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A NOTE ON COMPUTING THE CONTRIBUTION OF ROCKING EXCITATION TO EARTHQUAKE RESPONSE OF SIMPLE BUILDINGS

by

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INTRODUCTION

It has been recognized by a number of investigators that the rocking excitation during strong earthquake ground motion may contribute significantly to the overall response of structures. Studies by Bielak, 1978; Yim et al., 1980; Ishiyama, 1982; Psycharis and Jennings, 1983; Koh and Spanos, 1984; Kashefi and Trifunac, 1986 and Gupta and Trifunac, 1987 have indicated the importance of studying the rocking contribution to the total response of structures.

At present there are no recorded strong motion rocking accelerograms. However. by using the theory of elastic wave propagation in layered medium it can be shown that the rocking accelerations are related to the translational accelerations of ground motion (Trifunac, 1982). Therefore, it is possible to generate the rocking accelerograms from the recorded components of translational acceleration (Lee and Trifunac, 1987)

For multi-degree-of-freedom systems, the simplest and the commonly used practice for analyzing the structural response under translational excitation is the use of some response spectrum superposition method. Therefore, it is natural to extend the spectrum approach for analyzing the rocking contribution to the response also. Amini and Trifunac (1985) and Gupta and Trifunac (1987) have proposed a new statistical approach for response spectrum superposition, which give excellent agreement with the time history solutions for the case of translational response. This approach can also provide the amplitudes of all the significant peaks of the response which is not possible from other conventional methods, It will be shown here that their statistical method of spectrum superposition can be used for studying the rocking responses also. An example will be presented to illustrate the contributton of rocking excitations to total maximum response. Effects of varying the heights of building stories, the dimensions of the floors, and changes of the minimcm shear wave velocity in the layered ground, on the additional contribution to response due to free-field rocking

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excitations, will be investigated. This will provide an idea about the conditions under which rocking response is significant.

ROCKING

To use the statistical method of Gupta and Trifunac (198T) for spectrum superposition, which is based on the order statistics of peaks and to find the contribution of the rocking accelerations to the the total response of a structure, one should have the Fourier and the response spectra of the input rocking ground acceleration. Lee and Trifunace (1987) have proposed analytical procedures for finding the rocking ground motions and their spectra, directly from the translation ground motions Following tne ideas of Newmark (1969), Nathan and Mackenzie (1975) found the time history of rotational motion from the two translational components by using the differences between the translations at opposite edges of the foundation, and thus they accounted for the averaging effects of the foundation size. In this method for generating the rotational accelerograms, translational time histories have to be differentiated. Tso and Hsu (1978) presented a scheme for computing the rotational spectra which eliminates differentiation of the acceleration records. The methods of Newmark (1969), Nathan and Mackenzie (1975) and Tso and Hsu (1978) for computing rotational ground motions and their response spectra assume horizontally travelling seismic waves of constant shapes and velocities, and thus, neglect the dispersive nature of seismic waves In the present work we shall find the rocking spectra by using a a procedure based on the studies by Lee and Trifunac (1987). Considering the wave propagation in elastic layered medium, Lee and Trifunac have introduced the effects of wave dispersion and transient arrivals in their method of generating artificial rocking accelerograms. They have shown that the rocking Ψ_{12} Figures 1 and 2 is related to the translational component u_2, by

$$\Psi_{12} = i\frac{\omega}{C}u_2 \quad . \tag{1}$$

In this expression, ω is the circular frequency and C is the phase velocity of surface waves, $i = \sqrt{-1}$ and u_2 is the vertical motion associated with incident P and / or SV waves (Figures 1 and 2).

$$\frac{|\Psi_{12}(\omega)|}{|A_2(\omega)|} = \frac{\omega}{\beta_{max}}, \text{ as } \omega \rightarrow 0 \quad . \tag{2}$$

When $\omega \rightarrow \infty$ the phase velocities of all modes converge to be β_{min} and

$$\frac{|\Psi_{12}(\omega)|}{|A_2(\omega)|} = \frac{\omega}{\beta_{min}}, \text{ as } \omega \rightarrow \infty, \tag{3}$$

