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Internal Consistency Of Smooth Design Response Spectra

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INTRODUCTION

Specification of response spectra is a standard practice in seismic design and qualification. The design spectra are generally the curves of maximum response acceleration levels as functions of periods of single degree-of freedom systems for various values of damping. In literature several authors have addressed themselves to establish smooth design response spectra through statistical methods. Newmark, Blume and Kapur[1] have developed smooth design spectra with the help of thirty-seven earthquake records, recorded on different soil conditions. On the other hand, Mehraz[2] and Seed, Ugas and Lysmer[3] have established design spectra specific to certain soil conditions. Generally, the design spectra are smoothened to account for variations due to steep peaks and valleys. Prior to smoothing, the records are normalised with respect to their peak ground motion parameters. Smooth design response spectra (SDRS) are also specified at floor levels in the form of floor response spectra for design and qualifications of equipments in power plants. The SDRS, due to their simplicity, are very appealing to the design engineers. These are used commonly to find equivalent static forces or are used in estimating responses through the SRSS approach. Another very important application is in generating spectrum compatible accelerograms to be used in direct time-history response analysis. In the later case, it is now well recognized that not all spectra are internally consistent to produce a compatible accelerogram For a single spectrum, Levy and Wilkinson[4] point out that every spectrum need not have an admissible time-history. Scanlan and Sachs[5] have observed that it is inconsistent to simultaneously prescribe simplified spectra for several dampings that would have a single time-history compatible with all of them. However, at present what constitutes conistency between spectrum at different damping leveis is not clearly known. The same question arises when one has to estimate from a given spectrum the spectrum for another value of damping. The dependence of the response spectra on frequency and damping can perhaps be obtained analytically for simple excitation. But for real earthquake accelerograms, this can be obtained only through

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data analysis. With this in view, in the present study, a set of sixty-six earthquake records are selected for evolving a reference spectra and to compute inter-damping scaling relationships.

THE DATA

Sixty-six horizontal components of past californian earthquakes[6] have been selected as the data base. For sake of reference, these are listed in table I. The average peak acceleration of these records is 0 0713g. For further work, all the records are normalised with respect to their absolute peak acceleration value. The list includes several low amplitude records. It is known that these have a relatively less erratic evolution in comparison with large amplitude accelerograms. This would lead to a reduction in the scatter of data points which is desirable in studying inter-damping relationships.

SPECTRAL STATISTICS

The displacement spectra for all the sixty-six records are available from EERL reports. These are converted as relative acceleration spectra and averaged at ninty-one frequency points for five values of damping, namely, η=0 0, 0.02, 0.05, 0 1 and 0.2. The average plus standard deviation $(m+\partial)$ spectra so obtained are shown in Figure 1. These are also compared

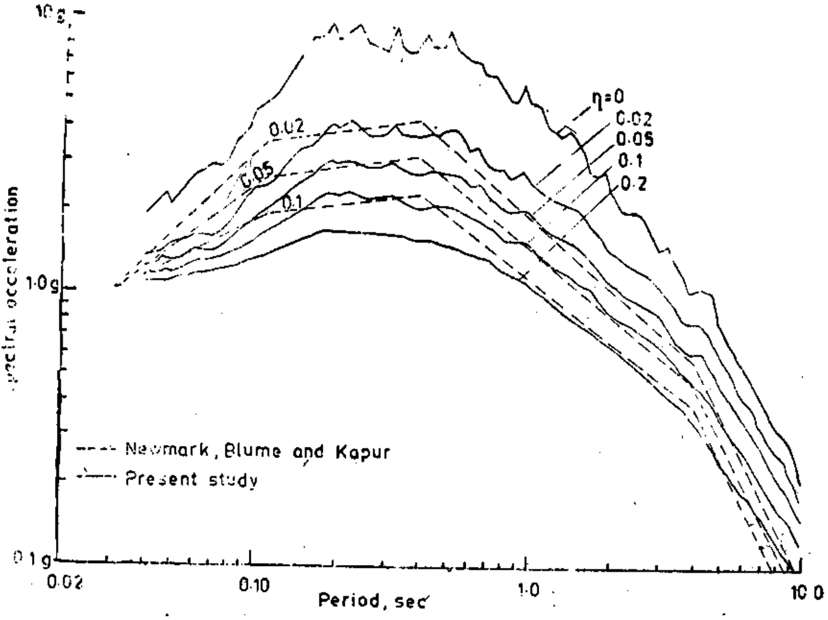

Fig. 1. Acceleration Spectrum. Mean + Std. Devn.

