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HYDRODYNAMIC EARTHQUAKE COEFFICIENTS OF BRIDGE PIERS

BY

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INTRODUCTION

Bridges in seismically active zones have to be designed with special considerations of the dynamic forces caused by an earthquake. These forces acting on the various components of a bridge and their effects on the bridge as a whole structure have to be carefully evaluated to ensure its stability and its availability for rescue operations during an earthquake. An earthquake or an explosion gives rise to unsteady flow at the bridge piers. This kind of flow occurs also when a flood wave or a tidal wave moves past a pier, an intake or a light-house tower. Evaluation of forces on structures in unsteady flow is an important consideration in their design. The problem is mathematically involved and exact solutions are available only for a few elementary shapes of these structures. Historically, Stokes[5] calculated this force on a circular cylinder in 1850. In 1920 Riabouchinsky[6] obtained expressions for these forces on rectangular cylinders. In 1933 Westergaard[10] gave a solution to determine the pressures on a rectangular vertical dam subjected to horizontal acceleration. Jacobsen[3] in 1949 gave the solution for a cylindrical pier taking into account the free-surface condition. In 1957 Stelson and Mavis[8] reported experimental results of the hydrodynamic forces on rectangular piers.

In the analysis of these problems the Laplace equation in the appropriate coordinate system is solved for the boundary conditions, namely, that the liquid in contact with a rigid wall sticks to it and that the pressure on the free surface is constant, and the solution implies an ideal fluid. Bharat Singh and A.K. Jain[7] compared their measurements with Jacobsen's theory for circular piers and with Westergaard's theory for rectangular piers. The American and the Indian[7] practices are based on Lamb's cylinder analogy (a misnomer !) and Westergaard's theory. Riabouchinskey's theory has been overlooked by the earthquake engineers although his analysis like that of Jacobsen's is rational and valid for rectangular piers. On the other hand, application of Westergaard's theory to rectangular piers is questionable. These aspects are examined in this paper. Dimensionless graphs are given to facilitate calculation of the hydro-dynamic forces on circular and rectangular piers.

ADDED-MASS APPROACH

When a bridge pier vibrates in a channel flow, it is subjected to two types of force; one is due to normal and tangential (frictional) stresses which make up the viscous drag force acting in the flow direction, and another due to acceleration of water which acts normally on the pier. The later force is in effect analogous to increasing the mass (inertia) of the pier. This apparent increase in the mass of the body is termed the added mass, and the ratio of the added mass to the mass of water displaced by the pier is known as the added-mass coefficient, denoted by A. It is also called as the hydro-dynamic inertia coefficient. It depends primarily on the geometry of the pier and its orientation to ground motion, and to some extent on the proximity of the boundary. Fluid viscosity has but inappreciable effect on it which may be neglected for all practical purposes. By virtual mass is meant the body's own mass plus the mass of the fluid which is accelerated by it. Some call the ratio of the virtual mass to the mass of water displaced by the body as virtual inertial factor or mass coefficient. There is, however, some confusion in the usage of these terms. It is suggested that the hydrodynamicist's notation-added-mass coefficient-may be used by the earthquake engineers also.

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There is an elegant mathematical theory in hydrodynamics (potential-flow theory) to compute the added-mass coefficients of regular bodies. For a translating sphere it is 0.5; for a long circular cylinder $A=1.0$; and for an elliptic cylinder of fineness ratio 4, $A=4$. The added-mass concept is very useful in calculating the unsteady flow forces on hydraulic structures oscillating in water with the ground motion during earthquakes. However, this concept cannot be extended to high frequencies when the compressibility effects of water become appreciable, because the added-mass calculations are based on the assumption of incompressibility of water. As an example, consider a pier in 30 metres height of water subjected to an earthquake of dominant frequency, 20 hertz. The velocity of pressure wave in water at 15°C and at atmospheric pressure is 1470 m/sec. The dimensionless parameter—a measure of elastic forces relative to inertial forces—for this oscillatory flow field is $n^2 L^2/c^2=(1/6)<1$. This shows that water behaves as an incompressible liquid at such a high frequency of 20 Hz uncommon even with destructive earthquakes. Furthermore, at such high frequencies the pier oscillates in its own wake of a complicated flow field.

Once the added-mass coefficient of a pier is known, the hydrodynamic force on it can be computed by multiplying it with the mass of water displaced by the pier and the acceleration. The variation of this coefficient with the height of water and with the fineness ratio (breadth/thickness) of the pier and the point of action of this force are discussed in the following paragraphs.

