Bull. Ind. Soc. Earth. Tech., Paper No. 311, Vol.28, No.4, Dec., 1991, pp. 1 to 34

12TH ISET ANNUAL LECTURE - 1991

PROBABILISTIC APPROACH TO EARTHQUAKE ENGINEERING - EMERGENCE OF A PRACTICAL THEORY

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INTRODUCTION

Earthquakes are a source of critical loading condition for structures located in the seismically active regions of the earth. A significant feature of the earthquake loading is a large measure of uncertainty associated with the earthquake phenomena. To establish the design seismic loading condition for a structure it is necessary to anticipate the number, size and location of future earthquakes in the region surrounding the site, during the service life of the structure. Further, this assessment should be coupled with the prediction of structural response, and damage, due to random vibration induced by ground motion of a given intensity. Both these steps involve uncertainty at several stages. The problem is compounded by the fact that potentially damaging strong motion earthquakes occur after long intervals of time and the available data for such events is statistically insufficient. A probabilistic treatment of earthquake engineering problems, which involves assessment of seismic risk and random vibration analysis, therefore, provides a rational and consistent basis for aseismic design. (Housner, 1947; Rosenblueth, 1951; Bolotin, 1960; Cornell, 1967; Vanmarcke, 1977; Whitman and Cornell, 1977; and Kiureghian, 1981).

Over the past four decades concurrent developments in the seismic risk analysis; the random vibration theory; stochastic modelling and simulation and the reliability based design have led to the emergence of a comprehensive probabilistic framework for practical aseismic design. In this lecture we shall discuss the present status of such an approach. The trestment is divided into three sections :

1. stochastic models of earthquake ground motion;
2. random vibration analysis for elastic and inelastic behaviour; and
3. probabilistic design for earthquakes.

In Appendix-A we discuss the probabilistic seismic risk analysis. Extensive bibliography is included.

STOCHASTIC MODELS OF EARTHQUAKE GROUND MOTIONS

Housner (1947) was first to suggest that "acceleration records of earthquake exhibit characteristics of randomness". If a large number of ground motion records were available for a particular site, the parameters of a stochastic process model could be determined directly by statistical analysis. However, this approach is not possible at the present time, in any part of the world, due to availability of only a few strong-motion records. It, therefore, becomes necessary to use considerable judgement in constructing, and validating, stochastic models of ground motion on the basis of a few available records at the site, or at comparable locations coupled with seismological and geological data, and local site conditions.

Let $Z_i(t)$, $i = 1,2,$ and 3, represent the three components of ground displacement at a point due to an earthquake. Consistent with general characteristics of strong-motion earthquakes, each component of the ground acceleration can be expressed as -

$$Z_i(t) = A_i(t)\, Y_i(t); \quad 0 \le t \le T', \qquad (1)$$
$$= 0; \quad \text{Otherwise,}$$

where $i = 1,2$ and 3; $A_i(t)$ are slowly varying random functions of time called the envelope; $Y_i(t)$ are a segment of stationary random processes; and T' is the duration of the motion. Thus each component of ground acceleration is a realisation of a nonstationary random process. The stochastic model represented by (1) may be described to varying degree of completeness by the joint distribution functions, the joint moment or spectral functions and other properties such as level crossing, peaks etc.

We shall first discuss the properties and models of individual components. We shall confine our discussion to specific aspects relevant to earthquake ground motion. Three types of basic models, and their minor variations, reflecting increasing level of complexity have been used to model earthquake ground acceleration :

(i) White Noise (Housner, 1947; Byrcroft, 1960; Hudson, Housner and Caughey, 1960)

(ii) Stationary Process (Tajimi, 1960; Housner and Jennings, 1964) ; and

(iii) Nonstationary Process (Bolotin, 1960; Cornell, 1964; Shinozuka and Sato, 1967; Amin and Ang, 1968; Boore, 1983).

If the short duration of initial rise and exponentially decaying tail are disregarded, the central strong-motion portion of acceleration records on firm ground can be treated as a segment of stationary random process. Bycroft (1960) has shown that properly scaled segment of white noise have response spectra similar to the response spectra of real earthquakes and can, therefore, be used to model the strong-motion portion of ground acceleration.

The stationary process model is an improvement over the white noise as it can be shaped to represent the frequency characteristics of the actual ground motions closely. The model makes it possible to incorporate the effect of local soil conditions explicitly. A segment of stationary process is, therefore, a more realistic model of the strong-motion portion of ground acceleration records. Following analytical expression due to Tajimi (1960) based on the work of Kanai (1957), called Kanai-Tajimi spectra, has been extensively used to represent the psd of ground acceleration :

$$\Phi(\omega) = \Phi_o \cdot \frac{1 + 4\,\zeta_g^2\,(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\,\zeta_g^2\,(\omega/\omega_g)^2} \qquad (2)$$

A filtered white noise with psd given by (2) can be generated by the following second order filter

$$\ddot{U} + 2\,\omega_g\,\zeta_g\,\dot{U} + \omega_g^2\,U = W(t) \qquad (3)$$

Where $W(t)$ is white noise with psd Φ_o, and $Z(t) = U(t) + W(t)$. From an analysis of actual records it is found that $\zeta_g = 0.6$ and $\omega_g = 5$ correspond closely to the spectral properties for firm ground. For a specific site the parameters ζ_g and ω_g should be chosen suitably to represent local site conditions.

The variance and other spectral parameters for the Kanai-Tajimi spectrum are given by

$$\sigma_g^2 = \frac{\pi \Phi_o \omega_g \; (1 + 4 \zeta_g^2)}{2 \zeta_g} , \tag{4}$$

$$\Omega_g = (\frac{\lambda_2}{\lambda_o})^{1/2} \doteq 2.1 \; \omega_g,$$

$$\delta_g = (1 + \frac{\lambda_1^2}{\lambda_o \lambda_2})^{1/2} = 0.67. \tag{5}$$

In (4) and (5), the cut-off frequency $\omega_o = 4 \omega_g$ and $\delta_g = 0.6$ (Vanmarcke, 1977). Bolotin (1960) proposed the following expression to represent a comparable auto-correlation function for stationary ground acceleration :

$$R(\tau) = \sigma_g^2 \; e^{-\alpha|\tau|} \; \cos \beta\tau \tag{6}$$

The nonstationarity in the ground acceleration records arises primarily through envelope functions $A_i(t)$ in (1). These are slowly varying random functions of time. A simple and adequate nonstationary model of ground acceleration can be constructed by assuming $A_i(t) = A(t)$ to be a deterministic function and $Y_i(t)$ a set of stationary random processes with specified psd. The model may be constructed either by multiplying a segment of filtered white noise by a modulating function (MFWN), or alternatively, by multiplying white noise by a modulating function first and then filtering the product process (FMWN). If the filter characteristics and the modulating functions are identical for the two models, the difference in their characteristics depends on the smoothness of $A(t)$. For earthquakes with long quasi-stationary motion both models yield similar characteristics.

Several functions have been used to model the envelope of the ground acceleration records. Following expressions are commonly used :

$$A(t) = (A_o + A_1 t) \, e^{-at^n} H(t); \quad n = 1 \text{ or } 2, \quad a > 0 \quad \text{(Iyengar and Iyengar, 1969)} \quad ...(7)$$

$$A(t) = A \, (e^{-at} - e^{-bt}) \, H(t), \quad b > a > 0 \quad \text{(Shinozuka and Sato, 1967)} \tag{8}$$

$$\begin{aligned} A(t) &= A \, (\frac{t}{t_1})^a \; H(t), && 0 \le t \le t_1 \\ &= A \; H(t - t_1), && t_1 \le t \le t_2 \quad \text{(Jennings et al., 1969)} \\ &= A \exp [-b \, (t - t_2)] \, H \, (t - t_2). && t_2 \le t \le t_3, \\ &= A_o \, [.05 \, C + d \, (T' - t)^2 \,] \; H \, (t - t_3), && t \ge t_3. \end{aligned} \tag{9}$$

where $H(t)$ is heaviside unit step function.

The envelope functions represented by (9) cover significant features of a complete range of recorded earthquake ground accelerations. Jennings et al. (1969) have suggested suitable values for the parameters of the envelope function to model four different types of earthquake motions (A, B, C and D) which are considered to be significant for engineering structures. The values of the parameters are given in Table 1. The envelope functions defined by (7) and (8) are analytically simpler and are used extensively in theoretical treatments. The nonstationary nature of ground motions has a significant influence on the tail value of the probability estimates, nonlinear response and soil behaviour. Further, the stationary models are inadequate for small near-field earthquake ground motions.

Table I Parameters of A(t) in (9) [Jennings et al., 1969]

Type	Target Magnitude	Target Spectrum Intensity	Duration T' (sec.)	t_1	t_2	t_3	a	b	c	d
A	≥8	1.5 (El-Centro, 1940)	120	4	35	80	2	.0357	1	$.938\times10^{-4}$
B	6 - 8	El-Centro, 1940	50	4	15	30	2	.0992	1	$.5\times10^{-2}$
C	5.5 - 6	Golden Gate,	12	2	4	-	2	.268	-	-
D	4.5 - 5.5	Parkfield 1966	10	2	2.5	3.5	3	1.606	2	$.237\times10^{-2}$

Multi-component ground motion

We have so far discussed the stochastic models of the individual components of ground acceleration during earthquakes. Since the three components of earthquake ground motion act simultaneously on a structure it is necessary to construct a stochastic model which incorporate their joint properties. Also, since the choice of axes along which components of an earthquake are recorded is arbitrary it is necessary to establish relations for the transformation of their properties due to the rotation of the coordinate system.

