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A STUDY OF A TWO STOREYED STEEL FRAME UNDER STATIC AND DYNAMIC LOADS

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Synopsis

This paper describes tests on two storeyed mild steel framed structures. Static lateral hysteretic load deflection measurements were obtained at various strain levels. Also, free vibration tests were carried out by giving initial displacements at different strain levels. The framed structure was mounted on a shake table which was given steadystate sinusoidal motions at various frequencies. Finally, the frame was tested to failure. Theoretical computations of spring constants and periods were compared with those of experiments. Damping obtained from different techniques were compared with each other.

Introduction

A design criterion for structures located in seismic zones is to have elastic behaviour during small size shocks which might occur more often in the area and to permit plastic deformations for the few large size shocks. Therefore, in order to find the response of structures due to severe ground motion it is necessary to determine the restoring forces and dissipative forces of systems under cyclic loads subjected to deformations beyond yield level.

The amount of experimental data available concerning hysteretic load deflection relationships at high strain levels is meagre. Jacobsen(1) has reviewed thirteen references dealing with reversed cyclic loading beyond the normal elastic conditions of joints, wood frames, built up beams and concrete frames, Hanson(2) has recently reported tests on a number

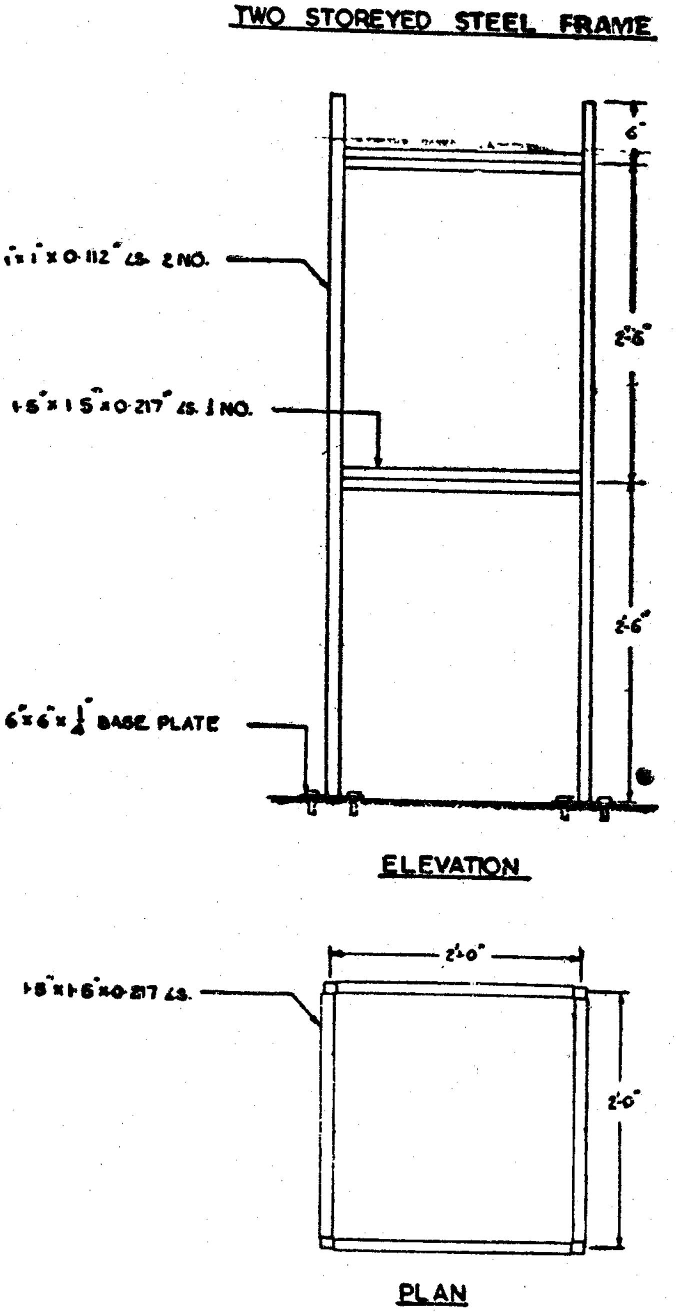

Fig. 1

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of single storeyed mild steel structures and showed that differences between the static and dynamic hysteretic load deflection curves were, in general, smaller than the changes in static curves caused by the deterioration of the material.

