Paper No. 23

DYNAMIC TEST OF A CONCRETE FRAME

A. R. Chandrasekaran*

Synopsis

Static and dynamic testing was performed on a four storeyed reinforced concrete frame. Theoretical analysis compared favourably with the experimental values. Experiments indicate a decrease in damping in higher modes.

Introduction

Natural periods of vibration and damping are the two important parameters which determine the behaviour of structures under dynamic loads. Damping could only be determined experimentally. Periods could be, theoretically determined, provided the mass and stiffness distributions are known. Mass distribution could be, relatively, easily estimated. Experimental determination of periods is necessary to verify the assumptions made in arriving at stiffness distribution.

Reinforced concrete was used as a construction material for the model, as this is the material generally used for construction of multi-storeyed frames of medium height. Static loads were applied horizontally to the frame, corresponding deflections were measured and the spring constants of the system have been, thereby, evaluated. The frame has been pulled horizontally, then let go and the consequent free vibrations have been measured to determine the natural period of the structure and damping in that system. The base of the structure was subjected to steady state sinusoidal excitations at various frequencies and amplitudes of vibration at various storey levels have been measured. Damping was obtained from experimental records. Theoretical analysis was made to obtain periods of vibration and these have been compared with values obtained experimentally.

Size of the Model

A shaking table of size 6′0″×4′0″ with a head room of 7′6″ above the table was available for doing experimental work. A four-storeyed reinforced concrete frame of

* Reader in Civil Engineering, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee (India).

dimensions as shown in Figure 1 was chosen such that it could be conveniently accommodated on the table.

The size of the column, $1\frac{1}{2}'' \times 1\frac{1}{2}''$ was the minimum that could be conveniently cast in situ. The thickness of the slab had to be increased at the supports to provide proper cover to top and bottom reinforcements. This model could be construed to be a geometrically similar model, with a length scale of eight, of a conventional four-storeyed frame.

Free Vibration Behaviour

Even neglecting the stiffening provided by the slab, the ratio of moment of inertia per unit length of girder to that of column works out to be four. Static deflection tests indicate that floor system acts as a rigid unit. The model, therefore, could be assumed to be represented by a four-degree of freedom system as shown in inset of Fig. 1

The equation of motion for free undamped vibration of the system is given by

$$[M]\{\ddot{x}\} + [K]\{x\}=0 \qquad (1)^*$$

where [M] and [K] are as defined below.

$$[M] = \begin{bmatrix} m & & & \\ & m & & O \\ & & m & \\ O & & & m \end{bmatrix} \quad \text{and} \quad [K] = \begin{bmatrix} k & -k & 0 & 0 \\ -k & 2k & -k & 0 \\ 0 & -k & 2k & -k \\ 0 & 0 & -k & 2k \end{bmatrix}$$

The solution of the above equation results in the following frequencies and mode shapes.