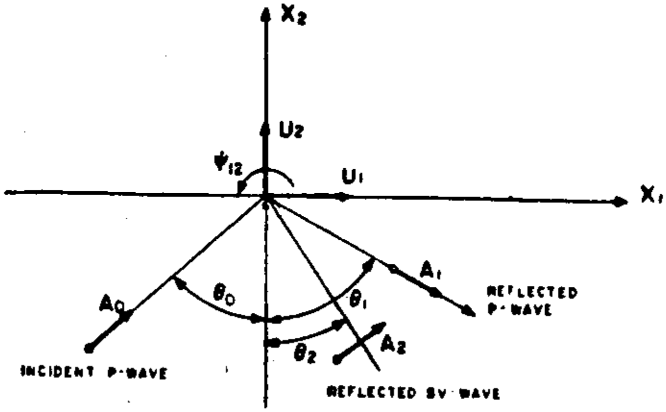

Fig. 1 Coordinate system for incident P-wave (from Trifunac, 1982).

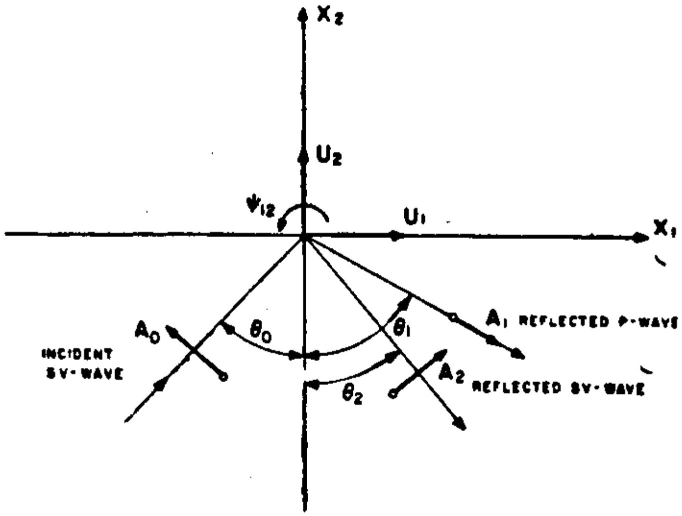

Fig. 2 Coordinate system for incident SV-wave (from Trifunac, 1982).

where A_2 (ω) is the Fourther spectrum of u_2.

From equation (2) and (3) it is seen that the velocity representing the overall trend of the ratios of rotational to translational spectra is not constant. It changes from β_{min} for ($\omega \rightarrow \infty$) to β_{max} (for $\omega \rightarrow 0$). A good approximation for the variation of velocity with frequency may be the harmonic mean of the phase velocities of various modes available at a particular frequency. Lee and Trifunac (1987) have shown that approximating the behavior of $\log_{10}$ [$|\Psi|/|A|$] versus $\log_{10}$ [Period] by a straight line joining the asymptotic levels, is a good approximation. Therefore, in the present study, we shall compute the rocking spectra, $\Psi_{12}(\omega)$, by using their approximation,

In the above discussion we employed the notation of Lee and Trifunac (1987). In the following equations, however, Ψ_{12} will be designated by θ_R, the horizontal motion u_1 by z and the vertical ground motion u_2 will be neglected in computing the structural response.

COMBINED TRANSLATIONAL AND ROCKING RESPONSE

To find the contribution from the rocking motion to the lateral response of a structure, we model the structure by concentrated masses, springs and dashpots. Figure 3 shows the lateral deformation of the structure due

