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DESIGN OF A TYPICAL MACHINE FOUNDATION BY DIFFERENT METHODS

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Synopsis

Procedures for evalution of soil constants from a resonance test, for the design of machine foundations have been illustrated by Barkan, Pauw and Richarts' methods. Based upon these soil constants, two typical foundations, resting on two different types of soils have been checked, by the three methods to illustrate the application of different design methods in practice.

Introduction

For evalution of dynamic soil constants a resonance test is recommended (Prakash and Gupta 1967). However there are several methods by which the results of this test can be interpretted (Prakash 1965). Resonance test data reported on two different sites (Gupta 1965 and Kondner 1964) has been interpretted by Barkan, Pauw and Richart's methods of analysis. Two typical foundations resting on two different soils have then been checked based on the three methods and comparison of the natural frequencies and amplitudes of motion has been made.

Notation

Symbol		*Units*
a	Length of the foundation	m
a_x	Dimensionless frequency Factor for sliding vibrations	
a_ϕ	Dimensionless frequency factor for rocking vibrations	
a_z	Dimensionless frequency factor for vertical vibrations	
A	Area of the foundation in contact with soil m^2	
A_s	Amplitude factor for sliding	
A_r	Amplitude Factor for rocking	
A_x	Amplitude in sliding	mm
A_ϕ	Amplitude in rocking	Radian
$A_{x\phi}$	Amplitude in combined rocking and sliding	mm
b	Width of foundation	m
C_u	Coefficient of elastic uniform compression of soil	kg/cm^3, t/m^3
C_ϕ	Coefficient of elastic non-uniform compression of soil	kg/cm^3
C_T	Coefficient of elastic uniform shear of soil	kg/cm^3, t/m^3

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E	Modulus of elasticity of soil	kg/cm^2, t/m^2
f_{nx}	Natural frequency in sliding	c.p.s.
$f_{n\phi}$	Natural frequency in rocking	c.p.s.
f_{nz}	Natural frequency in vertical vibration	c.p s.
$f_{nx\phi}$	Natural frequency in combined rocking and sliding	c.p.s.
g	Acceleration due to gravity	$m\text{-}sec^{-2}$
G	Shear modulus	kg/cm^2, t/m^2
h	Equivalent height of surcharge Height of foundation block.	m
I	Moment of inertia of the foundion contact area about an axis passing through the C.G. of the base perpendicular to plane of vibration	m^4
I_m	Mass moment of inertia of foundation and accessories about an axis through the C.G. of the system	$t\text{-}m\text{-}sec^2$
I_{mo}	Mass moment of inertia of foundation and machine about an axis through the C.G. of the base of the foundation	$t\text{-}m\text{-}sec^2$
I_{ms}	Mass moment of inertia of foundation and machine and soil mass about an axis through combined C.G.	$t\text{-}m\text{-}sec^2$
k_x	Spring constant for sliding vibration	t/m
k_z	Spring constant for vertical vibration	t/m
k_{xz}	Spring constant for rocking vibration	t/m
L	Height of C.G. of foundation and machine above the base of the foundation	m
m	Mass of foundation and machine	$t\text{-}m^{-1}\text{-}sec^2$
m_s	Apparent soil mass	$t\text{-}m^{-1}\text{-}sec^2$
M	Exciting moment	t-m
P	Exciting force	t
r_x	Equivalent radius for sliding vibration	m
r_ϕ	Equivalent radius for rocking vibration	m
r_z	Equivalent radius for vertical vibration	m
w_{nx}	Natural circular frequency in sliding	sec^{-1}
$w_{n\phi}$	Natural circular frequency in rocking	sec^{-1}
$w_{nx\phi}$	Natural circular frequency in combined rocking and sliding	sec^{-1}
w_{nz}	Natural circular frequency in vertical mode of vibration	sec^{-1}
ξ	Damping factor	
ρ	Density of soil	t/m^3
ε	Eccentricity factor	
a_x	Length of the element	m
a_y	Width of element	m
a_z	Height of the element	m
γ	Ratio I_m/I_{mo}	
q	Static soil pressure	kg/cm^2

Particulars of Available Test Data :

D. C. Gupta[1] (1965) performed resonance tests on four foundation blocks of one metre height resting on the surface of sandy soil and subjected to sinusoidally varying horizontal unbalance force. The tests were performd by mounting Lazan Oscillator on the top surface of the block and recording amplitudes of motion of the block at different frequencies. The results on a block of 1 m × 1 m × 1 m have been taken up for analysis. The particulars are given below and record of observations in Table I.

Weight of fonndation block	=	2.21 tonnes
Weight of oscillator assembly	=	0.062 tonnes
Density of soil	=	1.8 t/m^3
Base area of the block	=	1 m^2

Table I—Test Data Reported by D. C. Gupta

Eccentricity Factor $\varepsilon \times 10^{-5}$ cm	Observed Natural Frequeney $f_{nx\phi}$ c. p. s.	Peak Amplitude A_{max} mm	Unbalance Force at Resonance F kg
1.45	16.0	0.30	28.0
3.01	15.0	0.43	58.0
5.43	13.0	0.685	90.0
9.28	12.0	1.00	120.0

Konder (1964) reported data on circular footing resting on Silty Clay, tested under vertical vibrations. The particulars of thc test data are as given below :

Diameter of the footing	=	1.57 m
Weight of the footing including vibrator and ballast	=	14.02 tonnes
Unit weight of the soil	=	1.91 t/m^3
Compression modulus of soil at surface E_0	=	740 kg/cm^2
Compression modulus of soil at 8.85 m below surface E	=	1600 kg/cm^2
Shear modulus of soil at surface	=	G = 326 kg/cm^2
Shear modulus at 8.85 m below surface	=	G = 693 kg/cm^2

The compression modulus and shear modulus were determined by seismic methods.

Table II—Test Data Reported by Kondner

Eccentricity Factor $\varepsilon \times 10^{-3}$ cm	Natural Frequency f_{nz} c.p.s.	Force at Resonant Frequency F kg
1.77	15.2	23.0
3.60	13.6	38.5
5.50	12.8	50.0
7.20	12.0	58.5

The procedure for analysis of test data will now be illustrated.

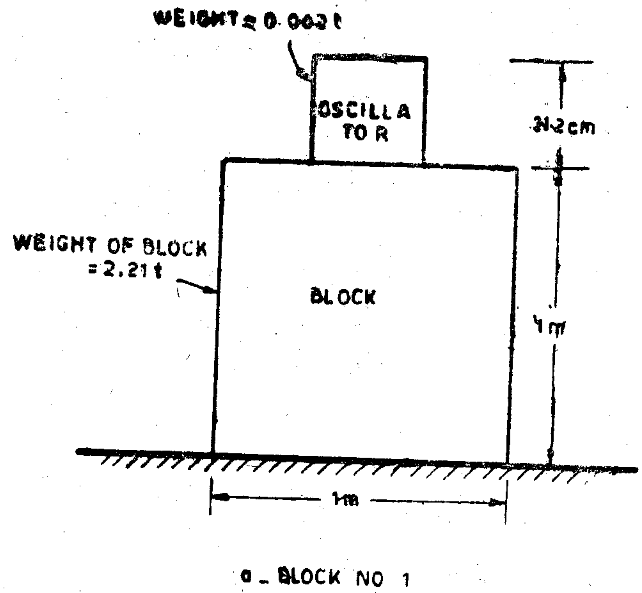

Fig. 1. Section of block

Analysis of Test Data

The test data on sandy soil will be analysed first.

