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FORCED HARMONIC VIBRATION OF SANDWHICH PLATES

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Summary

Static analysis of Sandwich plates by Eric Reissner [1] has been modified by including inertia terms in order to study the amplitude of forced harmonic vibrations of sandwich axisymmetric circular plates. Solution is found for the simply supported case and adapted for a viscoelastic core.

Notations

(2)	Reference No,
h	core layer thichness
ρ, ρ_c	the mass densities of the face and core respectively
t	each face layer thickness
q	axisymmetric load intensity
E_f, G_f, γ	Elastic modulii of isotropic face layer material
E_c, G_c	Elastic modulii of core layer material
w	deflection of the sandwich plate
W	amplitude of vibration
D	bending stiffness factor $= \frac{t\,(h+t)^2 E_f}{2\,(1-\gamma^2)}$
r	radial coordinate of any point in the plane of the flat plate
R	radius of the circular plate

$$\nabla^2 = \frac{d^2}{dr} + \frac{1}{r}\frac{d}{dr}$$

Assumptions

1. t/h is small compared with unity
2. $E_f\, t/E_c\, h$ is large compared with unity
3. Each face layer is a membrane without bending stiffness
4. Stresses in the core, which are parallel to the faces are negligible compared with transverse shear

Analysis

Eric Reissner(1) gives the following equation for the static analysis of a sandwich plate

$$D\,\nabla^2\,\nabla^2\,w = q - \frac{D\,\nabla^2 q}{(h+t)\,G_c} \qquad (1)$$

It is abvious that to modify the equation to describe the dynamical problem, it is necessary only to add the inertia terms to the equation. The deflection then becomes a function of both radius and time.

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If a load intensity $q = q_o \sin pt$ is assumed, the above equation gets modified to the following, to describe the vibration $w(r,t) = W(r) \sin pt$.

$$D \nabla^2 \nabla^2 W - Ap^2 W = q_o - \frac{D \nabla^2 q_o}{(h+t) G_c} - \frac{D \nabla^2 A p^2 W}{(h+t) G_c} \quad (2)$$

where $A = 2 \rho t + \rho_c h$

the time factor sin pt gets cancelled throughout in equation (2) confining attention to a circular axisymmetric plate, simply supported edges lead to the following boundry conditions :

$$\text{at } r = R, \quad W = 0$$

$$\text{and } \frac{d^2 W}{dr^2} + \frac{\gamma}{r} \frac{dW}{dr} = 0 \quad (3)$$

$$\text{at } r = 0, \frac{dW}{dr} = 0 \quad \text{(because of axisymmetry)}$$

W. Nowacki[2] modifies the edge moment condition (Equation 3) to

$$\frac{d^2 W}{dr^2} + \frac{1}{r} \frac{d W}{dr} = 0$$

This modification has been observed to have negligible effect on the results[4]. This approximation is taken.

Now introducing Hankel transforms.

$$W^* = \int_0^R r J_o (a_1 r) W \, dr$$

$$q_o^* = \int_0^R r J_o (a_1 r) q_o \, dr$$

$$\int_0^R r \nabla^2 W J_o (a_1 r) = \left[r \frac{dW}{dr} J_o (a_1 r) - a_1 r W J_o' (a_1 r) \right]_0^R - a_1^2 \int_0^R r W J_o (a_1 r) \, dr$$

The expression in brackets vanishes for upper limit provided $J_o (a_1 R) = 0$ and for the lower limit it always is zero.

Thus if parameter a_1 satisfies the equation $J_o (a_1 R) = 0$

$$\int_0^R r \nabla^2 W J_o (a_1 r) = - a_1^2 W^*$$

$$\int_0^R r \nabla^2 \nabla^2 W J_o (a_1 r) = a_1^4 W^*$$

$$\text{and} \quad \int_0^R r J_o (a_1 r) \nabla^2 q_o = - a_1^2 q_o^*$$

Substituting the above Hankel transforms in Equation (2)

$$D\,\alpha_1^4\,W^* - Ap^2\,W^* = q_o^* + \frac{D\,\alpha_1^2\,q_o^*}{(h+t)\,G_c} + \frac{DAp^2\,\alpha_1^2\,W^*}{(h+t)\,G_c}$$

or $$W^*\left(D\,\alpha_1^4 - \frac{D\,A\,p^2\,\alpha_1^2}{(h+t)\,G_c} - A\,p^2\right) = q_o^*\left[1+\frac{D\,\alpha_1^2}{(h+t)\,G_c}\right]$$

or $$W^* = \frac{q_o^*\left[1+\frac{D\,\alpha_1^2}{(h+t)\,G_c}\right]}{D\,\alpha_1^4 - \frac{D\,A\,p^2\,\alpha_1^2}{G_c\,(h+t)} - Ap^2}$$

By inverse transformation

$$W = \frac{2}{R^2}\sum_{i=1}^{i=\infty} \frac{q_o^*\left[1+\frac{D\,\alpha_1^2}{(h+t)\,G_c}\right]}{\left(D\,\alpha_1^4 - \frac{D\,A\,p^2}{(h+t)\,G_c} - Ap^2\right)} \frac{J_o(\alpha_1\,r)}{[J_1(\alpha_1\,R)]^2}$$

$$= \frac{2}{R^2}\sum_{i=1}^{i=\infty} \frac{\left[\int_0^R r\,J_o\,(\alpha_1\,r)\,q_o\right]\left[1+\frac{D\,\alpha_1^2}{(h+t)\,G_c}\right]}{\left(D\,\alpha_1^4 - \frac{D\,A\,p^2}{(h+t)\,G_c} - A\,p^2\right)} \frac{J_o\,(\alpha_1\,r)}{[J_1\,(\alpha_1\,R)]^2} \qquad (4)$$

where α_1 satisfies the equation

$$J_o\,(\alpha_1\,R) = 0$$

By eliminating the terms containing G_c, the equation (2) yields the expected form of solution for the homogeneous plate(2).

The solution is a series solution when each term is a constant multiplied by zero-th order Bessel function and as

$$\nabla^2\,J_o\,(\alpha_1\,r) = -\,J_o\,(\alpha_1\,r)$$

at $r = R$

$$\nabla^2\,J_o\,(\alpha_1\,r) = -\,J_o\,(\alpha_1\,R) = 0$$

This is the apprximate expression for the edge moment condition (Equation 3).

So the edge moment condition is satisfied.

Similarly it can be easily seen that the solution satisfies the other boundary conditions also.

To adapt equation (4) to a viscoelastic core, G_c can be made a complex modulus(3).

Let $G_c = G_{c1}\,(1 + j\,\beta)$

The solution for the viscoelastic core would modify to

$$W = \frac{2}{R^2}\sum_{i=1}^{i=\infty} \frac{[G_{c1}\,(1+j\,\beta)\,(h+t) + D\,\alpha_1^2]\int_0^R r\,J_o(\alpha_1\,r)\,q_o}{G_{c1}\,(1+j\,\beta)\,(h+t)\,[D\alpha_1^4 - A\,p^2] - D\,A\,p^2\,\alpha_1^2} \cdot \frac{J_o\,(\alpha_1\,r)}{[J_1\,(\alpha_1\,R)]^2}$$

References

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