

## A NOTE ON THE MAGNITUDE DETERMINATION OF EARTHQUAKES

Umesh Chandra\*

### SYNOPSIS

The various methods for the determination of earthquake magnitudes put forward during the last few decades, have been briefly reviewed and followed by a critical examination of the procedure involved. The task of extending the magnitude formula, from a knowledge of the absorption coefficient  $k$  for different periods and paths, for surface waves using periods deviating considerably from 20 sec., has been discussed. Further, in view of the revised work of Gutenberg and Richter (1956 a, equation 18) the magnitude formula for body waves needs a revision. An accurate magnitude determination for deep focus earthquakes also requires a more systematic investigation, than is yet available. An extension of the magnitude scale to other waves, especially the various waves encountering the core would be desirable. However, some problems, not yet solved theoretically, arise. It is suggested that a magnitude scale based on the measurements of a few (but not all) crests and troughs on either side of the maxima, rather than the maxima alone, would yield more accurate results. To some extent it obviates the difficulties due to the interference of waves and also takes account of the particular amplitude and period spectrum of a particular earthquake. However, the proposed idea requires a critical examination before being put to some practical use. The necessity of developing magnitude formulae and associated tables to Indian regions, similar to Southern California, is emphasized.

### INTRODUCTION

For many purposes in theoretical and practical seismology, it is desirable to have a scale for rating the various earthquakes in terms of the energy released in them which would be independent of the local effects produced at any particular point of observation. Following the suggestion by Wood, Richter (1935) devised such a scale called the, 'magnitude scale'. The magnitude thus assigned is characteristic of the earthquake as a whole and as such it differs from the intensity which varies from point to point of the affected area. The investigations of Gutenberg and Richter have simplified the practical determination of magnitude to such a straight forward procedure that it can be very easily applied in routine bulletin work for a given station. To day, many seismological stations, all over the world, regularly report magnitudes.

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\*Lecturer in Seismology, School of Research and Training in Earthquake Engineering, University of Roorkee (U.P.), India.

## MAGNITUDE FROM LOCAL SHOCKS

If we consider two shocks of different magnitudes originating from the same focus, all other circumstances being identical in both cases, a seismograph at a particular station should be expected to write two records one of which should be very closely an enlarged copy of the other. The ratio of this enlargement should be the same, independent of the recording seismograph, provided that we assume the instrumental constants to remain unaltered during the two events, and that the response of the registering apparatus is linear. Clearly this ratio may be taken to be a measure of the relative magnitudes of the two shocks. Richter (1935) studied a large number of earthquakes and for each case, he plotted the logarithm of the trace amplitude against epicentral distance. It was found that all the points for any shock tend to lie on a certain smooth curve, and these curves (corresponding to different shocks) were seen to be roughly parallel; in accordance with the proportional amplitude hypothesis. Following the general pattern of the curves, a curve parallel to these and passing through an arbitrarily selected point (epicentral distance=100 km., and maximum trace amplitude=1 micron) was taken to be a standard shock of Zero magnitude, and a table giving the logarithm (to the base 10) of the computed maximum trace amplitude in millimeter, with which the standard short-period torsion seismometer ( $T_0=0.8$  sec.,  $V_s=2800$ ,  $h=0.8$ ) should register the shock of zero magnitude at various distances, was prepared. Accordingly, Richter (1935) gave the following definition of the magnitude scale, for shocks of Southern California.

"The magnitude of any shock is taken as the logarithm of the maximum trace amplitude expressed in microns, with which the standard short-period torsion seismometer ( $T_0=0.8$  sec.,  $V_s=2800$ ,  $h=0.8$ ) would register that shock at an epicentral distance of 100 kilometers."

The definition may be expressed as follows :

$$M_1 - M_2 = \log B_1 - \log B_2 \quad (1)$$

Where,  $M_1$  and  $M_2$  are the magnitudes of two earthquakes and,  $B_1$  and  $B_2$  are their corresponding maximum trace amplitudes recorded on a standard torsion seismograph at an epicentral distance,  $\Delta$ :

This definition applies strictly only for  $\Delta = 100$  km. The zero of the scale is fixed by setting  $M=3$  when  $b=1$  mm. at the standard distance of 100km. Further, the definition is applicable only for local shocks of Southern California with a depth of focus of about 16 km., it being assumed that no special structure or material is involved either at the focus or at the recording station or in the wave path.