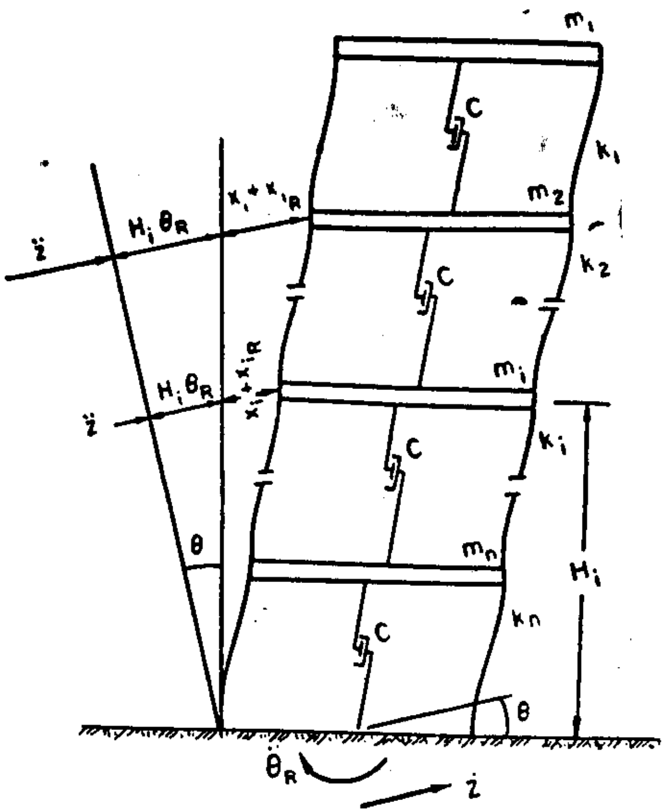

Fig. 3. Total Shear Deformation of a Multi-Story Structure due to Combined Translational and Rocking Excitation of the Ground.

to simultaneous action of the translational, z (t). and the rocking, $\theta_R(t)$, components of earthquake ground motion. The total relative lateral displacement at the i - th level. x_{itot}' is the sum of the relative displacement x_i due to translational acceleration z, and an additional relative displacement x_{iR} due to the acceleration $H_i\,\theta_R$ acting at the i-th level due to ground rocking, Thus the equations of motion for the structure of Figure 3 can be written in matrix from as

$$[m]\,\{\ddot{x}_{tot}\} + [C]\,\{\dot{x}_{tot}\} + [K]\,\{x_{tot}\} = -\ddot{z}[m]\,\{I\} - \ddot{\theta}_R\,[m]\,\{H\}. \quad (4)$$

In this equation, $\{x_{tot}\}$ is the total relative displacement vector, and { H } is the story height vector

$$\{x_{tot}\} = \begin{Bmatrix} x_{1\,tot} \\ x_{2\,tot} \\ \vdots \\ x_{n\,tot} \end{Bmatrix}, \quad \text{and } \{H\} = \begin{Bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{Bmatrix} \quad (5)$$

[m], [C] and [K] represent mass, damping and stiffness matrices. Equations (4) can be uncoupled by the transformation

$$\{x_{tot}\} = [A]\,\{\xi_{tot}\} \quad . \quad (6)$$

Using this transformation, the equation of motion for the j-th mode is written as

$$\ddot{\xi}_{jtot} + 2\zeta_j\omega_j\dot{\xi}_{jtot} + \omega^2_j\xi_{jtot} = -\ddot{z}\alpha_j - \ddot{\theta}_r\alpha H_j\,, \quad (7)$$

where α_j is the mode participation factor for the translation response,

$$\alpha_j = \frac{\{A^j\}^T\,[m]\,\{I\}}{\{A^j\}^T\,[m]\,\{A^j\}} \quad j = 1,2,\ldots,n\,. \quad (8)$$

and αH_j is the participation factor for rocking response

$$\alpha H_j = \frac{\{A^j\}^T\,[m]\,\{H\}}{\{A^j\}^T[m]\,\{A^j\}}\,, j = 1,2,\ldots,n\,. \quad (9)$$

From equation (7), the contribution of the j-th mode to the total displacement energy spectrum at the i-th level can be written as (e.g. see equation (23) in Amini and Trifunac, 1985 or equation A. 7 in Gupta and Trifunac, 1987)

$$E_{ij}(\omega) = \frac{1}{T}\;\frac{A^2_{ij}\,[\alpha_j\,\overline{z}\,(\omega_j) + \alpha Hj\,\overline{H_R}\,(\omega_j)]^2}{4\zeta\omega_j^3}\,\delta(\omega-\omega_j)\,, \quad (10)$$

where $\overline{H_R}$ is the averaoe value of the Fourier spectrum amplitudes, H_R, of the rocking accelerogram $\ddot{\theta}_R$, over the frequency band of width $\pi\zeta_j\omega_j$ centered at ω_j . $\overline{z}\,(\omega_j)$ is the average value for the Fourier spectrum of $\ddot{z}$.