Barkans Method

Fig. 1 shows a section of the block 1 m × 1 m × 1 m high. The axis of rotation of the block is perpendicular to the plane of the figure.

1. Moments of Inertia

(a) Base Area.

$$I^* = \frac{1 \times 1^3}{12} = 0.0834 \text{ m}^4$$

(b) Mass of oscillator and block.

For oscillator, $$I_{m1} = \frac{0\ 062}{9.81} (0.656)^2 = 0.00272 \text{ t-m-sec}^2$$

For foundation block $$I_{m2} = \frac{m}{12} (a_x^2 + a_y^2)$$

$$= \frac{2.21}{9.81 \times 12} (1+1) = 0.0376 \text{ t-m-sec}^2$$

$$I_m = I_{m_1} + I_{m_2} = 0.00272 + 0.0376 = 0.04032 \text{ t-m-sec}^2$$

$$I_{mo} = 0.0376 + \frac{2.21}{9.81} (0.5)^2 + \frac{0.062}{9.81} (1.156)^2 = 0.10237 \text{ t-m-sec}^2$$

$$\gamma = \frac{I_m}{I_{mo}} = 0.394$$

* All the symbols have been defined in the notation.

2. Determination of C_T

Frequency equation for combined rocking and sliding (Barkan, 1962) is

$$w^4{}_{nx\phi} - \frac{w^2_{nx} + w^2_{n\phi}}{\gamma} w^2_{nx\phi} + \frac{w^2_{nx} \cdot w^2_{n\phi}}{\gamma} = 0$$

$$w^2_{nx} = \frac{C_T \cdot A}{m} = \frac{C_T \times 1}{2.272/9.81} = 4.32\, C_T$$

$$w^2_{n\phi} = \frac{C_\phi \cdot I}{I_{mo}} = \frac{C_\phi \times 0.0833}{0.10237} = \frac{3.74\, C_T \times 0.0833}{0.10237} = 3.02\, C_T$$

assuming $C_\phi = 3.74 C_T$

Substituting in the frequency equation

$$w^4{}_{nx\phi} - \frac{4.32\, C_T + 3.02\, C_T}{0.394} w^2_{nx\phi} + \frac{4.32 \times 3.02}{0.394} \cdot C_T^2 = 0$$

$$33.1\, C_T^2 - 18.6\, C_T \cdot w^2_{nx\phi} + w^4_{nx\phi} = 0$$

$$C_T = 0.5\, w^2_{nx\phi}\, ;\ C_T = 0.06\, w^2_{nx\phi}$$

$$w_{nx\phi} = 2\,\pi\, f_{nx\phi}$$

For observed natural frequency, $f_{nx\phi} = 16$ c.p.s. the values of C_T come out to be 5.05 kg/cm³ and 0.595 kg/cm³. Substituting $C_T = 5.05$ Kg/cm³ in the frequency equation, we get $f_{nx\phi 1} = 16.2$ c.p.s.

$f_{nx\phi 2} = 46.0$ c.p.s.

when $C_T = 0.595$ is substituted we get

$f_{nx\phi 1} = 5.45$ c.p.s.

$f_{nx\phi 2} = 15.8$ c.p.s.

The value of C_T selected should be such that it satisfies the condition for two natural frequencies. The observed natural frequency is the lower natural frequency in the combined mode. The second natural frequency will have a higher value which is given only by $C_T = 5.05$ kg/w².

So out of the two values of C_T so obtained, only the higher value will satisfy the condition for two natural frequencies.

The values of C_T have been shown in col. 3, Table III.

Richarts' Method

1. Equivalent Radii

$$r_x = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{1}{\pi}} = 0.564 \text{ m}$$

$$\frac{W}{g} \left(\frac{r_\phi^2}{4} + \frac{h^2}{3}\right) = I_{mo}$$

$$\frac{2.272}{9.81} \left(\frac{r_\phi^2}{4} + \frac{1^2}{3}\right) = 0.10237 \therefore r_\phi = 0.64 \text{ m}$$

2. Mass ratio

$$bx = \frac{W}{\rho \, r_x^3} = \frac{2.272}{1.8 \times (0.564)^3} = 7.0$$

3. Inertia Ratio

$$b\phi = \frac{I_{mo}\, g}{\rho \, r\phi^5} = \frac{0.10237 \times 9.81}{(1.8) \times (0.64)^5} = 5.07$$

4. **Dimensionless Frequency Factors**

From Richarts charts, Fig. 2b and 2c.

$a_x = 0.85$; $a_\phi = 0.67$

5. **Determination of G**

$$w^4_{nx\phi} - \left[(w^2_{nx} + w^2_{n\phi}) + \frac{m}{I_{mo}} Z^2 \, w^2_{nx} \right] w^2_{nx\phi} + w^2 n\phi . w^2_{nx} = 0$$

$Z = 0.516$ m., where Z is the height of combined C.G. above the basis.

$$w^2_{nx} = \frac{(a_x)^2}{r^2_x} \cdot \frac{Gg}{\rho} = \frac{(0.85)^2 \times 9.81 \times G.}{(0.564)^2 \times 1.8} = 12.7\ G.$$

$$w^2 n\phi = \frac{a^2\phi}{r^2\phi} \times \frac{G.g}{\rho} = \frac{(0.67)^2 \times G \times 9.81}{(0.6.4)^2 \times 1.8} = 6.0\ G$$

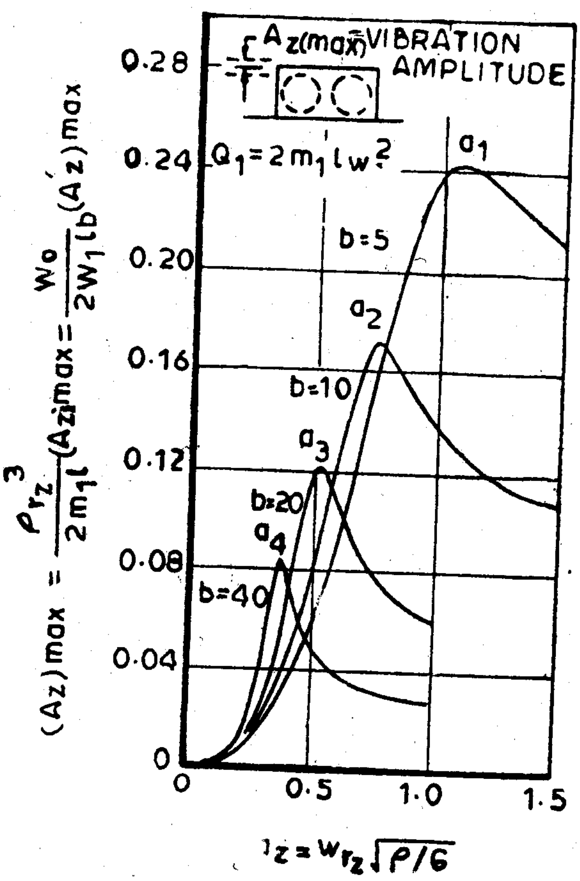

Fig. 2(a). For exciting force amplitude dependent upon the exciting frequency

2. Mass ratio

$$b_z = \frac{W}{r_o^3} = \frac{14.02}{1.91\times(0.785)^3} = 15.12$$

3. Dimensionless frequency factor from Richarts, Fig. 2(a) For bz = 15.12

$$a_z = 0.59$$

4. Determination of G

$$a_z = 2\pi f_{nz}\sqrt{\frac{\rho}{Gg}}\ ; \quad G = \frac{4\pi^2 f^2_{nz}\, r_z^2}{a_z^2}\ \frac{\rho}{g}$$

f_{nz}, r_z, a_z being known, G can be computed. The values of G so obtained have been given in col. 4, Table IV below:

Table IV. Soil Constants for Silty Clay

Eccentricity factor $\varepsilon \times 10^{-3}$ cm	Observed natural frequency c.p.s.	C_u kg/cm^3	G kg/cm^2
1	2	3*	4
1.77	15.2	6.75 (2.98)	322.0
3.60	13.6	5.40 (2.38)	256.0
5.40	12.8	4.76 (2.10)	230.0
7.20	12.0	4 20 (1.86)	220.0

* Values in the brackets show values of Cu for standard 10 m² area.

