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DYNAMIC ANALYSIS OF CHIMNEYS AND NATURAL DRAUGHT COOLING TOWERS

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ABSTRACT

Reinforced concrete chimneys and natural draught cooling towers are essential components of power plants. As these structures are flexible and prone to earthquakes & wind, their dynamic characteristics are prerequisite for rational and safer design. The paper reports the details of a truncated conical shell element to idealize the chimneys and cooling towers and the development of software for the dynamic response. Vibration characterstics of chimneys and cooling towers have been obtained using the element. The predicted results are compared against the theoretical as well as experimental values available in the literature.

INTRODUCTION

Reinforced concrete chimneys and Natural Draught Cooling Towers (NDCT) are essential components of a power plants and industrial set ups. Chimneys are required to disperse the unwanted gases of the processing plant into the atmosphere at a height such that when these reach the ground level sufficient dilution by diffusion would have occurred so that these become harmless; while NDCT are provided to evacuate the heat generated during various processes in a thermal or nuclear power plant into the atmosphere. With the advancement in the industrial sector and the strict conformity to the pollution control regulations, the chimneys and cooling towers are becoming taller and already chimneys as high as about 300m and NDCT of 140m have been constructed in our country. These structures being flexible are prone to earthquake and wind forces. The dynamic characteristics of these structures is, therefore, a prerequisite for their rational and safer design.

The chimneys are analysed by idealising them into the beam elements with lumped mass for the estimation of their mode shapes and frequencies. In such an idealisation the effect of taper of the chimney is neglected. It is also well known that taper of the meridian decreases the frequency of the chimney in comparison to the outer battering of the chimney. Based on the work of Vickery (1985), Bureau of Indian Standards recommends separate expressions for the acrosswind response of chimneys with taper

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less than 1 in 50 or greater than 1 in 50. Thus, consideration of the taper of the chimney in the free vibration analysis is required. Vanasup (1975) considered the effect of liners on the free vibration characterstics of the chimney by modelling its liner and shell as equivalent framed structure and the transverse members between the two as springs for shear transfer. In case of chimney with steel liners, the vibration analysis of chimney is performed for shell/shield only.

For the free vibration analysis of NDCT different methodologies are reported in the literature. In the beginning, the NDCT vibration analysis was performed considering the tower shell to be built-in or simply supported at the ground level. The techniques used were numerical integration method, finite difference approach and finite element method. For brevity, the literature review on free vibration analysis of NDCT is not included in the paper.

Presently, NDCTs are usually supported by a number of discrete columns at the base. These column supports influence the vibration characterstics of the NDCT significantly, hence their correct mathematical modelling is very important. Various techniques have been reported in the literature to model the discrete columns. Sen & Gould (1974) replaced the columns by a rotational shell element having averaged out property of the discrete columns. Discrete column elements were included in the analysis and parametric studies were conducted considering their geometry, number of columns, inclination and material properties. Earlier, thickness of the tower shell was increased to enhance the buckling strength of the tower whereas it reduces the fundamental frequency. The codes of NDCT specify that the fundamental frequency of NDCT should be greater than 1HZ, therefore, stiffening rings are provided for increasing the buckling strength as well as frequency rather than increasing thickness of shell. All these methods adopted the finite element technique for the analysis. Bhimaraddi et.al (1990) used an anisoparametric quadrilateral shell of revolution element for tower shell and isoparametric beam element for stiffeners and columns. They concluded that provision of ring stiffeners (i) alters frequency significantly of higher modes with the circumferential wave number greater than three, (ii) increase the load carrying capacity under wind excitation and (iii) may not increase the load carrying capacity under earthquake excitation since these rings have no influence on the mode shape.

The paper reports a truncated conical shell element to idealise chimneys and cooling towers for the free vibration analysis. Details of formulation of the element and the software developed are also included. Two examples each of RC chimneys and cooling towers have been analysed using this technique. The predicted results have been compared against the theoretical as well as experimental results published in the literature.

TRUNCATED CONICAL SHELL ELEMENT

Each element consists of a truncated conical shell with two end nodes and four degrees of freedom (u, v, w, and θ) per node (Fig.1a). Thus, each element has eight degrees of freedom. The thickness of the element varies linearly. The shell element is based on modified form of Novozhilov (1959) shell theory. The assumptions used in the derivation are:

1. The shell is thin and initially lines normal to mid surface remain straight and normal during deformation.
2. Meridians are assumed to be straight.
3. Mid-surface strains and curvature are expressed in terms of displacements.