Thus the energy density of the lateral displacement response of the i-th floor is given by

$$ED_i(\omega) = \sum_{j=1}^{n} E_{ij}(\omega) = \frac{1}{T}\sum_{j=1}^{n} \frac{A\omega_{ij}[\alpha_j z(\omega_j) + \alpha H^j H_R(\omega_j)]^2}{4\zeta_j \omega^3_j} \delta(\omega - \omega_j). \quad (11)$$

Similarly, using the results of Amini and Trifunac (1985) or Gupta and Trifunac (1987) the energy spectra $ES_i(\omega)$ and $EM_i(\omega)$ for the shear and bending moment responses of the i-th level can be written as

$$ES_i(\omega) = \frac{1}{T}\sum_{j=1}^{n} \frac{(m_1A_{1j} + m_2A_2 + \ldots + m_iA_{ij})^2[\alpha_j \bar{z}(\omega_j) + \alpha \bar{H}_j \bar{H}_R(\omega_j)]^2\omega_j}{4\zeta_j} \times \delta(\omega - \omega_j) \quad (12)$$

$$EM_i(\omega) = \frac{1}{T}\sum_{k=l}^{i}\sum_{j=l}^{n} \frac{h^2_k(m_jA_{1j} + m_2A_{2j} + \ldots + m_kA_{kj})^2[\alpha_j \bar{z}(\omega_j) + \alpha H_j \bar{H}_R(\omega_j)]^2\omega_j}{4\zeta_j}\delta(\omega - \omega_j). \quad (13)$$

Knowing the energy density spectra $ED_i(\omega)$. $ES_i(\omega)$ and $EM_i(\omega)$ for the displacement, base shear and overturning moment responses at the i-th level, due to the combined translational and rocking excitation, one can compute their zeroth, second and fourth order moments m_{0i}, m_{2i} and m_{4i}. Using these, one can find the parameters $\bar{a}_i$, ε_i and N_i for scaling the probability distribution function of the peaks of the responses. (Amini and Trifunac, 1985; Gupta and Trifunac, 1987. Values of $\bar{a}_B$ and $\bar{a}_\mu$ for the parameter $\bar{a}$ can also be determind by using the procedure of Amini and Trifunac (1985).

The maximum displcement, D_{ij}, at the i-th mode is given by

$$D_{ij} = A_{ij}[\alpha_j SD_j + \alpha H_j SD\theta_{jR}]. \quad (14)$$

The maximum shear force S_{ij} and maximum overturning moment M_{ij} are, similarly, given by

$$S_{ij} = (m_1A_{1j} + m_2A_{2j} + \ldots\ldots m_iA_{ij})\omega^2_j[\alpha_j SD + H_j SD\theta_{jR}] \quad (15)$$

$$M_{ij} = (h_1m_1A_{1j} + h_2(m_1A_{1j} + m_2A_{2j}) + \ldots\ldots\ldots + h_i(m_1A_{1j} + m_2A_{2j} + \ldots + m_iA_{ij})\omega^2_j(\alpha_j SD_j + \alpha H_j SD\theta_{jR}] \quad (16)$$

Using these results, the expected and the most probable values of maximum displacement, shear force and bending moment responses at various levels of a structure can be evaluated by using the order statistics formulation of Gupta and Trifunac (1987).

AN APPLICATION TO THE RESPONSE OF A THREE STORY BUILDING

The preceding analysis is illustrated here for a simple three story structure. The elevation of the building is shown in Figure 4. It is sssumed that the building has rigid floors. Further, without loss of genearlity, it is assumed that the structure is subjected to ground acceleration z (t) in x-direction ,only and has only translational (shear) degree of freedom in th's direction. The mass matrix of the structure is chosen to be

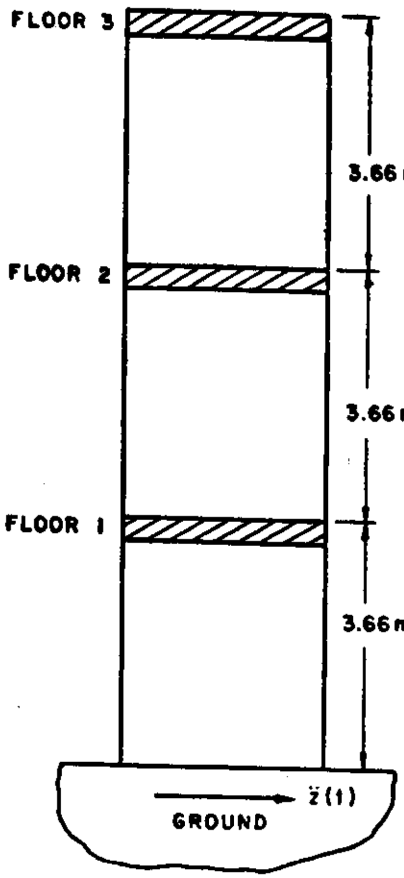

Fig. 4 Elevation of the Example Structure.