Pauw's method

From the data reported by Kondner for the silty clay under consideration.

Compression modulus of the soil at the surface Eo = 740 kg/cm²

Compression modulus of the soil at depth of 8.85 m E = 1600 kg/cm²

Shear modulus of the soil at surface Go = 326 kg/cm²

Shear modulus of the soil at depth of 8.85 m G = 693 kg/cm²

Rate of increase of compression modulus with depth $\beta = \frac{1600-740}{8.85} = 0.9725$ kg/cm³

Rate of increase of shear modulus with depth $\beta' = \frac{693-326}{8.85} = 0.415$ kg.cm³

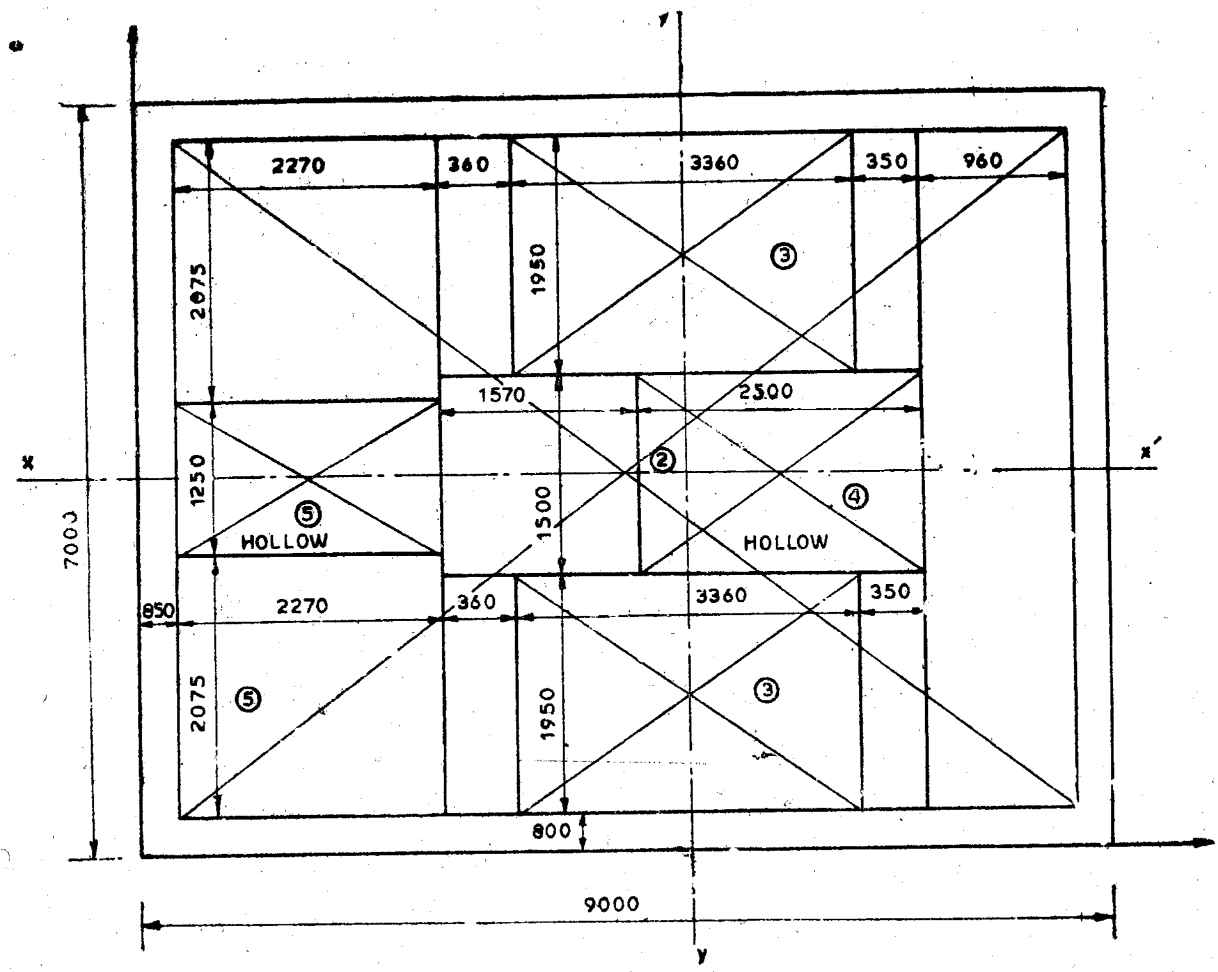

Fig. 3 (a) Plan

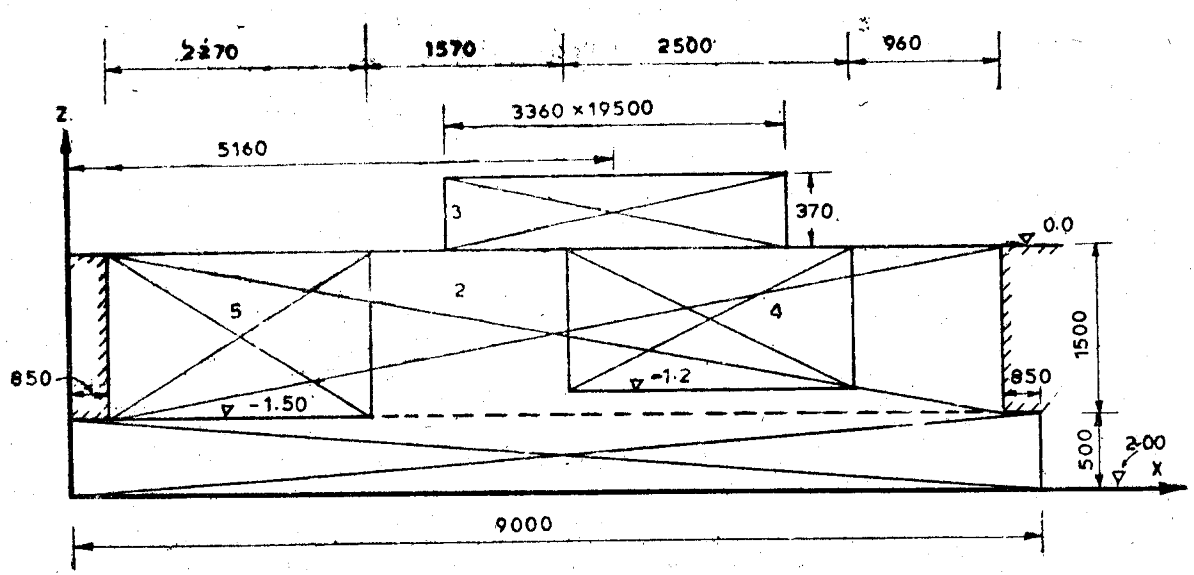

Fig. 3 b—Section along longitudinal axis section x x′

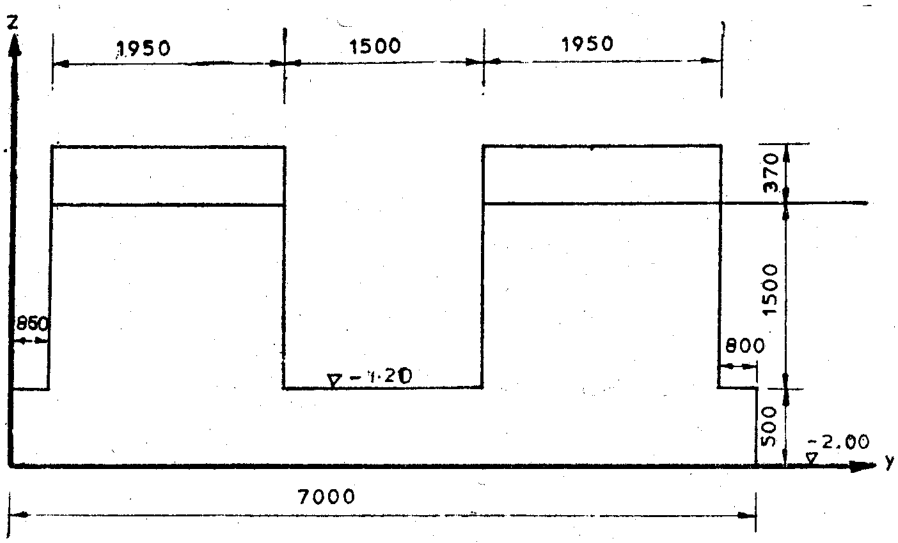

Fig 3 c—Section along the axis of main shaft section y y'

Design Examples

Design a foundation for a reciprocating horizontal compressor, the following data being given,

Operating speed of the engine	= 120 R.P.M.
Horizontal unbalance force PSinwt	= 2.5 Sin wt tonnes
Weight of compressor	=26.0 tonnes
Weight of motor	=14.8 tonnes
Density of soil	= 1.8 t/m³

The horizontal unbalance force acts at a height of 0.5 metre above the top surface of the foundation. The soil is sandy having bearing capacity of 1.5 kg/cm². Use soil constants obtained in Table III.

Barkan's Method

Cofficient of elastic uniform shear of soil C_τ = 0.85 kg/cm³. Use foundation of the type shown in Fig. 3.

1. Determination of combined C.G.

Density of concrete = 2.2 t/m³

Let x_0, y_0, z_0 denote co-ordinates of the centre of gravity of the whole system w.r.t. co-ordinate axes.

$$x_0 = \frac{111.76}{24.02} = 4.64 ; \quad y_0 = \frac{84.67}{24.02} = 3.53 ; \quad z_0 = \frac{28.34}{24.02} = 1.19$$

$$\% \text{ eccentricity in } x - \text{direction} = \frac{4.64 - 4.50}{9.0} \times 100 = 1.56\%$$

$$\% \text{ eccentricity in } y - \text{direction} = \frac{3.52 - 3.50}{7.0} \times 100 = 0.0281$$

Table V

Computation of Combined Center of Gravity

Element of the System	Dimensions of the Element			Mass t–m^{-1} sec^2 m_1	Coordinates of C–G of the element wrt x,y,x axes			Static moment of mass of element wrt x,y,z			Moment of Inertia of the element wrt axes passing through CG. t–m—sec^2	Distance between CG. of the element and comined CG. (m)		$m_1(x^2_{01}+z^2_{01})$