CIRCULAR PIERS

Viscous drag:—The dimensionless drag coefficient, C_D, when multiplied by $\rho A_1 V^2/2$ gives the drag force on the pier in the flow direction. For a given geometry of the pier it depends on the Reynolds number (R) and the roughness of pier. For circular pier $C_D=1.2$ for $R=4\times10^4$. There is a transition in the boundary-layer flow as a consequence of which C_D drops to 0.4. It was only recently that Roshko[6] reported a second upward transition at $R=4\times10^6$ which increases C_D to 0.78. Due to uncertainties in predicting C_D in transitional flow it is suggested that the following criteria may be adopted:

$C_D=1.2$ ... $10^3<R<4\times10^5$

$C_D=0.78$ ... ... $R>4\times10^5$

Since the Reynolds number of a prototype pier is usually greater than 4×10^5, a single value of $C_D=0.78$ may suffice for design purposes.

Force fluctuations due to turbulence:—The drag coefficients given above correspond to temporal mean flow velocity. Instantaneous value of the drag force can be obtained by adding to the mean drag the fluctuation, F'_D, for which the coefficient is C'_D. The force fluctuations are random in nature and closely follow the normal law of error. Hence it is possible to obtain the values of F'_D from the R.M.S. values reported by Vickery[9] from model tests in a water tunnel with turbulence of intensity approximately 10 percent and of scale of the order of the lateral dimension of the pier. The variation of F'_D/F_D with the slenderness ratio is shown in Fig. 1.

Hydrodynamic earthquake forces:—Jacobsen's analysis for circular piers yields a complex formula as a series of transcendental functions. The solution has been plotted in Fig. 2 as a variation of the added-mass coefficient with the slenderness ratio H/D, and in this form it facilitates the design engineers an ease in application. It can be seen that A increases with slenderness ratio and attains the Stokes value of 1.0 at $H/D=10$. The channel bottom has an obvious effect on the added mass only for shallow depths of water because of the changes in the irrotational flow patterns. Evidently the height of water at which the added mass is no longer affected is about 10 times the pier thickness (i.e. $H/D>10$).

The series in Jacobsen's expression was approximated by a power law which upon integration, yielded the point of application of the resultant at 0.41 height of water above

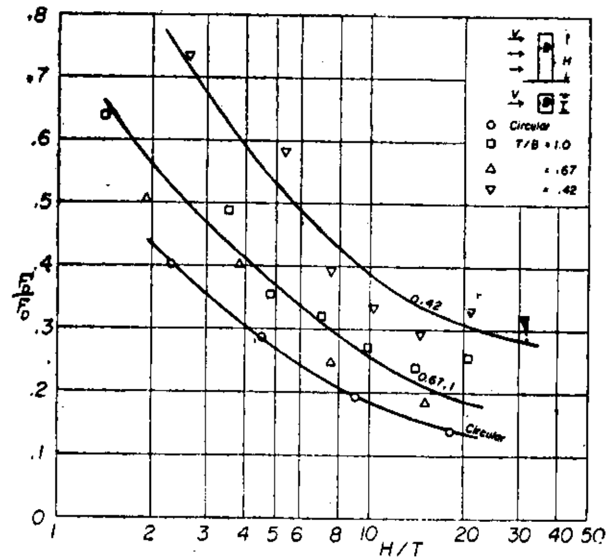

Fig. 1. Variation of force fluctuation due to turbulence with slenderness ratio for circular and rectangular piers

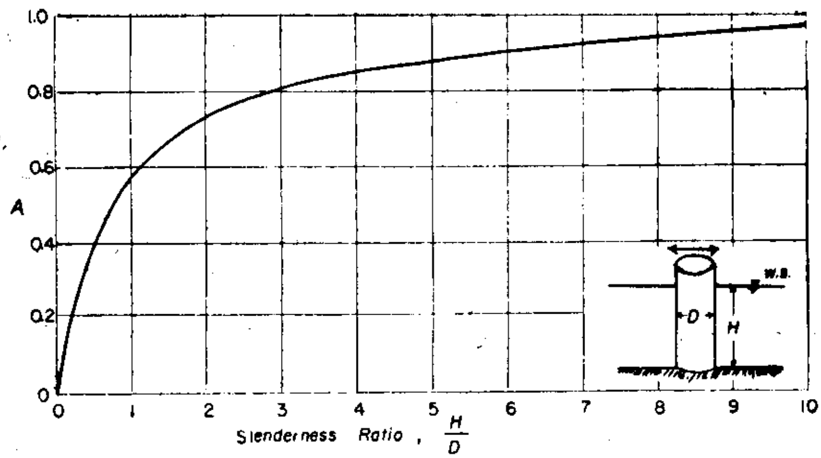

Fig. 2. Variation of added-mass coefficient with slenderness ratio for circular piers

the bed level. To find the design force on the pier in the flow direction, one has only to add all the three forces so computed (Fig. 3). It may, however, be mentioned here that the added mass of a pier in accordance with the potential-flow theory, is invariant with the flow velocity in the channel at the bridge.