Experimental Structure

A two storeyed steel frame model, as shown in fig. 1, was used for tests. A head room of 2.30m was available over a 1.80m × 1.20m size shake table and the dimensions of the frame were chosen such that it could be conveniently accommodated on the table. The size of the columns were the minimum sections that could be fabricated from available angle sections 1″×1″×1/8″ size. The ratio of moment of inertia per unit length of beam to that of column was kept as four.

Theoretical Analysis of Model Frame

Theoretical analysis was carried out to determine the influence coefficients due to lateral loads and to predict the frequencies of vibrations and mode shapes. Also, from the measured lateral force deflection characteristics, hysteretic damping values have been calculated.

The mass of the structure has been considered to be concentrated at the storey levels by assuming half the weight of the columns between adjacent floors to be acting at the floor levels. Two different floor masses are used and the mass matrices [M], are as follows:—

(a) Without floor slabs

$$[M] = \begin{bmatrix} 0.148 & 0 \\ 0 & .318 \end{bmatrix} \times 10^{-2} \quad \text{Kg. sec.}^2/\text{mm} \qquad (1)$$

(b) With slabs placed on the frame at the two floor levels

$$[M] = \begin{bmatrix} 1.073 & 0 \\ 0 & 1.303 \end{bmatrix} \times 10^{-2} \quad \text{Kg. sec.}^2/\text{mm} \qquad (2)$$

Considering joint rotation, theoretical values of influence coefficients [G], and stiffness coefficients, [K], for lateral motion are as follows :—

$$[G] = \begin{bmatrix} 19.5 & 21.6 \\ 21.6 & 45.4 \end{bmatrix} \times 10^{-3} \quad \text{mm/kg.} \qquad (3)$$

$$[K] = \begin{bmatrix} 108.4 & -51.6 \\ -51.6 & 46.6 \end{bmatrix} \quad \text{Kg/mm} \qquad (4)$$

Using standard procedures, the undamped frequencies and mode shapes could be determined and they are as follows :

(a) Without slabs

$$f_1 = 12.55 \text{ c.p.s} \ ; \ f_2 = 45.52 \text{ c.p.s.} \qquad (5)$$

$$\{\Phi^{(1)}\} = \begin{Bmatrix} 1.000 \\ 1.922 \end{Bmatrix} \ ; \quad \{\Phi^{(2)}\} = \begin{Bmatrix} 1.000 \\ -2.2414 \end{Bmatrix} \qquad (6)$$

(b) With floor slabs

$$f_1 = 5.93 \text{ c.p.s.} \quad ; \quad f_2 = 17.65 \text{ c.p.s.} \tag{7}$$

$$\{\Phi^{(1)}\} = \begin{Bmatrix} 1.000 \\ 1.813 \end{Bmatrix} \quad ; \quad \{\Phi^{(2)}\} = \begin{Bmatrix} 1.000 \\ -0.455 \end{Bmatrix} \tag{8}$$

Hysteretic Lateral Load Deflection Relationships

These were experimentally determined by applying a horizontal force in turn at each storey level and measuring the deflections at both the storey levels. The deflections were measured at various stages of a complete reversal of the applied loads so that hysteretic load deflection relationships could be obtained over a full cycle of loading. Hysteretic curves for various intensities of lateral loads, are shown in figs. 2, 3 and 4. These were used to evaluate influence matrix, frequencies and damping.