Consider the three components of ground acceleration defined by (1) along the axes (1 2 3). Let $A_i(t) = A(t)$ be a deterministic function, and $Y_i(t)$ a zero-mean, stationary random process. Then the elements of the covariance matrix of $Z_i(t)$ are given by

$$K_{\ddot{Z}_i\ddot{Z}_j}(t,\tau) = E[\ddot{Z}_i(t)\,\ddot{Z}_j(t+\tau)] = A(t)\,A(t+\tau)\,E[\ddot{Y}_i(t)\,\ddot{Y}_j(t+\tau)]$$
$$= A(t)\,A(t+\tau)\,R_{\ddot{Y}_i\ddot{Y}_j}(\tau); \qquad i,j = 1,2,3. \qquad ...(10)$$

Since the correlation time of the accelerogram is usually very small, the effect of changing the coordinate directions on the covariance functions can be investigated by considering (10) for $\tau = 0$, that is

$$E[\ddot{Z}_i(t)\,\ddot{Z}_j(t)] = A^2(t)\;E[\ddot{Y}_i(t)\,\ddot{Y}_j(t)], \qquad (11)$$

which can be expressed in the matrix form

$$R_{\ddot{Z}\ddot{Z}}(t) = A^2(t)\;R_{\ddot{Y}\ddot{Y}} \qquad (12)$$

Since $\ddot{Y}_i(t)$ are stationary, the elements of variance matrix $R_{\ddot{Y}\ddot{Y}}$ are constants. Also, $R_{\ddot{Y}\ddot{Y}}$ is a real, symmetric and positive definite matrix. It is, therefore, possible to find a canonical transformation matrix P, such that

$$\ddot{Y}_p(t) = P\,\ddot{Y}(t) \tag{13}$$

and

$$R_{\ddot{Y}_p\ddot{Y}_p} = P^T R_{\ddot{Y}\ddot{Y}} P = \text{diag.}\ (R_{11}, R_{22}, R_{33}). \tag{14}$$

where $R_{11} > R_{22} > R_{33}$ are the principal variances of the matrix $R_{\ddot{Y}\ddot{Y}}$, and the columns of P define the three orthogonal principal directions (Penzien and Watabe, 1975). Further, from (12)

$$R_{\ddot{Z}_p\ddot{Z}_p}(t) = A^2(t)\ \text{diag.}\ (R_{11}, R_{22}, R_{33}). \tag{15}$$

Hence the principal variance of $\ddot{Z}$ are given by $A^2(t)\,R_{ii}$ (no sum), $i = 1,2,3$. Note that variances are functions of time but the principal directions are time-independent*. Under the assumpotions stated above, it follows that the components of ground motion are correlated, but it is always possible to find at least one set of three principal directions along which the components are uncorrelated. The variances along these three principal directions represent maximum, minimum and intermediate values.

By analysing six different strong motion records Penzien and Watabe (1975) concluded that :

(i) the ratios of the minor and intermediate principal variances to major principal variance are of the order of 1/2 and 3/4 respectively;

(ii) the principal values of the cross-correlation coefficients ($\rho_{ij} = (R_{ii} - R_{jj})/(R_{ii} + R_{jj})$) ρ_{12}, ρ_{23} and ρ_{13} are approximately 0.14, 0.2 and 0.33 respectively.

(iii) the major principal axis lies generally in the direction of the epicentre and the minor principal axis is vertical; and

(iv) the directions of principal axes are reasonably stable over successive time intervals. It is, therefore, reasonable to assume the same envelope function, A(t) in the three directions.

A significant conclusion of the above discussion is that the three components of ground acceleration can be modelled as mutually uncorrelated random processes, provided they are directed along a set of principal axes with major principal axis directed towards the expected epicentre, and the minor principal axis directed vertically. Hadjian (1981) has analysed the correlation properties of the horizontal components of recorded ground motion to obtain the probability density function of the correlation coefficient. Recognising that the uncertainties in the value of the correlation coefficient arise due to two distinct sources- geometric and seismological, Hadjian has synthesised a probability density function shown in Figure 1. He has suggested that an 'equivalent' rectangular distribution, may be used for design codes and simulation purposes.

RANDOM VIBRATION DUE TO EARTHQUAKES

All structures resting on the ground, or connected to ground based structures, are subjected to random excitation during earthquakes. The response of such structures

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* If different modulation functions, $A_i(t)$, are used for each component, the principal directions will be time-dependent.

to earthquakes is, therefore, a random process. A reliability based design, in the framework of random vibration analysis, provides a rational and consistent basis for aseismic design of structures. To implement such a design procedure, it is necessary to establish the seismic risk at the site, construct a random process model of the ground motion, and carry out a random vibration analysis of the structure. Further, it is necessary to identify the response parameters of interest, the constraints on these parameters to maintain the integrity of the system, and the distribution of 'failure-events' or damage to the structure. In this section we shall discuss the random vibration analysis of structures for elastic and inelastic behaviour for a specified ground motion intensity.

We have noted that the ground acceleration at a point on the earth's surface during an earthquake is a random vector which may be modelled by three non-stationary random processes representing ground motion along two orthogonal horizontal directions and a vertical direction. We have also noted that along principal directions, these random processes are uncorrelated. A structure resting on the ground is, therefore, subjected to simultaneous action of three components at each point of contact. In earthquake engineering, it is generally assumed that same motion takes place at all points of contact, implying that foundation soil is rigid and the excitation is completely defined by a single random vector. If the base dimensions of a structure are small as compared to the wavelength of the ground motion, the assumption does not result in significant error. However, for long structures, such as, dams, bridges, pipelines, it is necessary to assume multiple support excitations (Clough and Penzien, 1975). We shall confine our discussion to single support excitation.

We assume that the nonstationary nature of each component of ground acceleration is adequately modelled by a uniformly modulated stationary random process. A segment of stationary random process is a special case of such a model with a box-car function as the modulating function. We assume that the ground acceleration is gaussian, and each component is partially, described by one or more, of the parameters, such as, peak ground acceleration, r.m.s. value, psd, response spectra.

The civil engineering structures, such as, buildings, chimneys, nuclear power plants, dams, bridges etc. cover a wide range of shapes and forms. Some structures, such as, dams and water-tanks involve special effects, such as, fluid structure interaction. Almost all structures resting on the ground involve soil-structure interaction which causes the free-field ground motion to be modified. In all cases, an analytical treatment of random vibration requires an idealized model of the structure and interacting media. We shall confine our discussion to discrete models exhibiting linear and nonlinear behaviour. Extension of the results to continuous models is generally straight forward, if normal mode method can be used.

The general theory of random vibration of discrete and continuous system, both for linear and nonlinear behaviour, is covered in the texts on random vibration (Lin, 1967; Nigam, 1983). We shall confine our discussion to special aspects associated with earthquake excitations. Our primary objective will be to determine the design values of response parameters of interest for specified reliability and ground motion intensity.

Discrete Linear Elastic Systems

A single-degree-of-freedom (sdf) system is the simplest, and the most important, of discrete linear systems. A large class of structures can be adequately modelled by an sdf system, and it is well known that analysis of mdf and continuous systems can, in most cases, be reduced to the analysis of a series of sdf systems. We shall first consider the response of an sdf system to a single component of ground acceleration. The equation of motion can be expressed as

$$\ddot{X} + 2\zeta\omega_o\dot{X} + \omega_o^2 X = -m\,\ddot{Z}(t) \tag{16}$$

where $X(t)$ is the relative displacement, and $\ddot{Z}(t)$ the ground acceleration during an earthquake. Since the system is at rest when the earthquake occurs, $X(0) = \dot{X}(0) = 0$.

The displacement response of an sdf system to a typical earthquake is shown in Figure 2. From the figure, and the general theory of the response of sdf systems to random excitation (Nigam, 1983), we conclude that for the range of damping values of interest $(0.01 < \zeta < 0.1)$, the response is a narrow-band, nonstationary random process. The nonstationary character is due to sudden start, and also due to nonstationary nature of ground acceleration. However, if the ground acceleration is modelled as a segment of stationary random process, the response tends to become stationary for large times, $t >> 1/\zeta\,\omega_o$. (Caughey and Stumpf, 1961). For example, if $\zeta = 0.1$ the response becomes stationary within 2-3 natural periods ($T_o = 2\pi / \omega_o$), where as for $\zeta = 0.025$, stationarity is attained after about 10 natural periods. The envelope of the response is a slowly varying function of time and the nonstationary spectral characteristics can be closely modelled by evolutionary psd.

The second order statistics and other properties of the response, such as, spectral moments, level crossings, peaks and the maximum value in a specified period, for the sdf system can be obtained from random vibration theory (Nigam, 1983). These properties can be used to determine the maximum values of the response parameters, such as, Displacement Spectra SD (ζ, ω_o), Velocity Spectra SV (ζ, ω_o) and Absolute Acceleration Spectra SA (ζ, ω_o).

Since the ground motion is treated as a random process, the response spectra, SD, SV and SA, are random variables and we are interested in their values for specified reliability levels. This is a classical problem in random process theory and is closely related to D-type first-passage problem. The exact solution of this problem is not available, but several approximate solutions have been attempted which yield results of practical interest (Crandall, 1970; Lutes, Chen and Tsuang, 1980; Crandall and Zhu, 1983).

Let

$$SD(\zeta, \omega_o) = X_m \quad \underset{0 \le t \le T}{\text{Max}} \; (|X(t)|). \tag{17}$$

The distribution function for SD can be expressed as :

$$F_{SD}(\alpha) = P\,[S\,D \le \alpha] = P\,[T_f > T] = p \tag{18}$$

where T_f is the first-passage time of D-type barrier $|X(t)| = \alpha$, p the probability that the level α will not be exceeded, and $T = T' + T_o/2$.

Stationary Ground Acceleration

The ground acceleration during earthquakes is nonstationary. However, stationary models, which are analytically much simpler to treat, give acceptable results for linear systems under fairly general conditions (Byrcroft, 1960; Rosenblueth and Bustamente, 1962). Further, the results for stationary models can be used to take into account the nonstationary effects by empirically modifying a few parameters (Vanmarcke, 1977; Kiureghian, 1980).