$$[m] = \begin{bmatrix} 175.13 & 0 & 0 \\ 0 & 262.70 & 0 \\ 0 & 0 & 350.26 \end{bmatrix} \left(\frac{kN-S^2}{m}\right). \quad (17)$$

The stiffness matrix is assumed to be

$$[K]=105\times10^{3}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix} \text{ (kN/m).} \qquad (18)$$

Solving the eigen value problem, the modal frequencies and eigenvectors have been found. Modal frequency vector is

$$\{\omega\}=\begin{Bmatrix} 14.5 \\ 31.1 \\ 46.1 \end{Bmatrix} \text{ (rad/s).} \qquad (19)$$

The matrix of modal eigenvectors is

$$[A]=\begin{bmatrix} 1.0 & 1.0 & 1.0 \\ 0.644 & -0.601 & -2.57 \\ 0.300 & -0.676 & 2.47 \end{bmatrix}. \qquad (20)$$

The mode participation factors are found to be

$$\{\alpha\}=\begin{Bmatrix} 1.424 \\ -0.509 \\ 0.090 \end{Bmatrix}, \qquad (21)$$

Using [m] and [A], one can find the values of ${}_{\alpha}H_j$ for any story height vector {H}. The fractions of critical dampings, ζ_j, have been chosen as 0.05 for all the modes of vibration.

Keeping the values of damping, mass and stiffness of various levels constant the effects of changing the story heights from H=3.66 m to 2H have been, studied for β_{min}=350 m/s, 1750m/s and 3500 m/s with β_{max} fixed 3500 m/s (see Lee and Trifunac, 1987). The expected and the most probable of maximum translational response have been also evaluated for excitation by only the translational component SOOE of the E1 Centro Earthquake. In these calculations, the rocking spectra have been evaluated spectra from the spectra of vertical component of the earthquake (see equations (1), (2) and (3)). The results are given in Tables 1 through 6. It is found that the contribution of rocking excitation increases as the story height are increased and with {ω} kept fixed Assuming that the mimimum shear wave velocity occnrs in the top layer, it can be coucluded that the contribution of rocking excitation is more significant for tall (but stiff) buildings of soft ground. Tables 1 through 6 provide a quantitative idea about this contribution, for the simple three story example structure.

CONCLUSIONS

From the combined translational and rocking repsonse analysis of the simple example structure, it is found that the rocking excitations may cause increase in maximum lateral displacement, shear force and bending moment responses. The contribution of rocking can increase with increasing the height of the structure (while keeping its natural frequencies fixed). However, the contribution of the rocking may also diminish with increasing the height of the structure because the amplitude of the rotational spectrum decreases like $1/T_n$ as the natural period of the structure, T_n, increases (i e. keeping the story heights fixed and increasing the number of stories) Further the effects of rocking excitations are more prominant for low values of the minimum shear wave velocity in the layered medium on which the structure is standing. Thus the contribution to the total response that comes from rocking will increase with increase of the β_{max}/β_{min}, total height of structure story height, and the story stiffness.

Though the above conclusions are not new, the present study provides a rational mathematical basis, by using statistical theory for response spectrum superposition, for estimating the actual increase in the translational response at any point of the structure. The method used for generating the spectra of rocking ground motion in this study is based on physical principles of wave propagation in layered ground and it take into account the dispersive nature of seismic waves, which is typically ignored by most of the methods used for this purpose so far.

REFERENCES

Amini, A and Trifunac, M.D. (1981). "Distribution of Peaks in Linear Earthquake Response," Journal of the Engineering Mechanics Division, ASCE, Vol. 107, No. EMI, 207 227.

Amini, A. and Trifunac, M D. (1985). "Statistical Extension of Response Spectrum Superposition," International Journal of Soil Dynamics and Earthquake Engineering, Vol. 4, 54-63.

