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# ON A SYSTEM OF CREEPING AND INTERACTING VERTICAL STRIKE-SLIP FAULTS IN A MODEL OF THE LITHOSPHERE-ASTHENOSPHERE SYSTEM

By

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## ABSTRACT

A system of aseismically creeping and interacting strike-slip faults, situated in a visco-elastic half space, representing the lithosphere-asthenosphere system, is considered. The effects of aseismic fault creep across faults of the system are investigated. The dynamics of the model in the aseismic state is considered, both before any fault movement occurs, and also after the commencement of fault creep. For a system of long vertical strike-slip faults, consisting of any finite number of faults, the effect of aseismic creep across faults of the system on the shear stresses near the other neighbouring faults is investigated in detail, together with the effect of fault movement on the surface displacement and surface shear strain in the model, using analytical methods involving the use of integral transforms and Green's functions, as well as computational algorithms and computer programmes developed for the purpose.

The investigations show that aseismic creep across any fault generally tends to release shear stress near the fault itself. Continuous aseismic stress release near the fault due to creep across the fault is found to occur under suitable conditions. This reduces progressively the possibility of a sudden earthquake-generating seismic fault movement. The effect of aseismic creep across any fault on the neighbouring faults of the fault system is found to depend on the model parameters, including the creep velocity, the dimensions of the faults as well as the relative positions of the faults and the distances between them. An interesting result of the interaction between the faults is that if the faults of the fault system are of similar dimensions and more or less at the same level, then aseismic creep across any fault tends to release the shear stress near the neighbouring faults, thus reducing gradually the possibility of future seismic fault movements. However, if the faults of the system are more or less vertically above or below one another, then creep across any fault tends to increase the shear stress accumulation near the neighbouring faults, resulting in an increase in the possibility of earthquake-generating fault movements in future. The effect of aseismic creep across any fault on the neighbouring faults is found to decrease quite rapidly with increase in the distance between the faults.

## INTRODUCTION

The problem of earthquake prediction has attracted widespread attention among seismologists in recent years, and the steady accumulation of relevant seismological data and improvements in the techniques of their analysis and interpretation, together with the development of relevant theoretical models and computer simulation techniques have made it possible to hope that effective programmes of earthquake prediction may become feasible in the near future. In this connection, it is realised that, effective programmes of earthquake prediction would require a better understanding of the process of stress accumulation and release in seismically active regions, and their relation to the dynamics of the lithosphere-asthenosphere system and seismic activity. Observations in seismically active regions in recent years indicate that, during apparently quiet aseismic periods, there are usually slow quasi-static aseismic surface movements of the order of a few cms. per year, or less, resulting in the accumulation of stress and strain in some cases, which may eventually lead to a sudden fault movement, generating an earthquake, if the stress accumulation reaches sufficiently high levels. The quantitative estimation of this stress accumulation would be facilitated if it is possible to develop suitable theoretical models which incorporate the essential features of the mechanism of stress accumulation in the regions concerned. Such theoretical models would enable seismologists to estimate the stress accumulation below the surface near the fault from the observed aseismic surface movements. In this connection, observational data also indicates continuous, slow, aseismic fault creep across some active faults, including the central part of the San Andreas fault in North America. The effect of this aseismic fault creep on the accumulation and release of stress in the region concerned is of great interest in the study of the dynamics of the lithosphere-asthenosphere system in seismically active regions during aseismic periods.

In recent years, some theoretical models of the lithosphere-asthenosphere system in seismically active regions during aseismic periods have been developed, starting with the early theoretical models of Nur and Mavko (1974) and others. The general features of the theoretical models of this type, developed till now, have been discussed by Cohen et al. (1984) and by Mukherji et al. (1984, 1986, 1988). In most of these theoretical models, simple models of the lithosphere-asthenosphere system containing single faults have been considered. However, active seismic fault systems often consist of several neighbouring faults which may interact when creep or sudden seismic fault movement occurs across one or more of these faults. For example, in the San Andreas fault system in North America, the Hayward and Calaveras faults are close to and roughly parallel to the main San Andreas fault. Keeping this in view, the authors of this paper have been trying to develop theoretical models of interacting faults in the lithosphere-asthenosphere system. Theoretical models of this type, with two interacting strike-slip faults, have been developed earlier, by Mukherji et al. (1984, 1986, 1988). Keeping in view the fact that the number of interacting faults in active faults systems may be greater, the case of a system of several long vertical interacting strike-slip faults in a simple model of the lithosphere-asthenosphere system has been considered in this paper.

## FORMULATION

We consider a simple model of the lithosphere-asthenosphere system, consisting of a visco-elastic half space. The material of the half-space is taken to be linearly visco-elastic, and of the Maxwell type. The reasons for assuming such a rheology for the model have been explained by Mukherji and Mukhopadhyay (1984, 1986, 1988). We consider a system of $n$ long plane strike-slip faults in the model, where is any finite positive integer. Both surface-breaking faults and buried faults are considered. We assume that the traces of the faults, (i.e., the lines of intersection of the free surface of the model and the planes of the faults) are parallel to each other. We introduce rectangular cartesian coordinates $(y_1, y_2, y_3)$ with the free surface of the half-space as the plane $y_3 = 0$, and the $y_3$-axis pointing into the half-space. We take the $y_1$-axis parallel to the traces of the faults, and assume that the displacements and stresses are independent of $y_1$, taking the lengths of the plane faults to be large compared to their widths and depths.

Fig. 1 shows the section of the theoretical model by the plane $y_1 = 0$. In this case, as explained by Mukherji and Mukhopadhyay (1984, 1986, 1988), the diaplcement components in the model, associated with strike-slip movement is parallel to the $y_1$-axis, and is represented by $u_1$. The relevant stress components for the half-space, associated with $u_1$, are represented by $(\tau_{12}, \tau_{13})$ as in Mukhopadhyay et al. (1984, 1986, 1988). Since the displacements and stresses in this case are independent of $y_1$, $(u_1, \tau_{12}, \tau_{13})$ are independent of the other components of displacement and stress as explained by Mukherji and Mukhopadhyay (1984, 1986, 1988).

