Paper No. 229, Bull. ISET, Vol. 20, No. 2, June 1983, pp. 41–51

INERTIA CHANGES FOR A MULTILAYERED SPHERE DUE TO A DISPLACEMENT DISLOCATION

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ABSTRACT

The displacement field due to an arbitrary tangential dislocation buried in a layered sphere has been derived by Wason and Singh (1972). Using the displacement field at any arbitrary point, the changes in the components of the inertia tensor due to a buried displacement dislocation are theoretically calculated for a multilayered spherical earth model. The inertia changes are first derived for the special case in which the epicentre lies on the earth's axis of rotation. Then, through the orthogonal transformation, the epicentral axes are carried to the geographical axes and the inertia changes have been calculated with reference to the geographical system. The results have been derived for three cases of shear dislocations: vertical strike-slip, vertical dip-slip and dip-slip on a 45 degree dipping plane.

INTRODUCTION

Press (1965) calculated the displacements, strains and tilts due to a static dislocation in a half-space. Mansinha and Smylie (1967) computed the changes in the products of inertia of the Earth due to a rearrangement of masses associated with major earthquakes and then calculated their contribution to the excitation of the Chandler wobble and the secular polar shift. As a mathematical model, they used vertical, rectangular strike-slip and dip-slip faults in a uniform half-space.

The extension to spherical Earth models was first reported by Ben-Menahem and his co-workers. Starting from the results of Ben-Menahem and Singh (1968), Ben-Menahem and Israel (1970) obtained the displacement field at any point within a sphere and then calculated the inertia changes due to an arbitrary displacement dislocation in a uniform sphere. These authors found that a spherical Earth model yields higher inertia changes than the corresponding half-space model. Smylie and Mansinha (1971) computed the changes in the inertia tensor of a realistic Earth model taking account of self-gravitation and initial hydrostatic stress. They showed that the changes in the inertia tensor computed for a real Earth model were higher than both for a uniform spherical model and for a half-space model. Dahlen (1971, 1973) computed the changes in the products of inertia of a real Earth model caused by the displacement on

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a fault of arbitrary location and orientation. Israel, Ben-Menahem and Singh (1973) calculated the changes in the inertia tensor for a real Earth model due to an arbitrary displacement dislocation. These authors treated in detail the singular case corresponding to the Legendre polynomial of the first order.

In this paper the changes in the inertia tensor for a layered spherical Earth model due to a buried arbitrary displacement dislocation are computed theoretically.

THE DISPLACEMENT FIELD DUE TO A BURIED TANGENTIAL DISLOCATION

The displacement field at any point of a layered sphere due to an arbitrary tangential dislocation was evaluated by Wason and Singh (1972). We express the displacement field at any point (r, θ, ϕ) in the ith layer for three fundamental shear dislocations in the following form.

Case I—Vertical Strike-Slip $(m = 2)$

$$u_r(r, \theta, \phi) = -\sum_{n=2}^{\infty} x_i^{2,n}(r) P_n^2(\cos\theta)\sin 2\phi,$$

$$u_\theta(r, \theta, \phi) = \sum_{n=2}^{\infty}\left[\frac{4}{\sin\theta} z_i^{2,n}(r) - y_i^{2,n}(r)\frac{\partial}{\partial\theta}\right]P_n^2(\cos\theta)\sin 2\phi, \tag{1}$$

$$u_\phi(r, \theta, \phi) = 2\sum_{n=2}^{\infty}\left[z_i^{2,n}(r)\frac{\partial}{\partial\theta} - \frac{1}{\sin\theta} y_i^{2,n}(r)\right]P_n^2(\cos\theta)\cos 2\phi.$$