The assumed displacement functions for an element, which allow for sinusoidal distribution of displacement in circumferetial direction to cater for various circumferential wave patterns (Fig.1b) are given below:

$$u = (u_i(1-Ç) + u_jÇ)\ \mathrm{Cos}n\phi \qquad \text{--------- (1)}$$

$$v = (v_i(1-Ç) + v_jÇ)\ \mathrm{Sin}n\phi \qquad \text{--------- (2)}$$

$$w = (w_i(1-3Ç^2+2Ç^3)+l(Ç-2Ç^2+Ç^3)\theta_i+w_j(3Ç^2-2Ç^3)+l(-Ç^2+Ç^3)\theta_j)\mathrm{Cos}n\phi \qquad \text{--------- (3)}$$

Writing in Matrix form

$$\{U\} = [N]\ \{U_e\} \qquad \text{--------- (4)}$$

Where

$\{U\} = \{u\ v\ w\}^T$ = the displacement vector

$\{U_e\} = [u_i\ v_i\ w_i\ \theta_i\ u_j\ v_j\ w_j\ \theta_j]^T$ = A vector of nodal displacements

[N] = Matrix of shape functions

$$= \begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} & N_{17} & N_{18} \\ N_{21} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} & N_{27} & N_{28} \\ N_{31} & N_{32} & N_{33} & N_{34} & N_{35} & N_{36} & N_{37} & N_{38} \end{bmatrix}$$

N_{11} = (1-Ç) Cosnφ ; N_{15} = Ç Cosnφ ; N_{22} = (1-Ç) Sinnφ ;

N_{26} = Ç Sinnφ ; $N_{33} = (1-3Ç^2+2Ç^3)$ Cosnφ ; $N_{34} = l(Ç-2Ç^2+Ç^3)$ Cosnφ

$N_{37} = (3Ç^2-2Ç^3)$ Cosnϕ ; $N_{38} = 1(-Ç^2+Ç^3)$ Cosnϕ.

Other elements of [N] are zero.

The vector of strains is given by

$(\epsilon) = [\ \epsilon_x\ \epsilon_\phi\ \epsilon_{x\phi}\ K_x\ K_\phi\ K_{x\phi}\ ]^T$ ------- (5)

where,

$\epsilon_x = u_{,x}$

$\epsilon_\phi = 1/r\ (v_{,\phi}) + 1/r\ (u\ \mathrm{Sin}\alpha - w\ \mathrm{Cos}\alpha)$

$\epsilon_{x\phi} = 1/r\ (u_{,\phi}) + (v_{,x}) - 1/r\ (v\ \mathrm{Sin}\alpha)$

$K_x = w_{,xx}$

$K_\phi = 1/r^2\ (w_{,\phi\phi}) + 1/r^2\ (v_{,\phi})\ \mathrm{Cos}\alpha + 1/r(w_{,x})\ \mathrm{Sin}\alpha$

$K_{x\phi} = 2(1/r\ (w_{,x\phi}) - 1/r^2\ (w_{,\phi})\ \mathrm{Sin}\alpha + 1/r\ (v_{,x})\mathrm{Cos}\alpha$
$-1/r^2\ v\mathrm{Sin}\alpha\ \mathrm{Cos}\alpha)$

i.e. $(\epsilon) = [B]\ (U_e)$ ----------- (6)

Substitution of displacement functions and its derivatives in equation 6 results in $[B]_{6x8}$ matrix whose elements are

$B_{11} = (-1/1)$ Cosnϕ ; $B_{15} = (1/1)$ Cosnϕ

$B_{21} = 1/r\ (1-Ç)$ Sinα Cosnϕ ; $B_{22} = n/r\ (1-Ç)$ Cosnϕ

$B_{23} = -1/r\ (1-3Ç^2+2Ç^3)$ Cosα Cosnϕ

$B_{24} = -1/r\ (Ç-2Ç^2+Ç^3)$ Cosα Cosnϕ ; $B_{25} = Ç/r$ Sinα Cosnϕ

$B_{26} = (nÇ/r)$ Cosnϕ ; $B_{27} = -1/r\ (3Ç^2-2Ç^3)$ Cosα Cosnϕ

$B_{28} = -1/r\ (-Ç^2+Ç^3)$ Cosα Cosnϕ ; $B_{31} = -n/r\ (1-Ç)$ Sinnϕ

$B_{32} = ((-1/1) - 1/r\ (1-Ç)$ Sinα) Sinnϕ ; $B_{35} = (-nÇ/r)$ Sinnϕ

$B_{36} = -((1/1)-(Ç/r)$ Sinα) Sinnϕ ; $B_{43} = 1/1^2\ (-6+12Ç)$ Cosnϕ

$B_{44} = 1/1\ (-4+6Ç)$ Cosnϕ ; $B_{47} = 1/1^2\ (6-12Ç)$ Cosnϕ

$B_{48} = 1/1\ (-2+6Ç)$ Cosnϕ ; $B_{52} = n/r^2\ (1-Ç)$ Cosα Cosnϕ

$B_{53} = (-n^2/r^2\ (1-3Ç^2+2Ç^3) + 1/r1\ (-6Ç+6Ç^2)$ Sinα) Cosnϕ

$B_{54} = (-n^2/r^2\ (Ç-2Ç^2+Ç^3)1 + 1/r\ (1-4Ç+3Ç^2)$ Sinα) Cosnϕ

$B_{56} = n/r^2\ Ç\ \text{Cos}\alpha\ \text{Cos}n\phi$

$B_{57} = (-n^2/r^2\ (3Ç^2-2Ç^3) + 1/rl\ (6Ç-6Ç^2)\ \text{Sin}\alpha)\ \text{Cos}n\phi$