For the faults $F_k$ $(k = 1,2,\ldots,n)$, the planes of the faults are given by $y_2 = P_k$ $(k = 1,2,\ldots,n)$. The upper and lower edges of the faults $F_k$ are at depths $(d_k, D_k)$ below the free surface and are parallel to the free surface and the $y_1$-axis. For a surface-breaking fault, $d_k = 0$ and for a buried fault $d_k > 0$, while $D_k > d_k$.

The relevant components $(u, \tau_{12}, \tau_{13})$ of displacement and stress, associated with strike-slip movement, satisfying the following stress-strain relations for the visco-elastic half-space with material of the Maxwell type :

$$\left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{13} = \frac{\partial^2 u_1}{\partial t\,\partial y_3}$$

and

$$\left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t}\right)\tau_{12} = \frac{\partial^2 u_1}{\partial t\,\partial y_2}$$

$$(-\infty < y_2 < \infty, \quad y_3 \geqslant 0, \quad t \geqslant 0) \qquad \ldots\ldots(1)$$

where $\mu$ is the effective ridigity and $\eta$ is the effective viscosity, as in Mukherji et al. (1984, 1986, 1988).

We consider the model during aseismic periods, leaving out the relatively small periods (if any) following sudden fault movements, when seismic disturbances are present in the model. For the slow, aseismic, quasi-static displacements we consider, the inertial forces are very small and are neglected. Hence the relevant stresses satisfy the relation

$$\frac{\partial}{\partial y_2}(\tau_{12}) + \frac{\partial}{\partial y_3}(\tau_{13}) = 0$$

$$(-\infty < y_2 < \infty, \quad y_3 \geqslant 0, \quad t \geqslant 0) \qquad \ldots\ldots(2)$$

From (1) and (2), we find that

$$\frac{\partial}{\partial t}(\nabla^2 u_1) = 0$$

which is satisfied if $\nabla^2 u_1 = 0$ ......(3)

$$(-\infty < y_2 < \infty, \quad y_3 \geqslant 0, \quad t \geqslant o).$$

At the free surface $y_3 = 0$, we have the boundary condition

$$\tau_{13} = 0, \quad \text{on} \quad y_3 = 0 \qquad .....(4)$$

$$(t \geqslant 0, \quad -\infty < y_2 < \infty)$$

We assume that the tectonic forces maintain a constant shear stress for away from the faults, while stresses near the fault may change with time, due to fault movement (including fault creep). We then have the boundary conditions

$$\tau_{13} \to 0 \quad \text{as} \quad y_3 \to \infty \qquad ......(5)$$

$$(\text{for} \quad -\infty < y_2 < \infty, \quad t > 0)$$

and

$$\tau_{12} \to \tau_\infty \quad \text{as} \quad |y_2| \to \infty \qquad ......(6)$$

$$(\text{for} \quad y_3 \geqslant 0, \quad t \geqslant 0).$$

## DISPLACEMENTS AND STRESSES IN THE ABSENCE OF FAULT MOVEMENT

In the absence of any movements across the faults, the displacements and stresses are continuous throughout the model. In this case, we measure the time $t$ from the instant at which the relations (1) - (6) become valid for the model. Let $(u_1)_0$, $(\tau_{12})_0$, $(\tau_{13})_0$, which may be functions of $(y_2, y_3)$, be the values of $(u_1)$, $(\tau_{12})$, $(\tau_{13})$ at $t = 0$. $(u_1)_0$, $(\tau_{12})_\theta$, $(\tau_{13})_0$ also satisfy the relations (1)-(6).

The initial and boundary value problem is solved, following exactly the same method as in Mukherji and Mukhopadhyay (1984, 1986, 1988), using Laplace transforms with respect to time and we have,

$$u_1(y_2, y_3, t) = (u_1)_0 + \frac{\tau_\infty y_2 t}{\eta}$$

$$\tau_{12}(y_2, y_3, t) = (\tau_{12})_0 \, e^{-\frac{\mu t}{\eta}} + \tau_\infty \left(1 - e^{-\frac{\mu t}{\eta}}\right)$$

$$\tau_{13}(y_2, y_3, t) = (\tau_{13})_0 \, e^{-\frac{\mu t}{\eta}} \qquad .....(7)$$

As in Mukherji and Mukhodhyay (1984, 1986, 1988) we find from (7), that if the shear stress $\tau_{12}$ near the faults, tending to cause strike-slip movement, is less than $\tau_\infty$ at $t = 0$, then there would be a continuous accumulation of shear stress near the faults for $t > 0$, with $\tau_{12} > \tau_\infty$ near the faults, as $t \to \infty$. We assume, as in Mukherji et al. (1984, 1986, 1988) that aseismic creep commences across $F_k$ when $\tau_{12}$ reaches critical values $\tau_{ck}$ near $F_k$.

If $\tau_{ck} < \tau_\infty$, then seismic creep would commence across $F_k$ after a finite time, from (7), we find that the fault across which aseismic creep commences first would be determined by the values of $\tau_{ck}$ for different faults $F_k$, and the values of $(\tau_{12})_0$ near the faults $F_k$ ($k = 1,2,....,n$). Assuming that $\tau_{ck} < \tau_\infty$, for a fault $F_k$ of the system, we consider next the situation after aseismic creep commences across $F_k$.