Bielak, J. (1978). "Dynamic Response of Nonlinear Bulding-Foundation Systems," Journal of Earthquake Engineering and Structural Dynamics, Vol. 6, 17-30,

Gupta, I D. and M.D. Trifunac (1987). "Statistical Analysis of Response Spectra Method in Earthquake Engineering, "Report No. 87 - 03, Dept. of Civil Eng., Univ. of Southern Calif., Los Angeles, Calif.

Ishiyama, Y. (1982). "Motions of Rigid Bodies and Criterion for Overturning by Earthquake Excitations," Journal of Earthquake Engineering and Structural Dynamics, Vol. No. 5, 635 - 650.

Kashefi, I. and M.D. Trifunac (1986). "Investigation of Earthquake Response of Simple Bridge Structure," Report No. 86 - 02, Dept. of Civil Eng., Univ. of Southern California, Los Angeles, Calif,

Kho, A S. and Spanos, P. - T.D. (1984). "Seismically Induced Rocking of Rigid Structures," Proc. 8th world Conf. on Earthquake Engineering, Vol. IV, 251 - 258.

Lee, V.W. and Trifunac, M.D. (1987). "Rocking Strong Earthquake Accelerations, "Journal of Soil Dynamics and Earthquake Engineering (to be published).

Nathan, N. D. and MacKenzie, J.R. (1975). "Rotational Components of Earthquake Motion, " Canadian Journal of Civil Engineering, Vol, 2, 430-436

Newmark, N.M. (1969). "Torsion in Symmetrical Buildings," Proc 4th World Conf. on Earthquake Engineering, Santiago, Chile, Vol 2, A-3 19-30.

Psycharis, I N. And Jennings. P. C. (1983). "Rocking of Slender Rigid Blocks Allowed to Uplift: "Journal of Earthquake Engineering and Structural D namics, Vol. 11, 57 - 76.

Trifunac, M D (1982). "A note on Rotational Components of Earthquake Motions for Incident body Waves, Int. Journal Soil Dynamics and Earthquake Eng , Vol. I, 11-19.

Iso, W.K. and Hsu, T, I. (1978). "Torsional Spectrum for Earthquake Motions," Journal of Earthquake, and Structurarl Dynamics, Vol. 6, 375-382.

Yim, C S Chopra, A K. and Penzien, J. (1980). "Rocking Response of Rigid Blocks to Earthquakes," Journal of Earthquakes," Journal of Earthquakes," Engineering and Structural Dynamics, Vol. 8, No. 6, 565-587.

APPENDDIX II : NOTATION

The following symbols have been used in this work :

$\bar{a}$	r.m.s. amplitude of peaks or the respone
$\bar{a}_1$	parameter $\bar{a}$ for i-th story response
$\bar{a}_E$	modified $\bar{a}$ for the expected peak amplitudes

$\bar{\mu}^{a}$	modified $\bar{a}$ for the most procable peak amplitudes
$A_2(\omega)$	transform of u_2 ground motion
A_{ij}	j^{th} element of matrix [A]
[A]	modal transformation matrix
C	phase velocity
[C]	damping matrix
D_{ij}	maximum dispacement at i^{th} level due to j^{th} mode
$E_{ij}(\omega)$	function E (ω) for i^{th} story sesponse due to j^{th} floor
$ED_i(\omega)$	energy spectrum for displacement response at i^{th} floor
$ES_i(\omega)$	energy spectrum for base shear response at i^{th} floor
$EM_i(\omega)$	energy spectrum for overturning moment response about i^{th} floor
{H}	floor height vector
{I}	unit column vector
[K]	stiffness matrix
M_{ij}	maximum overturning moment about the i^{th} floor due to j^{th} mode of vibration
[m]	mass matrix
S_{ij}	maximum shear force response at i^{th} floor due to j^{th} mode
SD_j	spectral displacement at j^{th} modal frequency
$SD\theta_R(\omega)$	rocking spectral displacement
$SD\theta_{jR}$	rocking spectral displacement at j^{th} modal frequency
u_1, u_2, u_3	components of ground motion
x_{itot}	relative displacement of i^{th} floor due to combined translational and rocking motion of the ground
{Xtot}	total relative displacement vector
$\ddot{}_{s}(t)$	ground acceleration
$\bar{Z}(\omega)$	mean velue of Z (ω) over the inteval $\pi\zeta_j\omega_j$
α_j	jth mode participation factor
$\{\alpha\}$	mode participation vector
αH_j	j^{th} mode participation factor for the lateral response due to ground rocking
β_{min}, β_{max}	minimum and maximum shear wave velocities in the layered ground
ζ_j	critical damping ratio for j^{th} mode
$\theta_R(t)$	rocking acceleration
$H_R(\omega)$	transform of θ_R (t)
$H_R(\omega)$	mean value of $H_R(\omega)$ over the interval $\pi\zeta_j\ \omega_j$
$\{\zeta_{tot}\}$	generalized coordinate vector for total lateral response due to combined translational and rocking motion of the ground
Ψ_{12}	rotation about axis 3 from axis 1 to 2 (rocking)
ω	circular frequency
$\{\omega\}$	modal frequency vector