	a_{x1}	a_{y1}	a_{z1}		x_1	y_1	z_1	$m_1 x_1$	$m_1 y_1$	$m_1 z_1$	$\frac{m_1}{12}(a^2_{x1}+a^2_{z1})$	x_{01}	z_{01}	
Compressor	—	—	—	2.23	4.11	3.34	2.50	9.17	7.45	5.57	—	0.53	1.31	+4.36
Motor	—	—	—	1.47	5.94	3.68	2.50	8.71	5.40	3.68	—	1.30	1.01	+5.00
1	9.0	7.0	0.50	7.50	4.50	3.50	0.25	33.8	27.0	1.87	50.80	0.14	0.94	+6.75
2	7.30	5.40	1.50	13.30	4.50	3.50	1.25	59 80	46.5	16.60	61.50	0.14	0.06	+0 031
3(2nos)	3.36	1.95	0.370	1.50	5.16	3.50	2.185	7.75	5.25	3.28	1.43	0.52	0.995	+1.85
—4	2.50	1.50	1.20	—1.02	5.94	3.50	1.40	—6.05	—3.57	—1.46	—0.63	1.30	0.21	—1.73
—5	2.270	1.25	1.50	—0.96	1.475	3.50	1.25	—1.420	—3.36	—1.20	—0.59	3.175	0.06	—9.70
				24.02				111.76	84.67	28.34	112.51			+6.56

Static soil pressure $= \frac{W}{A} = \frac{24.02 \times 9.81}{9 \times 7} = 3.75\ t/m^2$

$= 0.375\ kg/cm^2 < 1.5\ kg/cm^2$

Operating frequency of the engine $w = 120$ R.P.M. $= 2$ c.p.s.

$= 12.6$ rad/sec.

$w^2 = 158\ sec^{-2}$

Height of the force axis above the combined C.G., $h = 2+0.5-1.19 = 1.31$ m

Exciting momet about C.G. of the combined system $M = 2.50 \times 1.31 = 3.28$ t-m

2. Moment of Inertia

(a) Base Area

$$I = \frac{7 \times 9^3}{12} = 425\ m^4$$

(b) $I_m = \frac{1}{12}(a_{x1}^2 + a_{z1}^2)\, m_1 + m_1 (x_{o1}^2 + Z_{o1}^2)$

$I_m = 112.51 + 6.56 = 119$ t-m-sec^2

$I_{mo} = I_m + mL^2 = 119 + 24.02 \times 1.19^2 = 153$ t-m-sec^2 $(L = Z_o)$

$$\gamma = \frac{I_m}{I_{mo}} = \frac{119}{153} = 0.78$$

3. C_T for 63 m^2 area $= 0.85 \sqrt{\frac{10}{63}} = 0.34\ kg/cm^3$, $C\phi = 1.20\ kg/cm^3$

4. Natural frequency in sliding

$$w_{nx} = \sqrt{\frac{C_{TA}}{m}} = \sqrt{\frac{0.34 \times 10^3 \times 63}{24.02}} = 30 \text{ rad/sec.}, \quad f_{nx} = 4.8 \text{ c.p.s.}$$

5. Natural frequency in Rocking

$$w_{n\phi} = \sqrt{\frac{C\phi . I - WL}{I_{mo}}} = \sqrt{\frac{1.20 \times 10^3 \times 425 - (24.02 \times 9.81) \times 1.19}{153}} = 57.8 \text{ rad/sec.}$$

$$\therefore f_{n\phi} = 9.2 \text{ c.p.s.}$$

6. Natural frequency in combined mode

$$w^4_{nx\phi} - w^2_{nx\phi}\left(\frac{w^2_{n\phi} + w^2_{nx}}{\gamma}\right) + \frac{w^2_{nx} \cdot w^2_{n\phi}}{\gamma} = 0$$

$$w^2_{nx\phi} - \frac{(30)^2 + (57.8)^2}{0.78} w^2_{nx\phi} + \frac{(30)^2 \times (57.8)^2}{0.78} = 0$$

$w^2_{nx\phi_{1,2}} = 2.71 \times 10^3 (1 \pm 0.69)\ sec^{-2}$

$w_{nx\phi_1} = 29\ sec^{-1}$; $f_{nx\phi_1} = 4.6$ c.p.s.