## DISPLACEMENT, STRESSES AND STRAINS AFTER THE COMMENCEMENT OF FAULT CREEP

We consider the case in which there is aseismic creep across one or more faults in the system. Let $T_k$ ($\geq 0$) be the time of commencement of creep across the kth fault $F_k$. Here $u_1$, $\tau_{12}$, $\tau_{13}$ satisfy the relation (1) - (6). $\tau_{12}$, $\tau_{13}$ are continuous everywhere in the model. $u_1$ is continuous everywhere in the model, except for discontinuities across $F_k$ ($k = 1,2,....,n$) which are given by -

$$[u_1]_{FK} = \nu_k(t)_k f_k(y_3) H(t_k) \quad \text{across } F_k$$

$$(k = 1,2,.....,n)$$

$$(d_k \leq y_3 \leq D_k, \quad y_2 = P_k) \qquad .....(8)$$

Here $[u_1]_{F_k}$ = discontinuity of $u_1$ across $F_k$

$$= \lim_{y_2 \to D_{1k}+0} (u_1) - \lim_{y_2 \to D_{1k}-0} (u_1)$$

$$(k = 1,2,..........,n).$$

$t_k = t - T_k$

$\nu_k(t_k) = 0$ for $t_k \leq 0$ i.e., $t \leq T_k$ ($k = 1,2,.....,n$)

$d_k = 0$ if $F_k$ is a surface-breaking fault and $d_k > 0$ if $F_k$ is a buried fault.

$y_2 = P_k$ is the plane of $F_k$.

We try to find the values for $u_1$, $\tau_{12}$, $\tau_{13}$ in the form

$$u_1 = (u_1)_p + \sum_{k=1}^{n} (u_1)_k$$

$$\tau_{12} = (\tau_{12})_p + \sum_{k=1}^{n} (\tau_{12})_k$$

$$\tau_{13} = (\tau_{13})_p + \sum_{k=1}^{n} (\tau_{13})_k \qquad ......(9)$$

Here, $(u_1)_p$, $(\tau_{12})_p$, $(\tau_{13})_p$ satisfies (1)-(6), and are continuous throughout the model. Hence they have the same expressions as those for $u_1$, $\tau_{12}$, $\tau_{13}$ in (7). $(u_1)_k$, $(\tau_{12})_k$, $(\tau_{13})_k$ vanish for $t_k \leq 0$ i.e., $t \leq T_k$, satisfies (1)-(5) and the condition $(\tau_{12})_k \to 0$ as $|y_2| \to \infty$ [which replaces (6)]. $(u_1)_k$ is continuous everywhere in the model except across $F_k$, where we have the discontinuoity of $(u_1)_k$, across $F_k$, given by

$$[(u_1)_k] = U_k(t_k)\, H(t_k)\, f_k(y_3)$$

$$[y_2 = P_k, \qquad d_k \leq y_3 \leq D_k].$$

$(\tau_{12})_k$, $(\tau_{13})_k$ are continuous everywhere in the model.

We introduce the variables

$$y_{2k} = y_2 - P_k \; (k = 1,2,......,n)$$

$$y_{3k} = y_3$$

$$t_k = t - T_k.$$

Since $(u_1)_p$, $(\tau_{12})_p$, $(\tau_{13})_p$ satisfy the relations (1)-(6), the relation for them are given by (7).

$(u_1)_k$, $(\tau_{12})_k$, $(\tau_{13})_k$ satisfy the relations

$$\left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t_k}\right)(\tau_{12})_k = \frac{\partial^2 (u_1)_k}{\partial t_k \, \partial y_{2k}}$$

$$\left(\frac{1}{\eta} + \frac{1}{\mu}\frac{\partial}{\partial t_k}\right)(\tau_{13})_k = \frac{\partial^2 (u_1)_k}{\partial t_k \, \partial y_{3k}} \qquad ......(10)$$

$$\frac{\partial}{\partial y_{2k}}(\tau_{12})_k + \frac{\partial}{\partial y_{3k}}(\tau_{13})_k = 0$$

$$\nabla^2 (u_1)_k = 0$$

(all valid in $-\infty < y_{2k} < \infty$, $y_{3k} \geq 0$).

$$(\tau_{13})_k = 0 \quad \text{at} \quad y_{3k} = 0$$

$$(\tau_{13})_k \to 0 \quad \text{as} \quad y_{3k} \to \infty$$

$$(\tau_{12})_k \to 0 \quad \text{as} \quad |y_{2k}| \to \infty$$

$$[(u_1)_k] = u_k(t_k)\ f_k(y_3 k) \quad \text{acrosss} \quad F_k\ [y_2 = P_k,\ d_k \leq y_3 \leq D_k].$$

$(u_1)_k$ is continous everywhere in the model, except across $F_k$, $(\tau_{12})_k$, $(\tau_{13})_k$, are continous everywhere in the model, $U_k(t_k)$, $(u_1)_k$, $(\tau_{12})_k$, $(\tau_{13})_k$, are all $= 0$ for $t_k < 0$ (i.e., $t \leq T_k$). The creep velocity across $F_k$ is $V_k(t_k)$ $f_k(y_{3k})$,

where $$V_k(t_k) = \frac{d}{d\,t_k} V_k(t_k) \qquad ......(11)$$

To obtain the solutions for $(u_1)_k$, $(\tau_{12})_k$, $(\tau_{13})_k$ satisfying the relations (10), we take Laplace transforms of the relation with respect to $t_k$. This gives a boundary value problem which is exactly similar to a boundary value problem solved earlier by Mukherji and Mukhopadhyay (1984,, 1986, 1988), and following the same method, involving the use of Green's function technique developed by Maruyama (1966), we obtain the solution of the boundary value problem. Inversion of the Laplace transforms, as in Mukherji and Mukhopadhyay (1984, 1986, 1988), gives us solutions for $(u_1)_k$, $(\tau_{12})_k$, $(\tau_{13})_k$. Finally, we obtain, using (9) and (7).