Reductions, of the maximum trace amplitudes at different distances, to the standard distance of 100 km. involves an empirically determined table for the logarithm of the maximum trace amplitude, for a shock of magnitude zero, as a function of epicentral distance (Table 1, Richter 1935, or table 1, Gutenberg and Richter 1956a, revised and extended to cover the range 0 to 25 km., also). Nordquist (1942) designed a nomogram by which these tables are more conveniently represented and magnitude, from the trace amplitude in mm. of a standard

torsion seismometer, is determined directly. Station corrections (see table 3, Gutenberg and Richter 1956 a) are applied to magnitudes so determined.

The initial success of Richter (1935) is attributable to a set of fortunate circumstances. The method was developed for magnitude determinations in Southern California region, where the focal depths are fairly constant. Uncertainties arising from this factor are thus largely eliminated. The Wood-Anderson Torsion Seismometer is quite stable and writes a very legible record for small or moderately large shallow shocks. The prevailing periods corresponding to maximum waves for shocks in this magnitude range are such that the magnification of the torsion instrument is nearly constant. However, for large shocks difficulties arise and use is made of other methods to be described later.

The magnitude scale just described was applied by Richter (1935) to earthquakes recorded in California region within an epicentral distance of 600 km. For magnitude determination of distant shocks (i.e. for teleseisms), the method was extended by Gutenberg and Richter (1936), Gutenberg (1945 a) and M. Bath (1952), who set up further empirical tables whereby observations made at distant stations and on seismographs other than the standard torsion seismograph ( $T_0=0.8$  Sec.,  $V_s=2800$ ,  $h=0.8$ ) could be reduced to correspond with the standard conditions in Richter's definition. Gutenberg (1945 b, c) produced further empirical tables to cover earthquakes of significant focal depth from amplitude and period measurements of body waves (P, PP, S).

#### MAGNITUDE FROM SURFACE WAVES (DISTANT EARTHQUAKES)

For magnitude determinations on the same scale from distant earthquake records Gutenberg and Richter (1936) also made use of the trace amplitude of surface waves, of course restricted to shallow shocks only. In this case magnitude is expressed as

$$M = \log A - \log B + C + D \quad (2)$$

where,

**A**=The total horizontal component of ground movement in microns caused by surface waves having periods of about 20 seconds,

**B**=A quantity similar to **A** corresponding to a shock of zero magnitude. **B** depends only on the distance of the station from the epicentre, for a given focal depth;  $\log B$  being always negative, and

**C**=a constant for each station, correcting for the effects of the special conditions of the ground near the station and of the instrumental equipment (Table 1, Gutenberg 1945 a).

**D**=A factor depending upon the depth of focus, the original distribution of energy in various azimuths, the absorption of waves, and on the effect of irregularities lying in the path of the wave.

According to Gutenberg and Richter (1936), maximum trace amplitude **b** (measured

in mm.), corresponding to epicentral distances greater than  $20^\circ$ , as recorded by a standard Wood-Anderson torsion seismograph is given as

$$\log b = \log B - 2.5 \quad (3)$$

The formula does not hold for epicentral distances less than  $20^\circ$  and no attempt has been made to find the values of  $B$  for these distances. The zero of scale is fixed by taking  $\log B = -5.04$  (Gutenberg 1945 a, previously it was taken as  $\log B = -5.0$  for  $\Delta = 90^\circ$ , see Gutenberg and Richter 1936). For  $\Delta > 20^\circ$ , corresponding to a shock of magnitude zero, the maximum horizontal ground amplitude  $B$  of surface waves having periods of about 20 sec. may be obtained,

(i) From a table of observed values giving  $-\log B$  as a function of  $\Delta$ ,

(ii) From the formula

$$-\log B = 5.04 + \frac{1}{2} [48.25 k (\Delta - 90) + \log \sin \Delta + \frac{1}{3} (\log \Delta - 1.954)] \quad (4)$$

Where  $k$  is the absorption factor per kilometer for surface waves with periods of about 20 sec.

or (iii) more easily from the empirically determined formula

$$-\log B = 1.818 + 1.656 \log \Delta \quad (5)$$

(for  $\Delta$  between  $15^\circ$  and  $130^\circ$ )

The revised values, which were finally adopted, of  $-\log B$  as a function of  $\Delta$  are given in table 4 (Gutenberg 1945 a) and hold for an average depth of about 20-25 km.