$$p_1 = 0.3473\sqrt{\frac{k}{m}} \qquad (2)$$

$$p_2 = 1.000\sqrt{\frac{k}{m}} \qquad (3)$$

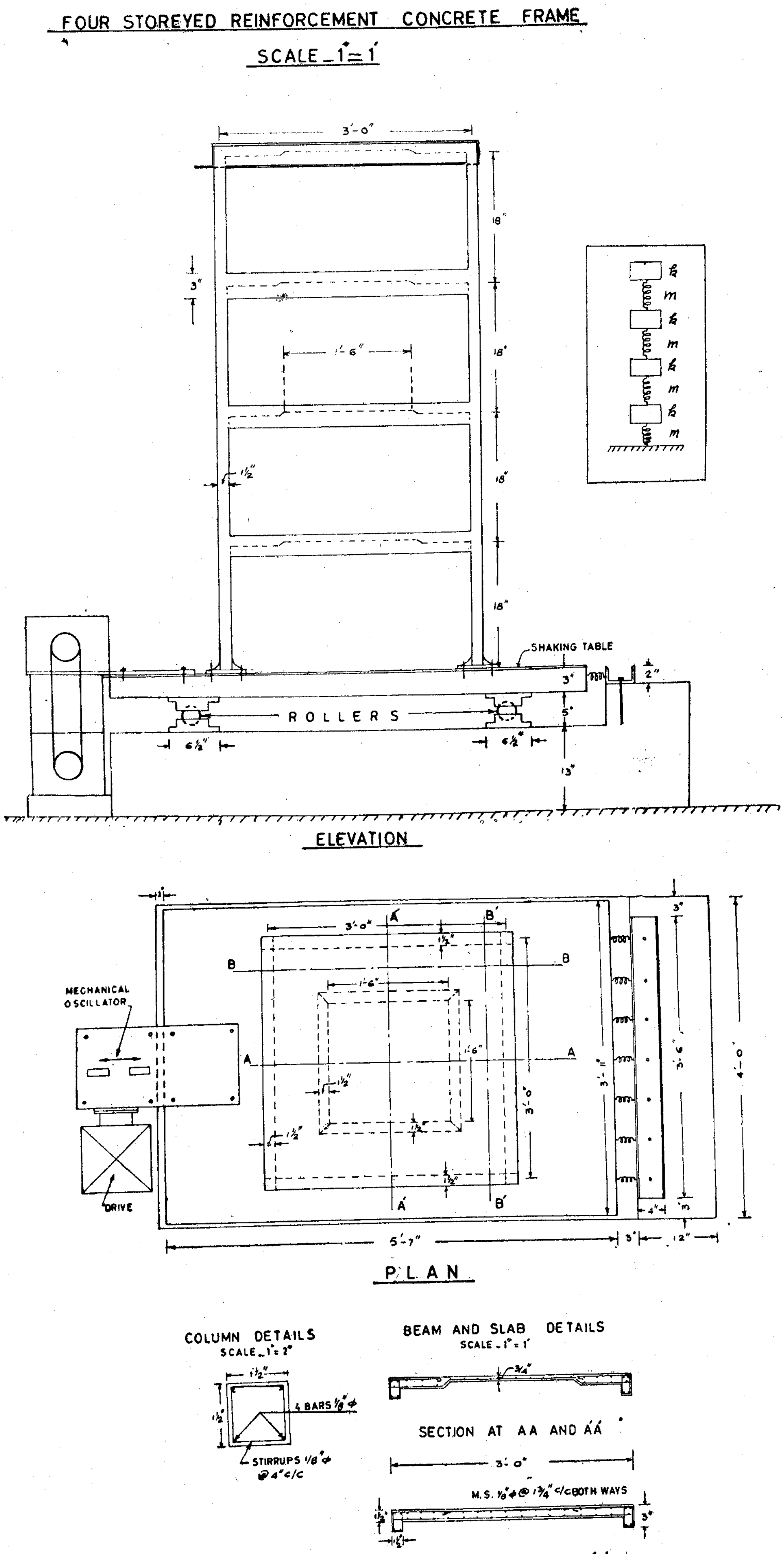
$$p_3 = 1.5321\sqrt{\frac{k}{m}} \qquad (4)$$

$$p_4 = 1.8794\sqrt{\frac{k}{m}} \qquad (5)$$

TABLE 1

No. i	1st Mode $\phi_1^{(1)}$	2nd Mode $\phi_1^{(2)}$	3rd Mode $\phi_1^{(3)}$	4th Mode $\phi_1^{(4)}$
1	0.6565	0.5774	0.4285	0.2280
2	0.5774	0.0000	−0.5774	−0.5774
3	0.4285	−0.5774	−0.2280	0 6565
4	0.2280	−0.5774	0.6565	−0.4285

* Notations are defined as they first appear in text.



FOUR STOREYED REINFORCEMENT CONCRETE FRAME
SCALE _ 1" = 1'
3'-0"
18"
3"
1'-6"
1½"
SHAKING TABLE
ROLLERS
6½"
13"
ELEVATION
MECHANICAL OSCILLATOR
DRIVE
3'-0"
1'-6"
3'-11"
3'-6"
4'-0'
5'-7"
PLAN
COLUMN DETAILS
SCALE_1" = 2"
4 BARS ⅛" φ
STIRRUPS ⅛" φ @ 4" c/c
BEAM AND SLAB DETAILS
SCALE _ 1" = 1'
SECTION AT AA AND A'A'
3'-0"
M.S. ⅛" φ @ 1¾" c/c BOTH WAYS
SECTION AT BB AND B'B'

Fig. 1

$\phi_1^{(r)}$'s have been calculated such that $\sum_{j=1}^{n} m_j(\phi_j^{(r)})^2=1$

The weight of each floor including the weight of columns works out to be 200 lbs.

Discussion of Results of free Vibration Tests

For the purpose of estimation of stiffness k, static deflections were measured for known horizontal loads. Horizontal loads were applied to the topmost mass and horizontal deflections at various floor levels were measured. For small loads, (refer Table 2 and Fig. 2) load deflection curves were sensibly linear. In this range, k of each floor works out to be 6667 lbs/in. This value of k corresponds to a modulus of elasticity, E_1 equal to 1.92×10^6 psi assuming moment of inertio. I, to be evaluated on the basis of gross cross-sectional area of concrete and an effective length of column equal to distance between centre to centre of floors. Using the above value of k and m in the frequency equation, $p=0.3473\sqrt{k/m}$, the fundamental frequency of vibration works out to be 6.27 cps. The experimentally determined fundamental frequency of vibration, from free vibration tests, is 6.25 cycles per second.