$B_{58} = (-n^2 l/r^2\ (-Ç^2+Ç^3) + 1/r\ (-2Ç+3Ç^2)\ \text{Sin}\alpha)\ \text{Cos}n\phi$

$B_{62} = 2((-1/rl)\ \text{Cos}\alpha - 1/r^2(1-Ç)\ \text{Cos}\alpha\ \text{Sin}\alpha)\ \text{Sin}n\phi$

$B_{63} = 2(-n/rl\ (-6Ç+6Ç^2) + n/r^2\ (1-3Ç^2+2Ç^3)\ \text{Sin}\alpha)\ \text{Sin}n\phi$

$B_{64} = 2(-n/r\ (1-4Ç+3Ç^2) + nl/r^2\ (Ç-2Ç^2+Ç^3)\ \text{Sin}\alpha)\ \text{Sin}n\phi$

$B_{66} = 2((1/rl)\ \text{Cos}\alpha - (Ç/r^2)\ \text{Cos}\alpha\ \text{Sin}\alpha)\ \text{Sin}n\phi$

$B_{67} = 2((-n/rl)\ (6Ç-6Ç^2) + (n/r^2)(3Ç^2-2Ç^3)\ \text{Sin}\alpha)\ \text{Sin}n\phi$

$B_{68} = 2((n/r)(-2Ç + 3Ç^2) + (nl/r^2)(-Ç^2+Ç^3)\ \text{Sin}\alpha)\ \text{Sin}n\phi$

other elements of [B] are zero.

The element stiffness matrix in local co-ordinates is given by

$$[k_e] = l\pi \int r\ [B]^T\ [D]\ [B]\ dÇ \qquad \text{----------- (7)}$$

In global co-ordinates element stiffness matrix is given by

$$[k^o] = [C]^T\ [k_e]\ [C] \qquad \text{----------- (8)}$$

where [D] is a matrix of elastic constants

$$[D] = Et/(1-\mu^2) \begin{bmatrix} 1 & \mu & & & & \\ \mu & 1 & & & & \\ 0 & 0 & (1-\mu)/2 & & & \\ 0 & 0 & 0 & t^2/12 & \mu t^2/12 & \\ 0 & 0 & 0 & \mu t^2/12 & t^2/12 & \\ 0 & 0 & 0 & 0 & 0 & (1-\mu)t^2/24 \end{bmatrix}_{6x6}$$

[C] = Transformation matrix between local and global co-ordinates

$$[C] = \begin{bmatrix} dc & 0 \\ 0 & dc \end{bmatrix}_{8x8}$$

$$[dc] = \begin{bmatrix} \text{Cos}\alpha & 0 & -\text{Sin}\alpha & 0 \\ 0 & 1 & 0 & 0 \\ \text{Sin}\alpha & 0 & \text{Cos}\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4x4}$$

Similarly, the element mass matrix in local co-ordinates and global co-ordinates are

$$[m_e] = l\pi \int [N]^T \int [N] \; tr \; d\zeta \quad \text{---------} \quad (9)$$

$$[m^o] = [C]^T \; [m_e] \; [C] \quad \text{---------} \quad (10)$$

COMPUTATIONAL TECHNIQUE

With the availability of high speed digital personal computers, it is possible to carry out the free vibration analysis of these structures more accurately. An interactive software for PC in Fortran 77 has been developed. Since, only displacement w is required to define the meridional mode shapes, u, v & θ are removed from all nodes except the last one using Iron's technique (1974). The condensed matrix so obtained is used for determination of eigen values and eigen vectors using power method.

The above methodology has been used in the vibration analysis of tall free standing chimneys by Agarwal et.al (1992).

EXAMPLE 1

A single flue chimney 150m high of Fig.2 was analysed using the truncated conical shell element as well as beam element. The computed frequencies by the two approaches are compared in Table 1. The mode shapes are given in Fig. 3 and 4 for shell alone as well as completed case of chimney. The frequency values corresponding to first, second and third mode using the beam element are lower by 9%, 6% and 1.4% obtained by the reported conical shell element, respectively. The mode shape for the first and second modes are practically the same by the two approaches and the variation in the third mode shape is shown in Fig. 3. A comparison of fundamental frequency using the truncated conical shell element against the reported value by Prabhakar (1985) and is well within 3% in both the cases.

TABLE. 1 FREQUENCY COMPARISON

	Frequency (Hz)	
Mode Number	Conical Shell Element	Beam Element
1	0.6378	0.5787
2	2.4090	2.2564
3	5.5040	5.4240

EXAMPLE 2

A 200m tall reinforced concrete chimney of the Hirono Power plant (Fig.5) earlier solved by Tamura et. al (1986) has been analysed. Frequencies and mode shapes are given in Fig. 5. The lowest natural frequency and the damping ratio of the chimney as per field measurement are reported by Sanda et. al.(1982) as 0.472 Hz and 0.005 respectively. The reported approach predicts the lowest frequency as 0.517 Hz. This variation of 10% can be due to damping which has not been considered in the reported formulation.