Seeking to investigate the effect of attenuation of surface waves due to absorption and the variation of velocity along the wave path, Gutenberg (1945 a) has concluded that for shocks arriving in Southern California along the critical azimuths, e.g., from Southern Japan and from Ecuador-Peru, a term at least as big as 0.5 should be added to the calculated magnitudes. If the surface waves have crossed the Pacific Basin without being tangent to its boundary, 0.1 or 0.2 should suffice, whereas for paths completely outside the Pacific Basin 0.1 or 0.2 should be subtracted. Special researches on regional characteristics are needed to determine the corresponding corrections for waves reaching other stations.

Markus Bath (1952) has developed the following magnitude formula from the vertical component of surface waves with periods of about 20 seconds from the records obtained at Pasadena,

$$M = \log A_z - \log B + \delta(h) + m_r + C(M_0 - M_{cat}) \quad (6)$$

where,

$A_z$  = the vertical component of the maximum ground movement in microns for surface waves of about 20-sec. period. This term takes account of the amplitude.

$B$  = A quantity similar to  $A_z$  corresponding to a shock of magnitude zero, and is here assumed to be the same as for the horizontal component (the constant term

in the expression of  $B$  is different, but that is taken into account by  $\bar{\delta}(h)$ . This term accounts for the amplitude variation with epicentral distance.

$\bar{\delta}(h) = 0.0082 h$  (determined empirically) is a function of focal depth down to 100 km.

$m_r$  = regional correction, depending upon the properties of the path of surface waves.

$C(M_0 - M_{calc})$  = a correction term, which corrects for the variation of the ratio ( $A_z/A_h$ ) of the vertical to the horizontal amplitudes of surface waves with magnitude ( $M$ ).

It has been shown by Bath (1952) that because of certain factors such as the earthquake mechanism, distribution of energy in different azimuths etc., it is not possible to obtain the magnitude from the records at one station to an accuracy higher than  $\pm 1/4$ .

The use of vertical component of ground amplitude in surface waves is advantageous, as it is necessary to measure only one record, whereas the total horizontal component requires two records (N-S and E-W) corresponding to two mutually perpendicular directions in the horizontal plane. From the theoretical view point also the vertical component has an advantage, as it represents the vertical component of Rayleigh waves alone, whereas the horizontal component generally is a combination of both Rayleigh and Love waves.

On the other hand disadvantages in the use of vertical components are the following: (1) that the number of stations supplying data for this component is smaller than those supplying the horizontal component records, and (2) that often the accuracy of the constant seems to be less than that for the horizontal instruments.

The above magnitude formulae are applicable to shallow earthquakes only. In order to correct for the depth  $h$  a term approximately equal to  $0.01 h - 0.2$  is added (Gutenberg and Richter, 1956 a).

### MAGNITUDE FROM BODY WAVES

Gutenberg (1945 b, c) further developed methods for the determination of magnitudes of distant deep focus shocks from the amplitude and period measurements of body waves (P, PP, S).

From the original theory of Zoeppritz, Geiger and Gutenberg (1912), the expression for the ground displacement, in terms of epicentral distance  $\Delta$ , during a single body wave is given as follows: (see Appendix: Gaur and Chandra 1964).

$$u \text{ (or } w) = K T N \sqrt{E} \quad (7)$$

$$\text{where } N = Z \sqrt{(F_1 F_2 \dots F_n) a \frac{\cos i \, di/d\Delta}{\sin e \sin \Delta}} \quad (8)$$

$K$  = a constant, its value is dependent on the fraction of energy  $E$  passing into the wave under consideration, and is different for P, SH, and SV waves.

$T$  = Period of the observed wave.

$U$  and  $W$  = The horizontal and vertical components of  $N$ .

$Z$  = ratio of the ground displacement to the incident amplitude, having different values for the horizontal and vertical components ( $u$  and  $w$ , respectively) of the ground displacement, being a function of  $e$  and Poisson's ratio just below the surface.

$F$  = ratio of transmitted or reflected energy to incident energy at each point where the wave has encountered a discontinuity in density or velocity or both. Its value at a given discontinuity depends upon the angle of incidence thereon, the densities and wave velocities on both sides of the discontinuity and the type of wave-longitudinal ( $P$ ) or transverse ( $SV$  in the plane of propagation, or  $SH$  with vibrations perpendicular to the plane of propagation only).