$$u_1 = (u_1)_p + \sum_{k=1}^{n} (u_1)_k,$$

$$\tau_{12} = (\tau_{12})_p + \sum_{k=1}^{n} (\tau_{12})_k$$

$$\tau_{13} = (\tau_{13})_p + \sum_{k=1}^{n} (\tau_{13})_k \qquad .....(11a)$$

where

$$(u_1)_p = (u_1)_0 + \frac{\tau_\infty y_2 t}{\eta}$$

$$(\tau_{12})_p = (\tau_{12})_0\, e^{-\frac{\mu t}{\eta}} + \tau_\infty \left(1 - e^{-\frac{\mu t}{\eta}}\right) \qquad ....(12a)$$

$$(\tau_{13})_p = (\tau_{13})_0 \, e^{-\frac{\mu t}{\eta}}$$

and

$$(u_1)_k = H(t - T_k)\frac{U_k(t_k)}{2\pi}\Psi_k(y_{2k}, y_{3k}) \quad .....(12)$$

$$(\tau_{12})_k = H(t - T_k)\frac{\mu}{2\pi}\left(\int_0^{t_k} V_k(\tau)\, e^{-\frac{\mu(t_k-\tau)}{\eta}}\, d\tau\right)\phi_{1k}(y_{2k}, y_{3k}) \quad .....(13)$$

$$(\tau_{13})_k = H(t - T_k)\frac{\mu}{2\pi}\left(\int_0^{t_k} V_k(\tau)\, e^{-\frac{\mu(t_k-\tau)}{\eta}}\, d\tau\right)\phi_{2k}(y_{2k}, y_{3k}) \quad .....(14)$$

where

$$H(t - T_k) = 0, \quad t \leqslant T_k$$
$$= 1, \quad t \geqslant T_k \quad ......(15)$$

$$\Psi_k(y_{2k}, y_{3k}) = \int_{d_k}^{D_k} f_k(x_{3k})\, y_{2k}\left[\frac{1}{(x_{3k}-y_{3k})^2 + y_{2k}^2} + \frac{1}{(x_{3k}+y_{3k})^2 + y_{2k}^2}\right] dx_{3k} \quad .....(16)$$

$$\phi_k(y_{2k}, y_{3k}) = \int_{d_k}^{D_k} f_k(x_{3k})\left[\frac{(x_{3k}-y_{3k})^2 - y_{2k}^2}{\{(x_{3k}-y_{3k})^2 + y_{2k}^2\}^2} + \frac{(x_{3k}+y_{3k})^2 - y_{2k}^2}{\{(x_{3k}+y_{3k})^2 + y_{2k}^2\}^2}\right] dx_{3k} \quad .....(17)$$

$$\phi_{2k}(y_{2k}, y_{3k}) = \int_{d_k}^{D_k} f_k(x_{3k})\left[\frac{2\, y_{2k}(x_{3k}-y_{3k})}{\{(x_{3k}-y_{3k})^2 + y_{2k}^2\}^2} - \frac{2\, y_{3k}(x_{3k}+y_{3k})}{\{(x_{3k}+y_{3k})^2 + y_{2k}^2\}^2}\right] dx_{3k} \quad .....(18)$$

Here $d_k = 0$, if $F_k$ is a surface-breaking fault, and $U_k(t_k) = 0$ for $t_k \leqslant 0$, i.e. $t \leqslant T_k$.

The shear strain $e_{12}$ is given by

$$e_{12} = \frac{\partial u_1}{\partial y_2} = (e_{12})_0 + \frac{\tau_\infty t}{\eta} + \sum_{k=1}^{n} \frac{\partial (u_1)_k}{\partial y_2}$$

$$= (e_{12})_0 + \frac{\tau_\infty t}{\eta} = \sum_{k=1}^{n} H(t - T_k) \frac{U_k (t_k)}{2\pi}$$

$$\phi_{1k} (y_{2k}, y_{3k}) \qquad ......(19)$$

The expressions for the displacements, stresses and strains we have obtained are valid everywhere in the model, and for all time, as long as the aseismic state is maintained, and there is no sudden seismic fault movement in the model. The solutions are valid for all types of fault creep in the model. By suitable choice of $T_k$ and $U_k(t_k)$, we can represent all types of fault creep including the case in which fault creep occurs for some time and then stops, taking $U_k(t_k) = 0$ for the period when there is no fault creep. If no creep occurs across a fault, say $F_k$, at any time, we take $U_k(t_k) = 0$ for all $t_k$ for that fault. The solutionscease to be valid during periods of seismic disturbances, since the inertial terms cannot be neglected, and the relations (2) and (3) cease to be valid.

The integrals in the expressions for $\phi_k$, $\phi_{1k}$, $\phi_{2k}$ can be expressed in closed form in terms of elementary functions if $f_k(y_3)$ is a polynomial. However, in some cases, and in particular if $f_k(y_3)$ = a constant, we find, as in Mukherji et al. (1984, 1986, 1988), that the integral $\phi_{1k}$ does not remain finite as we approach the upper and lower edges of the fault $F_k(y_3 = d_k,\ y_3 = P_k$ and $y_3 = D_k, y_2 = P_k)$. Hence, in this case, $\tau_{12}$ and $e_{12}$ do not remain finite as we approach the upper and lower edges of the fault.

The conditions under which the displacements, stresses and strains are finite everywhere in the model for all finite $(y_2, y_3, t)$ can be found by the following exactly the same method as in Mukherji et al. (1984, 1986, 1988).

The actual analytical conditions to be satisfied are found to be somewhat different for surface-breaking and buried faults. These conditions are to be satisfied by the functions $f_k(y_3)$ which specify the dependence on depths of the relative displacements across the faults $F_k$, and they have to be satisfied simultaneously across all the faults to ensure finite displacements, stresses and strains everywhere in the model, including the regions near the edges of the faults. If $F_k$ is a surface-breaking fault, the conditions to be satisfied by $f_k(y_3)$, are given by :

(i) $f_k(y_3)$, $f'_k(y_3)$, $f''_k(y_3)$ are continuous in $0 \leq y_3 \leq D_k$

(ii) $f_k(D_k) = 0 = f'_k(D_k)$, $f'_k(0) = 0$ .....(20)

If $F_k$ is a buried fault, $f_k(y_3)$ has to satisfy the following conditions :

(i) $f_k(y_3)$, $f'_k(y_3)$, $f''_k(y_3)$ are continuous in $d_k < y_3 < D_k$

(ii) $f_k(d_k) = 0 = f_k(D_k)$ .....(21)

$$f'_k(d_k) = 0 = f'_k(D_k).$$

These conditions imply that the relative displacements across the faults vary smoothly with the depth, and approach the value zero smoothly as we approach the edges of the faults within the model.