Let $\ddot{Z}(t)$, $0 \le t \le T$ be a segment of stationary random process with psd $\Phi_{\ddot{Z}\ddot{Z}}(\omega)$. The response of sdf system to $\ddot{Z}(t)$ will be initially nonstationary, but will tend to become stationary for $t >> 1/\zeta\omega_o$. Consider the case when $X(t)$ is assumed to be stationary. For this case, (18) can be expressed as (Crandall, 1970)

$$F_{SD}(\alpha) = L_{Tf}(T) = L_o e^{-\nu(\alpha)T} = p \tag{19}$$

where $\nu(\alpha)$ is called the limiting decay rate, and L_o is the probability of survival at $t = 0$. Since the system starts from rest, $L_o = 1$. Let

$$SD_{T;P} = \alpha = \eta_{T;P}\, \sigma_X(T) ; \tag{20}$$

$$\overline{SD} = E\,[SD] = \bar{\eta}\sigma_X(T) ; \tag{21}$$

$$\widetilde{SD} = Var\,[SD] = \tilde{\eta}\sigma_X\, 2(T) \tag{22}$$

where α is obtained by solving (19) for a specified p. $\eta_{T;p}$ is the peak factor for the response spectra, and $\bar{\eta}$ and $\tilde{\eta}$ are the peak factors for its mean and variance. $\sigma_X(T)$ is the standard deviation of the displacement at $t = T$. For stationary response

$$\sigma_X(T) = \sigma_X = \pi\, \Phi_{\ddot{Z}\ddot{Z}}(\omega_o)/2\zeta\, \omega_o^3 \tag{23}$$

The distribution of displacement spectra given by (19) depends on the choice of limiting decay rate ν. We shall consider two cases corresponding to independent X-crossing, and two-state Markov Crossing assumptions. Expressions for peak factor, $\eta_{T;p}$; mean and variance of the peak factor represented by $\bar{\eta}, \tilde{\eta}$ are also given for each case.

i) Independent X-crossings

$$\nu = 2\, N_o \exp(-\eta^2/2) \tag{24}$$

$$F_{SD}(\eta) = \exp(-2\, N_o T \exp(-\eta^2/2)) \tag{25}$$

$$\eta_{T;p} = [2 \ln(-2\, N_o T/\ln p)]^{1/2} \tag{26}$$

$$\simeq C_1 - \frac{\ln(-\ln p)}{C_1} \tag{27}$$

$$\bar{\eta} = C \tag{28}$$

$$\tilde{\eta} = \frac{\pi^2}{6\, C_1^2} \tag{29}$$

where

$$N_o = \frac{\Omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\lambda_2}{\lambda_o}} = \frac{\omega_o}{2\pi} \tag{30}$$

$$\lambda_i = \int_0^\infty \omega^i\, G_{XX}(\omega)\, d\omega , \qquad i = 0,1,2,\ldots\ldots \tag{31}$$

$$C_1 = [2 \ln(2\, N_o T)]^{1/2} \tag{32}$$

$$C = C_1 + \frac{0.5772}{C_1} \tag{33}$$

ii) Two-state Markov Crossings (Vanmarcke, 1969; 1977)

$$\nu = 2 N_0 \frac{1 - \exp(\sqrt{\frac{\pi}{2}} \delta_e \eta)}{\exp(-\eta^2/2) - 1}, \tag{34}$$

where

$$\delta = (1-\lambda_1^2 / \lambda_o \lambda_2)^{1/2} = k/\sqrt{2\pi} = \frac{2}{\sqrt{\pi}} \zeta^{1/2} \tag{35}$$

and

$$\delta_e = \delta^{1+b}; \quad b \simeq 0.2; \tag{36}$$

$$F_{SD}(\eta) = \exp\left[-2 N_o T \frac{1-\exp(-\sqrt{\pi}/2\ \delta_e \eta)}{\exp(-\eta^2/2) - 1}\right] \tag{37}$$

$$\eta_{T;p} = [2 \ln(-2 N_o T/\ln p\ [1 - \exp(-\delta_e \sqrt{\ln(2N_o T/\ln p)}])]^{1/2} \tag{38}$$

$$\bar{\eta} = \sqrt{2 \ln(2N_{oe}T)} + \frac{0.5772}{\sqrt{2 \ln(2N_{oe}T)}}, \quad \text{(Kiureghian 1980; Smeby and Kiureghian, 1985)} \tag{39}$$

$$\tilde{\eta} = \left[\frac{1.2}{\sqrt{(2 \ln 2N_{oe}T)}} \quad \frac{5.4}{13 + (2 \ln 2N_{oe}T)^{3.2}}\right]^2 \tag{40}$$

where

$$N_{oe}T = \begin{cases} \max(1.05,\ 2\delta N_o T), & 0.00 < \delta < 0.1 \\ (1.63\delta^{0.45} - 0.38) N_o T; & 0.1 < \delta < 0.69 \\ N_o T, & 0.69 < \delta < 1.00 \end{cases} \quad \text{....(41)}$$

Note that $\tilde{\eta}\,\sigma_X$, gives the average response spectra, and $\eta_{T;0.5}\,\sigma_X$, its median value.

A detailed discussion of the independent X-crossing, and two-state Markov crossing assumptions is given in Vanmarcke (1969) and Nigam (1983). The distribution of response spectra based on independent X-crossing, (25), are asymptotically exact for high reliability (high α - level), but are overly conservative for intermediate and low reliability. Two-state Markov crossing assumption, (37), which takes into account the effect of clumping, gives relatively accurate results at all levels. Several other approximations are available in the literature (Crandall, 1970; Iwan and Mason, 1980), including some empirical expressions (Shinozuka and Yang, 1971; Lutes, Chen and Tzuang, 1980).

The response of sdf system to a segment of stationary random process is initially nonstationary. In the preceding treatment we neglected the nonstationary behaviour of initial response. Corotis, Vanmarcke and Cornell (1972) have shown that this assumption may lead to overly conservative estimates of design values for intermediate and low reliability levels. The nonstationary behaviour can be easily incorporated in the analysis by taking into account the time-dependent nature of response statistics and a time-dependent limiting decay rate $\nu(t)$. The distribution function of the displacement spectra can be expressed as

$$F_{SD}(\alpha, T) = L_{T_f}(T) = L_o \exp\left(-\int_o^T \nu(t)dt\right). \tag{42}$$

Since the system starts from rest, $L_o = 1$.

The limiting decay rate, $\nu(t)$, is given by the following relations for the independent X-crossings, and two-state Markov Crossing assumptions :

i) Independent X-crossings

$$\nu(t) = 2N_o(t) \exp\left[-\frac{1}{2}\left(\frac{\alpha}{\sigma_X(t)}\right)^2\right], \tag{43}$$

(ii) Two-state Markov crossings

$$\nu(t) = 2N_o(t) \frac{1 - \exp\left(-\sqrt{\frac{\pi}{2}}\ \delta_e \alpha / \sigma_X(t)\right)}{\exp\left[\frac{1}{2}(\alpha/\sigma_X(t))^2\right] - 1} \tag{44}$$

where $\sigma_X(t)$ is time-dependent standard deviation of the displacement given by

$$\sigma_X(t) = \frac{\pi\, \Phi_{\ddot{Z}\ddot{Z}}(\omega_o)}{2\zeta\, \omega_o^3} \left(1 - \frac{e^{-2\zeta\omega_o t}}{\omega_d^2}\left[\omega_d^2 + 2(\zeta\omega_o \sin\omega_d t)^2 + \zeta\omega_o\omega_d \sin 2\omega_d t\right]\right) \quad ...(45)$$

$$\simeq \frac{\pi\, \Phi_{\ddot{Z}\ddot{Z}}(\omega_o)}{2\zeta\, \omega_o^3} \left(1 - e^{-2\zeta\omega_o t}\right) = \frac{\pi\, \Phi_{\ddot{Z}\ddot{Z}}(\omega_o)}{2\zeta_t\, \omega_o^3}, \quad \text{if } \zeta << 1 \tag{46}$$

where

$$\zeta_t = \left(1 - e^{-2\zeta\omega_o t}\right)^{-1} \tag{47}$$

$$N_o(t) = \frac{1}{2\pi}\left(\frac{\lambda_2(t)}{\lambda_o(t)}\right)^{1/2} = \frac{\Omega(t)}{2\pi} \tag{48}$$

$$\delta(t) = \left(1 - \frac{\lambda_1^2(t)}{\lambda_o(t)\lambda_2(t)}\right)^{1/2} \tag{49}$$

and $\lambda_j(t)$, $j = 0,1,2$ are time-dependent spectral moments given by

$$\lambda_j(t) = \int_0^\infty \omega^j G_{XX}(\omega, t)d\omega \quad (50)$$

where

$$G_{XX}(\omega, t) = |\tilde{H}(\omega, t)|^2 G_{\ddot{Z}\ddot{Z}}(\omega) \quad (51)$$

is the evolutionary spectral density of the displacement. Exact and approximate ($\zeta << 1$) expressions for $\lambda_j(t)$, $j = 0,1,2$, are available in Corotis, Vanmarcke, Cornell (1972) and Nigam (1983).

The displacement spectrum for reliability level p, $\alpha_{T;p}$, can be obtained by solving for α the equation.

$$F_{SD}(\alpha, T) = \exp\left[-\int_0^T \nu(t)\,dt\right] = p \quad (52)$$

where $\nu(t)$ is given by (43) or (44).

A solution of (52) to determine $SD_{T;p}$ requires a numerical procedure. Vanmarcke (1977) has proposed an approximate procedure based on 'equivalent stationary response' which permits a closed form solution of (52) to a reasonable accuracy. Since the system starts from rest, $\sigma_X(t)$ increases with time from 0 to $\sigma_X(T)$, and the decay rate $\nu(t)$ increases more rapidly. The major contribution to the integral in (52), therefore, comes from values of time t close to T. Hence, we can define an 'equivalent stationary response duration', T_e, such that

$$\int_0^T \nu(t)\,dt = T_e\,\nu(T) \quad (53)$$

where $T_e \leq T$. The ratio T_e/T can be obtained from the relation (Vanmarcke, 1977).

$$\frac{T_e}{T} \simeq \exp\left[-2\left(\frac{\sigma_X^2(T)}{\sigma_X^2(T/2)} - 1\right)\right] \quad (54)$$

which is obtained by approximating the area under $\nu(t)$ from 0 to T, in terms of $\nu(T)$ and $\nu(T/2)$.

The distribution of displacement spectra, the peak factor $\eta_{T;p}$, its mean and variance can now be obtained from (26) to (51) by replacing T by T_e and σ_X by $\sigma_X(T)$. Simulation studies (Vanmarcke, 1977) show that this procedure yields sufficiently accurate response spectra.