Table 1 Increase in Maximum Lateral Displacement Response Due to Ground Rocking for Different Story Heights and Different β_{min} and β_{max} in the Layered Ground. (Most Probable Values)

STORY HEIGHTS	FLOOR	TRANSLATIONAL DISPL. (Cm)	β_{max} = 3500 m/s, β_{min} = 350 m/s		β_{max} = 3500 m/s, β_{min} = 1750 m/s		β_{max} = 3500 m/s, β_{min} = 3500 m/s	
			TOTAL DISPL. (Cm)	INCREASE DUE TO ROCKING (%)	TOTAL DISPL. (Cm)	INCREASE DUE TO ROCKING (%)	TOTAL DISPL. (Cm)	INCREASE DUE TO ROCKING (%)
3.66 m	1	4.593	5.187	12.9	4.819	4.9	4.742	3.3
	2	2.958	3.341	12.9	3.104	4.9	3.055	3.3
	3	1.396	1.574	12.9	1.464	4.9	1.442	3.3
7.32 m	1	4.593	5.781	25.8	5.046	9.9	4.892	6.5
	2	2.958	3.721	25.8	3.250	9.9	3.151	6.5
	3	1.396	1.750	25.8	1.531	9.9	1.486	6.5

Table 2 Increase in Maximum Base Shear Response Due to Ground Rocking for Different Story Heights and Different β_{min} and β_{max} in the Layered Ground. (Most Probable Values)

STORY HEIGHTS	FLOOR	TRANSLATIONAL BASE SHEAR (10^6N)	β_{max} = 3500 m/s, β_{min} = 350 m/s		β_{max} = 3500 m/s, β_{min} = 1750 m/s		β_{max} = 3500 m/s, β_{min} = 3500 m/s	
			TOTAL SHEAR (10^6N)	INCREASE DUE TO ROCKING (%)	TOTAL SHEAR (10^6N)	INCREASE DUE TO ROCKING (%)	TOTAL SHEAR (10^6N)	INCREASE DUE TO ROCKING (%)
3.66 m	1	1.793	2.748	53.3	2.557	42.6	2.518	40.4
	2	3.333	5.338	60.2	4.963	48.9	4.883	46.5
	3	4.398	6.987	58.9	6.493	47.6	6.388	45.2
7.32 m	1	1.793	3.058	70.5	2.675	49.2	2.596	44.8
	2	3.333	5.953	78.6	5.197	55.9	5.036	51.1
	3	4.398	7.784	77.0	6.797	54.6	6.593	49.9

Table 3 Increase in Maximum Bending Moment Response Due to Ground Rocking for Different Story Heights and Different β_{min} and β_{max} in the Layered Ground. (Most Probable Values)

STORY HEIGHTS	FLOOR	TRANSLATIONAL BENDING MOMENT (10^6N.m)	β_{max} = 3500 m/s, β_{min} = 350 m/s		β_{max} = 3500 m/s, β_{min} = 1750 m/s		β_{max} = 3500 m/s, β_{min} = 3500 m/s	
			TOTAL MOMENT (10^6N.m)	INCREASE DUE TO ROCKING (%)	TOTAL MOMENT (10^6N.m)	INCREASE DUE TO ROCKING (%)	TOTAL MOMENT (10^6N.m)	INCREASE DUE TO ROCKING (%)
3.66 m	1	6.559	10.053	53.3	9.353	42.6	9.208	40.4
	2	18.505	29.524	59.5	27.444	48.3	27.018	46.0
	3	34.226	55.019	60.7	51.148	49.4	50.319	47.0
7.32 m	1	13.119	22.373	70.5	19.570	49.2	18.992	44.8
	2	37.009	65.812	77.8	57.475	55.3	55.734	50.6
	3	68.453	122.63	79.1	107.11	56.5	103.79	51.6