Case II—Vertical Dip-Slip $(m = 1)$

$$u_r(r, \theta, \phi) = -2\sum_{n=1}^{\infty} x_i^{1,n}(r) P_n^1(\cos\theta)\sin\phi,$$

$$u_\theta(r, \theta, \phi) = 2\sum_{n=1}^{\infty}\left[\frac{1}{\sin\theta} z_i^{1,n}(r) - y_i^{1,n}(r)\frac{\partial}{\partial\theta}\right]P_n^1(\cos\theta)\sin\phi, \tag{2}$$

$$u_\phi(r, \theta, \phi) = 2\sum_{n=1}^{\infty}\left[z_i^{1,n}(r)\frac{\partial}{\partial\theta} - \frac{1}{\sin\theta} y_i^{1,n}(r)\right]P_n^1(\cos\theta)\cos\phi.$$

Case III—Dip-Slip on a 45° Plane $(m = 0, 2)$

$$u_r(r, \theta, \phi) = 3\sum_{n=0}^{\infty} x_i^{0,n}(r) P_n(\cos\theta) + \frac{1}{2}\sum_{n=2}^{\infty} x_i^{2,n}(r) P_n^2(\cos\theta)\cos 2\phi,$$

$$u_\theta(r, \theta, \phi) = 3\sum_{n=1}^{\infty} y_i^{0,n}(r)\frac{\partial P_n(\cos\theta)}{\partial\theta} - \frac{1}{2}\sum_{n=2}^{\infty}\left[\frac{4}{\sin\theta} z_i^{2,n}(r) - y_i^{2,n}(r)\frac{\partial}{\partial\theta}\right]P_n^2(\cos\theta)\cos 2\phi, \tag{3}$$

$$u_\phi(r,\theta,\phi)=\sum_{n=2}^{\infty}\left[z_l^{2,n}(r)\frac{\partial}{\partial\theta}-\frac{1}{\sin\theta}y_l^{2,n}(r)\right]P_n^2(\cos\theta)\sin 2\phi.$$

Case III corresponds to a fault with the slip angle $\lambda^*=90°$ and dip angle $\delta=45°$. The displacement field due to an arbitrarily oriented tangential dislocation is given by a linear combination of these three fundamental fields:

$$\mathbf{u}=\cos\lambda^*[\sin\delta\mathbf{u}_\mathrm{I}+\cos\delta\mathbf{u}_\mathrm{II}^{**}]+\sin\lambda^*[\sin 2\delta\mathbf{u}_\mathrm{III}-\cos 2\delta\mathbf{u}_\mathrm{II}], \tag{4}$$

where $\mathbf{u}_\mathrm{I}$, $\mathbf{u}_\mathrm{II}$, $\mathbf{u}_\mathrm{III}$ are the displacement fields for cases I, II and III respectively. $\mathbf{u}_\mathrm{II}^{**}$ is obtained from $\mathbf{u}_\mathrm{II}$ by replacing ϕ by $(\phi-\pi/2)$. The radial functions $x_l^{mn}(r)$,$y_l^{mn}(r)$ and $z_l^{mn}(r)$ are given by

$$x_l^{mn}(r)=\sum_{q=1}^{4}[M_{11}M_{1q}^{-1}+M_{12}M_{2q}^{-1}+M_{13}M_{3q}^{-1}+M_{14}M_{4q}^{-1}](J_l^{mn}(r_l))_q,$$

$$y_l^{mn}(r)=\sum_{q=1}^{4}[M_{21}M_{1q}^{-1}+M_{22}M_{2q}^{-1}+M_{23}M_{3q}^{-1}+M_{24}M_{4q}^{-1}](J_l^{mn}(r_l))_q, \tag{5}$$

$$z_l^{mn}(r)=\sum_{q=1}^{2}[{}_LM_{11}\,{}_LM_{1q}^{-1}+{}_LM_{12}\,{}_LM_{2q}^{-1}]({}_LJ_l^{mn}(r_l))_q,$$

where M_{kl}, ${}_LM_{kl}$, M_{kl}^{-1} and ${}_LM_{kl}^{-1}$ represent the (kl)th element of matrices $M_l^n(r)$, ${}_LM_l^n(r)$, $[M_l^n(r_l)]^{-1}$ and $[{}_LM_l^n(r_l)]^{-1}$ respectively.