RECTANGULAR PIERS

Viscous drag:—For different fineness ratios the mean drag coefficients are plotted in Fig. 4 for rectangular piers with a blunt leading edge and with rounded ends. These curves are prepared from experimental data[2]. It is evident that C_D for a blunt edged section is

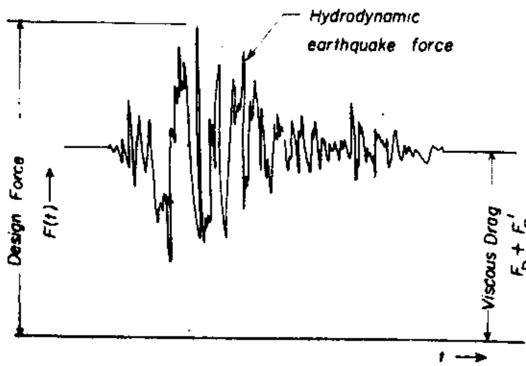

Fig. 3. Superposition of hydrodynamic earthquake force on viscous drag of a pier

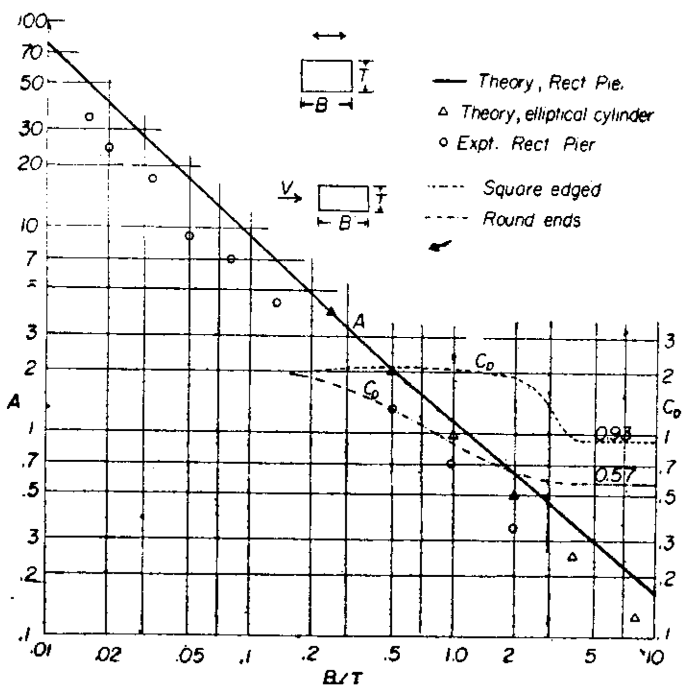

Fig. 4. Variation of added-mass coefficient and drag coefficient with aspect ratio for rectangular piers

larger than for a pier with rounded ends. In both the cases the value of C_D decreases as the fineness ratio is increased, and attains at $B/T=4$ constant values of $C_D=0.93$ for a pier with square edges and $C_D=0.57$ for a pier with rounded ends. In the case of piers with larger fineness ratios the separated flow reattaches at some distance downstream from the leading edge with almost identical wake characteristics and hence the drag coefficients remain constant.

Turbulence fluctuations:—In Fig. 1 are shown the variations of the fluctuation component of force due to turbulence (of the same description as given under circular piers) for different slenderness ratios and for fineness ratios 1.0, 0.67 and 0.42. Due to the inherent nature of turbulence the experimental data are not consistent. Yet, there are definite trends which enable extrapolation for any given pier shape. It can be also seen that the fluctuation force becomes a constant proportion of the mean drag at large ratios of slenderness.

