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CAUSALITY RATHER THAN CONSTANT Q FOR SEISMIC WAVES

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Introduction

The principle of causality was invoked in connection with seismic wave propagation by Futterman (1962) and Lamb (1962). It follows from this principle that a transient seismic disturbance can be felt at some distance from the source only after it has started to radiate. The principle is therefore an important constraint on mathematical simulations of seismic wave propagation. Within the framework of linear filter theory, the interpretation of the observational evidence (see for example Kanamori and Anderson, 1977) that the intrinsic quality factor Q for earth materials is frequency independent is incompatible with the principle of causality. The usual proof of the incompatibility (e.g., Strick, 1970) is based on the Paley-Wiener condition (Papoulis, 1962) for transfer functions of causal (i.e., phygical) linear systems. The condition is in the form of an inequality involving integration along the entire real angular frequency axis and is a consequence of the requirement that the transfer function of a causal linear system must be analytic over one half of the complex angular frequency plane. We prove the incompatibility using the Cauchy-Riemann conditions on analytic functions.

Background

Let w be dimensionless angular frequency and x dimensionless distance along the direction of propagation of plane seismic waves in a linear lossy medium of infinite extent. The transfer function H(w, x) for a thickness x of the medium may be written as

$$H(w, x) = \exp. (-\gamma(w)\, x) \tag{1}$$

The dimensionless spatial propagation factor $\gamma(w)$ is given by

$$\gamma(w) = \alpha(w) + i\,\beta(w) + i\, w/v_e \tag{2}$$

It is made up of spatial attenuation ($\alpha(w)$) and phase shift ($\beta(w)$) factors due to lossy nature of the medium and the spatial phase shift factor w/v_e due to elasticity of the medium, v_e being the frequency independent dimensionless elastic wave speed. $i = (-1)^{1/2}$.

Futterman (1962) showed that the intrinsic quality factor

$$Q(w) = (w/v_e + \beta(w))/2\,\alpha(w), \tag{3}$$

if $Q(w)$ is very much larger than 2π. Let

$$\gamma(w) = u(w) + i\, v(w)$$

$$u(w) = v(w)/2\, Q(w)$$

$$v(w) = w/v_e + \beta(w).$$

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Let w_i and w_e be real and complex variables such that $w_e = w + i\,w_i$. (w, w_i) define a point on the complex w_e-plane. Let $H'(w_e, x)$ and $\gamma'(w_e)$ be complex valued functions which reduce to $H(w, x)$ and $\gamma(w)$ for $w_i = 0$. Similarly, let $u'(w, w_i)$, $v'(w, w_i)$, $\beta'(w, w_i)$, and $Q'(w, w_i)$ be real valued functions which reduce to $u(w)$, $v(w)$, $\beta(w)$, and $Q(w)$ when $w_i = 0$. It may be shown that

$$H'(w_e, x) = \exp.\,(-\gamma'(w_e)\,x)$$

$$\gamma'(w_e) = u'(w, w_i) + i\,v'(w, w_i)$$

$$u'(w, w_i) = v'(w, w_i)/2\,Q'(w, w_i) \tag{4}$$

$$v'(w, w_i) = w/v_e + \beta'(w, w_i). \tag{5}$$

Incompatibility

The principle of causality requires that $H'(w_e, x)$ be analytic over the positive or negative imaginary half of the w_e-plane (Papoulis, 1962) depending upon the definition of the Fourier transform integral. The region $w_i \leqslant 0$ is pertinent in our case in view of (1). Also, analyticity of $H'(w_e, x)$ depends upon analyticity of $\gamma'(w_e)$. Thus $\gamma'(w_e)$ must satisfy the Canchy-Riemann conditions for $w_i \leqslant 0$.

$$\partial u'/\partial w = \partial v'/\partial w_i \tag{6}$$

$$\partial u'/\partial w_i = -\,\partial v'/\partial w \tag{7}$$

If $Q'(w, w_i)$ is constant then it follows from (4) to (7) that

$$\partial\beta'/\partial w_i = 0$$

$$\partial\beta'/\partial w = -\,1/v_e$$

Hence $\beta'(w, w_i) = -\,w/v_e + C$, where C is a constant of integration. In turn,

$$\gamma'(w_e) = C/2\,Q + i\,C = \gamma(w)$$

and

$$H(w, x) = \exp.\,(-\,(C/2\,Q + i\,C)\,x).$$

In other words, $H(w, x)$ as well as the spatial attenuation and phase shift factors are independent of w. This is contrary to the observations that these factors do depend on w. Hence we conclude that causality and constant Q are mutually incompatible within the framework of linear filter theory.

Conclusion

Adherence to causality is a rational necessity in the mathematical simulations of seismic wave propagation, while constant Q is a convenient generalization based on limited observational data.

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