The expressions for these matrices and the column matrices $J_l^{mn}(r_l)$ and ${}_LJ_l^{mn}(r_l)$ are given elsewhere (Wason and Singh, 1972).

CHANGES IN THE COMPONENTS OF THE INERTIA TENSOR IN THE EPICENTRAL COORDINATE SYSTEM

We shall follow the procedure adopted by Ben-Menahem and Israel (1970) for the case of homogeneous sphere. The inertia dyadic $\bar{J}$ for the undeformed sphere about its axis of rotation is given by the volume integral

$$\bar{J}=\int_V[\bar{I}(\mathbf{r}\cdot\mathbf{r})-\mathbf{rr}]dm, \tag{6}$$

where $\bar{I}$ is the unit dyadic and $\mathbf{r}$ is the position vector at any point before the deformation. The integration is to be carried out over all mass elements *dm* in the relevant volume V. To the first order in small quantity $|\mathbf{u}|/|\mathbf{r}|$, the change in the inertia dyadic, $\bar{\Theta}$, is given by

$$\bar{\Theta}=\int_V[2(\mathbf{r}\cdot\mathbf{u})\,\bar{I}-(\mathbf{ru}+\mathbf{ur})]\,\mathrm{dm}. \tag{7}$$

We set up the epicentral coordinate system with unit vectors ($\mathbf{e}_{\theta_0}$, $\mathbf{e}_{\phi_0}$,

$\mathbf{e}_{r_0}$) in such a way that the $\mathbf{e}_{r_0}$ axis coincides with the rotation axis, the $\mathbf{e}_{\theta_0}$ axis points in the direction of the fault's azimuth ($\Phi = 0$) and the $\mathbf{e}_{\Phi_0}$ axis in a direction opposite to the normal $\mathbf{n}_0$ to the source element dS on a vertical fault (see Fig. 1). Denoting by $\Theta_{ij} = \Theta_{ji}$, the cartesian elements of the inertia change dyadic with respect to this coordinate system, we have

$$\Theta_{ij} = \bar{\Theta} : \mathbf{e}_i \mathbf{e}_j, \tag{8}$$

where the indices 1, 2, 3 correspond to $\mathbf{e}_{\theta_0}$, $\mathbf{e}_{\Phi_0}$, and $\mathbf{e}_{r_0}$, respectively and the symbol (:) signifies the double dot product. The unit vectors $\mathbf{e}_{\theta_0}$, $\mathbf{e}_{\Phi_0}$, $\mathbf{e}_{r_0}$ are related with the unit vectors $\mathbf{e}_\theta$, $\mathbf{e}_\Phi$, $\mathbf{e}_r$ in the directions of θ, Φ, r, respectively, through the relations

$$\begin{aligned} \mathbf{e}_{\theta_0} &= \mathbf{e}_r \sin\theta \cos\Phi + \mathbf{e}_\theta \cos\theta \cos\Phi - \mathbf{e}_\Phi \sin\Phi, \\ \mathbf{e}_{\Phi_0} &= \mathbf{e}_r \sin\theta \sin\Phi + \mathbf{e}_\theta \cos\theta \sin\Phi + \mathbf{e}_\Phi \cos\Phi, \\ \mathbf{e}_{r_0} &= \mathbf{e}_r \cos\theta - \mathbf{e}_\theta \sin\theta. \end{aligned} \tag{9}$$