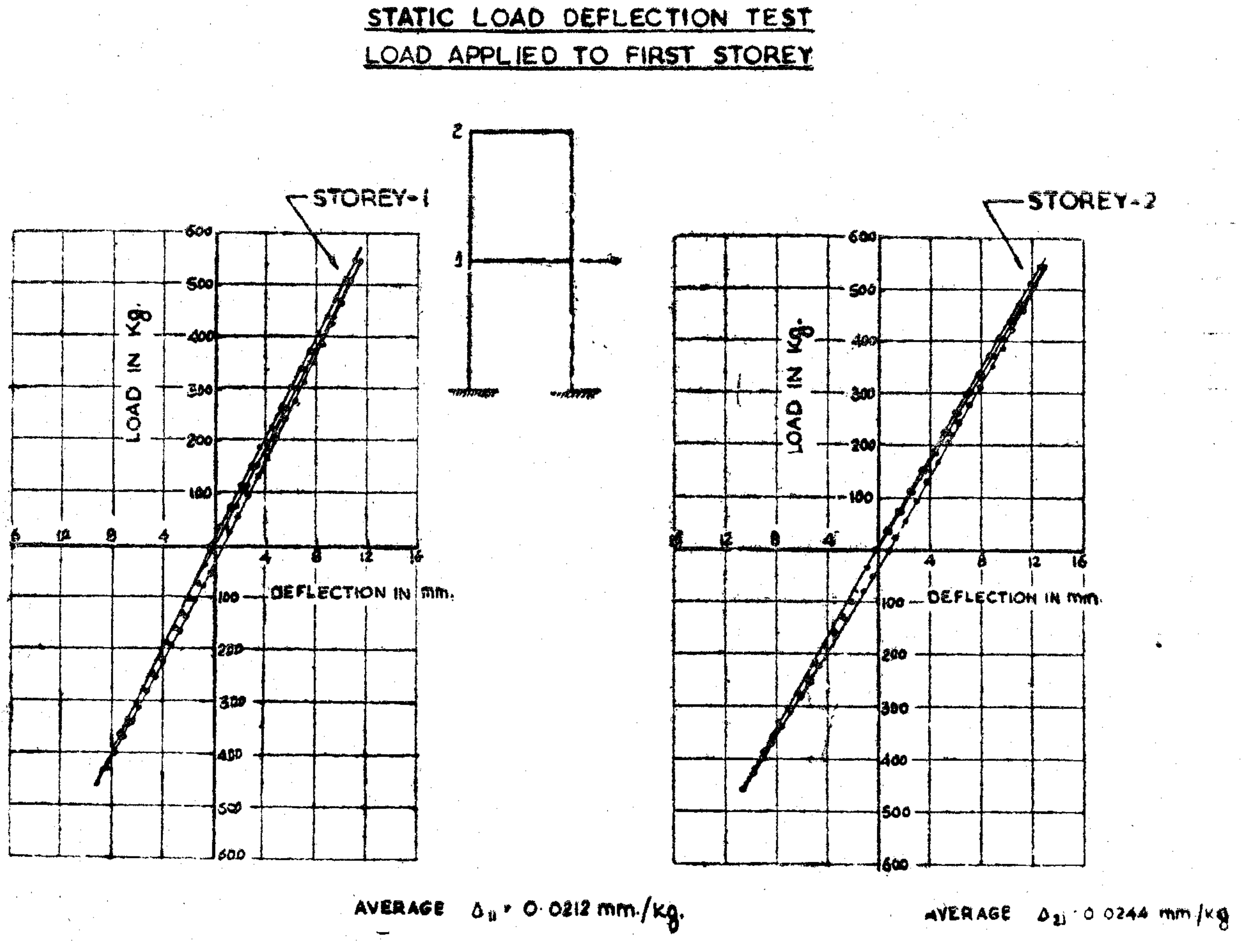

Fig. 2

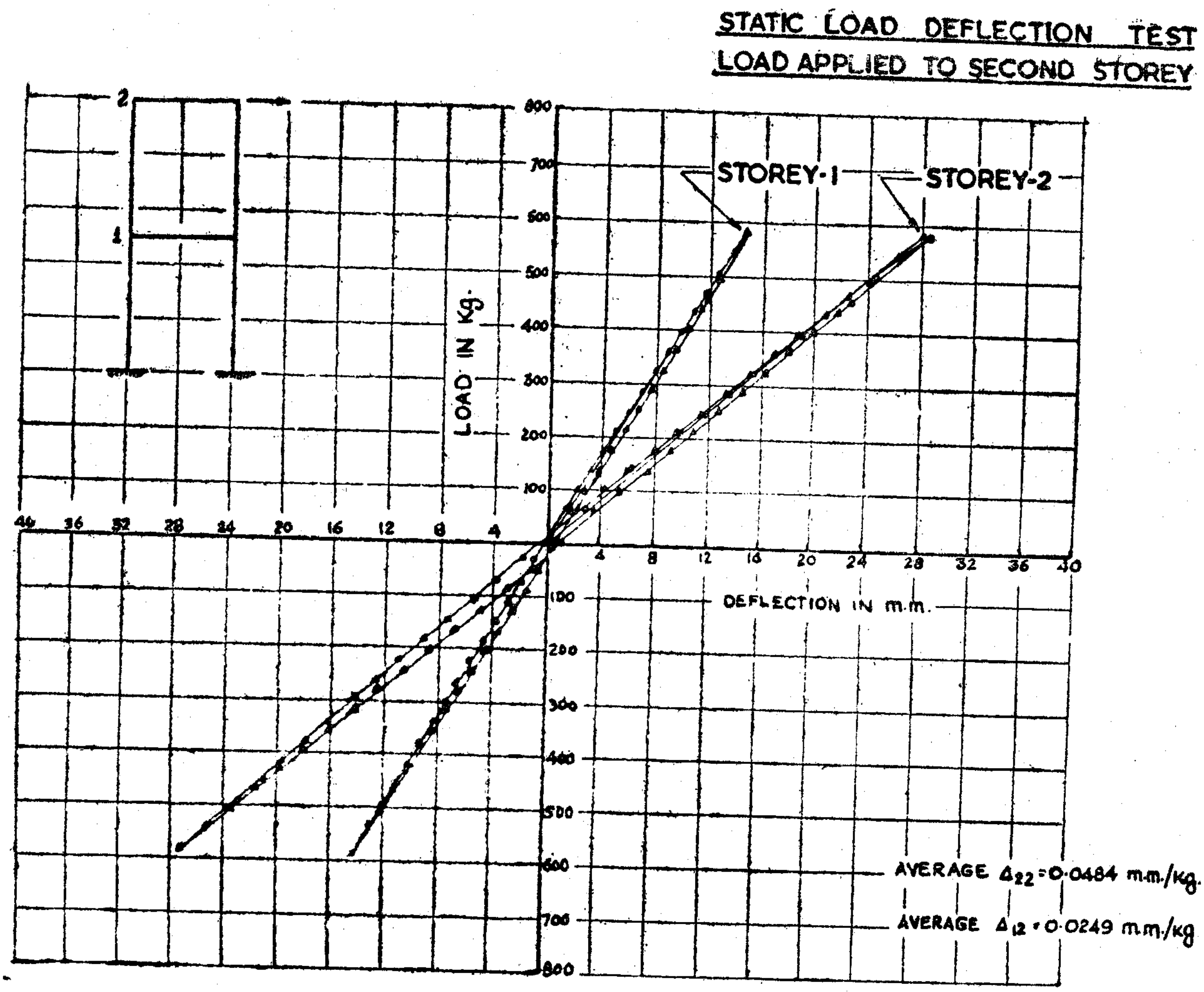

Fig. 3

For small intensities of lateral loads, where the structure essentially behaves linearly, the influence coefficients were obtained as the average of the slope of force deflection curve on each side of the loading. The influence matrix evaluated from experiments is as follows:—

$$[G] = \begin{bmatrix} 21.1 & 24.8 \\ 24.3 & 48.2 \end{bmatrix} \times 10^{-3} \quad \text{mm/Kg.} \qquad (9)$$

This matrix compares favourably with that obtained from theory.

Using the above matrix, frequencies and mode shapes would be as follows :

(a) Without slabs.

$$f_1 = 12.10 \text{ c.p.s.} \; ; \; f_2 = 47.50 \text{ c.p.s.} \qquad (10)$$

$$\{\phi^{(1)}\} = \begin{Bmatrix} 1.000 \\ 1.795 \end{Bmatrix} \; ; \; \{\phi^{(2)}\} = \begin{Bmatrix} 1.000 \\ -0.253 \end{Bmatrix} \qquad (11)$$

