspended systems, the free vibration characteristics must be studied for consideration in design. The rigorous dynamic analysis of such systems is quite tedius and time consuming. Therefore, for preliminary designs, it is desirable to develop approximate formulae for determining the natural frequencies of cable nets and trusses. This will facilitate the computation work of the designer in determining the preliminary dimensions of structural members which may be checked later with rigorous analysis. The present paper discusses the use of approximate formulae for determination of natural frequencies for both, networks as well as trusses.

ANALYSIS

An analytical solution is developed using the Rayleigh-Ritz approach (4) for arriving at the approximate formulae for determining natural frequencies of elastic cable structures. The analysis considers only vertical load and displacements and neglects edge displacements and horizontal loads. Analysis of cable nets and trusses thus carried out is described in the following sections.

CABLE NET

The cable net considered for illustrating the method is a translational surface of two parabolic curves and is shown in Fig. 1. For a surface having the horizontal com-

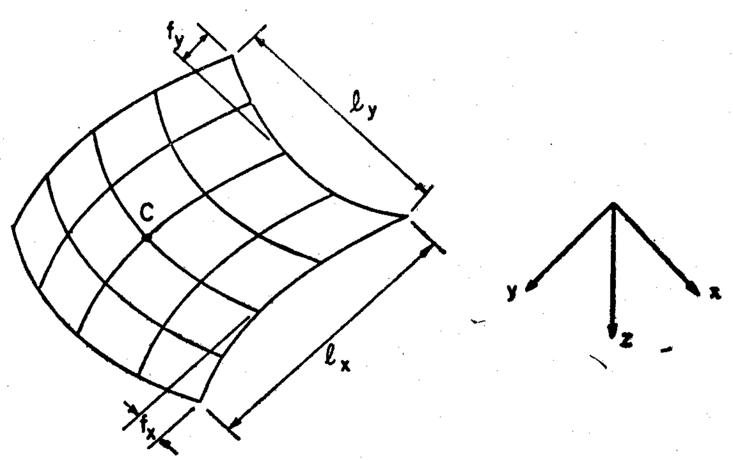

Fig. 1.

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component of tensile force in cables as $(H_x + h_x)$ and $(H_y + h_y)$ in x and y directions respectively, the equilibrium equations at any point can be written as follows:

$$\bar{h}_x(y)\frac{\partial^2 z}{\partial x^2}+\bar{h}_y(x)\frac{\partial^2 z}{\partial y^2}+\bar{H}_x(y)\frac{\partial^2 w}{\partial x^2}+\bar{H}_y(x)\frac{\partial^2 w}{\partial y^2}=-m\frac{\partial^2 w}{\partial t^2} \quad \ldots(1)$$

in which

$\bar{H}_x(y)$, $\bar{H}_y(x)$=horizontal component of tension in x and y directions per unit width of strip=H_x, H_y

$\bar{h}_x(y)$, $\bar{h}_y(x)$=change in horizontal component of tension in x and y directions per unit width of the strip=h_x, h_y

a =spacing of the cables in x and y directions,

m =mass per unit area,

w =vertical displacement of a point at any instant of time

Considering the change in length of cable and assuming the curve of cable to be parabolic, $\bar{h}_x(y)$ and $\bar{h}_y(x)$ can be expressed as follows:

$$\left.\begin{aligned}\bar{h}_x(y)&=-\frac{EA_x}{l_x}\int_0^{l_x} w\frac{\partial^2 z}{\partial x^2}dx\\ \bar{h}_y(x)&=-\frac{EA_y}{l_y}\int_0^{l_y} w\frac{\partial^2 z}{\partial y^2}dy\end{aligned}\right\} \quad \ldots(2)$$

in which

A_x, A_y=cross-sectional area of cables in x and y directions per unit width of the strip,

E =modulus of elasticity

l_x, l_y =span of cable in x and y directions.