Nonstationary Ground Acceleration

The nonstationary character of earthquake ground acceleration can be modelled by a random process with evolutionary spectra. Let the evolutionary psd of the ground acceleration be expressed as

$$\Phi_{\ddot{Z}\ddot{Z}}(\omega, t) = |A(\omega, t)|^2 \Phi(\omega) \quad (55)$$

where $A(\omega, t)$ is the modulating function and $\Phi(\omega)$ is the psd of an associated broad-band random process. A uniformly modulated, or a separable, random process, with $A(\omega, t) = A(t)$, is a special case of the evolutionary process.

The response of an sdf system to an excitation having evolutionary psd $\Phi_{\ddot{Z}\ddot{Z}}(\omega,t)$ given by (55), is also a nonstationary random process with evolutionary psd

$$\Phi_{XX}(\omega,t) = |M(\omega,t)|^2 \Phi(\omega) \tag{56}$$

where

$$M(\omega,t) = \int_0^t h(t-\tau) A(\tau,\omega) e^{-i\omega(t-\tau)} d\tau . \tag{57}$$

and $h(t)$ is the impulse response function of the sdf system.

The psd of the response (56) to nonstationary excitation is of the same form as the psd of the nonstationary response to stationary excitation treated earlier (51). The distribution of the response spectra and associated response statistics can therefore be obtained as before. Markov property of the response of an sdf system to evolutionary excitation (Spanos, 1980a; 1980b; Spanos and Lutes, 1980) provides an alternative method to obtain the distribution of the response spectra.

Multi-degree-of-freedom (mdf) systems

A large class of structural/mechanical systems can be modelled as mdf system. The equations of motion of mdf systems excited by the three components of ground motion during an earthquake can be expressed as

$$M \ddot{\bar{X}} + C \dot{\bar{X}} + K \bar{X} = \bar{F}(t) = -M I \ddot{\bar{Z}} \tag{58}$$

where M, C and K are mass, damping and stiffness matrices respectively; $\bar{X}(t)$ is the relative displacement vector; $\bar{Z} = [Z_1 \; Z_2 \; Z_3]^T$ is the vector of ground displacement components, in which Z_1 and Z_2 are horizontal and Z_3 is the vertical component; and I is the influence matrix such that its kth column I_k couples the degrees of freedom of the structure to ground motion component $Z_k(t)$.

We first consider the case for which M, C and K satisfy the conditions for the existence of classical normal modes (Nigam, 1983). Let $U = [\bar{u}^1, \bar{u}^2, \ldots \bar{u}^n]^T$ be the modal column matrix and $\bar{Y}$ the vector of normal coordinates. Setting

$$\bar{X} = U \bar{Y}, \tag{59}$$

the jth uncoupled modal equation is given by

$$\ddot{Y}_j + 2 \zeta_j \omega_j \dot{Y}_j + \omega_j^2 Y_j = - \bar{\Gamma}_j^T \ddot{\bar{Z}}, \tag{60}$$

where ζ_j and ω_j are the damping and natural frequency in the jth mode; $\bar{\Gamma}_j = [\Gamma_j(1) \; \Gamma_j(2) \; \Gamma_j(3)]^T$ is the vector of participation factors for mode j with elements

$$\Gamma_j(k) = \frac{(\bar{u}^j)^T M \bar{I}_k}{(\bar{u}^j)^T M \bar{u}^j} ; \quad k = 1,2,3. \tag{61}$$

The design of a system involves response quantities such as displacement, member forces, stresses, which can be expressed as a linear combination of nodal displacements. Let $Q(t)$ be a response quantity of interest, and let

$$Q(t) = \bar{C}^T \bar{X} = \sum_{i=1}^{n} C_i X_i \tag{62}$$

where $\bar{C}^T$ is a constant vector. Then following the standard techniques (Nigam, 1983), the evolutionary psd of $Q(t)$ can be expressed as

$$G_{QQ}(\omega,t) = \sum_k \sum_l \sum_i \sum_j L_i(k)\, L_j(l)\, \tilde{H}_i(\omega,t)\, \tilde{H}_j(\omega,t)\, G_{\ddot{Z}_k \ddot{Z}_l}(\omega,t) \tag{63}$$

where $L_i(k) = \Gamma_i(k)\, \bar{c}^T \bar{\phi}^i$, is the effective participation factor associated with mode i; $\tilde{H}_i(\omega,t)$ is the frequency response function for mode i; and $G_{\ddot{Z}_k \ddot{Z}_l}(\omega,t)$ is k, l element of the psd matrix, $G_{\ddot{Z}\ddot{Z}}(\omega,t)$. Note that ground motion components are modelled as evolution random processes.

Under the assumptions stated below (63) reduces to

(a) $\ddot{Z}(t)$ is modelled as a stationary random vector

$$G_{QQ}(\omega,t) = \sum_k \sum_l \sum_i \sum_j L_i(k)\, L_j(l)\, \tilde{H}_i(\omega,t)\, \tilde{H}_j(\omega,t)\, G_{\ddot{Z}_k \ddot{Z}_l}(\omega,t) \tag{64}$$

As $t \to \infty$, $Q(t)$ becomes stationary, and

$$G_{QQ}(\omega) = \sum_k \sum_l \sum_i \sum_j L_i(k)\, L_j(l)\, H_i(\omega)\, H_j^*(\omega)\, G_{\ddot{Z}_k \ddot{Z}_l}(\omega) \tag{65}$$

(b) $\ddot{Z}(t)$ are uncorrelated (principal directions)

$$G_{QQ}(\omega,t) = \sum_k \sum_i \sum_j L_i(k)\, L_j(k)\, \tilde{H}_i(\omega,t)\, \tilde{H}^*_j(\omega,t)\, G_{\ddot{Z}_k \ddot{Z}_k}(\omega,t) \tag{66}$$

Response to single earthquake component

For a single earthquake component, $\ddot{Z}(t)$, (63) reduces to

$$G_{QQ}(\omega,t) = G_{\ddot{Z}\ddot{Z}}(\omega,t) \sum_i \sum_j L_i L_j\, \tilde{H}_i(\omega,t)\, \tilde{H}^*_j(\omega,t) \tag{67}$$

If $\ddot{Z}(t)$ is modelled as a stationary random process

$$G_{QQ}(\omega,t) = G_{\ddot{Z}\ddot{Z}}(\omega) \sum_i \sum_j L_i L_j H_i(\omega,t)\, H^*_j(\omega,t) \tag{68}$$

and the spectral moments of $Q(t)$ are

$$\lambda_{m,Q}(t) = \int_0^\infty \omega^m G_{QQ}(\omega,t)\, d\omega \tag{69}$$

$$= \sum_i \sum_j L_i L_j \int_0^\infty \omega^m \tilde{H}_i(\omega,t)\, \tilde{H}_j^*(\omega,t)\, G_{\ddot{Z}\ddot{Z}}(\omega) d\omega \tag{70}$$

Vanmarcke (1972) has shown that significant contributions to the spectral moments in (70), usually come from the terms for which $i = j$, particularly when modal frequencies are well separated and damping is low. Following his treatment, (70) can be expressed as

$$G_{QQ}(\omega,t) = G_{\ddot{Z}\ddot{Z}}(\omega) \sum_i |\tilde{H}_i(\omega,t)|^2 \left(L_i^2 + \sum_{i\neq j} L_i L_j A_{ijt}\right) \tag{71}$$

in which A_{ijt} is a function of frequency ratio $\gamma = \omega_j/\omega_i$ and the equivalent damping values ζ_{it}, ζ_{jt}

$$A_{ijt} = \frac{8\gamma \zeta_{it}(\zeta_{jt} + \zeta_{it}\gamma)(1-\gamma^2)^2 - 4\gamma(\zeta_{it} - \zeta_{jt}\gamma)(\zeta_{jt} - \zeta_{it}\gamma)}{8\gamma^2 [(\zeta_{it}^2 + \zeta_{jt}^2)(1-\gamma^2) - 2(\zeta_{jt}^2 - \zeta_{it}^2\gamma^2)(\zeta_{it}^2 - \zeta_{jt}^2\gamma^2) + (1-\gamma^2)^4]} \tag{72}$$

A_{ijt} is plotted in Figure 3 as a function of γ for different pairs of ζ_{it} and ζ_{jt}. It is seen that A_{ijt} vanishes when γ is either very small or very large, and is sharply peaked for small values of damping at $\gamma = 1$.

Substitution of (71) in (69) and integration over all frequencies gives

$$\lambda_{m,Q}(t) = \sum_{i=1}^{n} \alpha_{it} L_i^2 \lambda_{m,ii}(t) \tag{73}$$

where

$$\alpha_{it} = 1 + \sum_{i\neq j} (L_j/L_i) A_{ijt} \tag{74}$$

and $\lambda_{m,ii}(t)$ is the mth spectral moment of modal coordinate $y_i(t)$. For $m = 0$.

$$\sigma_Q^2(t) = \lambda_{o,Q}(t) = \sum_i \alpha_{it} L_i^2 \sigma_{yi}^2(t). \tag{75}$$

Clearly the contribution of cross-terms ($i \neq j$) in (73) and (75) will be insignificant if $\alpha_{it} \approx 1$, which happens when i) modal frequencies are well separated; ii) damping is small; and iii) time t is sufficiently large. Under these conditions :

$$\lambda_{m,Q}(t) = \sum_i L_i^2 \lambda_{m,ii}(t) \tag{76}$$

The spectral parameters of $Q(t)$, that is, $\Omega_Q(t)$ and $\delta_Q(t)$ required for determining the characteristics, such as, level-crossing, peak, first-passage time, peak-response factor can be determined as follows. Equation (71) can be written as

$$G_{QQ}(\omega,t) = \sum_i G_{QiQi}(\omega,t) = \sum_i G_{\ddot{Z}\ddot{Z}}(\omega)\, |\tilde{H}_i(\omega,t)|^2\, L_i^2\, \alpha_{it} \tag{77}$$

Let,

$$p_i = \frac{\int_0^\infty G_{Q_iQ_i}(\omega, T)d\omega}{\int_0^\infty G_{QQ}(\omega, T)d\omega} = \frac{\alpha_i T \; L_i^2 \sigma_{yi}^2 (T)}{\sigma_Q^2 \; (T)} \tag{78}$$

and let $\Omega_i(t)$ and $\delta_i(t)$ be the spectral parameters of $Q_i(t)$. Clearly $\sum_i p_i = 1$, and it can be shown that (Vanmarcke, 1977)

$$\Omega_Q(t) = (\sum_i p_i \Omega_i^2)^{1/2} \tag{79}$$

$$\delta_Q(t) = [1 - \{\sum_i p_i (\Omega_i/\Omega_Q) \sqrt{1 - \delta_i^2}\}^2]^{1/2} \tag{80}$$

The maximum value of $|Q(t)|$, $o \leq t \leq T$, and associated probability p, its expected value and variance can be expressed as

$$Q_{m;T;p} = \eta_{T;p} \sigma_Q(T) \tag{81}$$

$$E[Q_m] = \bar{\eta} \sigma_Q (T) \tag{82}$$

$$Var [Q_m] = \tilde{\eta} \sigma_Q^2(T) \tag{83}$$

where $\eta_{T;p}$, $\bar{\eta}$ and $\tilde{\eta}$ are given by (26) to (41) for X-crossing and Markov crossing assumptions respectively.