Earthquake forces:—It is rational to compute the hydrodynamic forces on rectangular piers by using Riabouchinsky's analysis based on potential-flow theory. His equations are complex and require evaluation of special functions. His expressions are plotted in this paper on a log-log graph (Fig. 4) showing the variation of the added mass coefficient with the fineness ratio of the pier. On the same graph are shown the experimental data obtained by Stelson and Mavis, and the theoretical values for elliptical cylinders of different fineness ratios. The experiments were performed with water and the results are consistently lower than what ideal-fluid theory predicts, although the effect of fluid viscosity should be in accordance with Stokes' formula[6] to increase very slightly the values of the added-mass coefficient. It is interesting to note that there is little difference in the value of the added-mass coefficient of a bridge pier of the same fineness ratio whether it is streamlined or not. It is also not an uncommon practice in aerodynamics to approximate elongated bodies by elliptical shapes. For this reason it is recommended here that the hydrodynamic forces on bridge piers streamlined or not, may be computed from the theoretical curve in Fig. 4. If an earthquake is assumed to occur in the thickness direction of a pier, then the hydrodynamic force on it can be easily found from Fig. 4 by reading the added-mass coefficient for the reciprocal of the fineness ratio of the pier. Westergaard's theory considers a two-dimensional plate retaining water on one side of it, and it represents in no way a bridge pier surrounded by water for which the hydrodynamic force has to be obtained by integrating the pressure around its body contour. It is, therefore, fallacious to apply Westergaard's theory to rectangular piers even as an approximation. Photographs of bridges[4] that failed during an earthquake show that in some cases (See Figs. 2a & 2b in Ref. 4) one end of the deck slab slipped down from the pier into the water while the other end was resting on the pier. This can be caused by the pier set into vibration in a transverse direction. The pier is weaker in its thickness direction and the vibration effects from an earthquake in this direction need to be carefully examined by the design engineer. The flexibility of the pier which is not considered in this paper becomes very important in this case.

THE INDIAN SPECIFICATIONS

For circular or rectangular bridge piers the specifications[7] recommended the use of Lamb's cylinder analogy if the transverse dimension of pier is less than half the height of water (i.e. H/T, $H/B>2$), and Westergaard's theory otherwise.

Consider a circular pier. Theory predicts that $A=0.74$ (Fig. 2) at $H/D=2$ while the specifications stipulate $A=1.0$, nearly 50 percent in excess of the actual force.

Consider a rectangular pier with $B=15$m, $T=2$m and $H=30$m for which H/T, $H/B>2$. As per the specification, $A=6.07$ for an earthquake occurring in any direction, while theoretically $A=0.2$ for an earthquake occurring parallel to the pier and $A=7.0$ in the transverse direction. It is obvious that the hydrodynamic force in one direction is overestimated, and it is underestimated in another direction (thickness-wise). The pier can vibrate adversely and fail in the later case. If the water depth, $H=20$m, $H/B<2$, then using Westergaard's theory, $A=5.83$ in the transverse direction. It decreases as H/T is further decreased. In view of the above discrepancies it is suggested that the following specification may be considered.

For circular piers with $H/D > 10$, $A = 1.0$, and the hydrodynamic force acts at the midpoint of the height of water. For $H/D \leqslant 10$, the value of A should be read from Fig. 2, and the force acts at 0.41H above the river bed.

For rectangular piers, A should be read from Fig. 4 for the corresponding fineness ratio, T/B of the pier section and it may be taken to act at 0.41 H above the river bed. The hydrodynamic force in the thickness-wise direction has to be found out for fineness ratio B/T, and the safety of the pier against the force and vibration due to an earthquake in this direction has to be carefully examined. If $H/T > 10$, the point of application of the hydrodynamic force may be taken at 0.5H above the river bed.

CONCLUSIONS

The following conclusions are given on the basis of the above discussion:

1. In the design of bridge piers in seismic areas, the moment due to hydrodynamic force on the pier generated by a horizontal earthquake has to be added to the moment due to viscous drag which should also include the turbulence fluctuation.
2. The use of Jacobsen's analysis for circular piers and Riabouchinsky's theory for rectangular piers is recommended for the determination of the hydrodynamic forces on bridge piers. Necessary graphs are furnished in this paper for an easy calculation of these forces using the added-mass approach.
3. The transverse hydrodynamic force on a bridge pier needs special consideration while examining its safety against an earthquake in this direction.

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NOTATION

A = added mass coefficient,
A_1 = area of a pier projected on a plane normal to flow,
B = breadth of pier,
c = velocity of propagation of pressure wave,
C_D = coefficient of drag,
D = diameter of circular pier,
F_D = viscous drag force,
F'_D = turbulence fluctuation part in drag force,
H = height of water,
L = H,
n = frequency of vibration,
R = Reynolds number,
T = thickness of pier,
V = mean velocity of flow in river,
ν = kinematic viscosity of water,
ρ = mass density of water,
$\frac{B}{H}, \frac{D}{H}, \frac{T}{H}$ = slenderness ratios, and
$\frac{B}{T}, \frac{T}{B}$ = fineness ratios.