EXAMPLE 3

A 99.422m tall cooling tower shown in Fig. 6 has been analysed using 13 truncated conical shell elements. The predicted values for circumferential wave number n=1to7 and meridional modes m=1to3 are compared against the reported values of Bhimaraddi et. al (1990) in Table. 2. Bhimaraddi used quadrilateral shell element based on higher order shear deformation theory with 64 d.o.f and a mesh size of 16x11 (16 elements in meridional direction and 11 in circumferential direction) on a quarter circle of the tower. The predicted results are within 8% for the lowest frequency. The mode shapes are shown in Fig. 7.

EXAMPLE 4

Free vibration analysis of ISAR II Cooling tower (Fig. 8) is carried out using 30 truncated conical shell elements and the lowest frequency so obtained is 0.68 Hz against the reported value of Form (1984) of 0.67 Hz. Thus the frequency matches very well.

CONCLUSION

The reported technique can be used with confidence and reasonable accuracy for the analysis of both chimneys and cooling towers at very little computational effort on PC's. Addition of a ring stiffner element to the formulation will help to analyse cooling towers for better dynamic response.

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TABLE. 2

COMPARISON OF NATURAL FREQUENCIES OF 99.24m HIGH NDCT

n	m	Quadrilateral shell element [16x11 element in a quarter tower]	Truncated conical shell element using 13 elements
1	1	2.55677	2.7581
1	2	6.27744	7.0404
1	3	10.33951	12.6778
2	1	1.58803	1.6745
2	2	2.82911	3.2164
2	3	6.02512	7.0945
3	1	1.37364	1.556
3	2	1.63588	1.7156
3	3	3.39453	4.0012
4	1	1.14386	1.2491
4	2	1.72112	1.9103
4	3	2.28921	2.5879
5	1	1.46236	1.709
5	2	1.54180	1.5587
5	3	2.41561	2.708
6	1	1.75613	1.9161
6	2	2.06611	2.1522
6	3	2.23791	2.5883
7	1	2.05964	2.3050
7	2	2.57100	2.7365
7	3	2.74711	2.8936

n - circumferential wave number
m- meridional mode

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NOTATIONS

B	-	Strain displacement matrix
C	-	Transformation matrix between local and global co-ordinates
D	-	Matrix of elastic constants
E	-	Young's modulus of elasticity
k_e, k^o	-	Element stiffness matrix in local and global co-ordi nates respectively
l	-	Length of truncated conical shell element
m_e, m^o	-	Element mass matrix in local and global co-ordinates respectively
m	-	Meriodional mode number
N	-	Matrix of shape functions
n	-	Circumferential wave number
r	-	Radius of the element
t	-	Thickness of the element
R_i, R_j	-	Radius of element at ith and jth node
u, v, w	-	Displacements in meriodional, circumferential and transverse direction
U_e	-	Vector of nodal displacements for an element
$u_{,x}$	-	Partial derivative of u w.r.t x
$u_{,\phi}$	-	Partial derivative of u w.r.t ϕ
$v_{,x}$	-	Partial derivative of v w.r.t x
$v_{,\phi}$	-	Partial derivative of v w.r.t ϕ
$w_{,x}$	-	Partial derivative of w w.r.t x
$w_{,\phi}$	-	Partial derivative of w w.r.t ϕ
$w_{,xx}$	-	Partial derivative of $w_{,x}$ w.r.t x
$w_{,x\phi}$	-	Partial derivative of $w_{,x}$ w.r.t ϕ
ς	-	Non dimensional element co-ordinate
μ	-	Poissons ratio
ϵ	-	Vector of strains
ρ	-	Density of material
ϕ	-	Angle in circumferential direction
α	-	Angle of meridian with vertical