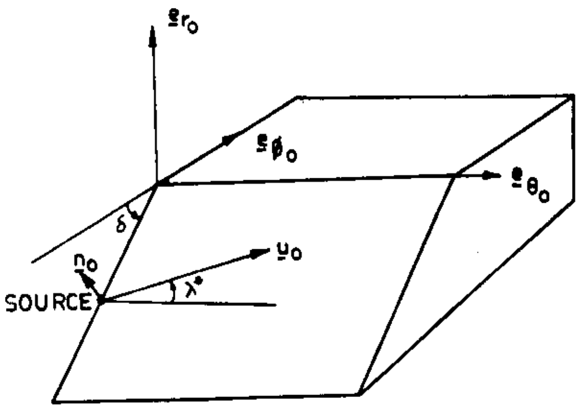

Fig. 1 Geometry of a Tangential Dislocation

In the ith layer, we take

$$\begin{aligned} \mathbf{u} &= u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta + u_\Phi \mathbf{e}_\Phi \\ dm_i &= \rho_i r^2 \sin\theta \, dr \, d\theta \, d\Phi, \end{aligned} \tag{10}$$

where dm_i and ρ_i denote, respectively, mass element and the density of the ith layer. From equations (7)–(10), we obtain the following integrals for the inertia-change dyadic.

$$\Theta_{11} = \sum_{i=0}^{p} \int_{r_i}^{r_{i+1}} \rho_i \, [2 \int_0^\pi \int_0^{2\pi} u_r(r, \theta, \Phi) \sin\theta \, d\theta \, d\Phi$$

$$- 2 \int_0^\pi \int_0^{2\pi} u_r(r, \theta, \Phi) \sin^3\theta \cos^2\Phi \, d\theta \, d\Phi$$

$$- \int_0^{\pi}\int_0^{2\pi} u_\theta(r, \theta, \Phi) \sin\theta \sin 2\theta \cos^2\Phi \, d\theta \, d\Phi$$

$$+ \int_0^{\pi}\int_0^{2\pi} u_\Phi(r, \theta, \Phi) \sin^2\theta \sin 2\Phi \, d\theta \, d\Phi] \, r^3 \, dr, \qquad (11)$$

$$\Theta_{22} = \sum_{i=0}^{p} \int_{r_i}^{r_{i+1}} \rho_i [2 \int_0^{\pi}\int_0^{2\pi} u_r(r, \theta, \Phi) \sin\theta \, d\theta \, d\Phi$$

$$- 2 \int_0^{\pi}\int_0^{2\pi} u_r(r, \theta, \Phi) \sin^3\theta \sin^2\Phi \, d\theta \, d\Phi$$

$$- \int_0^{\pi}\int_0^{2\pi} u_\theta(r, \theta, \Phi) \sin\theta \sin 2\theta \sin^2\Phi \, d\theta \, d\Phi$$

$$- \int_0^{\pi}\int_0^{2\pi} u_\Phi(r, \theta, \Phi) \sin^2\theta \sin 2\Phi \, d\theta \, d\Phi] \, r^3 \, dr, \qquad (12)$$

$$\Theta_{33} = \sum_{i=0}^{p} \int_{r_i}^{r_{i+1}} \rho_i [2 \int_0^{\pi}\int_0^{2\pi} u_r(r, \theta, \Phi) \sin^3\theta \, d\theta \, d\Phi$$

$$+ \int_0^{\pi}\int_0^{2\pi} u_\theta(r, \theta, \Phi) \sin\theta \sin 2\theta \, d\theta \, d\Phi] \, r^3 \, dr, \qquad (13)$$

$$\Theta_{12} = - \sum_{i=0}^{p} \int_{r_i}^{r_{i+1}} \rho_i [\int_0^{\pi}\int_0^{2\pi} u_r(r, \theta, \Phi) \sin^3\theta \sin 2\Phi \, d\theta \, d\Phi$$

$$+ \frac{1}{2} \int_0^{\pi}\int_0^{2\pi} u_\theta(r, \theta, \Phi) \sin\theta \sin 2\theta \sin 2\Phi \, d\theta \, d\Phi$$

$$+ \int_0^{\pi}\int_0^{2\pi} u_\Phi(r, \theta, \Phi) \sin^2\theta \cos 2\Phi \, d\theta \, d\Phi] \, r^3 \, dr, \qquad (14)$$