If the conditions (20) and (21) are satisfied, then integration by parts of the integrals $\Psi_k$, $\phi_{1k}$, $\phi_{2k}$ in (16)-(18), which occur in the expressions for the displacements, stresses and strains, reduces them to the following forms, which remain finite everywhere in the model :

$$\Psi_k(y_{2k}, y_{3k}) = - \int_{d_k - y_{3k}}^{D_k - y_{3k}} f_k(v_{3k} + y) \tan^{-1} (y/y_{2k})\, dy - \int_{d_k + y_{3k}}^{D_k + y_{3k}} f_k(x - y_{3k}) \tan^{-1} (x / y_{2k})\, dx \qquad \text{.....(22)}$$

$$\phi_{1k}(y_{2k}, y_{3k}) = -\frac{1}{2} \left[ \int_{d_k - y_{3k}}^{D_k - y_{3k}} f_k(y + y_{3k}) \log_e (y^2 + y_{2k}^2)\, dy + \int_{d_k + y_{3k}}^{D_k + y_{3k}} f_k(x - y_{3k}) \log_e (x^2 + y_{2k}^2)\, dx \right] \qquad \text{.....(23)}$$

$$\phi_{2k}(y_{2k}, y_{3k}) = - \int_{d_k - y_{3k}}^{D_k - y_{3k}} f_k(y_{3k} + y) \tan^{-1} (y/y_{2k})\, dy - \int_{d_k + y_{3k}}^{D_k + y_{3k}} f_k(x - y_{3k}) \tan^{-1} (x/y_{2k})\, dx \qquad \text{......(24)}$$

The integrals in (22)-(24) can be expressed in closed form in terms of elementary functions if $f_k(y_3)$ are polynomials. If $f_k(y_3)$ are not polynomials, but satisfy the conditions (20) and (21), the integrals are convergent, and can be evaluated approximately by numerical methods. As we approach the faults, i.e., $y_2 \to P_k$, the integral can be evaluated by taking the limit inside the integral, which is valid in this case, by the conditions of interchange of limit and integral, given in Carslaw (1950).

## DISCUSSION OF THE RESULTS AND CONCLUSIONS

We now study in greater detail the displacements, stresses and strains in the model, including the effect of fault creep and interaction between the faults. We compute the changes in the displacement $u_1$ and the shear strain $e_{12}$ on the free surface $y_3 = 0$ in the region above the faults, and the shear stress $\tau_{12}$ near the faults, tending to cause strike-slip movement. We consider, for the computation, a system of four faults-two buried and two surface-breaking. We assume simple forms of the relative creep displacement $u_1$ across the buried faults $F_1$, $F_2$ and the surface-breaking faults $F_3$, $F_4$, satisfying the conditions (20) and (21) in the form

$$[u_1] = V_k t_k f_k(y_3) H(t - T_k)$$

$$(k = 1,2,3,4).$$

across $F_k$ $(k = 1,2,3,4)$, where $V_k$ are constants and $t_k = t - T_k$. For the buried faults $F_1$, $F_2$ we assume the depths of the upper and lower edges of the fault below the free surface to be $d_k$, $D_k$ $(k = 1,2)$ and for the surface-breaking faults $F_3$, $F_4$ we take $D_k$ $(k = 3,4)$ to be the depths of the lower edges. For $(F_1, F_2)$ we assume, as in Mukherji et al. (1984, 1986, 1988) that

$$f_k(y_3) = \frac{16 (y_3 - d_k)^2 (D_k - y_3)^2}{(D_k - d_k)^4}$$

$$(k = 1,2)$$

so that the maximum creep displacement across the buried fault $F_k$ $(k = 1,2)$ is at $y_3 = (d_k + D_k)/2$, and for $(F_3, F_4)$ we assume

$$f_k(y_3) = 1 - \frac{3 y_3^2}{D_k^2} + \frac{2 y_3^3}{D_k^2}$$

$$(k = 3,4).$$

These forms of $f_k(y_3)$ ensure that the relative creep movement approaches zero smoothly as we approach the edges of the faults within the model, and that the displacements, stresses and strains are finite everywhere in the model, including the regions close to the edges of the faults, for all finite values of $(y_2, y_3, t)$, as explained in Mukherji et al. (1984, 1986, 1988).

The planes of the faults $F_k$ $(k = 1,2,3,4)$ are taken to be $y_2 = P_k$ $(k = 1,2,3,4)$ where $P_1 = 0$ and $P_4 > P_3 > P_2 > 0$. This fault system is shown in Fig. 2. In the computation, keeping in view shallow strike-slip faults in the lithosphere, we have taken values of $d_k$, $D_k$, $P_k$ in the range 0 to 40 kms. (with $d_k < D_k$ in all cases). We have taken values for $\mu$ in the range (3 to 4) x $10^{11}$ dynes/cm$^2$ and for $\eta$ in the range $10^{21}$ - $10^{22}$ poise, following Cathles (1976) and Mukherji et al. (1984, 1986, 1988). For $\tau_\infty$, we consider values in the range 50 to 200 bars, and for $(\tau_{12})_0$ near the faults, we consider values from 0 to 100 bars, with $(\tau_{12})_0 < \tau_\infty$ in all cases. The computed values obtained be using the computational algorithms and computer software developed for the purpose, lead to the following interesting conclusions :

i) In the absence of any fault movement, the shear strear $\tau_{12}$ near the faults increases with time, with a slowly decreasing rate of increase. This increase in the value of $\tau_{12}$ would be expected to increase its possibility of movement across the faults. If no fault movement occurs at any subsequent time, $\tau_{12} \to \tau_\infty$ ultimately as $t \to \infty$. But if fault creep commences across one or more fault, the displacements, stresses and strains are influenced significantly by the fault creep.

ii) The effects of fault creep on the surface displacement, surface shear strain and shear stress the faults are found to depend on the following factors :

a) The creep velocities across the faults and their variation with depth, characterised by $V_k$ and $f_k(y_3)$.

b) The depths and dimensions of the faults and the distances between them, characterised by $d_k$, $D_k$ and $P_k$.

c) The relative positions of the faults.

d) The shear stress $\tau_\infty$, maintained by tectonic forces away from the faults.

e) The displacements, stresses and strains present initially, at $t = 0$.

f) The material rheology of the model, characterised by $\mu$ and $\eta$.

iii) For reasonable values of the model parameters. There is a slow aseismic accumulation of shear strain on the surface $y_3 = 0$ in the region of the faults in the absence of fault movement.