Stationary Response

If the response is assumed to be stationary, the relations for the nonstationary response presented above get modified by setting :

$$H(\omega, t) = H(\omega) e^{i\omega t}; \tag{84}$$

$$\zeta_t = \zeta;$$

$$G_{QQ}(\omega, t) = G_{QQ}(\omega); \quad \text{and}$$

$$\lambda_{m,Q}(t) = \lambda_{m,Q}, \text{ in particular } \sigma_Q(T) = \sigma_Q.$$

Discrete Inelastic System

So far we have considered systems modelled by linear, elastic force-deformation relationship. It is well known that both steel and concrete structures have a large reserve of strength beyond the elastic range (Housner, 1956, 1959; Nigam, 1970; Sues et al., 1983). In the design of structures to resist earthquakes, inelastic behaviour is of special interest due to reduction in the intensity shaking caused by dissipation of energy through hysteresis. A structure can be designed to remain elastic during more frequent small earthquakes, and allowed to undergo inelastic deformation, without collapse, during infrequent large earthquakes. This basic approach to aseismic design is incorporated in most codes. The random vibration analysis of inelastic systems is, therefore, of considerable interest.

The force-deformation relationships for inelastic behaviour are both nonlinear and hysteretic. Commonly used models of inelastic behaviour are shown in Figure 4. Wen (1976, 1980), Baber and Wen (1981) have proposed models of inelastic behaviour which, besides nonlinear hysteretic behaviour, incorporate degradation of stiffness and/or strength. The mathematical difficulties in a nonlinear random vibration analysis of inelastic behaviour are considerable (Nigam, 1983). Approximate methods, such as, equivalent linearization (Caughey, 1960, 1963); Gaussian closure technique (Iyengar and Dash, 1978) and Markov vector formulation (Wen, 1976) have been used to obtain the response statistics of such systems. Numerical integration of the equation of motion to real or simulated, ground has received considerable attention for the study of inelastic behaviour due to analytical difficulties, and has formed the basis of validating approximate methods.

Karnopp and Scharton (1965), developed a simple procedure for random vibration analysis of elasto-plastic sdf systems to determine statistics of accumulated plastic deformation and its application to low cycle fatigue. Vanmarcke (1969, 1977) improved upon this method and extended it to bilinear systems (Vanmarcke and Venizians, 1973). We shall discuss this procedure in the sequel.

Figure 4(b) shows the force deformation behaviour of an elasto-plastic system. Before and after each plastic excursion, the system behaves as a linear elastic sdf system oscillating about the immediate past permanent set, X_p. The equation of motion of an sdf elasto-plastic (EP) system can be expressed as

$$m\ddot{X} + c\dot{X} + g(X, \dot{X}) = -m\ddot{Z} \tag{85}$$

where restoring force function $g(X, \dot{X})$ is given by

$$g(X,\dot{X}) = k(X - \dot{X}_{pn}), \text{ if } |X - X_{pn}| < X_y \text{ or } |X - X_{pn}| = X_y \text{ and } \dot{X} = 0 \tag{86a}$$

$$= g_y \, \mathrm{sgn}(\dot{X}), \quad \text{if } |X - X_{pn}| = X_y \text{ and } \dot{X} > 0. \tag{86b}$$

(86a) holds during elastic-state after the nth inelastic excursion and, (86b) holds when the system is in the inelastic state.

The elasto-plastic system can be replaced by an associated linear sdf system on the basis of the following simplifying assumptions :

i) the elasto-plastic system spends a small fraction of the total in the inelastic state; and

ii) the inelastic deformation is 'instantaneous' after which the system becomes linearly elastic about a shifted equilibrium state.

Let $X'(t) = X(t) - X_{pn}$, denote the relative displacement of an 'associated linear sdf system', in which the elastic segments of elasto-plastic system are joined together. The equation of motion of the associated system is given by

$$m\ddot{X}' + cX' + kX' = -m\ddot{Z} \tag{87}$$

A sample function of the time-history of $X'(t)$; and its behaviour in the phase-plane are shown in Figure 7. It must be noted that besides shifting the equilibrium-state to the permanent-set after each inelastic excursion, the time scale of associated system is also changed by joining the elastic segments. Thus if, T is the duration of $X(t)$, the duration of $X'(t)$,

$$T' = T - \sum_{i=1}^{n} \Delta t_{pi} \tag{88}$$

where n is the number of inelastic excursions, and Δt_{pi} is the time spent during the ith excursion.

The subsequent treatment is based on the plausible assumption that the response statistics of the elastic behaviour of EP system and associated linear elastic system are not significantly different, and the system response during inelastic excursions can be determined on the basis of following physical arguments

During an elastic excursion, the kinetic energy of the mass at the beginning of the excursion is dissipated through plastic work, and the system returns to elastic-state with the initial conditions $X'(t) = 0$.

Thus

$$g_y\,|\delta| = \frac{1}{2} m\,(\dot{X}')^2 \Big|_{X(t) = X_y} \tag{89}$$

where δ is the plastic displacement during an inelastic excursion. Setting

$$g_y = k\,X_y,$$

$$|\delta| = \frac{1}{2\,\omega_o^2\,X_y}\,(\dot{X}')^2\Big|_{X(t) = X_y} \tag{90}$$

For random excitation, δ is random and its expected value is given by

$$E[|\delta|] = \frac{1}{2\,\omega_o^2\,X_y} \int_{-\infty}^{\infty} \dot{x}'^2\; p(\dot{x}'|\,X = X_y)\, d\dot{x}' \tag{91}$$

For practical aseismic design, it is realistic to assume that inelastic excursion will be infrequent. If so, the response can be assumed to be stationary at the entry to the next excursion, and, therefore, $E[X\,\dot{X}'] = 0$. The conditional probability in (91) becomes the probability of $\dot{X}'$ alone, and

$$E[|\delta|] = \frac{1}{2\,\omega_o^2\,X_y}\,\sigma_{\dot{X}'}^2 \tag{92}$$

Assuming $\sigma_{\dot{X}'} = \sigma_{\dot{X}}$ and $\sigma_{\dot{X}} = \sigma_X/\,\omega_o^2$ for broad-band excitation,

$$E[|\delta|] = \frac{\sigma_X^2}{2\,X_y} = \frac{\sigma_X}{2\,n_y}, \tag{93}$$

where

$$n_y = X_y/\,\sigma_X.$$

The response of an sdf system is a narrow-band random process for which excursions above a level tend to occur in 'clumps' particularly if the level is not too high and the damping is small in each clump, there may be several successive elastic excursions during which energy is dissipated, followed by relatively long elastic regime. As an approximation the cluster of excursions in a clump may be treated as a point in time at which plastic jumps, D_i, take place. The number of clumps can be treated as a Poisson process to define the following response parameters of practical interest

Plastic Drift

$$D(T) = \sum_{i=1}^{N(T)} D_i \tag{94}$$

Total Energy Dissipated Due to Yielding

$$DA(T) = \sum_{i=1}^{N(T)} |D_i| \tag{95}$$

Ductility Ratio

$$\nu = \frac{1}{X_y} [\max_{0 \le t \le T} |D(T)|] + 1 - \frac{X_{pm}}{X_y} + 1 \tag{96}$$

where $N(t)$ is a Poisson process with mean arrival rate,

$$\nu_c = \frac{2 N_X^+ (X_y)}{E [M_1]} \tag{97}$$

and $E[M_1]$ is the expected clump-size. The tendency for clumping is expected to be less in elasto-plastic systems, as compared to corresponding elastic systems due to energy dissipation during each inelastic excursion.

Total Energy Dissipated

Let the excitation be a gaussian random, process. We assume that the response of the elasto-plastic system is also gaussian. Under this assumption, $\dot{X}/\sigma_{\dot{X}}$ is a standard gaussian random variable, and therefore $\dot{X}^2/\sigma^2_{\dot{X}}$ has χ^2 - distribution with one-degree-of-freedom, with mean and variance equal to 1 and 2 respectively. Since, $|\delta|$ and, therefore, $|D_i|$ are proportional to $\dot{X}^2/\sigma^2_{\dot{X}}$, we have

$$E[|D_i|] = \frac{\sigma_{\dot{X}}^2}{2 X_y} E[\frac{\dot{X}^2}{\sigma^2_{\dot{X}}}] = \frac{\sigma_{\dot{X}}^2}{2 X_y} \tag{98}$$

$$Var[|D_i|] = (\frac{\sigma_{\dot{X}}^2}{2 X_y})^2 \quad Var[\frac{\dot{X}^2}{\sigma^2_{\dot{X}}}] = 2 (\frac{\sigma_{\dot{X}}^2}{2 X_y})^2 \tag{99}$$

Assuming $|D_i|$ in (95) to be mutually independent, identically distributed and also independent of $N(s)$, we have (Parzen, 1962)

$$E[DA(T)] = E\,[N(T)]\;\; E[|D_i|]$$

$$= \nu_c\, T\, \frac{\sigma_X^{\;2}}{2\, X_y} \;=\; 2\, N_X^+\,(X_y)\, \frac{\sigma X^2}{2\;X_y}\,,$$

$$= \frac{\omega_0}{2\pi}\, T\, \frac{\sigma_X}{\eta_y}\; e^{\,-\eta^2 y/2} \qquad \text{...(100)}$$

$$Var\,[DA(T)] = E\,[N(T)]\;\; Var[|D_i|] + Var\,[N(T)]\, E^2\,[|D_i|]$$

$$= \frac{3}{2}\, \frac{\omega_o}{2\pi}\, T\, \frac{\sigma_X^2}{\eta_y^2}\; e^{\,-\eta_y^2/2} \qquad (101)$$

Total plastic drift

The total plastic deformation $D(t)$ defined by (94) has a zero-mean, as each inelastic excursion D_i is equally likely to be positive or negative with mean and variance equal to 0 and $2\,\delta^2$ respectively. Hence

$$E[D(T)] = \nu_c\, T\, E[D_i] = 0 \qquad (102)$$

$$Var\,[D(T)] = \nu_c\, T\, Var\,[D_i] + E^2\,[D_i] = 2\,(\nu_c\, T)\,\delta^2 \qquad (103)$$

$$= \frac{\omega_o\, T}{4\pi}\, \left(\frac{\sigma_y}{\eta_y}\right)^2\; e^{-\eta_y^2/2} \qquad (104)$$

Ductility ratio

To establish the distribution of ductility ratio, γ, defined by (96), it is necessary to establish the distribution of peak inelastic deformation X_{pm}, the maximum value of $D(t)$ in the interval $0 < t \leq T$. Vanmarcke (1977) has shown that the distribution function of X_{pm} can be approximated as

$$F_{X_{pm}}(\alpha) = \exp\,[-\,(\exp\,(\,\nu_c\, T) - 1)\;\; \exp\,(-\alpha\,/\delta\,) \qquad (105)$$

The distribution of ductility ratio follows from (96).