$w_{nx\phi_2} = 67.8\ sec^{-1}$; $f_{nx\phi_2} = 10.8$ c.p.s.

7. Amplitudes

$\triangle(\omega^2) = m\, I_m (w_{nx\phi_1} - w^2)(w_{nx\phi_2} - w^2)$

$= 24.02 \times 119 ((29)^2 - (12.6)^2)((67.8)^2 - (12.6)^2)$

$= 8.6 \times 10^9$

$$A_x = \frac{C\phi \,.\, I - WL + C_T.AL^2 + I_m\, w^2}{\triangle(\omega^2)} \times P + \frac{C_T.AL.}{\triangle(\omega^2)} \,.\, M$$

$$= \frac{1.20 \times 425 - 236.19 + 0.34 \times 10^3 \times 63 \times (1.19)^2 + 119\ (12.6)^2}{8\ 6 \times 10^9} \times 2.5 +$$

$$\frac{0.34 \times 10^3 \times 63 \times 1.19}{8.6 \times 10^9} = 0.0143 + 0.0975 = 0.1118 \text{ mm.}$$

$$A\phi = \frac{C_T.AL}{\triangle\ (\omega^2)} \,.\, P + \frac{C_{TA} - mw^2}{\triangle\ (\omega^2)}\ M$$

$$= \frac{0.34 \times 10^3 \times 63 \times 1.19}{86 \times 10^9} \times 2.5 + \frac{0.34 \times 10^3 \times 63 - 24.02 \times (12.6)^2}{8.6 \times 10^9} \times 3.28$$

$$= 7.4 \times 10^{-6} + 6.2 \times 10^{-6} = 13.6 \times 10^{-6} \text{ radian}$$

$$Ax\phi = Ax + hA\phi = 0.1118 + 13.6 \times 10^{-6} \times 0.81$$

$$= 0.188 + 0.011 = 0.129 \text{ mm where } h = 2 - 1.19 = 0.81 \text{ m}$$

Richart's Method

$G = 1740\ t/m^2$

For the Foundation shown in Fig. 3.

1. Equivalent radii

$$r_x = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{63}{\pi}} = 4.48 \text{ m}$$

$$m\left(\frac{r^2\phi}{4} + \frac{h^2}{3}\right) = I_o = I_{mo} - \text{where } h = \text{height of block} = 2\text{m.}$$

$$24.02\left(\frac{r\phi^2}{4} + \frac{2^3}{3}\right) = 153 \ \therefore\ \ r\phi = 4.9 \text{ m}$$

2. Mass ratio

$$bx = \frac{mg}{\rho\ (rx)^3} = \frac{24.02 \times 981}{1.8\ (4.48)^3} = 1.45$$

Inertia Ratio

$$b\phi = \frac{I_o \times q}{\rho(r\phi)^5} = \frac{153 \times 9.81}{1.8 \times (4.9)^5} = 0.3$$

3. Frequency Factors

From Richarts charts Fig. 2b and 2c.

$a_x = 1.4$; $a\phi = 1.4$

4. Natural frequency in sliding.

$$w^2_{nx} = \frac{a^2_x\ Gg}{\rho ro^2} = \frac{(1.4)^2 \times 1740 \times 9.81}{1.8 \times (4.48)^2} = 930 \text{ sec}^{-2}$$

$$w_{nx} = 30\ 5 \text{ sec}^{-1} \ ;\ f_{nx} = 4.8 \text{ c. p. s.}$$

5. Natural frequency in Rocking

$$w^2_{n\phi} = \frac{a^2_\phi\, Gg}{\gamma\,(r_\phi)^2} = \frac{(1.4)^2 \times 1740 \times 981}{1.8 \times (4.9)^2} = 775 \text{ sec}^{-2}$$

$w_{n\phi} = 27.8 \text{ sec}^{-1}$; $f_{n\phi} = 4.35$ cp.s.

6. Natural Frequency in combined mode

$$w^4_{nx\phi} - \left[(w^2_{nx} + w^2_{n\phi}) + \frac{mz^2}{I_o} w^2_{nx} \right] w^2_{nx\phi} + w^2_{nx} \cdot w^2_{n\phi} = 0$$

$z = 1.19$

$$w^4_{nx\phi} - \left[(27.8)^2 + (30.5)^2 + \frac{24.02 \times (1.19)^2}{15^3} \times (30.5)^2 \right] w^2_{nx\phi} + (27.8)^2 \times (30.5)^2 = 0$$

$w^2_{nx\phi_1} = 725 \text{ sec}^{-2}$; $w_{nx\phi_1} = 26.8 \text{ sec}^{-1}$; $f_{nx\phi_1} = 4.2$ c.p,s.

$w^2_{nx\phi_2} = 1425 \text{ sec}^{-2}$: $w_{nx\phi_2} = 37.5 \text{ sec}^{-1}$; $f_{nx\phi_2} = 6$ c.p.s.

7. Amplitudes

Amplitude factors

$As = 0.27$; $Ar = 1.3$

$$A_x = \frac{P}{Gr_x} \times As = \frac{2.5}{(1740 \times (4.9)^3} \times 0.27 = 0.08 \text{ mm}$$

$$A\phi = \frac{M \times Ar}{G \times r_\phi^3} = \frac{3.28 \times 13}{1740 \times (4.9)^3} = 2.1 \times 10^{-5} \text{ radian}$$

$$A_x\phi = A_x + hA\phi = 0.08 + (2.1 \times 10^{-5}) \times 0.8 \times 10^{-3}$$
$$= 0.08 + 0.0168 = 0.0068 \text{ mm}$$

Pauw's Method

From Table III, using smallest values

$\beta = 3.87 \text{ kg/cm}^3$

$\beta' = 1.42 \text{ kg/cm}^3$

Assume $\alpha = 1$

For the foundation shown in Fig. 2,

$a = 9$ m ; $b = 7$ m

$$q = \frac{24.02 \times 9.81}{63} = 3.75 \text{ t/m}^2; \; h = \frac{q}{\rho} = \frac{3.75}{1.8} = 2.08 \text{ m}$$

$$S = \frac{\alpha h}{b} = \frac{1 \times 2.08}{7} = 0.297; \; r = \frac{a}{b} = \frac{9}{7} = 1.29$$

1. Spring constants

From Pauw's charts ; for above values of S and r.

$$\frac{\gamma_x^{xy}}{r} = 0\,95 \; ; \; \gamma_x^{xy} = 1.29 \times 0.95 = 1.23$$

$$k_x^{xy} = \beta' b^2 \gamma_x^{xy} = 1.42 \times (10)^3 \times (7)^2 \times 1.23 = 85100 \text{ t/m}$$

$$\frac{\gamma_{xy}^{xy}}{r} = 0.18 \ ; \quad k_{xz}^{xy} = \beta b^4 \gamma_{xz}^{xy} = 3.87 \times 10^3 \times (7)^4 \times 1.29 \times 0.18 \text{ t/m}$$
$$= 2160000 \text{ t/m}$$

2. Apparent Soil mass

$$\frac{Cm}{r} = 0.62 \ ; \quad Cm = 0.62 \times 1.29 = 0.795$$

$$ms = \frac{\rho\, b^3\, Cm}{g\alpha} = \frac{1.8 \times 7^3}{9.81 \times 1} \times 0.795 = 63.0 \text{ t-m}^{-1}\text{-sec}^2$$