Table 4 Increase in Maximum Lateral Displacement Response Due to Ground Rocking for Different Story Heights and Different β_{min} and β_{max} in the Layered Ground. (Expected Values)

STORY HEIGHTS	FLOOR	TRANSLATIONAL DISPL. (Cm)	β_{max} = 3500 m/s, β_{min} = 350 m/s		β_{max} = 3500 m/s, β_{min} = 1750 m/s		β_{max} = 3500 m/s, β_{min} = 3500 m/s	
			TOTAL DISPL. (Cm)	INCREASE DUE TO ROCKING (%)	TOTAL DISPL. (Cm)	INCREASE DUE TO ROCKING (%)	TOTAL DISPL. (Cm)	INCREASE DUE TO ROCKING (%)
3.66 m	1	4.591	5.185	12.9	4.818	4.9	4.741	3.3
	2	2.957	3.340	12.9	3.103	4.9	3.054	3.3
	3	1.393	1.571	12.8	1.461	4.8	1.439	3.3
7.32 m	1	4.591	5.779	25.9	5.045	9.9	4.891	6.5
	2	2.957	3.720	25.8	3.250	9.9	3.150	6.5
	3	1.393	1.748	25.5	1.528	9.7	1.483	6.5

Table 5 Increase in Maximum Base Shear Response Due to Ground Rocking for Different Story Heights and Different min and max in the Layered Ground. (Expected Value)

STORY HEIGHTS	FLOOR	TRANSLATIONAL BASE SHEAR (10^6N)	β_{max} = 3500 m/s, β_{min} = 350 m/s		β_{max} = 3500 m/s, β_{min} = 1750 m/s		β_{max} = 3500 m/s, β_{min} = 3500 m/s	
			TOTAL SHEAR (10^6N)	INCREASE DUE TO ROCKING (%)	TOTAL SHEAR (10^6N)	INCREASE DUE TO ROCKING (%)	TOTAL SHEAR (10^6N)	INCREASE DUE TO ROCKING (%)
3.66 m	1	1.776	2.746	54.6	2.555	43.9	2.515	41.6
	2	3.333	5.339	60.2	4.964	48.9	4.884	46.5
	3	4.388	6.987	59.2	6.492	47.9	6.388	45.6
7.22 m	1	1.776	3.056	72.1	2.673	50.5	2.594	46.0
	2	3.333	5.954	78.6	5.197	55.9	5.036	51.1
	3	4.388	7.784	77.4	6.797	54.9	6.592	50.2

Table 6 Increase in Maximum Bending Moment Response Due to Ground Rocking for Different Story Heights and Different β_{min} and β_{max} in the Layered Ground. (Expected Values)

STORY HEIGHTS	FLOOR	TRANSLATIONAL BENDING MOMENT (10^6N.m)	β_{max} = 3500 m/s, β_{min} = 350 m/s		β_{max} = 3500 m/s, β_{min} = 1750 m/s		β_{max} = 3500 m/s, β_{min} = 3500 m/s	
			TOTAL MOMENT (10^6N.m)	INCREASE DUE TO ROCKING (%)	TOTAL MOMENT (10^6N.m)	INCREASE DUE TO ROCKING (%)	TOTAL MOMENT (10^6N.m)	INCREASE DUE TO ROCKING (%)
3.66 m	1	6.498	10.044	54.6	9.344	43.8	9.199	41.6
	2	18.483	29.525	57.7	27.446	48.5	27.020	46.2
	3	34.234	55.031	60.7	51.160	49.4	50.330	47.0
7.32 m	1	12.995	22.358	72.0	19.553	50.5	18.973	46.0
	2	36.966	65.818	78.0	57.480	55.5	55.739	50.8
	3	68.470	122.66	79.1	107.14	56.5	103.82	51.6