$$\Theta_{23} = - \sum_{i=0}^{p} \int_{r_i}^{r_{i+1}} \rho_i [\int_0^{\pi}\int_0^{2\pi} u_r(r, \theta, \Phi) \sin\theta \sin 2\theta \sin\Phi \, d\theta \, d\Phi$$

$$+ \int_0^{\pi}\int_0^{2\pi} u_\theta(r, \theta, \Phi) \sin\theta \cos 2\theta \sin\Phi \, d\theta \, d\Phi$$

$$+ \frac{1}{2} \int_0^{\pi}\int_0^{2\pi} u_\Phi(r, \theta, \Phi) \sin 2\theta \cos\Phi \, d\theta \, d\Phi] \, r^3 \, dr, \qquad (15)$$

$$\Theta_{31} = - \sum_{i=0}^{p} \int_{r_i}^{r_{i+1}} \rho_i [\int_0^{\pi}\int_0^{2\pi} u_r(r, \theta, \Phi) \sin\theta \sin 2\theta \cos\Phi \, d\theta \, d\Phi$$

$$+ \int_0^{\pi}\int_0^{2\pi} u_\theta(r, \theta, \Phi) \sin\theta \cos 2\theta \cos\Phi \, d\theta \, d\Phi$$

$$- \frac{1}{2} \int_0^{\pi}\int_0^{2\pi} u_\Phi(r, \theta, \Phi) \sin 2\theta \sin\Phi \, d\theta \, d\Phi] \, r^3 \, dr, \qquad (16)$$

where $i = 0$ corresponds to the liquid core and p is the number of concentric, homogeneous, isotropic and perfectly elastic spherical shells.

To evaluate the integration over Φ, we make use of the orthogonality relations

$$\int_0^{2\pi} \cos m\Phi \sin m\Phi \, d\Phi = 0, \tag{17}$$

$$\int_0^{2\pi} \cos m\Phi \cos n\Phi \, d\Phi = \int_0^{2\pi} \sin m\Phi \sin n\Phi \, d\Phi = \pi\delta_{mn}.$$

For the integration over the colatitude θ, we use the relations (Ben-Menahem and Israel, 1970)

$$\int_0^{\pi} \sin^2 \theta \cos \theta \, P_n^1 (\cos \theta) \, d\theta = \frac{4}{5} \delta_{2n},$$

$$\int_0^{\pi} \left[\cos 2\theta + \sin \theta \cos \theta \frac{\partial}{\partial\theta}\right] P_n^1 (\cos \theta) \, d\theta = 0,$$

$$\int_0^{\pi} \left[\cos \theta + \cos 2\theta \sin \theta \frac{\partial}{\partial\theta}\right] P_n^1 (\cos \theta) \, d\theta = \frac{24}{5} \delta_{2n}, \tag{18}$$

$$\int_0^{\pi} \sin^3 \theta \, P_n^2 (\cos \theta) \, d\theta = \frac{16}{5} \delta_{2n},$$

$$\int_0^{\pi} \left[\sin 2\theta + \sin^2 \theta \frac{\partial}{\partial\theta}\right] P_n^2 (\cos \theta) \, d\theta = 0,$$

$$\int_0^{\pi} \left[\sin^2 \theta \cos \theta \frac{\partial}{\partial\theta} + 2 \sin \theta\right] P_n^2 (\cos \theta) \, d\theta = \frac{48}{5} \delta_{2n},$$

$$\int_0^{\pi} \sin^3 \theta P_n (\cos \theta) \, d\theta = \frac{4}{3} \delta_{0n} - \frac{4}{15} \delta_{2n},$$

$$\int_0^{\pi} \sin \theta \, P_n (\cos \theta) \, d\theta = 2\delta_{0n}.$$

performing the integrations over Φ and θ in (11)–(16) and using equations (1)–(3), (17) and (18),we find that all contributions for cases I, II and III arise from the spherical harmonics of orders 0 and 2 only. The non-zero contributions to the intertia-change dyadic are given by