$a = e^{-kd}$  gives the effect of absorption,  $k$  is the absorption factor,  $d$  the distance.

$i$  = angle which the ray leaving the focus makes with level surface through it.

$e$  = angle between the emergent ray and the surface.

$\Delta$  = the angular distance from its source to its point of emergence. From equation (7) we get

$$\frac{1}{2} \log E = \log u - \log K - \log T - \log U \quad (9)$$

and similarly for the vertical component.

Again from Gutenberg and Richter (1942, equation 35, pp 180),

$$\log E = 11.3 + 1.8 M \quad (10)$$

Assuming that the duration  $t$  of a given phase increases with the distance  $\Delta$  proportionally with  $T$ , we get

$$E = q t E_0 / T = q t_0 E_0 / T_0 \quad (11)$$

The subscript  $0$  refers to the source and  $q$ , assumed to be constant, is the fraction of energy imparted to the phase under consideration. Assuming, further, that  $t_0/T_0$  does not depend appreciably on magnitude, we find from a combination of equations (9), (10) and (11) that for a given earthquake,

$$L = 0.9 M - \log u + \log T + \log U \quad (12)$$

should have a nearly constant value for all waves starting as  $P$ , another for all starting as  $SV$ , and a third for those starting as  $SH$ . It will be sufficient here to assume that there is one constant  $C$  in all shocks for  $P$  waves and another for all  $S$  waves (combined  $SV$  and  $SH$ ) given by

$$C = M - \log u + \log U - 0.1 (M - 7) + \log T \quad (13)$$

$$\therefore A = C - \log U = M - \log u - 0.1 (M - 7) + \log T \quad (14)$$

The value of  $C$  was found by Gutenberg to be 6.3.

$$M = A + 0.1 (M - 7) - \log T + \log u \quad (\text{or } \log w) \quad (15)$$

Gutenberg gave the value of  $A$  (1945 b, for shallow shocks table 4; and 1945 c, for deep focus shocks see figures 2, 3 and 4, the same has been revised by Gutenberg and Richter 1956 b, table 2, figures 3, 4 and 5) as a function of epicentral distance  $\Delta$  and depth of focus  $h$ . The value of  $A$  together with their ground amplitudes in microns (total horizontal  $u$ , vertical  $w$ , considering the station corrections given in table 2, 1945 b) and their periods  $T$  give the magnitude  $M$  from equation (15). For all longitudinal waves in great shocks or shocks of magnitude less than  $6\frac{1}{2}$ , a tentative additional correction  $+0.1(M-7)$  is applied.

Thus we see that so for three imperfectly consistent magnitude scales have been developed for current use. These are,

1.  $M_L$ , determined from the maximum trace amplitudes of local earthquake records, according to the original definition of Richter (1935).

2.  $M_S$ , determined from the maximum ground amplitudes (Horizontal components vectorially combined, Gutenberg 1945 a, and vertical component, Bath 1952) corresponding to 20 second period for shallow distant shocks.

3.  $M_B$ , calculated from the maximum of the ratio of amplitude and period ( $A/T$ ) for body waves (P, PP and S) for distant earthquakes (shallow shocks: Gutenberg 1945 b, shocks of any depth Gutenberg 1945 c).

Finally, Gutenberg and Richter (1956 b) introduced a 'unified magnitude'  $m$ , whose formal definition may be phrased as follows :

$$m - 7.0 = q \quad (16)$$

at a distance of  $90^\circ$  for normal shallow focal depth, where  $q = \log w/T$  refers to PZ, and the station ground conditions. It has the following relations to  $M_L$ ,  $M_S$  and  $M_B$ ,

$$m = 1.7 + 0.8 M_L - 0.01 M_L^2 \quad (17)$$

$$m = M_S - 0.37 (M_S - 6.76) \quad (18)$$

$$m = M_B \quad (\text{without correction}) \quad (19)$$

The unified magnitude is placed on a self consistent and independent basis as satisfactory for teleseisms as that of  $M_L$  for local earthquakes, and with the great advantage of being applicable directly to seismograms recorded on instruments of all types and at all stations. The relation (18) is based on a large body of data, but since the relation of  $M_L$  to  $m$  is not yet on a definitive basis, Gutenberg and Richter (1956 b) suggest that the 'Richter Scale' as defined in 1935 be retained for determining magnitudes of local shocks. They have preferred and strongly recommended the use of the unified magnitude scale  $m$ , for teleseisms. It appears possible, in very near future, to express the entire range of observed magnitudes in terms of the unified magnitude  $m$ .