In the above discussion we have treated the random vibration of elasto-plastic sdf system for infrequent excursions. Karnopp and Scharton (1965), have also given approximate results for frequent excursions taking into account the effect of clumping and nonstationary behaviour in excursions.

Gazetas (1976), Irschik (1986), Ziegler (1987) have extended the procedure due to Karnopp and Scharton (1965) to mdf systems, including the effects of nonstationary excitation.

PROBABILITY DESIGN FOR EARTHQUAKES

Aseismic design encompasses choice of design parameters to avoid, or limit, 'damage' to a system due to earthquakes during its service-life. The term 'damage' is used in a generic sense to denote any 'unfavourable behaviour' such as, loss of life, structural collapse, cost of repair or interruption of services, discomfort to occupants etc. Due to a large measure of uncertainty contributed by several sources, a probabilistic formulation provides a consistent and realistic basis for aseismic design. In aseismic design it is convenient to partition the total risk into two steps :

i) determination of conditional distribution of damage to a system due to an ensemble of ground-motions representing design earthquake(s) in terms of one, or more, given site-intensity parameters; and

ii) determination of the distribution of site-intensity parameters (seismic inputs) at the site during the service-life of the system.

The total risk is the combined effect of the different ways in which the system may respond to a given event, and all possible seismic events that may occur during its service-life. It is computed by a convolution of the conditional distribution of system response for given seismic input, with the distribution of seismic input. A general expression for the total risk can be written (Whitman and Cornell, 1977).

$$P[D_i] = \sum_{\text{all } j} P[D_i | Y_j] \; P[Y_j] \tag{106}$$

where D_i denotes the event that the damage state of the system is i, and Y_j denotes the seismic input at 'level' j; and $P[D_i | Y_j]$ is the conditional probability that the damage state of the system is D_i, given that seismic input Y_j has occurred. If the damage and seismic input represent continuous variables, (106) can be expressed as

$$F_D(d) = P[D \le d] = \int F_{D|Y}(d|y) \, p_Y(y) \, dy. \tag{107}$$

If the seismic input is specified in terms of more than one site-intensity parameters, say by an m-dimensional vector Y, total risk can be written as

$$F_D(d) = \int_m \cdots\cdots \int F_{D|\bar{Y}}(d|\bar{y}) \, p_Y(\bar{y}) \, d\bar{y}. \tag{108}$$

In the preceding section, we have derived the distributions of structural response for elastic and inelastic behaviours. These are conditional distributions for given seismic input. The damage to a structure during a seismic event, or a sequence of seismic events, can be expressed as a stochastic, and under certain conditions and approximations, as a deterministic function of response parameters. Its conditional distribution can be determined from the distribution of response parameters using the theory of random functions. The distribution of site-intensity parameters can be established through a determination of seismic risk treated in Appendix-A. We, therefore, have the necessary framework for evaluating total risk; or choosing design parameters, or design intensities for a specified total risk, if the functional relationship between damage and response parameters is established.

Damage to a Structure due to Earthquakes

The damage to a structure due to earthquakes may be treated under two broad classes :

i) instantaneous damage, which results when at an instant of time one, or more, response parameters cross the safe operating domain for the first time. This is the classical first passage problem; and

ii) cumulative damage , which accumulates during and over several seismic events, and the 'failure' occurs when damage function exceeds a specified limit.

As mentioned earlier, the damage is generally a stochastic function of the response. However, under certain conditions it may be treated as a deterministic function of response. We shall consider the latter case first.

Deterministic Damage-Response Relationship

We have seen that response of a structure due to an earthquake depends on the given value of site-intensity. If the damage is deterministically related to the response, it will still be a random function of the site-intensity. Let the damage measure, D, be a monotonic function of the site-intensity, Y, that is

$$D \;=\; f(Y); \quad \frac{df}{dy} > 0 \quad \text{for} \ \forall \ y, \quad \text{and} \quad D \geq o. \tag{109}$$

Inverting (109)

$$Y \;=\; g\,(D), \tag{110}$$

Let D_{max} be the maximum value of D in time interval t. Then the distribution of D_{max}

$$F^{(t)}_{D_{max}}(d,t) \;=\; p\,[D_{max} \leq d] \;=\; P[Y_{max} \leq g(d)], \tag{111}$$

$$= \; F_{Y_{max}}\,(g\,(d),\, t).$$

From (A.21),

$$F^{(t)}_{D_{max}}\;(d,t) \;=\; \exp\,[-C\,\hat{\nu}\;\;G(g\,(d))^{-\,\beta\,/b_2}\;t] \tag{112}$$

and

$$P[D^{(t)}_{max} > d] \;=\; 1 - \exp\,[-C\,\hat{\nu}\,G\,(g(d))^{-\,\beta\,/b_2}t\;,$$

$$\doteq \; C\,\hat{\nu}\,G\,(g(d))^{-\,\beta\,/b_2}t,$$

for the range of probabilities of practical interest in design.

If the damage is proportional to Y, that is D = aY, where a is constant, as in the case of expected peak response of high frequency sdf systems (Cornell, 1971)

$$P[D^{(t)}_{max} > d] \;=\; C\,\hat{\nu}\,G\,(d/a)^{-\,\beta\,/b_2}\;t, \tag{113}$$

It must be noted that the in the above derivation it has been assumed that there has been no deterioration due to previous lower intensity loads and the events are independent. If this condition remains true, that is either previous damage does not influence the response significantly, or the system is restored through repair, the cumulative total damage, TD, can be expressed as

$$TD = \sum_{j=1}^{N(t)} D_j = \sum_{j=1}^{N(t)} f(Y_j), \tag{114}$$

where N(t) is a Poisson process. For D = aY,

$$\mu_{TD} = E[TD] = \hat{\nu} t\, aY, \tag{115}$$

$$\sigma_{TD} = Var\,[TD] = \hat{\nu}\, t\, a^2\, E\,[Y^2], \tag{116}$$

The characteristic function of TD is given by

$$M_{TD}(\theta) = \exp\,[\hat{\nu}\, t\, (M_D(\theta) - 1)] \tag{117}$$

where $M_D(\theta)$ is characteristic function of D_j. If $\hat{\nu}t$ is not small, TD will be approximately normally distributed. A Markov model can be used if the incremental damage is stochastically dependent on the total damage todate (Vanmarcke, 1969).

A simple example where damage is directly related to peak ground motion occurs in nuclear power plants which sustain economic loss whenever accelerometer scramming control shuts down the plant if peak ground acceleration exceeds a threshold level. If damage due to each shut down is, d', the total expected damage in time t is

$$E[TD] = d'\, \hat{\nu} t\;\; CG\, y^{-\beta/b_2}, \tag{118}$$

from (A.21).

Stochastic Damage-Response Relationship

Due to random nature of earthquake ground motion, the response is a random process and the damage is therefore a stochastic function of system response. The conditional distribution of system response is obtained through random vibration analysis. The damage can be generally related to the peak response of the system. In the preceding section we have derived the approximate expressions for the distribution, and second order statistics, of the peak response of sdf and mdf systems. These are conditional distributions for given seismic input. The total risk, or design based on specified total risk, can be obtained by substituting these in (107) or (108).

CONCLUSION

In this lecture we have discussed several aspects of the probabilistic approach to earthquake engineering. We have demonstrated that uncertainties inherent at various stages in the process of aseismic design can be treated in a probabilistic framework. We have noted that our ability to construct realistic stochastic models is critical for the success of this approach, and paucity of available data in several seismically active regions of the world a major limitation. However, by supplementing the available seismic data with engineering judgement aseismic designs can be implemented meaningfully in a probabilistic framework. As the data accumulates in future the reliability of designs will improve.

APPENDIX A : SEISMOLOGY AND SEISMIC RISK

The occurrence of earthquakes, both temporal and spacial, involves a large measure of uncertainty. The multiple reflections, refractions and dispersions of seismic waves at irregular boundaries in the course of their travel to a site compound the level of uncertainty even further. This results in a high level of variability in the estimates of ground motion intensity at a site during the service life of structure and is the major contributor to total seismic risk which is determined by embedding the random vibration analysis into the seismic risk analysis. In this appendix we present a broad outline of the approach to seismic risk analysis and the principal results needed for our limited purpose. Idriss (1978), has given an excellent review or various approaches, definitions developed from time to time relating to earthquake ground motion.

A.1 Seismology

It is convenient to divide the study of seismicity under two broad headings : (i) local seismicity, which deals with the probabilities of earthquakes occurring in a given portion of earth's crust; and (ii) regional seismicity, which deals with the probabilities of earthquakes of given intensities shaking a given region of earth's surface (Newmark and Rosenblueth, 1971). The study of local seismicity basically involves knowledge of the geotectonic features and seismic history at micro- and macro-levels. Regional seismicity studies combine this information with the transmission of seismic waves, local geological information and statistical data on intensities of past earthquakes to establish estimates of seismic risk. The historical data for a site is seldom sufficient to permit direct statistical estimate of seismic risk. Seismic risk is, therefore, assessed through deductive probabilistic models using the available data. The Bayesian approach, which makes it possible to systematically supplement quantitative data with qualitative information is ideally suited for seismicity investigations (Esteva, 1969).