Case I

$$\Theta_{12} = \frac{16\pi}{5} \sum_{l=0}^{p} \int_{r_l}^{r_{l+1}} \rho_l r^3 [x_l^{2,2} (r) + 3y_l^{2,2} (r)] \, dr. \tag{19}$$

Case II

$$\Theta_{23} = \frac{16\pi}{5} \sum_{l=0}^{p} \int_{r_l}^{r_{l+1}} \rho_l r^3 [x_l^{1,2} (r) + 3y_l^{1,2} (r)] \, dr. \tag{20}$$

Case III

$$\Theta_{11} = \frac{8\pi}{5} \sum_{l=0}^{p} \int_{r_l}^{r_{l+1}} \rho_l r^3 [10x_l^{0,0}(r) + x_l^{0,2}(r) + 3y_l^{0,2}(r) - x_l^{2,2}(r) - 3y_l^{2,2}(r)]\, dr,$$

$$\Theta_{22} = \frac{8\pi}{5} \sum_{l=0}^{p} \int_{r_l}^{r_{l+1}} \rho_l r^3 [10x_l^{0,0}(r) + x_l^{0,2}(r) + 3y_l^{0,2}(r) + x_l^{2,2}(r) + 3y_l^{2,2}(r)]\, dr, \quad (21)$$

$$\Theta_{33} = \frac{16\pi}{5} \sum_{l=1}^{p} \int_{r_l}^{r_{l+1}} \rho_l [5x_l^{0,0}(r) - x_l^{0,2}(r) - 3y_l^{0,2}(r)]\, r^3\, dr.$$

From equations (19)-(21), it may be noted that the radial toroidal functions $z_l^{mn}(r)$ get eliminated during the integration over θ and Φ. Hence, it becomes clear that a change in the inertia tensor of the Earth due to a tengential dislocation is caused only by the zero frequency limit of the spheroidal modes of orders 0 and 2.

Now, we have to integrate over the radial functions according to equations (19)-(21). Using equations (5) and (17), we get

$$\int_{r_l}^{r_{l+1}} \rho_l r^3 [x_l^{m,2}(r) + 3y_l^{m,2}(r)]\, dr = \sum_{q=1}^{4} (J_l^{m,2}(r_l))_q \Big[M_{2q}^{-1} (r_{l+1}^5 - r_l^5) + \frac{\lambda_l + 8\mu_l}{2(\lambda_l + 2\mu_l)} M_{3q}^{-1} (r_{l+1}^3 - r_l^3) + \frac{3}{2} M_{4q}^{-1} (r_{l+1}^7 - r_l^7) \Big], \quad (22)$$

and

$$\int_{r_l}^{r_{l+1}} \rho_l r^3 x_l^{0,0}(r)\, dr = \sum_{q=1}^{4} (J_l^{0,0}(r_l))_q \Big[-\frac{1}{2} M_{1q}^{-1} (r_{l+1}^3 - r_l^3) - \frac{\mu_l}{5(\lambda_l + 2\mu_l)} M_{4q}^{-1} (r_{l+1}^5 - r_l^5) \Big]. \quad (23)$$

CHANGES IN THE COMPONENTS OF THE INERTIA TENSOR IN THE GEOGRAPHIC COORDINATE SYSTEM

Following Munk and MacDonald (1960), we set up a geographic system with unit vectors ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$) having its origin at the centre of mass of the Earth. The $\mathbf{e}_z$ axis coincides with the axis of rotation of the Earth, the $\mathbf{e}_x$ axis is drawn through the Greenwich meridian and the $\mathbf{e}_y$ axis points at 90° east of Greenwich.