Assuming the origin of hypar at the centre of the roof C (Fig. 1), equation of the surface is given by,

$$z=-\frac{4f_x}{l_x^2}x^2+\frac{4f_y}{l_y^2}y^2 \quad \ldots(3)$$

Then, curvatures in x and y directions are given by

$$\left.\begin{aligned}\frac{\partial^2 z}{\partial x^2}&=\rho_x=-\frac{8f_x}{l_x^2}\\ \frac{\partial^2 z}{\partial y^2}&=\rho_y=\frac{8f_y}{l_y^2}\end{aligned}\right\} \quad \ldots(4)$$

where f_x, f_y=sag of cables in x and y directions.

As the curvatures are invariant, the prestress forces are equal and the notations $\bar{H}_x(y)$ and $\bar{H}_y(x)$ may be replaced by H_x and H_y respectively in eq. (1).

For uniformly distributed load on the hypar, assuming origin at one corner, the deflection is given by,

$$w=\sum_i\sum_j C_{ij}.\sin\frac{i\pi x}{l_x}\sin\frac{j\pi y}{l_y} \quad \ldots(5)$$

in which i and j are integers.

However, eqn. (3) remains unaltered because of invariance of second derivative. Now substituting eqn. (4) and (5) in eqn. (2), we get

$$\bar{h}_x(y)=-\frac{EA_x}{l_x}\int_0^{l_x}\sum_i\sum_j C_{ij}\sin\frac{i\pi x}{l_x}\sin\frac{j\pi y}{l_y}\left(-\frac{8f_x}{l_x^2}\right)dx$$

$$=\frac{16EA_x}{\pi l_x}\frac{f_x}{l_x}\sum\sum\frac{C_{ij}}{i}\sin\frac{j\pi y}{l_y} \quad \ldots(6)$$

Similarly $\bar{h}_y(x)=\frac{16EA_y}{\pi l_y}\frac{f_y}{l_y}\sum\sum\frac{C_{ij}}{j}\sin\frac{j\pi y}{l_y}$ ...(7)

Now using these, the strain energy E_1 can be obtained as,

$$E_1=EA_x\rho_x^2\frac{l_xl_y}{\pi^2}\sum\sum\frac{C_{ij}^2}{i^2}+EA_y\rho_y^2\frac{l_xl_y}{\pi^2}\sum\sum\frac{C_{ij}}{j^2}$$

$$+H_x\frac{\pi^2 l_y}{8l_x}\sum\sum i^2C_{ij}^2+H_y\frac{\pi^2 l_x}{8l_y}\sum\sum j^2C_{ij}^2 \quad \ldots(8)$$

Now assuming harmonic solution for w, one could write,

$$w=w_{mx}\sin\eta t \quad \ldots(9)$$

in which w_{mx}=maximum deflection at a point

η=circular natural frequency of the net in radians/sec.

The maximum kinetic energy for vibrating structure is given by

$$E_2=\tfrac{1}{2}\text{ mass (velocity)}^2$$

$$=\frac{m}{2}\eta^2\int_0^{l_x}\int_0^{l_y}w_{mx}^2\,dx\,dy \quad \ldots(10)$$

Assuming that the maximum amlitude of vibration is given by

$$w_{mx}=\sum_i\sum_j C_{ij}\sin\frac{i\pi x}{l_x}\sin\frac{j\pi y}{l_y} \quad \ldots(11)$$

where i and j are integers.

Kinetic Energy E_2 is given by

$$E_2=\frac{m}{2}\eta^2\frac{l_xl_y}{4}\sum_i\sum_j C_{ij}^2 \quad \ldots(12)$$

According to Rayleigh-Ritz method (5) which proceeds by minimising (E_2-E_1),

$$\frac{\partial(E_2-E_1)}{\partial C_{ij}}=0 \quad \ldots(13)$$

Now,

$$E_1-E_2=H_x\left(\frac{\pi}{l_x}\right)^2\frac{l_xl_y}{8}\sum\sum i^2C_{ij}^2+H_y\left(\frac{\pi}{l_y}\right)^2\frac{l_xl_y}{8}\sum\sum j^2C_j^2$$