Table 1. The data base of 66 accelerograms

Serial No.	Earthquake, data and station	Component	Peak ground acc. in g
1	Imperial Valley, Apr. 12, 1938	S 00°E	0.0287
	ElCentrao (T 272)	S 90°	0 0497
2	Imperial Valley, May, June 5, 1938	S 00°E	0.0339
	ElCentro (T 276)	S 90°W	0.0265
3	Imperial Valley, May 18, 1940	S 00°E	0.3483
	ElCentro (A 001)	S 90°W	0.2142
4	Imperial Valley, May 18, 1940	S 00°E	0.0299
	ElCentro (T 277)	S 90°W	0.0246
5	Imperial Valley, May 18, 1940	S 00°E	0.0121
	Elcentro (T 278)	S 90°W	0.0147
6	Imperial Valley, May 18, 1940	S 00°E	0.0116
	ElCentro (T 279)	S 90°W	0 0189
7	Imperial Valley, May 18, 1940	S 00°E	0 0228
	ElCentro (T 280)	S 90°W	0.0097
8	Imperial Valley, May 18, 1940	S 00°E	0 0652
	ElCentro (T 283)	S 90°W	0.0787
9	Imperial Valley, May 18, 1940	S 00°E	0.0116
	ElCentro (T 284)	S 90°W	0.0164
10	Imperial College, May 18, 1940	S 00°E	0.0520
	ElCentro (T 285)	S 90°W	0 0725
11	Borrego Valley, Oct. 21, 1942	S 00°E	0.0596
	ElCentro (T 286)	S 90°W	0.0474
12	Imperial Valley, Jan. 23, 1951	S 00°E	0.0310
	ElCentro (T 287)	S 90°W	0.0281
13	Northwest Californa, Oct. 17, 1951	S 44 W	0.1040
	Ferndale (A 002)	N 46 W	0.1116
14	Kern county, July 21, 1952	S 00°E	0.0413
	Caltech. Athenaeum (A 003)	S 90°W	0.0531
15	Kern county, July 21, 1952	N 21°E	0.1557
	Taft Lincoln School (A 004)	S 69°E	0.1793
16	Kern county, July 21, 1952	N 42°E	0 0895
	Santa Barbara (A 005)	S 48°E	0.1311
17	Kern county, July, 21, 1952	S 00°W	0 0551
	Hollywood Storage Basement(A006)	N 90°E	0.0443