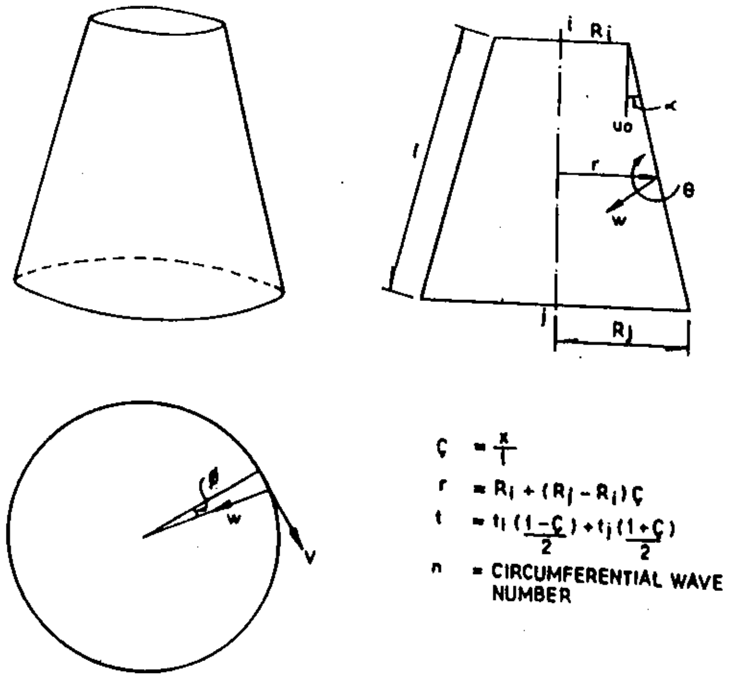


FIG.1 (a) CIRCULAR SHELL ELEMENT

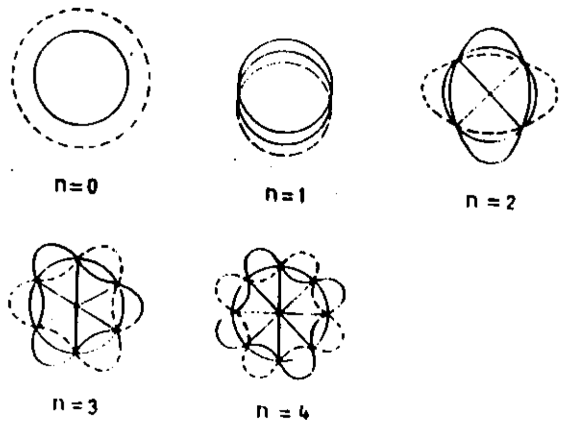
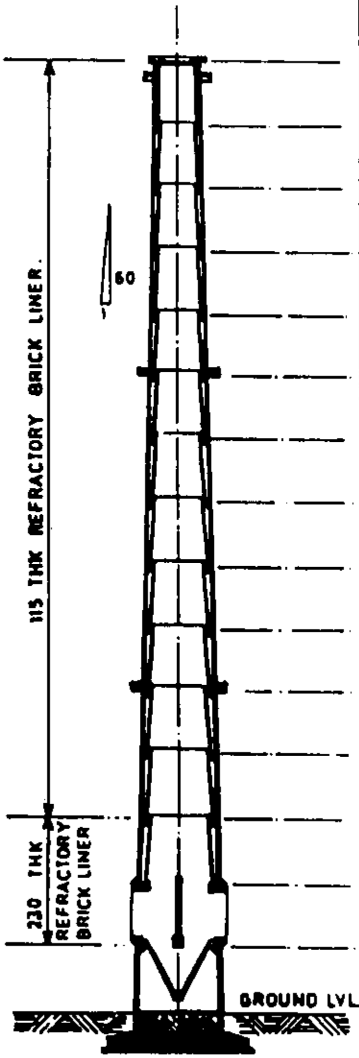


FIG.1(b) CIRCUMFERETIAL WAVE PATTERNS



SECTION

LEVELS IN METRES.	SHELL OVERALL DIA IN. M.	SHELL THICKNESS IN M.M.	LINER LOAD ON SHELL IN KN
150.0	7.750	165	30 (c.i.cap
140.0	8.083	165	670
130.0	8.417	200	560
120.0	8.750	200	585
110.0	9.083	250	610
100.0	9.417	250	770
90.0	9.750	300	650
80.0	10.083	300	670
70.0	10.417	350	690
60.0	10.750	350	715
50.0	11.083	450	895
40.0	11.417	500	745
30.0	11.750	600	760
20.0	12.083	650	1550
10.0	12.417	650	4420
0.0	12.750	650	0