The rate of accumulation of surface shear strain is of the order of $10^{-7}$ per day, which is of the same order of magnitude as the observed rate of shear strain accumulation on the surface near the locked parts of the San Andreas fault.

iv) Aseismic creep across any fault has significant influence on the shear stress near itself, and generally leads to decrease in the rate of accumulation of the shear stress near the fault itself. The magnitude of this effect depends on different model parameters, including the creep velocity. For suitable values of the model parameters (including sufficiently large creep velocities) there is a continuous slow aseismic release of shear stress near the creeping fault, reducing continuously the possibility of a sudden seismic movement across the fault.

v) The effect of aseismic creep across any fault on the neighbouring faults of the fault system depends to some extent on the displacements, stresses and strains present initially at $t = 0$, and values of the model parameters $\mu$, $\eta$ and $\tau_\infty$. But the effect depends mainly on the creep velocity and its dependence on the depth, the dimensions of the faults, the distances between them and their relative positions, which has a very important role.

vi) It is found that, for any surface-breaking fault, say $F_k$, there is a region $I_k$ of the model, mainly below the fault, where aseismic creep across $F_k$ increases the rate of accumulation of shear stress, and two other regions $(R_k, R'_k)$ on either side of the fault, where aseismic creep across $F_k$ is found to reduce this rate. Hence, if any other fault of the system, say $F'_k$ is situated in $I_k$, then creep across $F_k$ would tend to increase the rate of accumulation of shear stress near $F'_k$, thus increasing the possibility of a sudden seismic fault movement across $F'_k$. If $F'_k$ is situated in the region $R_k$ or $R'_k$, then creep across $F_k$ would reduce the rate of accumulation of shear stress near $F'_k$, and if $F'_k$ is in $R_i$ or $R'_i$, this rate will be reduced by seismic creep across $F_i$. Fig. 3 shows the general shape of these regions $T_k$, $(R_k, R'_k)$, $(I_i, I'_i)$ and $(R_i, R'_i)$ in the model, for the buried fault $F_i$ and the surface-breaking fault $F_k$. The actual positions of the boundaries between these regions depend on the characteristics of the faults $F_i$ and $F_k$, including the depths and dimensions of the faults and the dependence of the creep on the depth, which is characterised by the functions $f_k(y_3)$.

An interesting consequence of this result is that aseismic creep across any fault of the model tends to release the shear stress near neighbouring faults which are of similar dimensions and more or less at the same level as the creeping fault, and tends to increase the rate of accumulation of shear stress near neighbouring faults which are more or less vertically above or below the creeping fault, thus increasing the possibility of a sudden seismic fault movement across the neighbouring fault.

vii) The influence of creep across any fault on the shear stress near a neighbouring fault decreases fairly rapidly with increase in the distance between the creeping fault and the neighbouring fault.

viii) For a surface-breaking fault, aseismic creep across the fault reduces the rate of accumulation of surface shear strain on the surface near the fault, and for suitable values of the model parameters, including the creep velocity, the rate of accumulation becomes nearly zero. This result for the model resembles the situation near the creeping central part of the San Andreas fault, where the rate of accumulation of surface shear strain is nearly zero, while it has values of the order of $10^{-7}$ per year near the locked part of the San Andreas fault. However, for a buried fault, creep across the fault is found to increases, to some extent, the rate of accumulation of surface shear strain in the region vertically above the fault in our model.

We now show, in greater detail, the variation with time of the surface shear in the region above the faults and the shear stress near one of the faults for a typical set of values of the model parameters, with $F_1$ vertically above $F_2$, and with the relative positions of the faults as in Fig. 2, where,

$\eta = 10^{22}$ poise, $\mu = 3\times10^{11}$ dynes/cm$^2$, $\tau_\infty = 100$ bars,

$(\tau_{12})_0 = 25$ bars (in the region of the faults), $d_1 = 5$ kms, $D_1 = 15$ kms,

$d_2 = 20$ kms, $D_2 = 30$ kms, $d_3 = 0$, $D_3 = 10$ kms, $d_4 = 0$, $D_4 = 10$ kms,

$P_1 = P_2 = 0$, $P_3 = 10$ kms, $P_4 = 20$ kms, $T_1 = 50$ years, $T_2 = 100$ years,

$T_3 = 150$ years, $T_4 = 200$ years.

Fig. 4 shows the variation with time of the quantity

$$E_{12} = [e_{12} - (e_{12})_0]_{\substack{y_3=0 \\ y_2=0}} \times 10^7$$

$$= \text{(change in the surface shear strain vertically above } F_1) \times 10^7,$$

in the case where $V_1 = V_2 = V_3 = V_4 = 2$ cms/year, specify the creep velocitites across the system of faults shown in Fig. 2. Before the commencement of any fault creep, there is a steady accumulation of the surface shear strain above $F_1$. Commencement of creep across $F_1$ at time $T_1$ increases this rate. When $F_2$ also starts creeping at time $T_2$ ( > $T_1$ > 0), there is some further increase in the rate of accumulation of surface shear strain above $F_1$, although the effect is smaller. Later, the commencement of creep across $F_3$ and $F_4$ at times $T_3$ and $T_4$ ($T_4 > T_3 > T_2 > T_1 > 0$) leads to some reduction in the rate of accumulation of surface shear strain above $F_1$. The effect of fault creep on the changes of $E_{12}$ above $F_1$ is greatest for creep across $F_1$, followed by $F_2$, $F_3$ and $F_4$ in decreasing order.