18	Kern county, July 21, 1952	S 00°W	0.0592
	Hollywood Storage P.E. Lot (A 007)	N 90°E	0.0420
19	Imperial Valley, Jun. 13, 1953	S 00°E	0.0073
	ElCentro (T 288)	S 90°W	0 0363
20	Lower California, Nov. 12, 1954	S 00°E	0.0247
	ElCentro (T 289)	S 90°W	0.0275
21	Eureka, Dec. 21, 1954	N 11°W	0.1677
	Eureka Federal Bldg (A 008)	N 79°E	0.2576
22	Eureka, Dec. 21, 1954	N 44°E	0.1587
	Ferndale (A 009)	N 46°W	0.2011
23	San Jose, Sept. 4, 1954	N 59°E	0.1078
	Jan Jose Bank of America (A 010)	N 31°W	0.1021
24	Imperial County, Dec. 16, 1955	S 00°E	0.0310
	ElCentro (T 290)	S 90°W	0.0162
25	Imperial County, Dec. 16, 1955	S 00°E	0.0637
	ElCentro (T 292)	S 90°W	0.0725
26	El Alamo, Baja, Feb. 9, 1956,	S 00°E	0.0330
	ElCentro (A 011)	S 90°W	0.0510
27	San Francisco, Mar. 22, 1957	N 45°E	0.0468
	South Pacific Bldg. (A 013)	N 45°W	0.0458
28	San Francisco, Mar. 22, 1957	N 09°W	0.0426
	Alexander Bldg. (A 014)	N 81°E	0.0463
29	San Franciso, Mar. 22, 1957	N 10°E	0.0834
	Golden Gate Park (A 015)	S 80°e	0.1048
30	San Francisco, Mar. 22; 1957	S 09°e	0.0854
	State Bldg. (A 016)	S 81°W	0.0562
31	Hollister, Apri. 8, 1961	S 01°W	0.0646
	Hollister City Hall (A 018)	N 89w	0.1791
32	Gulf of California, Aug. 7, 1966	S 00°e	0 0138
	ElCentro (T 293)	S 90°W	0.0151
33	Borrego Mt., Apr. 8, 1968	S 00°W	0.1303
	ElCentro (A 019)	S 90°w	00.574

with the corresponding SDRS of Newmark, Blume and Kapur[1] (NBK) for three damping values. The general appearance of the tripartite spectra is well reproduced in Figure 1. However, except in the approximate central interval of 0.12 to 0.4 seconds, the NBK spectra and the present spectra differ quantitatively. For periods less than 0.12 seconds, the present results are consistently lower than the NBK values. Similarly for periods greater than 0.04 second, the computed spectra lie above the smooth NBK spectra. The reasons for this difference lie in the frequency contant of the records chosen for the analysis. However, in the 2 to 8 Hz range which is the dominant frequency range of Californian earthquake, the comparison is quite good. The (m+o) spectra is generally specified for the design of important structures like power plants. For less important structures, it is more common to select the average spectra as the basis for design. For example, the Indian Code for Earthquake Resistant Design IS-1893[7] recommends an average spectra which is based on Housner's[8] investigations. This has been further modified recently and in the draft revision of the above code, a set of new smooth design spectra have been recommended. It would be informative to compare this with the average of the present set of the sixty-six records. Such a comparison is shown in Figure 2.

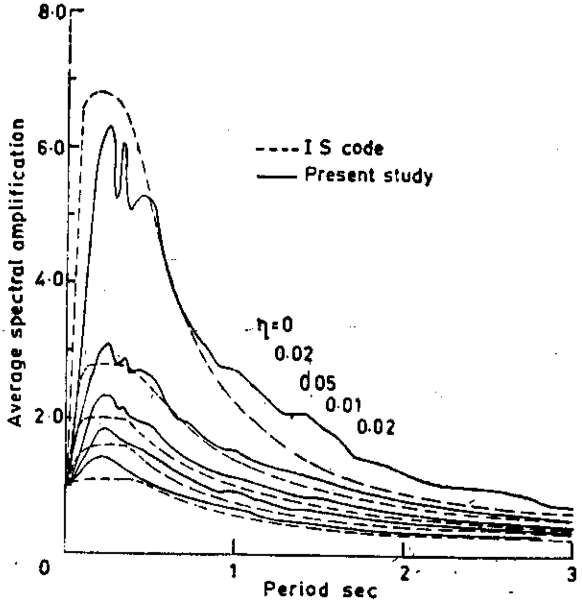

Fig 2. Average Spectrum IS-1893 Fourth Revision Draft And Present Result

It is observed that except for 2% damping, the two spectra differ considerably over a wide frequency range. It would seem that the I.S. Code draft provisions are less conservative than the present averages.