In the previous section we derived results for the particular case in which the epicentre lies on the Earth's axis of rotation For an arbitrarily situated source we must apply an orthogonal transformation which carries the epicentral axes to the geographical axes. To this end we consider a source at a point (r_0, θ_0, Φ_0), where θ_0, Φ_0, respectively, are the colatitude and the longitude drawn eastward from Greenwich. Let d_0 be the fault's azimuth, taken clockwise from north toward the fault strike (Fig. 2). The unit vectors in the two coordinate systems are related by the relation

$$\begin{bmatrix} \mathbf{e}_x \\ \mathbf{e}_y \\ \mathbf{e}_z \end{bmatrix} = [T] \begin{bmatrix} \mathbf{e}_{\theta_0} \\ \mathbf{e}_{\Phi_0} \\ \mathbf{e}_{r_0} \end{bmatrix} \tag{24}$$

where the elements T_{ij} of the matrix T are (Ben-Menahem and Isreal, 1970).

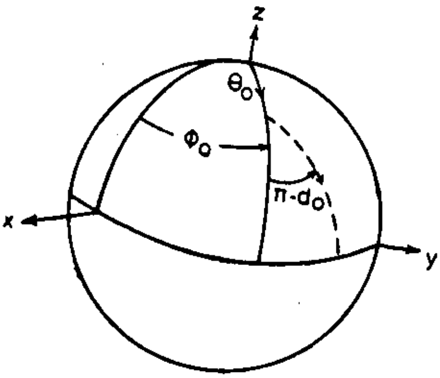

Fig. 2 Azimuth (Φ_0), Colatitude (θ_0) of the Source and Azimuth (d_0) of the Fault Taken as Euler's Angles.

$$\begin{aligned}
T_{11} &= -\sin\Phi_0 \sin d_0 - \cos\Phi_0 \cos\theta_0 \cos d_0, \\
T_{12} &= \sin\Phi_0 \cos d_0 - \cos\Phi_0 \cos\theta_0 \sin d_0, \\
T_{13} &= \cos\Phi_0 \sin\theta_0, \\
T_{21} &= \cos\Phi_0 \sin d_0 - \sin\Phi_0 \cos\theta_0 \cos d_0, \\
T_{22} &= -\cos\Phi_0 \cos d_0 - \sin\Phi_0 \cos\theta_0 \sin d_0, \\
T_{23} &= \sin\Phi_0 \sin\theta_0, \\
T_{31} &= \sin\theta_0 \cos d_0, \\
T_{32} &= \sin\theta_0 \sin d_0, \\
T_{33} &= \cos\theta_0.
\end{aligned} \tag{25}$$

From equations (8) and (24), the inertia-change dyadic $\overline{C}$ in the geographic system is related to the inertia-change dyadic $\overline{\Theta}$ in the epicentral system through the relation