Fig. 5 shows the variation with time of the quantity,

$$\tau_{1m} = \max_{d_1 \leq y_3 \leq D_1} (\tau_{12})_{y_2 \to 0}$$

which represents the maximum value of the shear stress $\tau_{12}$ near the fault $F_1$ at time t. Fig. 5 shows the variation of $\tau_{1m}$ with time in two cases - the case in which there is no creep across any fault ($V_k = 0$) and the case in which creep commences across $F_k$ at time $T_k$ with $V_k = 2$ cms/year (k = 1,2, 3,4). We find that, in the absence of fault movement, there is a gradual accumulation of shear stress near $F_1$. When creep commences across $F_1$ at t = $T_1$, the rate of increase of $\tau_{1m}$ with time is reduced to nearly zero. Later, when creep commences across $F_2$, which is vertically below $F_1$, the rate of increase of $\tau_{1m}$ is enhanced to some extent. Again, when creep commences still later across $F_3$ and $F_4$ at time $T_3$ and $T_4$, there is some reduction in the rate of increase of $\tau_{1m}$ with time. The effect of fault creep on $\tau_{1m}$ is greatest for creep across $F_1$, followed by $F_2$, $F_3$ and $F_4$ in decreasing order. These results bring out the significant influence of the relative positions of the faults on the effect of aseismic creep across a fault on the neighbouring faults of the system. The nature of the interaction between the faults in the model, therefore, depends significantly on the relative positions of the faults.

In conclusion, it is hoped that the theoretical model presented here will be useful in the study of interaction between neighbouring strike-slip faults in seismically active regions, and in gaining greater insight into the physics and dynamics of earthquake processes.

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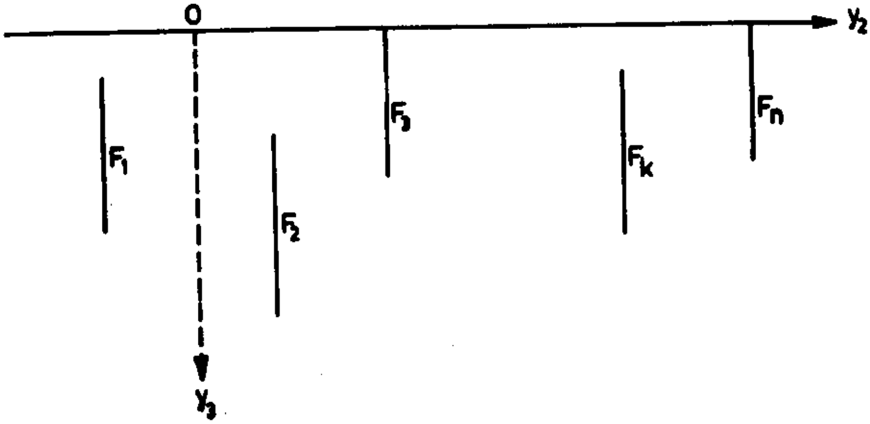


Fig.1: Section of the model by the plane $y_1 = 0$.

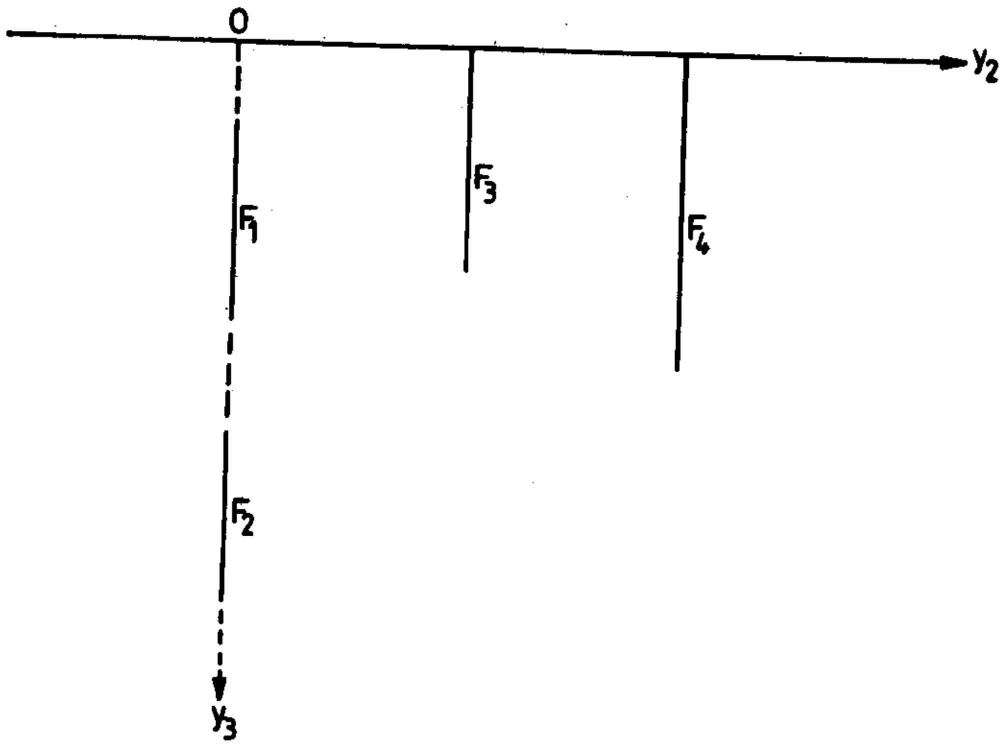


Fig. 2: Section, by the plane $y_1 = 0$, of the model used for computations. This figure shows only one set of relative positions of the faults. Other relative positions have also been considered, including some cases in which the faults are more or less at the same level in different parallel planes, and some other cases in which one c two faults are more or less vertically above or below other faults of the system.