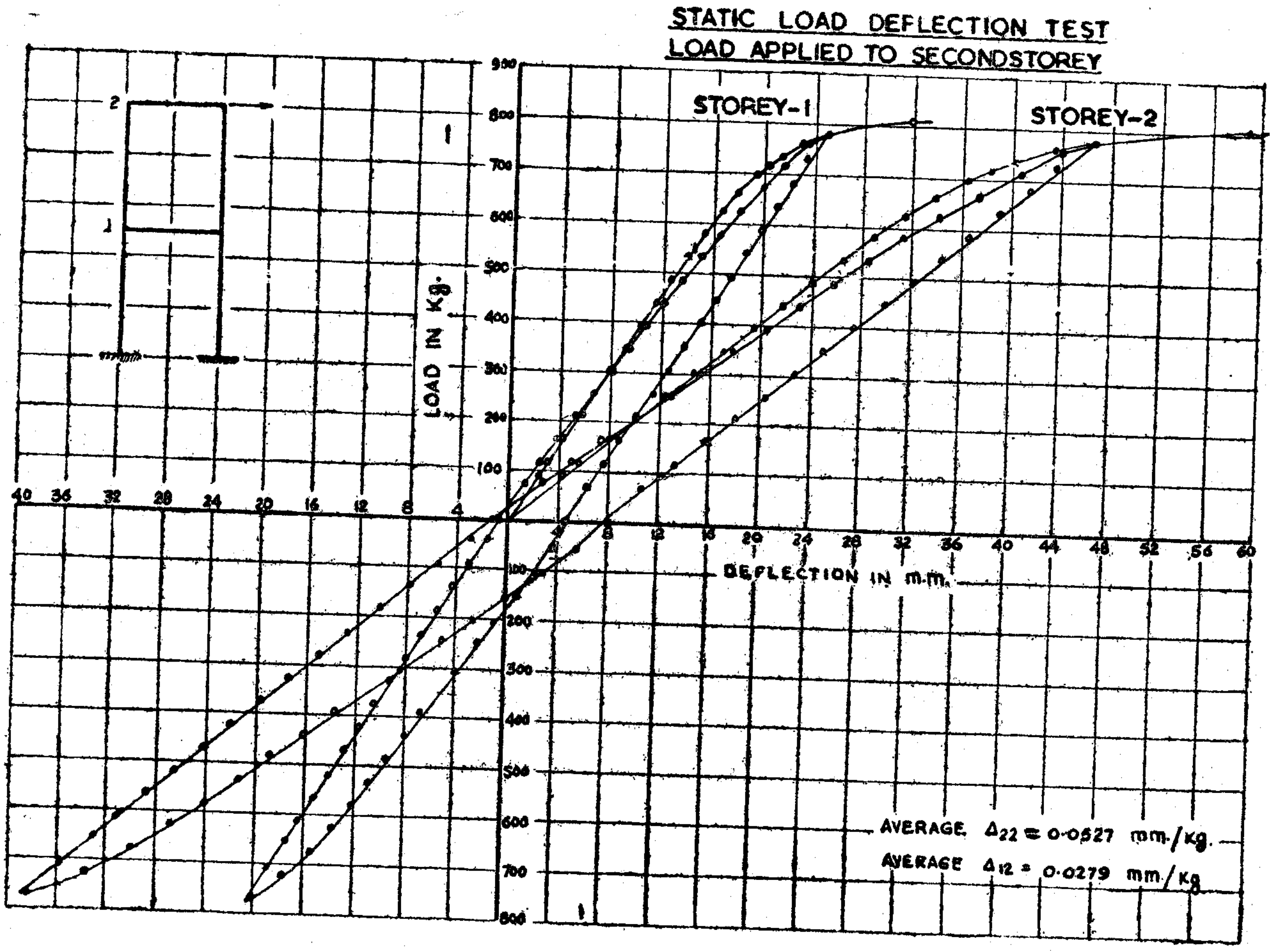

Fig. 4

(b) With floor slabs.

$f_1 = 5.70$ c.p.s. ; $f_2 = 18.50$ c.p.s. (12)

$$\{\Phi^{(1)}\} = \begin{Bmatrix} 1.000 \\ 1.700 \end{Bmatrix} \quad ; \quad \{\Phi^{(2)}\} = \begin{Bmatrix} 1.000 \\ -0.472 \end{Bmatrix} \tag{13}$$

These values also are in close agreement with those of theory.

For higher intensities of lateral loads, influence coefficients are obtained by considering the secant modulus of the curve. This equivalent linearisation, involving a reduction in stiffness, was used for evaluating frequencies in the nonlinear range. The computed values of frequencies at various levels of lateral loads are given in table 1.

The damping in a framed structure is mainly due to the hysteretic behaviour of the restoring force deflection relationship. For a single degree of freedom system, damping due to hysteresis is given by

$$\zeta = \frac{1}{2\pi} \frac{A}{k x^2} \tag{14}$$

where ζ = percentage of critical damping

A = area of the hysteresis loop

k = secant modulus corresponding to x

x = maximum amplitude of vibration

Though hysteresis damping is not exactly defined for multiple degree of freedom systems, damping has been calculated using force deflection relationship of the top storey. These values are given in Table 2.

An attempt was made to find whether Jennings's[3] equation could be fitted to the hysteretic curves obtained experimentally. It was found that the parameters were different for the various curves corresponding to different intensities of loading. For a load P_o of 630 Kg., a was equal to 0.2 and $\gamma=11$. This gave a damping of 1.15%. At that load level, from the area of hysteretic curve, damping worked out to be 1.3%.