SPECTRAL CONSISTENCY

From the way the spectra is computed, it is evident that the spectra for different damping values, are not independent. In other words, once the spectrum is given for a particular damping value, the spectra for other values of damping must get fixed upto a large extent. This property may be loosely called the internal consistency of spectra. It is difficult to make this statement more precise, since accelerograms are so varied and in fact can only be thought of as random processes. This hints at the necessity of searching the consistency in statistical terms. It may be noted here that this consistency or lack of it in a given SDRS will not be noticed when the spectra are used in conjunction with modal responce analysis employing either the SRSS or other summation procedures. 'However, when spectra compatible accelerograms have to be generated for time-history response analysis, inconsistent spectra pose many problems. Since response spectra are functions of damping and period (or frequency), there must be a consistency with respect to both these variables. Some constraints on the spectra with respect to frequency are well known. For example, as the period goes to zero, the spectra must approach the peak ground acceleration. However, the rate of approach differs with damping. Some other measures of consitency for a single spectrum have been mentioned by Levy and Wilkinson[4].

To study the consistency with respect to damping in actual earthquake records, the most natural method is to look at scattergrams at different periods. In figures 3,4, 5 and 6, a few typical scattergrams at periods of 0.1, 0.5, 1 and 2 seconds respectively are shown. The amplitudes at various dampings are referred as a_0, a_2, and a_5. It is observed that all these scattergram show a nearly linear relationship. At higher amplitude levels a slight nonlinearity is discernible which in a first order engineering analysis could be neglected. Thus, a linear equation of the form

$$y = mx + c$$

is appropriate to estimate the spectrum y at a new level of damping from the given spectrum x. The constant,m, in the above refers to the correlation coefficient between the two damped spectra and is a measure of linearity of the relationships.The regression constants m and c have been determined

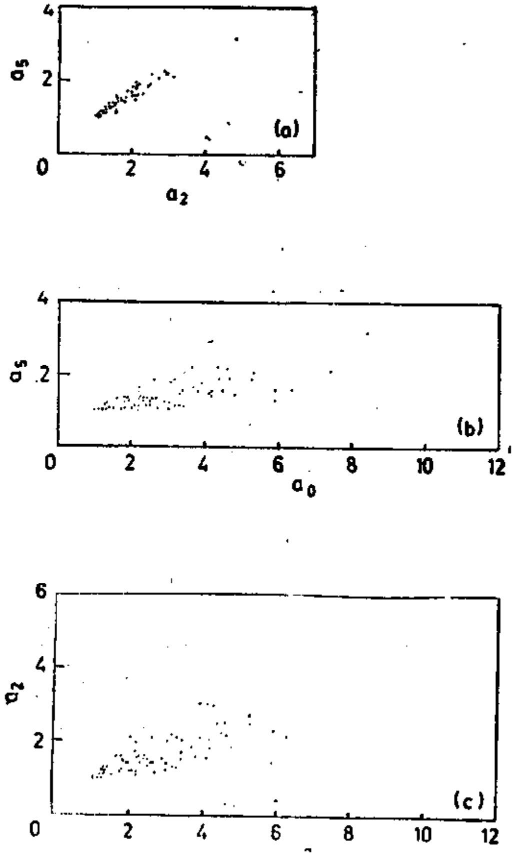

Fig. 3. Scattergram At 0.1 sec. (a) 2% Vs 5% Damping. (b) 0% Vs 5% Damping. (c) 0% Vs 2% Damping

between the various levels of damping, at all the 91 periods of the data set. A few typical correlation coefficient matrices of the spectral amplitudes, a_0, a_2, a_5, a_{10} and a_{20} are shown below.

Interdamping corre at on matrix

T = 0 1 second	1.0				
	0 787	1.0	Sym.		
	0.705	0 958	1.0		
	0 618	0 861	0 957	1.0	
	0 445	0 696	0 831	0 938	1.0