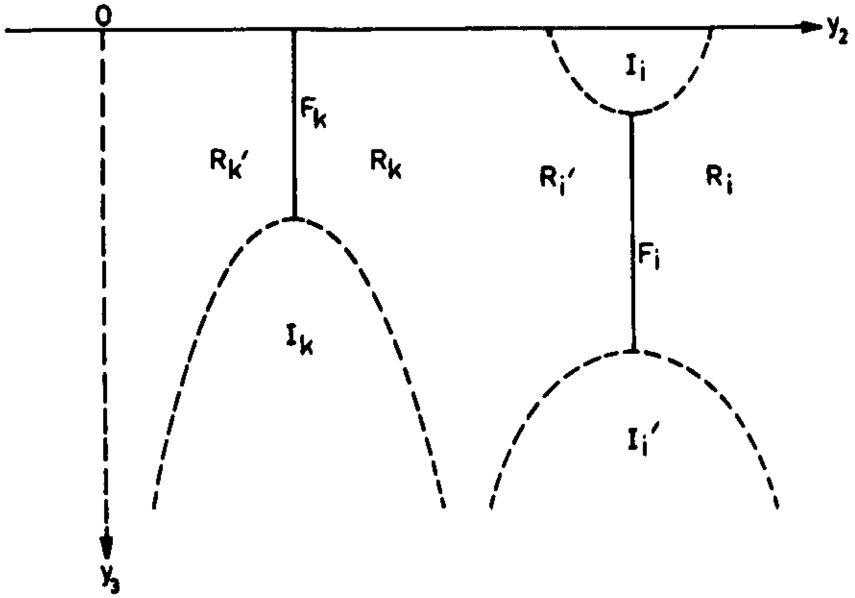


Fig.3: Regions of shear stress accumulation($I_k$), ($I_i$, $I_i'$) and shear stress release($R_k$, $R_k'$), ($R_i$, $R_i'$) due to creep across $F_k$ and $F_i$. The other faults of the system are not shown in this figure.

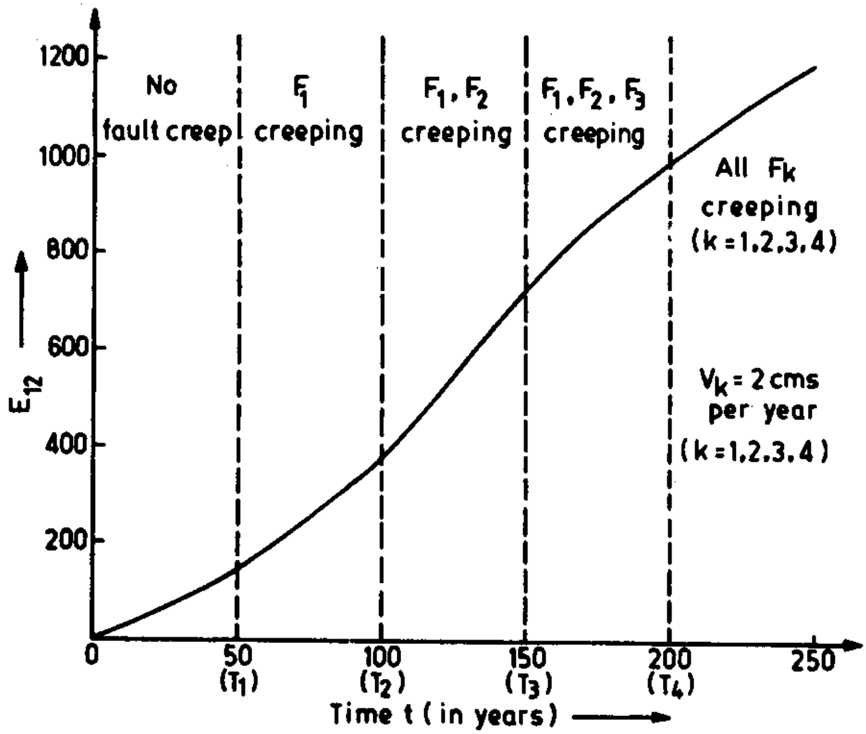


Fig. 4. Changes, with the time t, of the surface shear strain above $F_1$ and $F_2$ in the case in which the faults have relative positions as shown in figure 2.

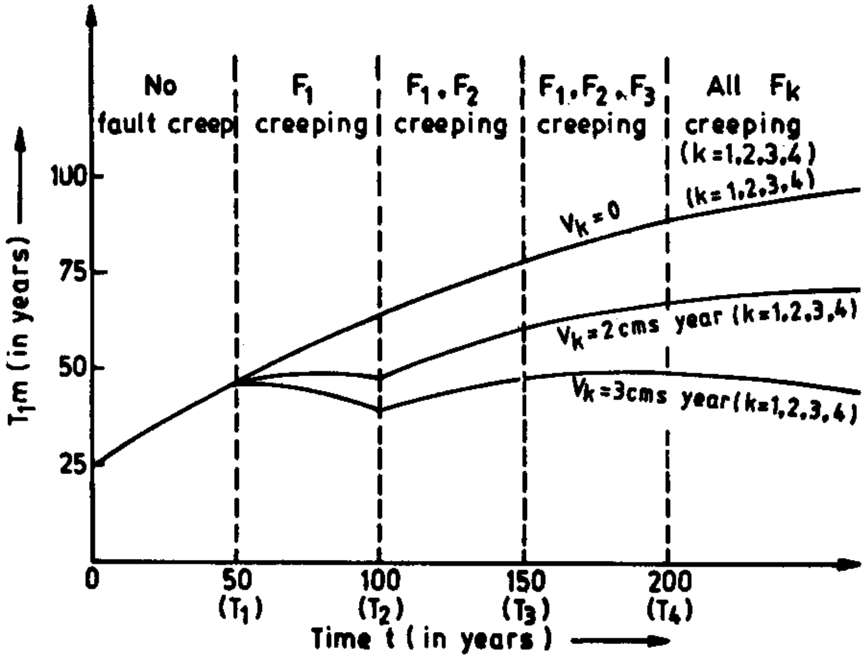


Fig.5: Changes with the time $t$, of $T_1m$, the maximum shear stress near $F_1$, where the faults have relative positions as shown in figure 2.