Free Vibration Tests

The structure was subjected to free vibration tests by applying a horizontal pull at the top storey and suddenly releasing it. The resulting vibrations were recorded. Such tests were carried for various intensities of initial pull.

The frequencies of vibrations and damping obtained from the free vibration records (based on the first one or two cycles of vibration) are given in Table 1 and Table 2 respectively.

It is seen from the tables that there is good correlation between values obtained from hysteretic curves and free vibration tests. It is observed that frequencies decrease with increase in lateral load intensity thereby confirming the softening behaviour of the restoring force. Damping increases with nonlinearity and this behaviour is also as expected. The maximum change in frequency was of the order of 10%.

Forced Vibration Tests

These tests were carried out by mounting the structure on a platform supported on rollers and subjecting the platform to a steady state sinusoidal excitation in a horizontal direction. Accelerations at the two floor levels as well as at the base were measured. The relationship between the acceleration at any floor level i and that at the base is given by

$$\frac{\ddot{x}_i}{\ddot{y}_b} = \left[\left\{1 + \left(\sum_{r=1}^{n} B_i^{(r)} \eta_r^2 \mu_r \cos\theta_r\right)\right\}^2 + \left\{\sum_{r=1}^{n} B_i^{(r)} \eta_r^2 \mu_r \sin\theta_r\right\}^2\right]^{1/2} \quad (15)$$

where

$\ddot{x}_i$ = maximum absolute acceleration at floor level

$\ddot{y}_b$ = maximum base acceleration

$B_i^{(r)}$ = mode participation factor in the rth mode of vibration

$$= \frac{\phi_i^{(r)} \cdot \sum_{j=1}^{n} m_j \phi_j^{(r)}}{\sum_{j=1}^{n} m_j (\phi_j^{(r)})^2}$$

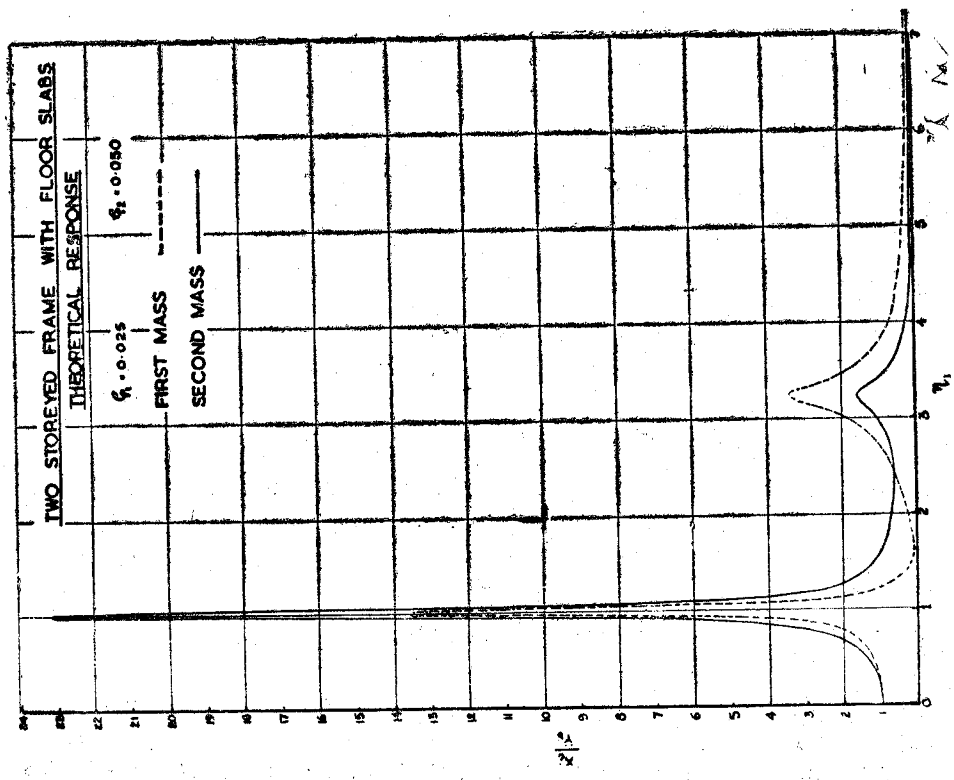
TWO STOREYED FRAME WITH FLOOR SLABS
THEORETICAL RESPONSE
$\xi_1 = 0.025$
$\xi_2 = 0.050$
FIRST MASS
SECOND MASS