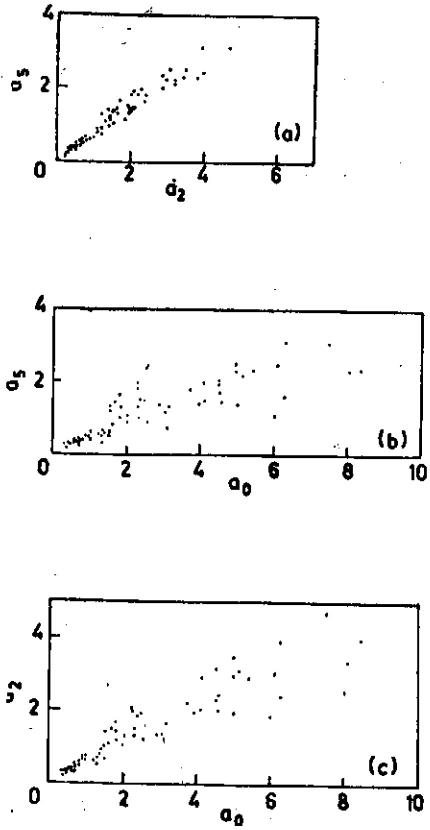

Fig. 5 Scattergram At 1.0 sec. (a) 2 % Vs 5 % Damping (b) 0 % Vs 5 % Damping (c) 0% Vs 2% Damping

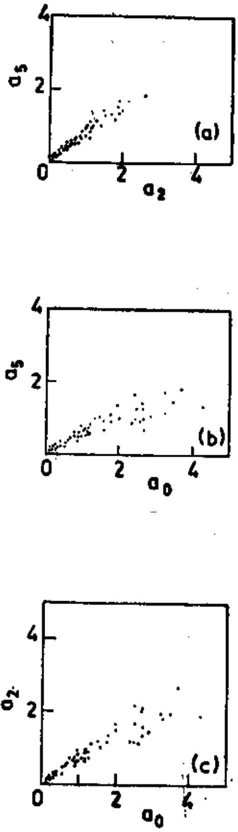

Fig. 6. Scattergram At 2.0 sec. (a) 2 % Vs 5% Damping (b) 0% Vs 5%(c) 0% Vs 2% Damping.

T = 2.0 second	1.0				
	0.941	1 0			
	0.914	0.987	1.0		
	0.872	0.943	0.981	1.0	
	0.891	0.896	0.947	0,988	1.0

In Figures 7(a) and 7(b), the regression constants m and c are presented as functions of the period The intercept, c, is seen to approach zero for long periods Similarly the slope, m, approaches a constant value as period increases. However, at the high frequency end both m and c show considerable fluctuations. This indicates that specifying a set of spectra which are smooth for all damping values may not be realistic. In other words, when a smooth design response spectra is selected at any particular value