FIG.2 150M HIGH SINGLE-FLUE CHIMNEY

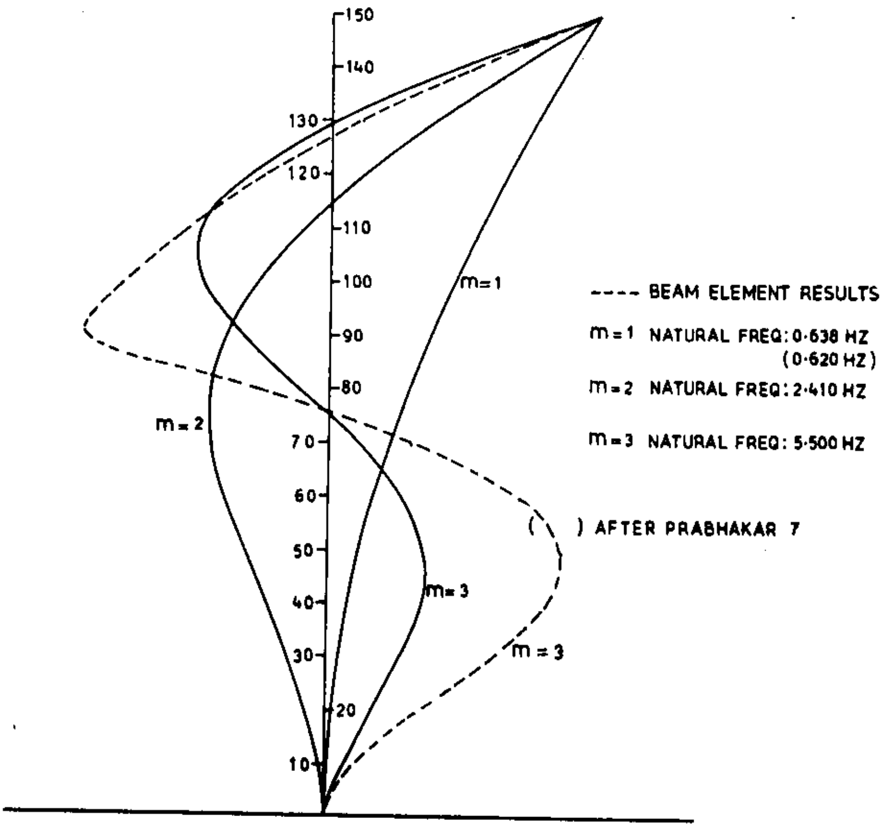


FIG. 3 FREQUENCY AND MODE SHAPE OF A 150 M HIGH CHIMNEY (SHELL ALONE CASE)

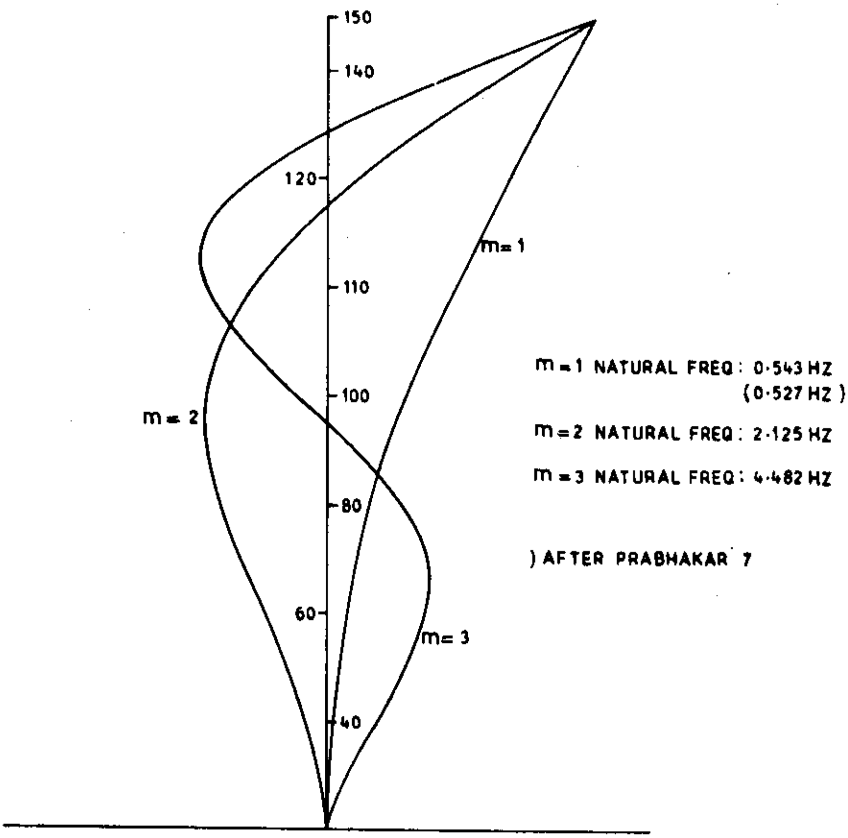


FIG. 4 FREQUENCY AND MODE SHAPE OF A 150 M HIGH CHIMNEY (COMPLETED CHIMNEY)