### ENERGY MAGNITUDE RELATIONS

The following magnitude energy relations have been given by Gutenberg and Richter (1956 a).

$$\text{Log } E = 9.4 + 2.14 M - 0.054 M^2 \quad (20)$$

For  $M$  ranging from 1 to 8.7, this is numerically equivalent to

$$\log E = 9.1 + 7 M/4 + \log (9-M). \quad (21)$$

The most reliable connection between the total energy  $E$  (in ergs) of seismic waves and the 'unified magnitude  $m$ ', is given as (Gutenberg and Richter 1956 b),

$$\log E = 5.8 + 2.4 m \quad (22)$$

Hence, substituting (22) in equations (17), (18) and (19), the total energy released in the form of elastic waves following a certain earthquake may be given in terms of magnitudes determined from local shocks ( $M_L$ ), body waves for distant shocks of any depth ( $M_B$ ) and surface waves ( $M_S$ ) for distant shallow shocks, as follows :

$$\log E = 9.9 + 1.92 M_L - 0.024 M_L^2 \quad (23)$$

$$\log E = 5.8 + 2.4 M_B \quad (24)$$

$$\log E = 11.8 + 1.5 M_S \quad (25)$$

By employing completely different methods and different material, M. Bath (1958) obtained the following energy formula.

$$\log E = 12.24 + 1.44 M_S \quad (26)$$

which is in very good agreement with the results of Gutenberg and Richter (compare with equation 25 above).

However, owing to the effect of absorption, which in the case of body waves may account for a factor of approximately 20 in total energy, any determination of energy obtained from P and S waves alone can not be expected to yield an absolute value. Hence an exact determination of absorption and its consideration is important in energy computations.

It appears that energy computations made by measuring surface waves, for which the absorption can fairly easily be determined, may give more reliable results.

### REAPPRAISAL OF THE METHODS OF MAGNITUDE DETERMINATION

Richter's magnitude scale (1935) involves a reduction of observed amplitudes at different distances to the expected amplitude at the standard distance of 100 km. and is accomplished by means of a tabulation of amplitude as a function of distance for a standard shock (magnitude zero). The method is based on the assumption that the ratio of amplitude at two given distances is the same for all shocks and in all azimuths. This does not strictly hold good, since the amplitude ratio, at two given distances, depends on various factors, which are in general not the same for different earthquakes. These are, (1) depth of focus, predominant period and fraction of energy passing into the wave under consideration (in general, for near shocks SH has the maximum trace amplitude), (2) variation of absorption factor with period and path (for body waves, the absorption factor is more or less constant, whereas for surface waves it varies considerably with period), (3) unequal distribution of energy in different azimuths due to anisotropy and the particular mechanism of energy release, and, (4) linear extent of faulting, its rate of progression.



Bath (1955 b) has presented a method for the computation of  $A_{20}$  corresponding to 20 sec. period from the amplitude  $A$  of surface waves with periods  $T$  different from 20 sec. He states that, "the underlying idea should be to define  $A_{20}$  such that the corresponding energy is the same for  $A$ , rather than to put their velocities equal", and that "we have to take account not only of the period difference but also of the difference in the absorption coefficient ( $k$ ) for different periods". Equating two energies in a given time interval, he obtained the following expression.

$$\frac{A_{20}^2}{20} e^{-k(20)\Delta} = \frac{A^2}{T} e^{-k(T)\Delta} \quad (27)$$

$$\text{or } \log A_{20} = \log A + 1/2 \log 20/T + 24.13 \Delta [k(T) - k(20)] \quad (28)$$

Instead of equating energies in a given time interval, one could, as an alternative, equate energies per wave length. This would mean to equate  $A^2 e^{k\Delta}$  instead of  $A^2/Te^{k\Delta}$ . The numerical difference between these two methods would not be of much significance for magnitude determinations, if  $10 \leq T \leq 40$  sec., which in fact covers all periods of importance in this case.

One could naturally argue which of these two methods of equating energies is the correct one. However, the essential thing is to equate energies and doing it one way or the other is mainly a matter of defining the corresponding  $A_{20}$ .