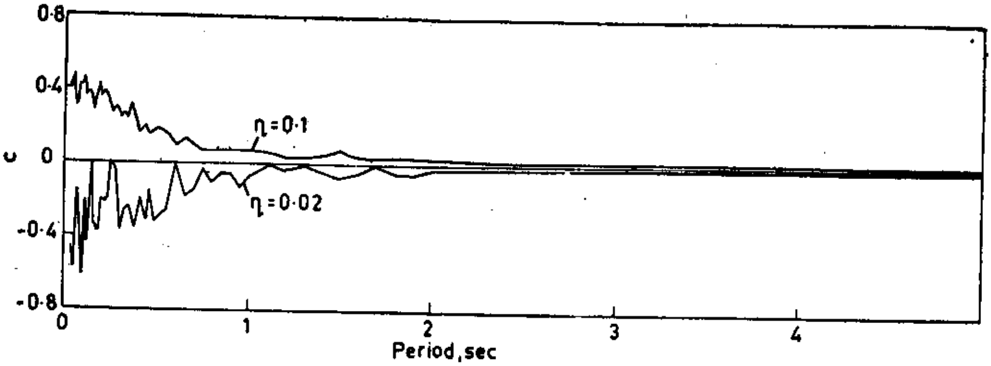

FIG.7a. VALUE OF C FOR η=0.02,0.1 FROM GIVEN 5% SPECTRUM

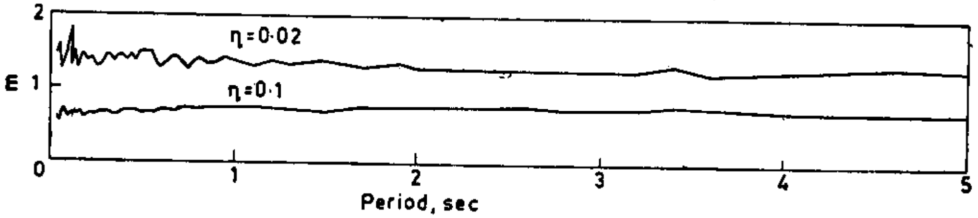

Fig. 7b. Value Of 'm' For η = 0.02, 0.1 From Given 5% Spectrum

of damping, the consistent spectra at other values of damping cannot also be smooth. This is particularly so in the high frequency range. In figure 7, the m and c values are given for finding the spectra for 2% and 10% damping from the given 5% damped spectrum. Numerical results have been obtained for other combinations of the dampings also. The major application of the regression analysis lies in the use of m and c to construct the spectra at different values of damping from a given damped spectrum. Hence it would be interesting to see how well the linear relation reconstructs the spectrum of a real earthquake. A comparison is presented in figure 8 between the actual 5% spectrum and the same spectrum derived from the present approach starting from the actual 2% spectrum. It is observed that the actual and the derived spectra match very well.

INTERNAL CONSISTENCY OF SDRS

The regression coefficients computed can be used to check the internal consistency of a given smooth design response spectra. Results of such an exercise are shown in figurees 9, 10 and 11 forstandared spectra of USNRG the Canadian Standards Association (CSA) and the Indian Standards Institute (IST). In all the three cases, the 2% spectrum derived from 5%, spectrum and 5% sepctrum derived from 2% spectrum are compared with the actual codal spectra.

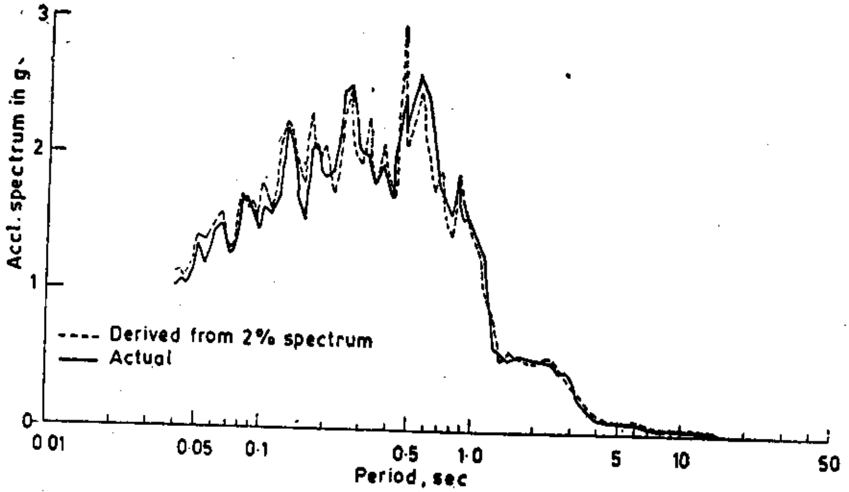

Fig. 8. Spectrum Of El Centro 1940, Sooe Accelerogram For 5% Damping.