Fig. 5

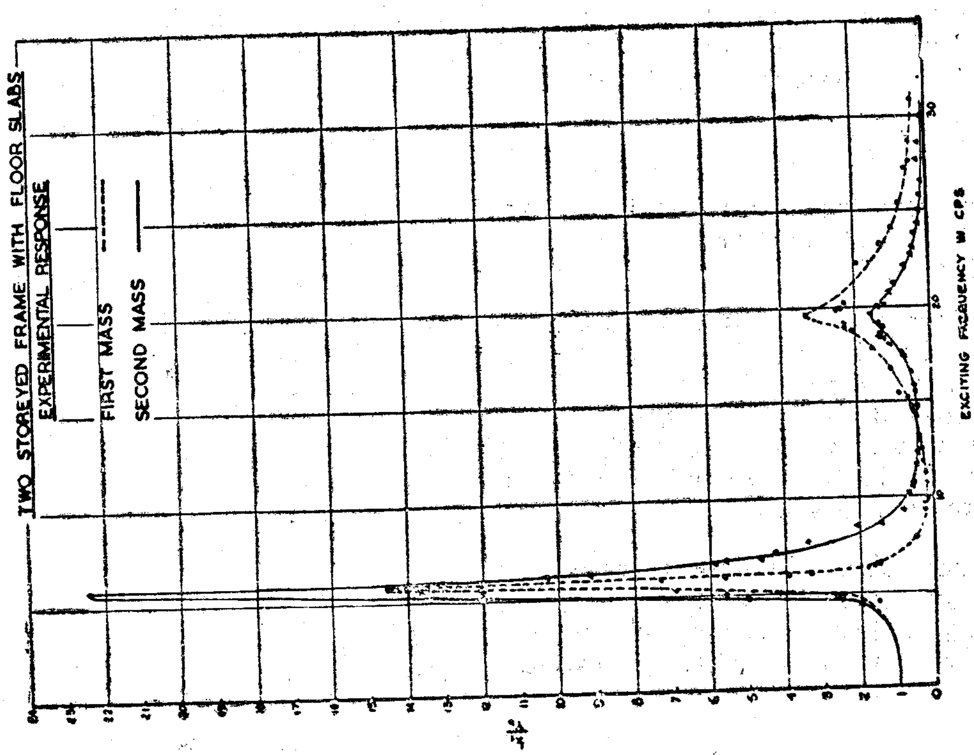
TWO STOREYED FRAME WITH FLOOR SLABS
EXPERIMENTAL RESPONSE
FIRST MASS
SECOND MASS
EXCITING FREQUENCY W CPS

Fig.6

μ_r = dynamic amplification factor in the r^{th} mode
$= [(1-\eta_r^2)^2 + (2\,\eta_r\,\zeta_r)^2\,]^{-1/2}$
θ_r = $\tan^{-1}[2\,\eta_r\,\zeta_r/1-\eta_r^2]$
η_r = ω/p_r
ζ_r = damping factor in r^{th} mode
p_r = undamped natural frequency of vibration in the r^{th} mode
ω = frequency of excitation

The theoretical response curves relating $\ddot{x}_1/\ddot{y}_b$ and ω have been given in fig. 5. The damping values have been assumed to be equal to those obtained from steady state response tests. The experimental response curves are plotted in fig. 6 and there is a close agreement between the theoretical and experimental response curves.

The excitation frequency at resonance correspond to the natural frequency of the system. The two resonant frequencies are 5.72 c p.s. and 19.56 c p.s. The damping factors worked out on the basis of measured acceleration values at resonance are equal to 0.025 and 0.050 for the first and second modes of vibration. These apparently higher values of damping, compared to the free vibration tests in the elastic range, may be due to instability of the vibrating device at resonance.

For systems with small damping, the total damping may be assumed to be a combination of absolute damping and relative damping[4]. Absolute damping would decrease with higher modal frequencies whereas relative damping would increase. For this system experiments show that damping is greater in the second mode compared to that of the first, indicating that damping is of relative type. This system ought to have greater interfloor damping and this behaviour is confirmed.

Ultimate Load Test

The structure was finally tested to failure by applying lateral loads. The frame failed at an ultimate load of 872 kg.