The investigations of Gutenberg and Richter (1936) and Gutenberg (1945 a), for the determination of magnitudes using maximum ground amplitudes during surface waves, make use of the formula;

$$\frac{A_2}{A_1} = \frac{T_2}{T_1} e^{-k(\Delta_2 - \Delta_1)/2} \sqrt{\frac{\sin \Delta_1}{\sin \Delta_2}} \sqrt[6]{\frac{\Delta_1}{\Delta_2}} \quad (29)$$

Originally developed by Jeffreys (1925).  $A_1$  and  $A_2$  are the amplitudes at distances  $\Delta_1$  and  $\Delta_2$  respectively;  $T_1$ ,  $T_2$  are the corresponding periods,  $k$  is the absorption coefficient. In developing the formula (29), the absorption coefficient  $k$  has been assumed constant along the whole path and also for the period range under consideration.

A little consideration of equation (29) and (4) shows that in the expression for  $-\log B$  on the right hand side in equation (4), the term  $\log(T_{90}/T_{\Delta})$  has been omitted.  $T_{90}$  was taken to be 20 sec., and  $\log(T_{90}/T_{\Delta})$  can vanish only if  $T_{\Delta} = 20$  sec. It is perhaps not necessary to give complete details here, as they are contained in the original papers by Gutenberg and Richter (1936) and Gutenberg (1945 a).

It is evident that the magnitude formulae and the associated tables (in Gutenberg 1945 a, Gutenberg & Richter 1936) hold only for surface waves with maximum ground amplitude corresponding to periods of about 20 sec. Hence there may be two ways of

extending the magnitude formula for surface waves using periods deviating considerably from 20 sec.

1. From the energy equality postulated between surface waves of period  $T$  and surface waves of period 20 sec., we can calculate what  $A_{20}$  should be, given  $A_T$ . Knowing  $A_{20}$  (e.g. from eq. 28), we can proceed as usual, that is with the tables and graphs which Gutenberg has given. This method would only require a knowledge of  $k(T)$ .

2. We may use  $A_T$  directly, that is not to pass over  $A_{20}$ . This would require new tables and graphs for each period to be considered.

Obviously, the first method would be simpler in practice, although the outcome of the two should be equivalent. The main trouble is that we still have too meagre information on  $k(T)$ . It may appear to be an interesting piece of investigation to reverse the procedure mentioned above, that is, combine known  $M$  with measured  $A_T$  and calculate  $k(T)$  for a given range of periods.

The magnitude determination from body waves (Gutenberg 1945 b, c) is based on the equation (8), which has been derived by means of several simplified assumptions such as a spherically symmetrical earth, zero depth of focus, the validity of the ray theory for the calculation of energy flux, equal distribution of energy in all directions from the source, conditions of perfect elasticity, deviations from which are quite significant in many cases. The value of  $L$  in equation (12) and hence of  $C$  in equation (13) depends on the fraction of energy distributed among the fundamental types of waves and thus is likely to differ from one shock to another. The value  $C = 6.3$ , found by Gutenberg (1945b), represents some sort of average condition. In arriving at the final expression (equation 15) for magnitude, equation (10) has been used for magnitude energy relationship. In view of the revised work of Gutenberg and Richter (1956 a, equation 18), this needs a revision. The simplified form of equation (11) may be responsible for some differences in the values of residuals.

In magnitude determination for deep focus earthquakes (Gutenberg 1945 c), the magnitude is defined in such a way that the energy released in two shocks of the same magnitude become equal, regardless of the focal depth. Contrary to the case of shallow shocks, it is not possible to find the residuals of the magnitude  $M$  calculated from the amplitudes of body waves, when compared with the magnitude found from that of the surface waves. The assumption has been made that the average magnitude found from the theory, assuming  $C=6.3$  and using data from various distances is substantially correct. From the average value of  $M$  thus obtained the residuals were plotted as a function of epicentral distance  $\Delta$  and focal depth  $h$ ; and corrections to the calculated values of  $U$  and  $W$  as a function of  $\Delta$  have been obtained. At present, it is not possible to evaluate the errors which affect  $M$  as the focal depth  $h$  increases. A more systematic investigation, is thus needed for an accurate evaluation of the magnitude of deep focus earthquakes.