$$\overline{C} = T\,\overline{\Theta}\,T^{-1}. \tag{26}$$

Thus the elements of the inertia-change dyadic $\overline{C}$ are given by

$$
\begin{aligned}
C_{11} &= T_{11}^2\Theta_{11} + T_{12}^2\Theta_{22} + T_{13}^2\Theta_{33} + 2T_{11}T_{12}\Theta_{12} \\
&\qquad + 2T_{12}T_{13}\Theta_{23} + 2T_{11}T_{13}\Theta_{31}, \\
C_{22} &= T_{21}^2\Theta_{11} + T_{22}^2\Theta_{22} + T_{23}^2\Theta_{33} + 2T_{21}T_{22}\Theta_{12} \\
&\qquad + 2T_{22}T_{23}\Theta_{23} + 2T_{21}T_{23}\Theta_{31}, \\
C_{33} &= T_{31}^2\Theta_{11} + T_{32}^2\Theta_{22} + T_{33}^2\Theta_{33} + 2T_{31}T_{32}\Theta_{12} \\
&\qquad + 2T_{32}T_{33}\Theta_{23} + 2T_{31}T_{33}\Theta_{31}, \\
C_{12} &= T_{11}T_{21}\Theta_{11} + T_{12}T_{22}\Theta_{22} + T_{13}T_{23}\Theta_{33} + (T_{11}T_{22} + T_{12}T_{21})\Theta_{12} \\
&\qquad + (T_{12}T_{23} + T_{13}T_{22})\Theta_{23} + (T_{13}T_{21} + T_{11}T_{23})\Theta_{31}, \\
C_{23} &= T_{21}T_{31}\Theta_{11} + T_{22}T_{32}\Theta_{22} + T_{23}T_{33}\Theta_{33} + (T_{21}T_{32} + T_{22}T_{31})\Theta_{12} \\
&\qquad + (T_{22}T_{33} + T_{23}T_{32})\Theta_{23} + (T_{21}T_{33} + T_{23}T_{31})\Theta_{31}, \\
C_{31} &= T_{11}T_{31}\Theta_{11} + T_{12}T_{32}\Theta_{22} + T_{13}T_{33}\Theta_{33} + (T_{11}T_{32} + T_{12}T_{31})\Theta_{12} \\
&\qquad + (T_{12}T_{33} + T_{13}T_{32})\Theta_{23} + (T_{13}T_{31} + T_{11}T_{33})\Theta_{31}.
\end{aligned} \tag{27}
$$

Here the indices 1, 2 and 3 of C correspond to the geographic axes $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ respectively. From equations (19)–(21), and (27), we have

Case I

$$
\begin{aligned}
C_{11} &= 2T_{11}T_{12}\Theta_{12}, \\
C_{22} &= 2T_{21}T_{22}\Theta_{12}, \\
C_{33} &= 2T_{31}T_{32}\Theta_{12}, \\
C_{12} &= (T_{11}T_{22} + T_{12}T_{21})\Theta_{12}, \\
C_{23} &= (T_{21}T_{32} + T_{22}T_{31})\Theta_{12}, \\
C_{31} &= (T_{11}T_{32} + T_{12}T_{31})\Theta_{12}.
\end{aligned} \tag{28}
$$

Case II

$$
\begin{aligned}
C_{11} &= 2T_{12}T_{13}\Theta_{23}, \\
C_{22} &= 2T_{22}T_{23}\Theta_{23}, \\
C_{33} &= 2T_{32}T_{33}\Theta_{23}, \\
C_{12} &= (T_{12}T_{23} + T_{13}T_{22})\Theta_{23}, \\
C_{23} &= (T_{22}T_{33} + T_{23}T_{32})\Theta_{23}, \\
C_{31} &= (T_{12}T_{33} + T_{13}T_{32})\Theta_{23}.
\end{aligned} \tag{29}
$$

Case III

$$
\begin{aligned}
C_{11} &= T_{11}^2\Theta_{11} + T_{12}^2\Theta_{22} + T_{13}^2\Theta_{33},\\
C_{22} &= T_{21}^2\Theta_{11} + T_{22}^2\Theta_{22} + T_{23}^2\Theta_{33},\\
C_{33} &= T_{31}^2\Theta_{11} + T_{32}^2\Theta_{22} + T_{33}^2\Theta_{33},\\
C_{12} &= T_{11}T_{21}\Theta_{11} + T_{12}T_{22}\Theta_{22} + T_{13}T_{23}\Theta_{33},\\
C_{23} &= T_{21}T_{31}\Theta_{11} + T_{22}T_{32}\Theta_{22} + T_{23}T_{33}\Theta_{33},\\
C_{31} &= T_{11}T_{31}\Theta_{11} + T_{12}T_{32}\Theta_{22} + T_{13}T_{33}\Theta_{33}.
\end{aligned} \tag{30}
$$

CONCLUSIONS

The changes in the components of the inertia tensor of the Earth due to a buried displacement dislocation are calculated theoretically. The inertia changes are independent of the toroidal modes and depend on the spheroidal modes of order zero and two only. The effect the displacement fields described have on the rotation of the Earth, the Chandler Wobble and the secular polar shift can be evaluated from these inertia-changes.

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