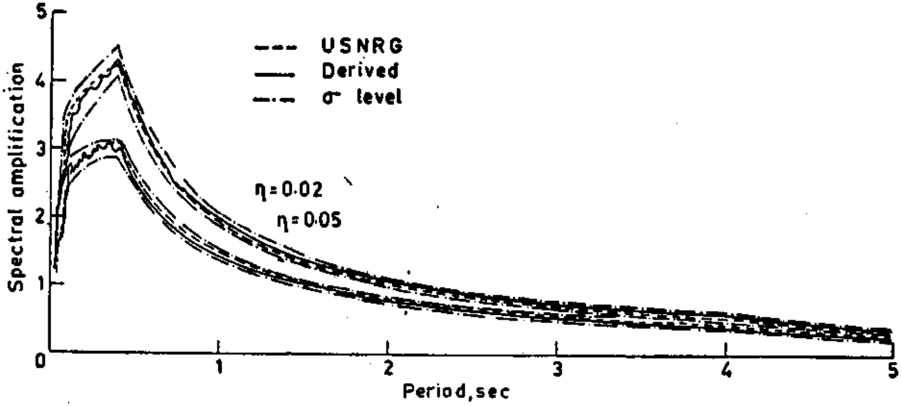

Fig. 10. Derived Spectra From Given CSA Spectra

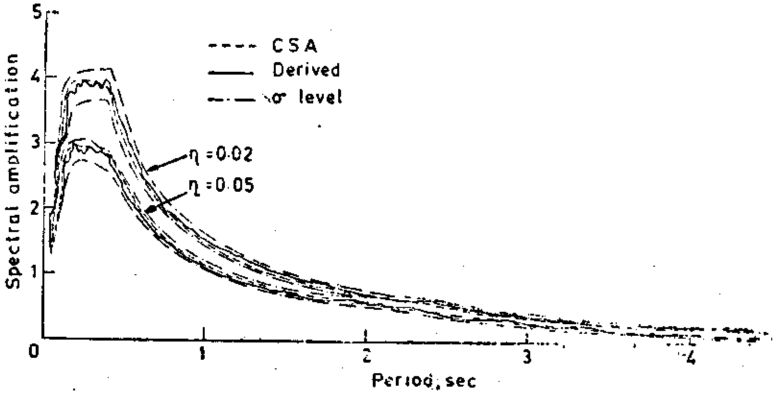

Fig. 9. Derivee Spectra From Given Usnrg Spectra

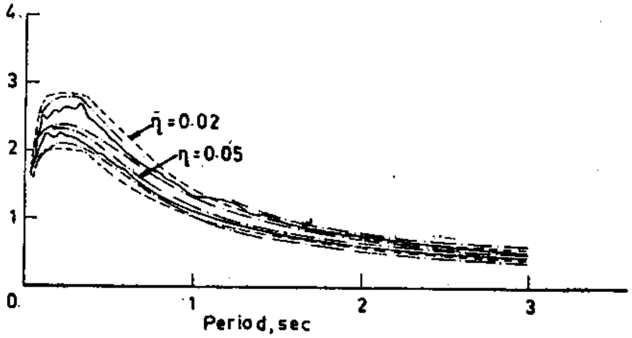

Fig. 11. Derived Spectra From Given Is-1893 Fourth Devision Draft Spectra

It is worthwhile to note that both USNRG and CSA spectra show a high level of consistency. In case of the ISI spectra, there are appreciable deviations between the derived and the actual results. Since the present regression is based on a finite number of sampels, the effect of the sample size will have to be studied. This is done by estimating the standard deviation of the regression coefficients (Benjamin and Cornell[9]) at several periods of the spectra. The standard deviation above and below the mean levels are obtained and plotted in figures 9, 10 and 11. These may be thought of as the consistency bands within which the derived spectrum should lie to be consistent with a given smooth spectrum at the other value of damping. It is seen that the USNRG and the CSA spectra follow the consistency bands except at a few isolated points. However, the proposed ISI spectra lie outside the consistency bands for both the 2% and 5% damping values

SUMMARY AND CONCLUSIONS

A fresh statistical analysis of response spectral data of sixty-six actual earthquake records has been presented. The mean plus standard deviaton results have been compared with the smooth spectra of Newmark, Blume and Kapur. Considerable differences are found particularly in the high frequency end of the spectra The variation of the spectra with respect to damping has been studied by regression analysis The relation between any two damped spectra are found to be well represented by a linear relationship. The regression coefficients given can be used in costructing a new spectrum which is consistent to a given damped spectrum. The internal consistency of the spectra provided in USNRG, CSA and ISI Codes are checked by using the results obtained here. It is found that while first two

Codes. have put forward smooth spectra which are internally consistent, the spectra of the draft Indian Code for different values damping are not internally consistent.

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