The ultimate load has been evaluated theoretically. The beam and column sections were separately tested to failure to determine the plastic moment capacity of these sections. The average values were found to be150750 mm. Kg. for the beam (M_{pb}) and 77000 mm. Kg. for the column (M_{pc}).

The mechanism of failure is as shown in fig. 7. This gives the least value of the lateral load F_u applied at the top storev level. Equating the energy dissipated to the external work done,

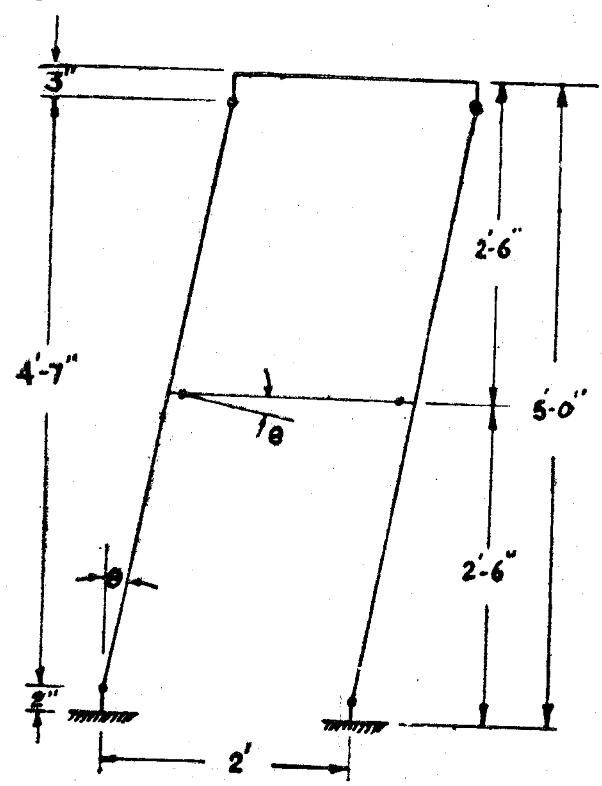

Fig. 7

$$4\, M_{pc}\, \theta + 2\, M_{pb}\, \theta = 1396 \times \frac{F_u}{2} \times \theta \tag{16}$$

and $F_u = 872$ Kg.

The axial column thurst at ultimate load is found to be less than 10% of the axial thrust capacity of the column, so that the thrust does not affect the plastic moment capacity of the column.

The ultimate load test results are in agreement with that of theory.

Conclusions

Frequencies were calculated theoretically for elastic behaviour. These compared well with those obtained from hysteresis curves and free and forced vibration tests. In the nonlinear range, the secant modulus method of estimating influence coefficients proved to be a good approximation. Frequencies decrease with increase in nonlinearity, confirming the softening restoring force characteristics of the system. There is a fair agreement between damping factors calculated by different methods. Damping increases with nonlinearity. From forced vibration tests it was observed that damping in second mode is greater than that in the first mode. This would indicate that damping is predominantly due to inter-floor damping. The ultimate load test results are also consistent with theory.

Acknowledgements

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References

1. Jacobsen, L.S., "Frictional Effects in Composite Structures Subjected to Earthquake Vibrations", Department of Mechanical Engineering, Stanford University, March 1959.

2. Hanson, R.D., "Comparison of Static and Dynamic Hysteresis Curves", Proc. A.S.C.E., Vol. 92, No. EM5, Oct. 1966.

3. Jennings, P. C., "Response of Simple Yielding Structures to Earthquake Excitation", Earthquake Engineering Research Laboratory Report, California Institute of Technology, California, June, 1963.

4. Berg, G.V., "The Analysis of Structural Response to Earthquakae Forces", Enginreering College Industry Programme, University of Michigan. Report IP-291, May 1958.

TABLE 1

Fundamental Frequency of Frame (Without Slab)

Sr. No	Load in Kg.	Frequency in cycles per second	
		Load—Defletion Curve	Free Vibration Records
1.	250	12.00	12.00
2.	500	11.90	12.00
3.	630	11.80	11.80
4.	680	11.70	11.60
5.	725	11.50	11.40
6.	770	11.10	11.20
7.	820	—	11.10
8.	860	—	11.00

TABLE 2

Damping Factors

Sr. No.	Load in Kg.	Percentage of critical Damping	
		Hysteresis Curves	Free vibration Records
1.	580	1.1	1.2
2.	630	1.3	1.5
3.	780	5.0	4.1