Some definitions and formulae

Magnitude (M) is a measure of the size of the earthquake in terms of the energy released. There are several definitions of magnitude (Idriss, 1978). The original definition due to Richter (1958) defines magnitude as the common logarithm of the trace amplitude, in microns, or a standard seismograph located 100 km from the epicentre. Intensity (I) is a measure of earthquake's local destructiveness. It is measured on a subjective scale, such as, the modified Mercalli (MM) scale. Focus or Hypocentre is the point in the earth's crust where the first seismic waves originate. Epicentre is the vertical projection of the focus on the earth surface. The location of a site in relation to an earthquake may be specified by the focal distance (R); or by epicentral distance (X) and focal depth (H). Clearly

$$R = (X^2 + H^2)^{1/2}. \qquad (A.1)$$

Peak ground acceleration (A_g), peak ground velocity (V_g), and peak ground displacement (D_g), are respectively the peak values of the absolute value of the ordinates in the ground acceleration, velocity and displacement records at a site. An empirical relationship between the peak ground motion parameters (A_g, V_g or D_g) and the magnitude and focal distance, called attenuation relations, can be expressed in the following general form (Esteva and Rosenblueth, 1964; Idriss, 1978).

$$Y = b_1 e^{b_2 M} R^{-b_3}, \qquad (A.2)$$

where Y may denote, A_g, V_g or D_g; and b_1, b_2 and b_3 are constants which depends on the quantity represented by Y. Esteva and Rosenblueth suggests that, for southern California, the average values of the constants (b_1, b_2, b_3) may be chosen as (2000, 0.8, 2), (16, 1.0, 1.7) and (7, 1.2, 1.6), if Y denotes A_g, V_g or D_g in units of centimeters and seconds, and R is measured in kms. It may be remarked that available data shows a large scatter around the mean curve represented by (A.2) (Esteva, 1970). We shall discuss this aspect later.

Duration (T_g) is the length in time of significant shaking at a site during an earthquake. Several definitions have been proposed for T_g (Idriss, 1978). The definitions based on Husid-plot (Husid, 1969) are being adopted increasingly. We shall express the relation between the duration, magnitude and focal distance in the form

$$T_g = C_1 e^{C_2 M} + C_3 R, \tag{A.3}$$

Where T_g is in seconds, R is in kms, and the average values of the constants (C_1, C_2, C_3) are (0.02, 0.74, 0.3) (Esteva and Rosenblueth, 1964).

The r.m.s. value (σ_g) of the ground acceleration can be expressed in terms of the peak median value of A_g in the following form (Vanmarcke, 1977)

$$\sigma_g = \hat{A}_g \left(2 \ln \left(\frac{2.8\, \Omega_g}{2\pi}\right)\right)^{-1/2}, \tag{A.4}$$

where $\hat{A}_g$ is the median value of A_g, and $\Omega_g \approx 2.1\ \omega_g$ for the Kanai-Tajimi psd with $\omega \leq 4\ \omega_g$ and $\zeta_g = 0.6$. Through (A.2), (A.4) σ_g gets related to M and R.

The half slipped length (S) of fault can be expressed in terms of magnitude in the following form.

$$S = \frac{1}{2} \exp\left[a_1 (M - m_o)^{a_2}\right], \tag{A.5}$$

where m_o is the lower limit on earthquakes of engineering interest, and a_1 and a_2 are constants.

A.2 Seismic Risk Analysis

The seismic risk at a site is usually expressed in terms of the probability of site 'intensity' exceeding a certain value in a given period of time. The term 'intensity' is used here in a generic sense to denote any one, or a function of several, ground motion parameters (Kiureghian 1981). To determine the seismic risk at a site it is necessary to construct the stochastic models of the source parameters, both temporal and spacial, and the travel path. A seismic source is characterised by its geometry, temporal characteristics and size of the seismic events. The geometric shape of the source on the surface of the earth can be idealized as a point, line or area based on the knowledge of the spacial distribution of past earthquakes and known geotectonic features. The spacial distribution of earthquakes can generally be assumed to be homogeneous in a source, and if necessary a source may be subdivided into homogeneous sources. The focal depth of earthquakes is usually assumed to be constant. However, if local depth data is available, a distribution can be fitted (Basu, 1977; Basu and Nigam, 1977). The occurrence of earthquakes in time, at a source is generally assumed to be Poisson, so that

$$P_N(n,t) = \frac{(\nu t)^n \exp(-\nu t)}{n!} \quad (A.6)$$

where $P_N(n,t)$ denotes the probability of occurrence of n earthquakes of magnitude greater than say m_o, during the time interval t, and $\nu = \nu(m_o)$ is the expected rate of occurrence of such earthquakes. A Poisson model assumes stationarity and independence of successive events. Both assumptions are not fully substantiated by data and evolutionary models of earthquake occurrence, specially if fore-and after- shocks are included. However, the model is considered adequate for the service life of most structures (Knopoff, 1964; Limnitz, 1966).

The probability density function of the magnitude of earthquakes may be expressed as

$$P_M(m) = \exp[-\beta(m - m_o)], \quad m \geq m_o, \quad (A.7)$$

where m_o, is the threshold magnitude below which the events are not of engineering interest and $\beta(1.5 - 2.3)$ is a constant for the source.

It is clear that the variable M and R in (A.2) are random variables, and therefore, the site intensity Y, is also a random variable. The distribution of M is given by (A.7) and the distribution of R can be derived for a given source-geometry, assumed spacial distribution of epicentres and focal depth. For example, for a line-source of length l, uniform distribution of epicentres along the line and constant focal depth, it can be shown (Cornell, 1967) that

$$P_R(r) = \frac{2r}{l(r^2 - d^2)^{1/2}}, \quad d \leq r \leq r_o, \quad (A.8)$$

where d and r_o are indicated in Figure A-1 . Assuming that R and M are independent random variables, it can be shown from the theory of functions of random variables that (Cornell, 1968)

$$P_Y = P[Y \geq y] = 1 - F_Y(y) = \frac{C}{l} G\, y^{-\beta/b_2}, \quad y \geq y'; \quad (A.9)$$

where

$$C = \exp(b_2 m_o)(b_1)^{\beta/b_2}, \quad (A.10)$$

$$G = \frac{2}{d^{\nu}} \int_o^{\sec^{-1}(r_o/d)} (\cos^{\nu} u)^{-1}\, du \quad (A.11)$$

$$\nu = \beta \frac{b_3}{b_2} - 1, \quad (A.12)$$

and

$$y' = b_1 \exp(b_2 m_o)\, d^{-b_3}, \quad (A.13)$$

Note that parameter G depends on the geometry of the source and can be similarly derived for the area source. For ~~points~~ source

$$G = (r)^{-(1+\nu)}$$

Equation (A.9) gives the probability that the site intensity, Y, will exceed the value, y, given that an event of magnitude $M \geq m_o$ has occurred on the source. Since the occurrence of such events at a source is assumed to be Poisson with arrival rate ν, it is clear that the events with site-intensity $Y \geq y$ are 'special events' with arrival rate P_y, and

$$P_{N*}(n,t) = \frac{(P_y \nu t)^n}{n!} \exp(-P_y \nu t), \quad n = 0,1,2,.... \tag{A.14}$$

where N* (t) is the counting process of special events, $(Y \geq y)$, at the site.

Let $Y_m^{(t)}$ denote the maximum value of the intensity over a t-year period. Since

$$P[Y_m^{(t)} \leq y] = P\,[\text{Zero special-events in the time interval, } t],$$

$$F_{Y_m(t)}(y) = P_{N*}(0,t) = \exp[-P_y \nu t], \tag{A.15}$$

Let

$$Y_m = Y_m^{(t)} \Big|_{t=1}$$

be the annual maximum intensity. Then

$$P_{N*}(0,1) = F_{Y_m}(y) = \exp[-P_y \nu], \tag{A.16}$$

Substituting for P_y from (A.9) in (A.16)

$$F_{Y_m}(y) = \exp[-\nu C G y^{-\beta/b_2}], \quad y \geq y_l \tag{A.17}$$

$$= 1 - \nu C G y^{-\beta/b_2}, \quad y \geq y_l,$$

if $C G y^{-\beta/b_2} << 1$, for probabilities of interest in design. The annual return period (T_y) is given by

$$T_y = \frac{1}{1 - F_{Y_m}(y)} = \frac{1}{\hat{\nu} C G} y^{\beta/b_2}, \tag{A.18}$$

where $\hat{\nu} = \nu/1$. The T-year intensity (y_T) is obtained from (A.18)

$$y_T = (\hat{\nu} C G T)^{-\beta/b_2}, \tag{A.19}$$

It may be noted that (A.17) represents Type-II asymptotic extreme value distribution of largest values.

The above results have been derived for a single source. If a site may experience shaking by more than one source, the above results can be easily extended, if the sources are assumed to be independent. Consider m such sources. It is clear that $p[Y_m(t) \le y]$ is the probability that the maximum value from each source is less than or equal to y, that is

$$F_{Y_m(t)}(y,t) = \prod_{j=1}^{m} F_{Y_{mj}(t)}(y,t),$$

$$= \exp\left[-\sum_{j=1}^{m} \hat{\nu}_j C_j G_j \, y^{-\beta_j/2b_{2j}} \, t\right]; \quad y \geq y' \qquad \ldots(A.20)$$

where $F_{Y_{mj}(t)}(y,t)$ is the distribution of the maximum, in time t, for the jth source, and y' is the largest y_j. If constants β_i, b_i, $i = 1,2,3$ are the same for each source, (A.20) reduces to

$$F_{Y_m(t)}(y,t) = \exp\left[-C\hat{\nu} G \, y^{-\beta/b_2} \, t\right], \qquad (A.21)$$

where,

$$\hat{\nu} G = \sum_{j=1}^{m} \hat{\nu}_j G_j \qquad (A.22)$$

Equations (A.21), (A.22) indicate that each source contributes approximately in an additive way to the risk. The design intensity for a specified probability $(y_{t;p})$, the return period for a specified intensity, and T-year intensity can be obtained from (A.20) or (A.21) for multiple source situation.