$$+EA_x\rho_x^2\,\frac{l_xl_y}{\pi^2}\sum\sum\frac{C_{ij}^2}{i^2}+EA_y\rho_y^2\,\frac{l_xl_y}{\pi^2}\sum\sum\frac{C_{ij}^2}{j^2}$$

$$-m\eta^2\,\frac{l_xl_y}{8}\,C_{ij}^2 \qquad \ldots(14)$$

Differentiating (14) with respect to C_{ij} and equating to zero, we get,

$$H_x\left(\frac{\pi}{l_x}\right)^2\frac{i^2}{4}\,C_{ij}+H_y\left(\frac{\pi}{l_y}\right)^2\frac{j^2}{4}\,C_{ij}+\frac{EA_2\rho_x^2}{\pi^2}\frac{2C_{ij}}{i^2}+\frac{EA_y\rho_y^2}{\pi^2}\frac{2C_{ij}}{j^2}$$

$$-\frac{m}{4}\,\eta^2C_{ij}=0 \qquad \ldots(15)$$

As an approximation, only the first term of the series in eqn. (11) is considered. Then, taking $w_{mx}=C_{11}\sin\frac{\pi x}{l_x}\sin\frac{\pi y}{l_y}$, we get,

$$\eta_{11}^2=\frac{\pi^2}{m}\left[\frac{H_x}{l_x^2}+\frac{H_y}{l_y^2}+\left(\frac{8}{\pi^2}\right)\frac{EA_x\rho_x^2+EA_y\rho_y^2}{\pi^2}\right] \qquad \ldots(16)$$

If i and j are even integers in eqn, (5), $\bar{h}_x(y)$ and $\bar{h}_y(x)$ are equal to zero, consequently the first and second term of eqn. (1) will vanish, reducing eqn. (15) to the form

$$H_x\left(\frac{\pi}{l_x}\right)^2\frac{i^2}{4}\,C_{ij}+H_y\left(\frac{\pi}{l_y}\right)^2\frac{j^2}{4}\,C_{ij}-\frac{m\eta^2}{4}\,C_{ij}=0 \qquad \ldots(17)$$

Taking $i=j=2$ (even integers), for C_{22} we get,

$$\eta_{22}^2=\frac{4\pi^2}{m}\left[\frac{H_x}{l_x^2}+\frac{H_y}{l_y^2}\right] \qquad \ldots(18)$$

Equation (18) gives the frequency of vibrations for an asymmetric mode, with asymmetry about both x and y axes.

If i is odd integer and j is even integer in eqn. (5), $\bar{h}(x)_y$ is equal to zero, vanishing the second term of eqn. (1). Thus eqn. (15) reduces to the form

$$H_x\left(\frac{\pi}{l_x}\right)^2\frac{i^2}{4}\,C_{ij}+H_y\left(\frac{\pi}{l_y}\right)^2\frac{j^2}{4}\,C_{ij}+\frac{EA_x\rho_x^2}{\pi^2}\frac{2C_{ij}}{i^2}-\frac{m\eta^2}{4}\,C_{ij}=0 \qquad \ldots(18)$$

Taking $i=1$, $j=2$, for C_{12} we get

$$\eta_{1,2}^2=\frac{\pi^2}{m}\left[\frac{H_x}{l_x^2}+\frac{4H_y}{l_y^2}+\left(\frac{8}{\pi^2}\right)\frac{EA_x\rho_x^2}{\pi^2}\right] \qquad \ldots(20)$$

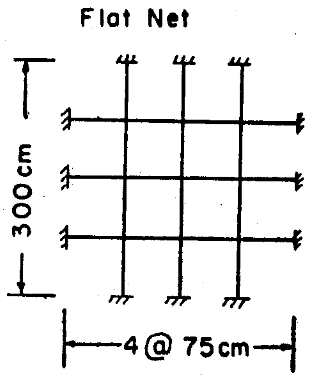
Following the above argument, for $i=2$, $j=1$, we get

$$\eta_{2,1}^2=\frac{\pi^2}{m}\left[\frac{4H_x}{l_x^2}+\frac{H_y}{l_y^2}+\left(\frac{8}{\pi^2}\right)\frac{EA_y\rho_y^2}{\pi^2}\right] \qquad \ldots(21)$$

Equation (20) and Eqn. (21) gives the frequency of vibration for an asymmetric mode, with asymmetry about either x axis or y axis.