The magnitude determinations from body waves has so far been confined to P, PP and S only. The extension of the method to other waves, especially the various core waves, would be desirable. The results of a very systematic study by Kazim Ergin (1953) for the ratios of (displacement/period) of  $P_c P$ ,  $P_c S$ ,  $S_c S$ , and  $S_c P$  to that of the corresponding incident wave e.g.,

$\left(\frac{\text{displacement}}{\text{period}}\right)_{P_c P} / \left(\frac{\text{displacement}}{\text{period}}\right)_P$ , using intermediate and deep focus earthquake seismograms, indicate that the observed ratios of the horizontal component of the waves that are reflected as P waves (i.e.,  $P_c P/P$  and  $S_c P/S$ ) and that of the vertical component of the waves that are reflected as the S waves (i.e.,  $S_c S/S$  and  $P_c S/P$ ) at the mantle-core boundary are considerably larger than the theoretical ones, whereas the observed ratios of the vertical component of the first group and that of the horizontal component of the second group are in fairly good agreement with the theoretical values. Further, he finds that the behaviour of the direct P and S waves is in accord with theory, but the vibration of the ground is not in the direction of propagation for  $P_c P$  and  $S_c P$ , and is not perpendicular to the direction of propagation for  $P_c S$  and  $S_c S$  as expected. From the foregoing, it is obvious that unless the difficulties, just mentioned, are resolved, it would be premature to attempt for any development of the magnitude scale based on these waves.

All the magnitude determinations have been made with the help of formulae which are essentially far more simplified than what actually occurs in case of the earth. The plane wave theory and point source assumption has been used throughout. In shocks of large magnitudes, which are generally characterised by a large linear extent of faulting and involve large dimensions of crustal blocks, the point source theory does not hold and line source assumption would be expected to yield better results although the resulting theory would be inevitably involved. In large shocks the change in elastic properties may be significant. Factors such as diffraction, scattering and internal friction do not seem to have been adequately taken into account owing of course to the mathematical difficulties in formulating these phenomena, as they depend on the particular geological structures and crustal irregularities involved. The effects of dispersion and observed oscillatory character of motion, appear to be quite significant, though here again a satisfactory theory is not available for the explanation of oscillatory character of motion on seismograms attending the body waves. Only the dispersion of surface waves has to some extent lent itself to mathematical formulation, but the effect does not seem to have been considered in developing the magnitude formulae. Further details are given by Gaur & Chandra (1964).

It is felt that a consideration of the complete spectrum of amplitude and periods rather than the maxima alone (maximum trace amplitude on torsion seismometer for local shocks, maximum ground amplitude corresponding to 20 sec. period for surface waves and maximum

amplitude/period ratio for body waves P, PP and S) would result in a quantity more representative of the shock as a whole. The foregoing idea suggests that the energies computed by the complete integration of seismogram would give more precise result. The method was applied by Bath (1955 a, 1958). But a further complication arises. The onset of any new phase follows interference with the preceding waves and thus far it has not been possible to evaluate its effect. From a study of seismograms it appears that, in general, just before the onset of a new phase, the preceding wave has significant amplitude, and neglect of the effect of interference may not be quite reasonable. It appears that the measurement of a few (but not all) crests and troughs on either side of the maxima would yield more useful information. Of course, the idea needs a critical examination before it may be put to some practical use. To a certain extent, it takes care of the particular energy distribution as represented by the amplitude and period spectrum, and also the effect of interference may be expected to die out or become negligible at this time i.e., the time by which the said maxima is reached.

However, the detailed statistical studies, the results of which are incorporated in terms such as station correction, regional correction, residuals of magnitude etc., eliminates in some cases, and averages out in others, the uncertainties due to many factors.

No systematic study, for the magnitude determinations from the records of Indian observatories, seems to have been undertaken so far. Some investigators report magnitude values for certain earthquakes recorded at Indian stations, but the formulae used by them are those which were originally developed by Gutenberg and Richter for the Southern California region. Further, the magnitude determinations involve station correction, corrections for the path and regional corrections. This requires a detailed statistical study in the absence of which all the available reports would appear to be misleading. Unfortunately the number of recording stations is very small in this part of the world, but from whatever is available it would be desirable to develop magnitude relations for this region as has been done for Southern California. This can be done by comparison with earthquake magnitudes which have been determined sufficiently accurately from the records of standard observatories, such as Pasadena etc.

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