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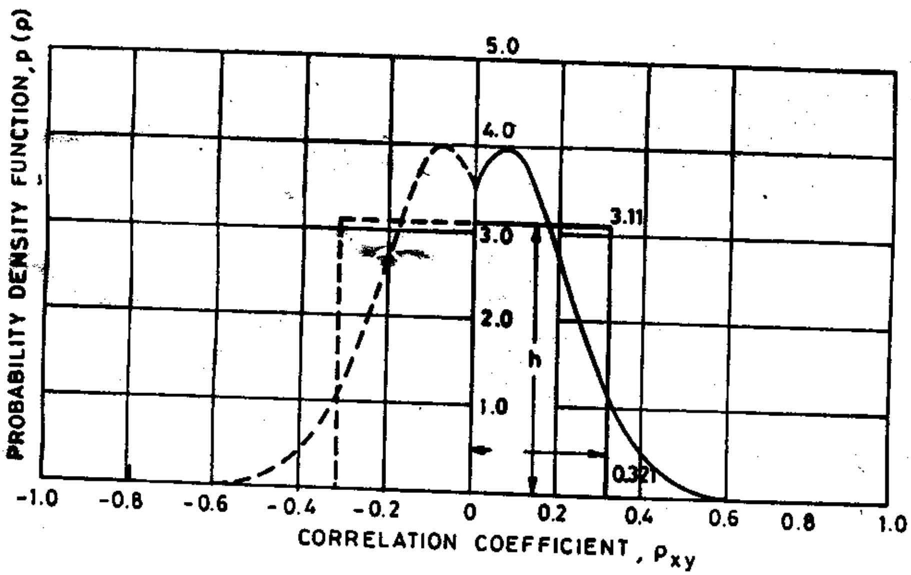

Fig. 1 Probability distribution of correlation coefficient and an 'equivalent' rectangular distribution. (Hadjian, 1981)

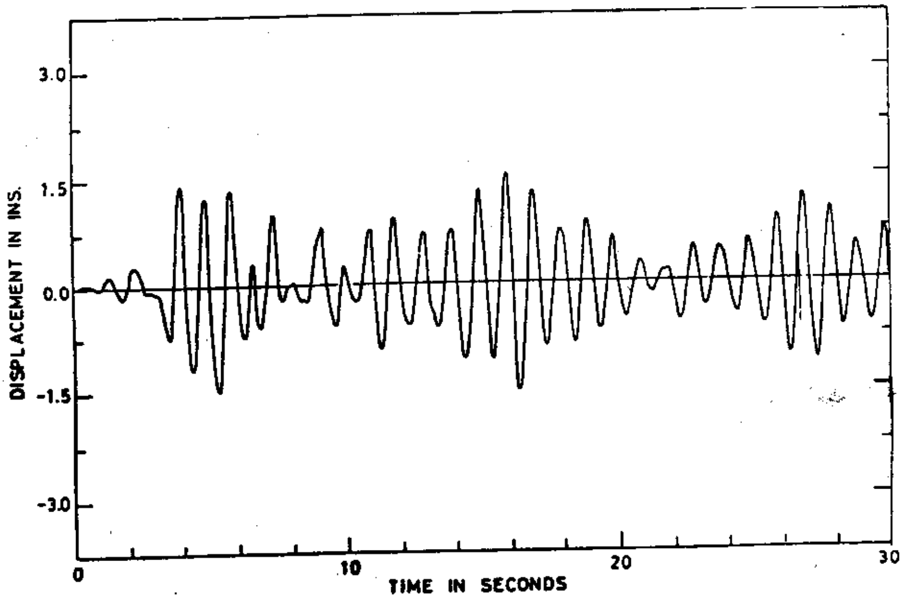

Fig. 2 Displacement response of a linear sdf system to earthquake excitation (Nigam, 1967)

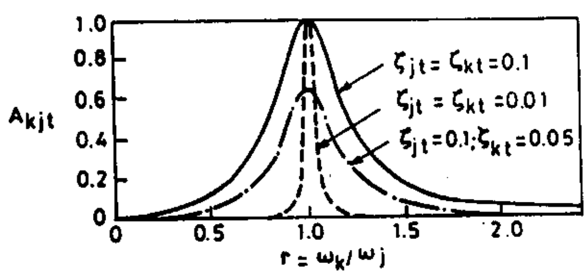

Fig. 3 Factors A_{ijt} showing ineraction between modes i and j of a mdf system (Vanmarcke, 1977)

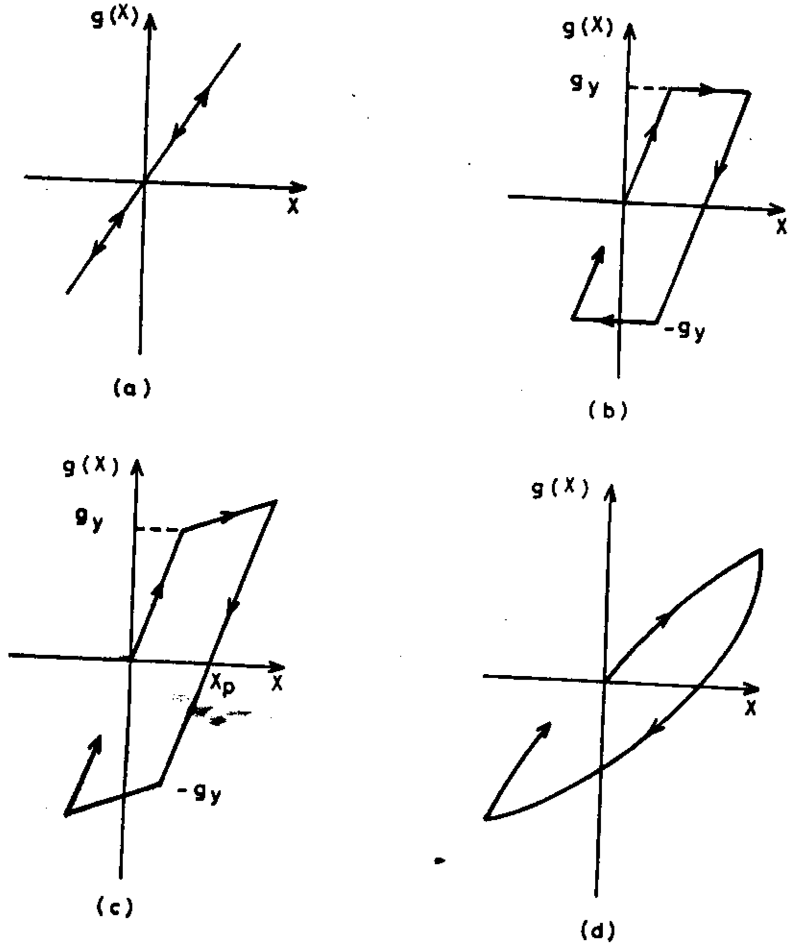

Fig. 4 Idealised models of inelastic force-displacement behaviour. (a) elastic, (b) elasto-plastic, (c) bilinear hysteretic, and (d) general yielding.

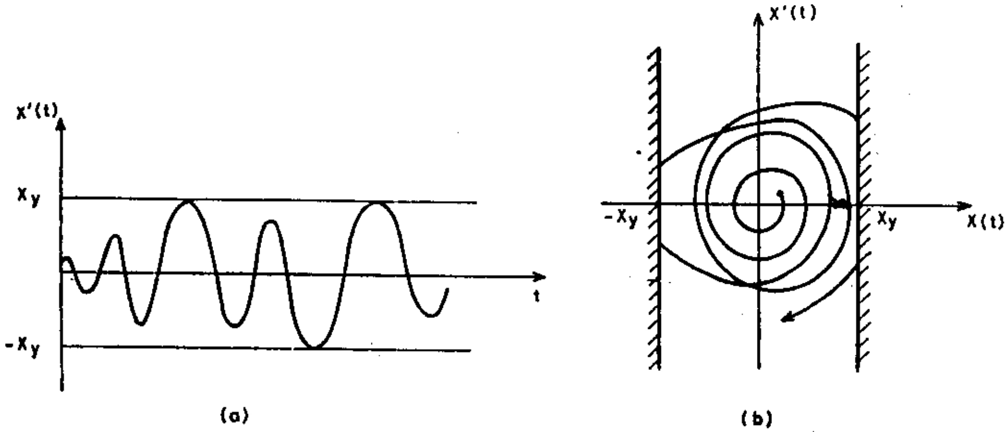

Fig. 5 A sample function of X'(t). (a) time history, (b) behaviour in phase plane.

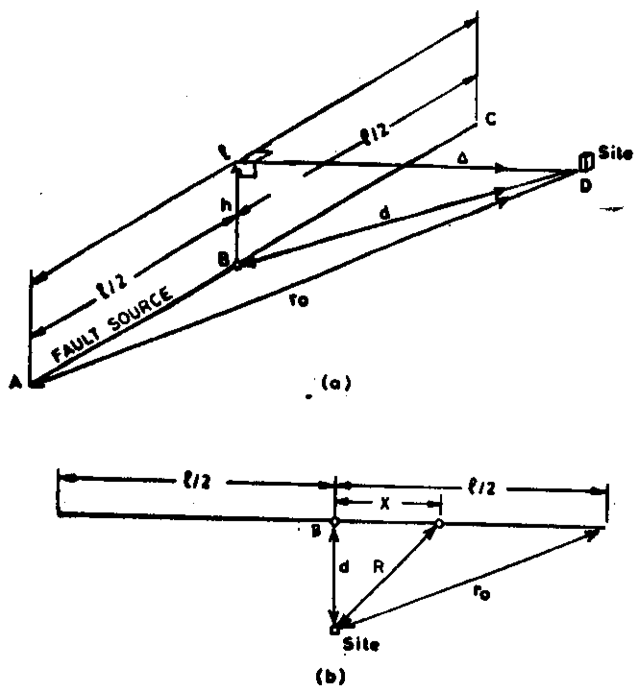

Fig. A-1 Line source (a) perspective, (b) plan (Cornell, 1967).