The frequencies of the cable net systems shown in Fig. 2 (Example 1) and Fig. 3 (Example 2) are computed using the expressions given by eqn. (16) to eqn. (21) and are compared with the frequencies obtain by Jacobi's transformation technique for lumped mass discrete models. Comparison of results is shown in Table 1 and is found to be close. It is of importance to point out that if larger number of terms is considered in Eqn. (5)

Example 1

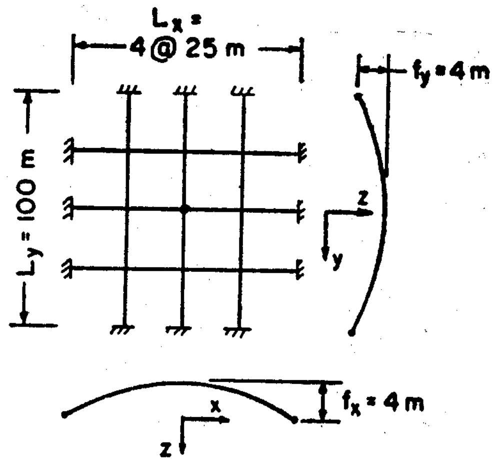


Data:
EA=6810,000 kg for each cable
H_x=H_y=34050 kg for each cable

Load at
each node=3.95 kg

Fig. 2

Example 2



Data:
EA=262500 Tonnes per cable
H_x=580 Tonnes per cable
H_y=326 Tonnes per cable
Load at
each node=18.75 Tonnes

Fig. 3

TABLE I

	Frequency of vibrations in c/s					
	Example 1 (Flat net)		Remarks	Example 2 (Hypar)		Remarks
	Addroi-x mate Method	Conven-tional Method		Approxi-mate Method	Conven-tional Method	
First Frequency	59.389	57.16	Symmetric Mode	1.08	0.98	Antisymmetric Mode
Second Frequency	93.90	85.02	Symmetric about x axis and antisym-metric about y axis	1.15	1.095	Symmetric about x axis and antisym-metric about y axis
Third Frequency	93.90	85.02	Symmetric about y axis and antisym-metric about x axis	1.259	1.17	Symmetric about y axis and antisym-metric about x axis
Fourth Frequency	118.778	105.74	Antisym-metric Mode	1.315	1.26	Symmetric Mode

greater accuracy shall be obtained in the computed results. Further, it shall be possible to obtain higher order frequencies. However, including more terms in Eqn. (5), increases the complexity of the numerical work.

CABLE TRUSS(3)

Using the approach adopted for the network, the equilibrium equation at any point for a truss can also be written as follows:

$$h_u \left[\frac{d^2z}{dx^2}\right]_u + hl \left[\frac{d^2z}{dx^2}\right]_l + (H_u + Hl)\frac{d^2w}{dx^2} = -m\frac{d^2w}{dt^2} \qquad \ldots(22)$$

where m=mass per unit length of span of cable,

On simplification, eqn. (22) reduces to

$$\left[\frac{d^2z}{dx^2}\right]_u \frac{(EA)_u}{u}(1+q^2r)\int\left[\frac{d^2z}{dx^2}\right]_u wdx - (H_u+Hl)\frac{d^2w}{dx^2} = m\frac{d^2w}{dt^2} \qquad \ldots(23)$$

where L=span

$$q = \frac{\rho_l}{\rho_u}$$

$$r = \frac{(EA)_l}{(EA)_u}$$

$(EA)_{l,u}$=cable rigidity for bottom, top cable respectively,

$\rho_{l,u}$=curvature for the bottom, top cable respectively.