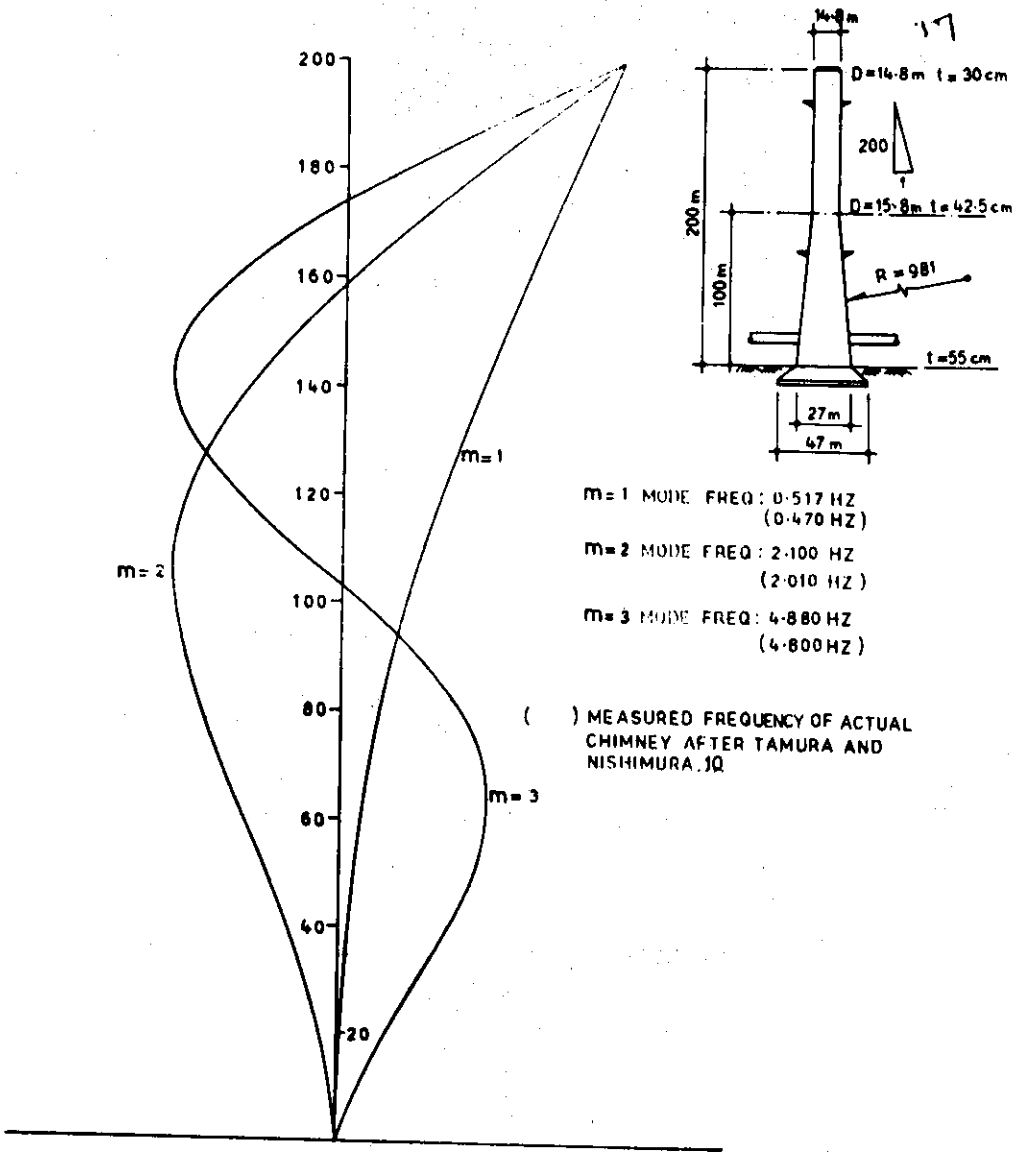
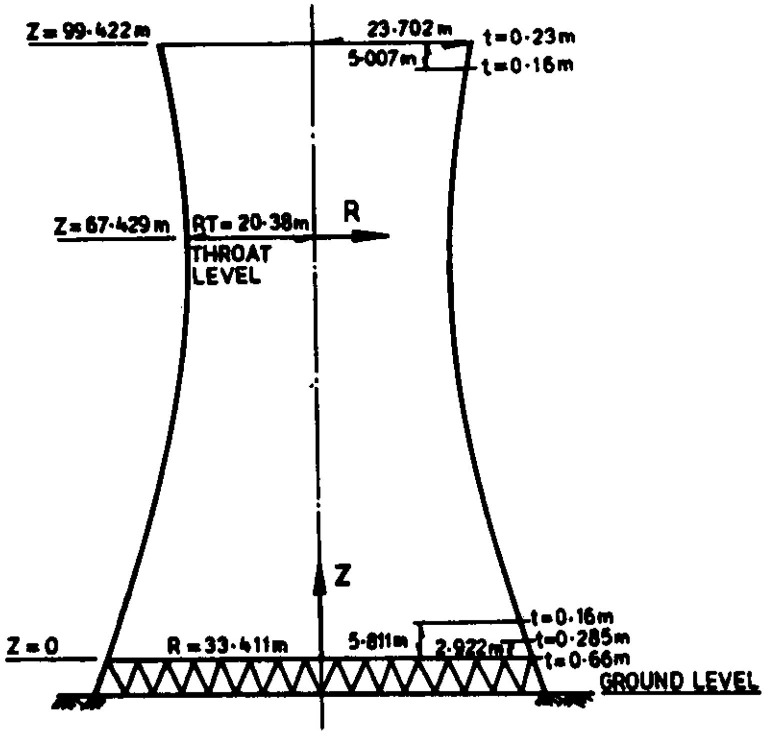


FIG.5 VIBRATION MODES OF 200M HIGH CHIMNEY



RADIUS BELOW THE THROAT ($0 \leq Z \leq 67 \cdot 429$)

$$R(Z) = 20 \cdot 38 \sqrt{1 + \left(\frac{67 \cdot 429 - Z}{51 \cdot 905}\right)^2}$$