3. Apparent mass moment of Inertia of soil I_s

$$\frac{C_1}{r^3} = 0.21 \ ; \quad C_1 = (1.29)^3 \times 0.21 = 0.451$$

$$I_s = \frac{C_1\, \rho\, b^5}{12\, g\alpha} = \frac{0.451 \times (7)^5 \times 1.8}{19 \times 9.81 \times 1} = 116 \text{ t-m-sec}^2$$

Height of C.g. of the soil mass and foundation and machine above the base

$$Z = \frac{24.02 \times 1.19}{63.0 + 24.02} = 0.328 \text{ m}$$

$$I_{ms} = I_m + m\,(Z_{01} - Z)^2 + I_s + m_s\,(Z)^2$$
$$= 119 + 24.02\,(1.19 - 0.328)^2 + 116 + 63\,(0.228)^2 = 256 \text{ t-m-sec}^2$$

4. Natural Frequency in Sliding

$$w_{nx} = \sqrt{\frac{kx}{m + m_s}} = \sqrt{\frac{85100}{24.02 + 63.0}} = 31.3 \text{ sec}^{-1}$$

$$f_{nx} = 5 \text{ c.p.s.}$$

5. Natural Frequency in Rocking.

$$w_{n\phi} = \sqrt{\frac{k_{xz}}{I_{ms}}} = \sqrt{\frac{2160000}{256}} = 92 \text{ rad/sec.}$$

$$f_{n\phi} = 14.5 \text{ c.p.s.}$$

6. Natural Frequency in Combined Mode

$$w^2_{nx\phi 1,2} = \frac{1}{2}\left[\left(\frac{kx}{m} + \frac{Z^2\, k_x + k_{zx}}{I_{ms}}\right) \pm \sqrt{\left(\frac{kx}{m} + \frac{z^2\, kx + kzx}{I_{ms}}\right)^2 - \frac{4kx\, kxz}{m\, I_{ms}}}\right]$$

$$w^2_{nx\phi 1,1} = \frac{1}{2}\left[\frac{85100}{87.02} + \frac{(0.328)^2 \times 85100 + 2160000}{256} \pm \sqrt{\left(\frac{85100}{87.02} + \frac{(0.328)^2 \times 85100 + 2160000}{256}\right)^2 - \frac{4 \times 85100 \times 2160000}{86.02 \times 256}}\right]$$

$$= 4715\,(1 \pm 0.90)$$

$w_{nx\phi_1} = 21.7 \text{ sec}^{-1}$; $f_{nx\phi_1} = 3.5$ c.p.s.

$w_{nx\phi_2} = 94.8 \text{ sec}^{-1}$; $f_{nx\phi_2} = 15.1$ c.p.s.

7. Amplitudes

Assume damping factor = 0.20

$$A_x = \frac{P}{k_x\sqrt{\left(1-\left(\frac{w}{w_{nx}}\right)^2\right)^2 + \left(2\xi\frac{w}{w_{nx}}\right)^2}}$$

$$= \frac{2.5}{85100\sqrt{(1-(0.4)^2)^2+(2\times0.2\times0.4)^2}} = 0.03 \text{ mm}$$

$$A\phi = \frac{Q}{k_{xz}\sqrt{\left(1-\left(\frac{w}{w_{n\phi}}\right)^2\right)^2 + \left(2\xi\frac{w}{w_{n\phi}}\right)^2}}$$

$$= \frac{3.28}{2160000\sqrt{(1-0.138^2)^2+(2\times0.2\times0.138)^2}} = 1.77\times10^{-6} \text{ radian}$$

$$A_{x\phi} = A_x + h\,A\,\phi$$

$$= 0.03 + (0.81\times1.77\times10^{-6})\times10^3$$

$$= 0.03 + 0.014 = 0.0443 \text{ mm}$$

Design a foundation for a horizontal reciprocating engine with following data.

Speed of the engine	= 200 R.P.M.
Total weight of engine	= 7000 kg
Weight of reciprocating parts	= 45 kg
Weight of eccentrically rotating parts	= 30 kg
Length of connecting rod	= 100 cms
Crank radius	= 30 cm
Height of horizontal unbalance force above the top surface of foundation	= 30 cm
Soil-Uniform Silty clay	
Allowable bearing capacity	= 1.0 kg/cm²

Use soil constants determined in Table IV

Solution

$$P = \frac{W_p}{g} rw^2 \left(\cos wt \frac{r}{l} \cos 2\,wt\right) + W_c\, rw^2 \cos wt$$

$$(P)_{max} = \frac{W_p}{g} rw^2 \left(1+\frac{r}{l}\right) + \frac{W_c}{g} rw^2,\quad w = \frac{200}{60}\times 2\pi = 21 \text{ rad/sec}$$

$$= \frac{0.045}{9.81}\times 1 \times (21)^2\,(1+0.3) + \frac{0.03}{9.81}\times 0.3\,(21)^2 = 1.2 \text{ t.}$$

Barkan's Method

Adopt a block foundation shown in Fig 4.

Density of concrete $= 2.2\ t/m^2$

Wt. of foundation block $= (4\times4\times0.5 + 3\times3\times1)\times2.2$

$= 37.4$ t.

$$\text{Soil pressure} = q = \frac{44.4}{4\times4} = 2.78\ t/m^2$$

$$= 0.278\ kg/cm^2 < 1.0\ kg/cm^2$$

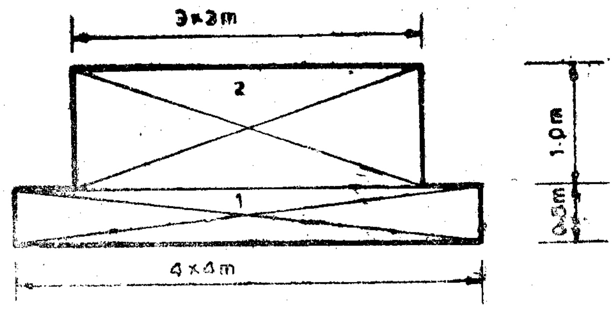

Fig. 4. Cross-section of the foundation block

1. Hight of C-G. above base of foundation

$$Z = \frac{3\ 57}{4.345}\ 0.825\ m$$

Exciting Moment $= M = 1.2\times0.975 = 1.17$ t-m

2. Moment of Inertia

$$I = \frac{4\times4^3}{12} = 21.3\ m^4$$

$$I_m = 3.95 + 1.68 = 5.63\ \text{t-m-sec}^2$$

$$I_{mo} = I_m + m\,L^2 = 5.63 + \frac{4.44}{9.81}(0.825)^2$$

$$= 8.58\ \text{t-m-sec}^2$$

$$\gamma = \frac{I_m}{I_{mo}} = \frac{5.63}{8.58} = 0.65$$

3. Soil Constants

Cu for 10 m^2 area (Table IV) $= 1.68\ kg/cm^3$

Cu for 16 m^2 area $= 1.86\sqrt{\frac{10}{16}} = 1.47\ kg/cm^3 = 1.47\ t/m^3$

$C_T = 0.735\times10^3\ t/m^3$

$C\phi = 2.5\times10^3\ t/m^3$

Table V Computation of Centre of Gravity

Element	Dimension of element			Mass t-m^{-1} —sec^2	Cordinates of C.G. of the element w.r.t. x,y,z axes (m)			Static moment w.r.t. z—axis t-m	Mass moment of inertia of element about an axis through its C.G.	Distance between C.G. of the element and combined C.G.		
A	a_{x1}	a_{y1}	a_{z1}	m_1	x_1	y_1	z_1	$m_1 z_1$	$\frac{m_1}{12}(a_{x1}^2+a_{z1}^2)$	x_{01}	z_{01}	$m_1(x_{01}^2+z_{01}^2)$
Enine	—	—	—	0.715	2	2	1.80	1.28	—	0	0.975	0.680
1	4.0	4.0	0.5	1.79	2	2	0.25	0.45	2.42	0	0.725	0.945
2	3.0	3.0	1.0	1.84	2	2	1.0	1.84	1.53	0	0.175	0.056
				4.345				3.57	3.95			1.681