For uniformly distributed load on truss, the deflection is given by

$$w = \sum C_i \sin\frac{i\pi x}{l} \qquad \ldots(23a)$$

in which i is an integer

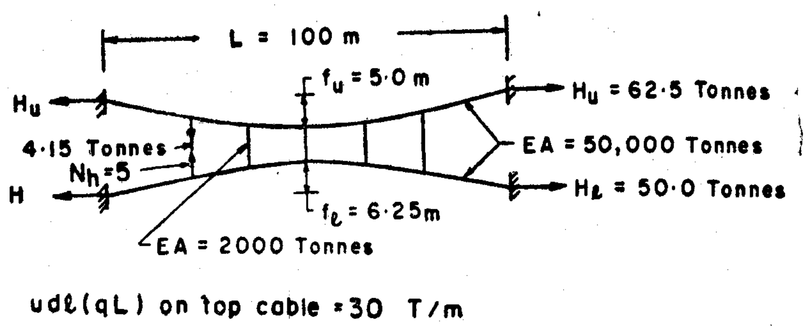
Solution of eqn. (23) is carried out using Eqn. (23a) and adopting the same procedure as adopted for the cable network in previous section. From this an approximate formulae for natural frequency of a dual cable system is derived. For a single wave symmetric mode the frequency η is given by,

$$\eta^2 = \frac{\pi^2}{mL^2}\left[(H_u+H_l) + \left(\frac{8}{\pi^2}\right)\left(\frac{\rho_u^2(EA)_u L^2}{\pi^2}\right)(1+q^2r)\right] \qquad \ldots(24)$$

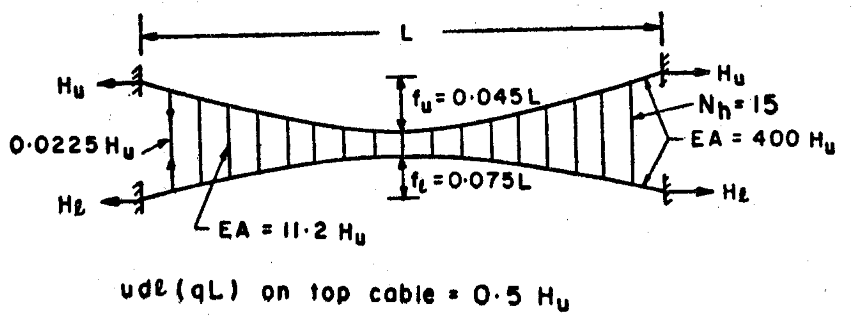
Expression for the frequency of vibration with double wave antisymmetric mode is obtained as,

$$\eta^2 = \frac{4\pi^2}{mL^2}\left[H_u + H_l\right] \qquad \ldots(25)$$

Results of two cases of dual cable systems have been computed by using eqn. (24) and (25) considering two terms in Eqn. (23a). The frequencies, thus, obtained are compared in Table 2 with that obtained from conventional analysis. From the Example 3 and 4, the frequencies have also been computed by taking three terms in Eqn. (23a). These results are given in Table 3. It may be seen that the comparison of the results is closer in this case. However, as before consideration of large number of terms entails greater numerical complexity.



Example 3



Example 4

TABLE 2

Problem		Frequency of vibration c/s		Remarks
		Approximate formulae	Conventional analysis	
Example 3	First Frequency	0.665	0.608	Antisymmetric mode
	Second Frequency	1.33	0.815	Symmetric mode
Example 4	First Frequency	0.577	0.567	Antisymmetric mode
	Second Frequency	0.959	0.743	Symmetric mode

TABLE 3

Problem		Frequency of vibration c/s		Remarks
		Approximate Method	Conventional Method	
Example 3	First Frequency	0.665	0.608	Antisymmetric mode
	Second Frequency	0.921	0.815	Symmetric mode
	Third Frequency	1.452	1.046	Three wave symmetric mode
Example 4	First Frequency	0.577	0.567	Antisymmetric mode
	Second Frequency	0.775	0.743	Symmetric mode
	Third Frequency	1.079	0.9995	Three wave symmetric mode

CONCLUSIONS

The approximate method as presented herein gives results close to those obtained using the conventional analysis and are recommended for the use in estimating the dynamic response of such systems for preliminary design.

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