RADIUS ABOVE THE THROAT ($67 \cdot 429 \leq Z \leq 99 \cdot 422$)

$$R(Z) = 20 \cdot 38 \sqrt{1 + \left(\frac{Z - 67 \cdot 429}{53 \cdot 88}\right)^2}$$

FIG. 6 COOLING TOWER OF EXAMPLE-3

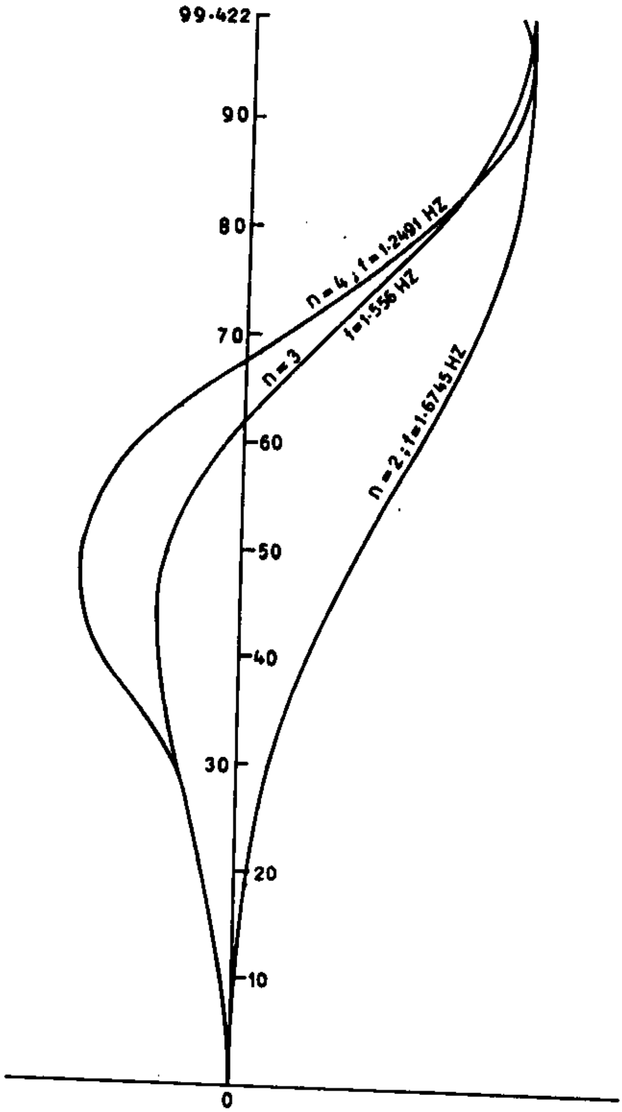
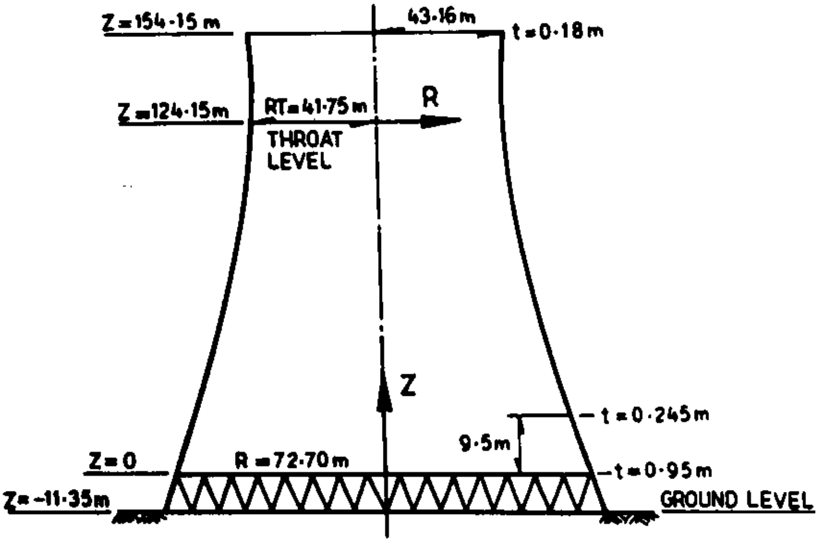


FIG. 7 VIBRATION MODES OF EXAMPLE-3 COOLING TOWER (m = 1)



RADIUS BELOW THE THROAT ($0 \leqslant Z \leqslant 124{\cdot}15$)

$$R(Z) = 28{\cdot}061 + \frac{13{\cdot}689}{40{\cdot}00}\sqrt{(Z-124{\cdot}15)^2 + (40{\cdot}0)^2}$$

RADIUS ABOVE THE THROAT ($124{\cdot}15 \leqslant Z \leqslant 154{\cdot}15$)

$$R(Z) = 41{\cdot}343 + \frac{0{\cdot}407}{6{\cdot}90}\sqrt{(Z-124{\cdot}15)^2 + (6{\cdot}90)^2}$$

FIG. 8 COOLING TOWER OF EXAMPLE-4