4. Natural frequency in Sliding

$$w_{nx} = \sqrt{\frac{C_T A}{m}} = \sqrt{\frac{0.735\times 10^3\times 16}{4.35}} = 54.8 \text{ sec}^{-1}$$

$$f_{nx} = 8.5 \text{ c.p.s. }; \quad w^2_{nx} = 3.0\times 10^3 \text{ sec}^{-2}$$

5. Natural frequency in Rocking

$$w_{n\phi} = \sqrt{\frac{C\phi.I-WL}{I_{mo}}} = \sqrt{\frac{2.5\times 10^3\times 21.3-44.4\times 0.825}{8.58}} = 78 \text{ sec}^{-1}$$

$$f_{n\phi} = 12.4 \text{ c/s }; \quad w^2_{n\phi} = 6.1\times 10^3 \text{ sec}^{-2}$$

6. Natural frequency in combined mode

$$w^4_{nx\phi} - \frac{w^2_{n\phi} + w^2_{nx}}{\gamma} w^2_{nx\phi} + \frac{w^2_{nx}+w^2_{n\phi}}{\gamma} = 0$$

$$w^4_{nx\phi} - \frac{6.1\times 10^3+3.0\times 10^3}{0.65} w^2_{nx\phi} + \frac{6.1\times 3.0\times 10^6}{0.65} = 0$$

$$w^2_{nx\phi_1} = 2.415\times 10^3 \text{ sec}^{-2} ; \; w_{nx\phi_1} = 49.1 \text{ sec}^{-1}; \; f_{nx\phi_1} = 7.8 \text{ c.p.s.}$$

$$w^2_{nx\phi_2} = 11.58\times 10^3 \text{ sec}^{-2} ; \; w_{nx\phi_2} = 108 \text{ sec}^{-1} ; \; f_{nx\phi_2} = 17.2 \text{ c.p.s.}$$

7. Amplitudes

$$\Delta(\omega^2) = m\, I_m\, (w^2_{nx\phi_1}-w^2)\,(w^2_{nx\phi_2}-w^2)$$

$$= 4.35\times 5.63\,(2.415\times 10^3-441)\,(11.58\times 10^3-441) = 5.4\times 10^8$$

$$A_x = \frac{C\phi\,.\,I-WL+C_T.AL^2+I_m\,w^2}{\Delta(\omega^2)} \times P + \frac{C_T.AL}{\Delta(\omega^2)} \,.\, M$$

$$= \frac{2.5\times 10^3\times 21.3-44.4\times 0.825+0.77\times 10^3\times 16\times 0.8-5.63(21)^2}{5.4\times 10^8}\times 1.2 +$$

$$\frac{0.77\times 10^3\times 16\times 0.825}{5.4\times 10^8}\times 1.17 = 0.152 \text{ mm.}$$

$$A\phi = \frac{C_T.AL}{\Delta(w^2)} .. P + \frac{C_T AL-mw^2}{\Delta(w^2)} M$$

$$= \frac{0.77\times 10^3\times 16\times 0.825}{5.4\times 10^8}\times 1.2 + \frac{0.77\times 10^3\times 16-4.35\times 441}{5.4\times 10^2}\times 1.17$$

$$= 4.5\times 10^{-5} \text{ radian}$$

$$Ax\phi = A_x+h\,A\phi = 0.152+0.675\times 4.15\times 10^{-5}\times 10^3$$

$$= 0.152+0.0302 = 0.182 \text{ mm}$$

Richart's Method

Try Foundation shown in Fig. 4.

Adopt value of $G = 2200$ t/m² (From Table IV)

1. Equivalent radii

$$rx = \sqrt{\frac{16}{\pi}} = 2.26 \text{ m}$$

$$m\left(\frac{r^2\phi}{4} + \frac{h^2}{3}\right) = I_{mo}$$

$$4.35\left(\frac{r\phi^2}{4} + \frac{1.5^2}{3}\right) = 8.58 \; ; \quad r\phi = 2.20 \text{ m}$$

2. Mass ratio

$$bx = \frac{mg}{\rho \, rx^3} = \frac{4.35 \times 9.81}{1.91 \times (2.26)^3} = 2$$

3. Inertia ratio

$$b\phi = \frac{I_{mo} \times g}{\rho (r\phi)^5} = \frac{8.58 \times 9.81}{1.91 \times (2.2)^5} = 1.0$$

4. Frequency Factors

From Richarts charts (Fig. 2)

$a_x = 1.7$; $a\phi = 1.38$

5. Natural frequency in sliding.

$$w_{nx} = \frac{a_x}{r_x}\sqrt{\frac{G.g}{\rho}} = \frac{1.7}{2.26}\sqrt{\frac{9.81 \times 2200}{1.91}} \; 80 \text{ sec}^{-1}$$

$f_{nx} = 12.7$ c.p.s.

6. Natural frequency in rocking

$$w_{n\phi} = \frac{a\phi}{r\phi}\sqrt{\frac{g\,G}{\rho}} = \frac{1.38}{2.2}\sqrt{\frac{9.81 \times 2200}{1.91}} = 67 \text{ sec}^{-1}$$

$f_{n\phi} = 10.7$ c.p.s.

7. Natural frequency in combined mode

$$w^4_{nx\phi} - \left[(w^2_{nx} + w^2_{n\phi}) + \frac{m}{I_{mo}} Z^2 \, w^2_{nx}\right] w^2_{nx\phi} + w^2_{nx} \cdot w^2_{n\phi} = 0$$

$$w^4_{nx\phi} - \left[(84)^2 + (67)^2 + \frac{4.35}{8.58}(0.825)^2 \times (84)^2\right] w^2_{nx\phi} + (84)^2 \times (67)^2 = 0$$

$w^2_{nx\phi_1} = 2800 \text{ sec}^{-2}$: $w_{nx\phi_1} = 53 \text{ sec}^{-1}$; $f_{nx\phi_1} = 8.45$ c.p.s.

$w^2_{nx\phi_2} = 11200 \text{ sec}^{-2}$: $w_{nx\phi_2} = 106 \text{ sec}^{-1}$; $f_{nx\phi_2} = 16.9$ c.p.s.

8. Amplitude factors

$A_s = 0.5$; $A_r = 1.2$

Amplitudes

$$Ax = \frac{P \times As}{G\, r_x^3} = \frac{1.2 \times 0.5}{2200 \times (2.26)^3} = 0.12 \times 10^{-3}\ m = 0.12\ mm$$

$$A\phi = \frac{A_r \times M}{G \times (r\phi^3)} = \frac{1.2 \times 1.17}{2200 \times (2.2)^3} = 0.06 \times 10^{-3}\ \text{radian}$$

$$A_x\phi = Ax + h\, A\phi = 0.12 + 0.06 \times 10^{-3} \times 0.675 \times 10^3 = 0.1605\ mm$$

Pauw's Method

Use the same foundation as above,

The values of β and β' to be used are as given in (Table IV)

$\beta = 0.97\ kg/cm^3 = 970\ t/m^3$

$\beta' = 0.42\ kg/cm^3 = 420\ t/m^3$

$E_o = 740\ kg\ cm^2$

Assume $\alpha = 1$

$$q = \frac{44.4}{4 \times 4} = 2.78\ t/m^2;\ h = \frac{E_o}{\beta} + \frac{q}{\rho} = \frac{740}{97.0} + \frac{2.78}{1.91} = 0.9\ m$$

$$r = \frac{a}{b} = 1;\ S = \frac{ah}{b} = \frac{1 \times 9}{4} = 2.25$$

1. Spring constants

$$\frac{\gamma_x^{xy}}{r} = 1.8;\ \gamma_x^{xy} = 1.8,\ k_x^{xy} = \beta^1 b^2\, \gamma_x^{xy} = 420 \times 16 \times 1.8 = 12100\ t/m$$

$$\gamma_{xz}^{xy} = 0.48;\quad k_{xz}^{xy} = \beta b^4\, \gamma_{xz}^{xy} = 970 \times 256 \times 0.48 = 120000\ t/m$$

2. Apparent Soil mass

$$\frac{Cm}{r} = 1.8;\quad Cm = 1.8;$$

$$ms = \frac{\rho\, b^3}{g\alpha} Cm = \frac{1.91 \times 4^3}{9.81 \times 1} \times 1.8 = 22.4\ t\text{-}m^{-1}\text{-}sec^2$$

3. Apparent mass moment of Inertia of soil

$$\frac{C_1}{r} = 0.45;\quad C_1 = 0.45$$

$$I_s = \frac{\rho\, b^5}{12\, g\alpha} C_1 = \frac{1.91 \times (4)^5}{12 \times 9.81 \times 1} \times 0.45 = 7.2\ t\text{-}m\text{-}sec^2$$

4. Height of Combined C.G. of the soil and foundation etc.

$$\bar{Z} = \frac{4.35 \times 0.825}{4.35 + 22.4} = 0.134\ m$$

$$I_{ms} = \frac{1.79}{12}(4^2+0.5^2) + 1.79\,(0.116)^2 + 1.84\,(3^2+1^2) + 1.84\,(0.866)^2 + 0.715\,(1.66)^2 + 7.2 + 22.4\,(0.134)^2 = 14.95 \text{ t-m-sec}^2$$

5. Natural Frequency in Sliding

$$w_{nx} = \sqrt{\frac{k_x}{m+ms}} = \sqrt{\frac{12100}{4.35+22.4}} = 67.2 \text{ sec}^{-1}$$

$$f_{nx} = 10.7 \text{ c p.s.}$$

6. Natural Frequency in Rocking.

$$w_{n\phi} = \sqrt{\frac{k_{xz}}{I_{ms}}} = \sqrt{\frac{120000}{14.95}} = 90 \text{ sec}^{-1}$$

$$f_{n\phi} = 14.3 \text{ c.p.s.}$$

7. Natural frequency in combined mode

$$w^2_{nx\phi 1,2} = \frac{1}{2}\left[\left(\frac{kx}{m} + \bar{Z}^2 \frac{k_x + k_{zx}}{I_{ms}}\right) \pm \sqrt{\left(\frac{k_x}{m} + \bar{z}^2 \frac{k_x + k_{zx}}{I_{ms}}\right)^2 - \frac{4kx\,kxz}{m\,I_{ms}}}\right]$$

$$w^2_{nx\phi 1,2} = \frac{1}{2}\left[\left(\frac{12100}{26.7} + \frac{(0.134)^2 \times 12100 + 120000}{14.95}\right) \pm \sqrt{\left(\frac{12100}{26.7} + \frac{(0.134)^2 \times 12100 + 120000}{14.95}\right)^2 - \frac{4 \times 120000 \times 12100}{14.95 \times 26.7}}\right]$$

$w_{nx\phi 1} = 3700 \text{ sec}^{-2}$; $w_{nx\phi_1} = 60.8 \text{ sec}^{-1}$; $f_{nx\phi 1} = 9.65$ c.p.s.

$w_{nx\phi_2} = 9553 \text{ sec}^{-2}$; $w_{nx\phi 2} = 98 \text{ sec}^{-1}$; $f_{nx\phi 2} = 15.6$ c.p.s.

8. Amplitudes

$$Ax = \frac{P}{k_x \sqrt{\left(1 - \left(\frac{w}{w_{nx}}\right)^2\right)^2 + \left(2\xi \frac{w}{w_{nx}}\right)^2}}$$

Assume $\xi = 0.2$; $\frac{w}{w_{nx}} = \frac{21}{67.8} = 0.312$

$$= \frac{1.2}{12100\sqrt{(1-(0.312)^2)^2 + (2 \times 0.2 \times 0.312)^2}} = 0.1 \times 10^{-5} \text{ m} = 0.10 \text{ mm}$$

$$A\phi = \frac{M}{k_{xz} \sqrt{\left(1 - \left(\frac{w}{w_{n\phi}}\right)^2\right)^2 + \left(2\zeta \frac{w}{w_{n\phi}}\right)^2}}$$

$$\frac{w}{w_{n\phi}} = \frac{21}{90}$$

$$= \frac{1.17}{120000\sqrt{\left(1-\left(\frac{21}{90}\right)^2\right)^2+\left(2\times0.2\times\frac{21}{90}\right)^2}} = 1.02\times10^{-5} \text{ radian}$$

$$\begin{aligned} Ax\phi &= Ax + h\,A\phi = 0.1\times10^{-3} + 1.02\times10^{-5}\times0.675 \\ &= 0.107 \times 10^{-3}\text{ m} \quad = 0.107\text{ mm} \end{aligned}$$

Comparison of Results

Example A

Table VI

Method	Barkan	Pauw	Richart
Natural frequency in sliding $f_{n\phi}$ c p.s.	4.8	5.0	4.8
Natural frequency in rocking $f_{n\phi}$ c.p s.	9.2	14.5	4.35
Natural frequency in combined mode $f_{nx\phi 1}$, $f_{nx\phi 2}$ c.p.s.	4.6, 10.8	3.5, 15.1	4.2, 6.0
Amplitude in sliding A_x mm	0.1118	0 03	0.08
Amplitude in rocking $A\phi$ radian	13.6×10^{-6}	1.77×10^{-6}	2.1×10^{-5}
Amplitude in combined mode $A_{x\phi}$	0.129	0.0443	0.0968

Example B

Table VII

Method	Barkan	Pauw	Richart
Natural frequency in sliding f_{nx} c p s.	8.5	10.7	13.4
Natural frequency in rockin $f_{n\phi}$ c p.s.	12.4	14.3	10.7
Natural frequency in combined mode, $f_{nx\phi 1}$, $f_{nx\phi 2}$ c.p.s.	7.8, 17.2	9.65, 15.6	8.45, 16.9
Amplitude in sliding A_x mm	0.152	0.100	0.120
Amplitude in rocking A_ϕ radian	4.5×10^{-5}	1.02×10^{-5}	6.0×10^{-5}
Amplitude in combined mode $Ax\phi$ mm	0.182	0.107	0.1605

Conclusions

A comparison of natural frequencies and amplitudes obtained in the two design problems for typical foundation shows that these three methods give results which are sufficiently in agreement. The slight difference is due to assumptions made in particular type of analysis.

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