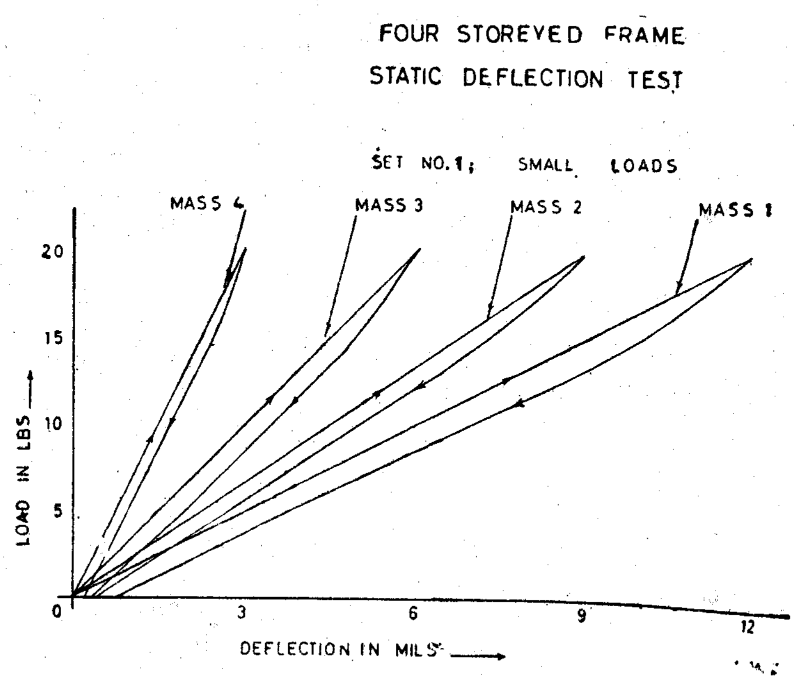


Fig. 2

When large loads are applied, (refer Table 3 and Fig. 3) static deflection tests indicate deflections increasing more rapidly with increase in load. The increased deflection would indicate that k decreases with increase in load. This means that E also decreases. This behaviour is typical of concrete. When k decreases, frequency also should decrease. This is also borne out by free vibration tests in which large initial displacement were given.

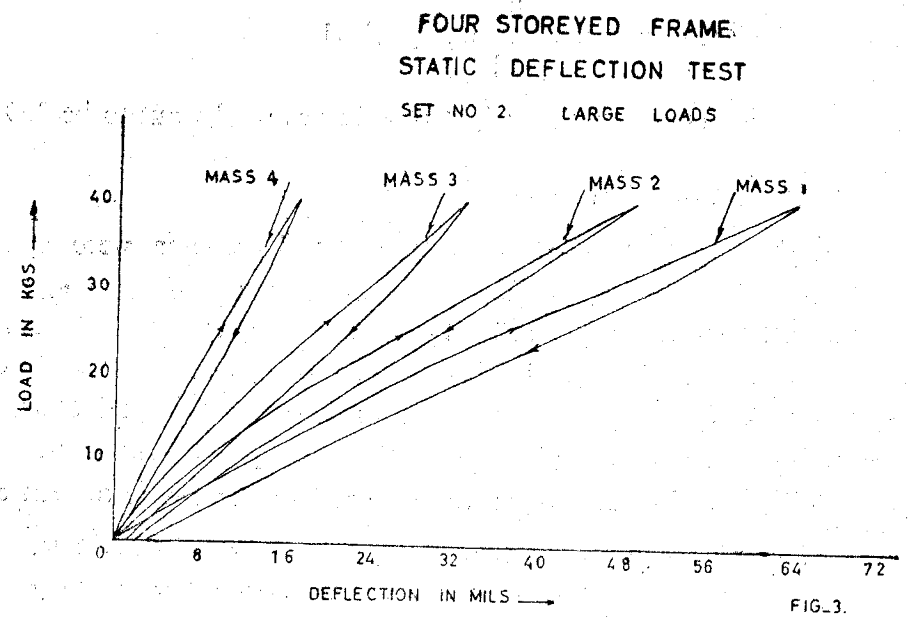


Fig. 3

Fig. 4 shows the free vibration records of the four storeyed frame. The fundamental frequency of vibration (f_1) and damping factor (ζ_1) could be obtained from the records.

Free vibration tests indicate that the experimental value of fundamental frequency of vibration is in accordance with the theory. Average values of damping factor ζ_1 as obtained from free vibration studies, is indicated in Fig. 4.

Forced Vibration Behaviour

To study the behaviour under forced vibration, the frame was subjected to steady state sinusoidal excitation. The frame was cast on top of a platform and the platform was subjected to sinusoidal excitation.

Analysis

Consider the building frame to be subjected to a sinusoidal ground motion represented by $\ddot{y} = \ddot{y}_b \times \sin wt$.

The equation of motion for such a system is given by

$$[M]\{\ddot{z}\} + [C]\{\dot{z}\} + [K]\{z\} = -[M]\{\ddot{y}\} \qquad (6)$$

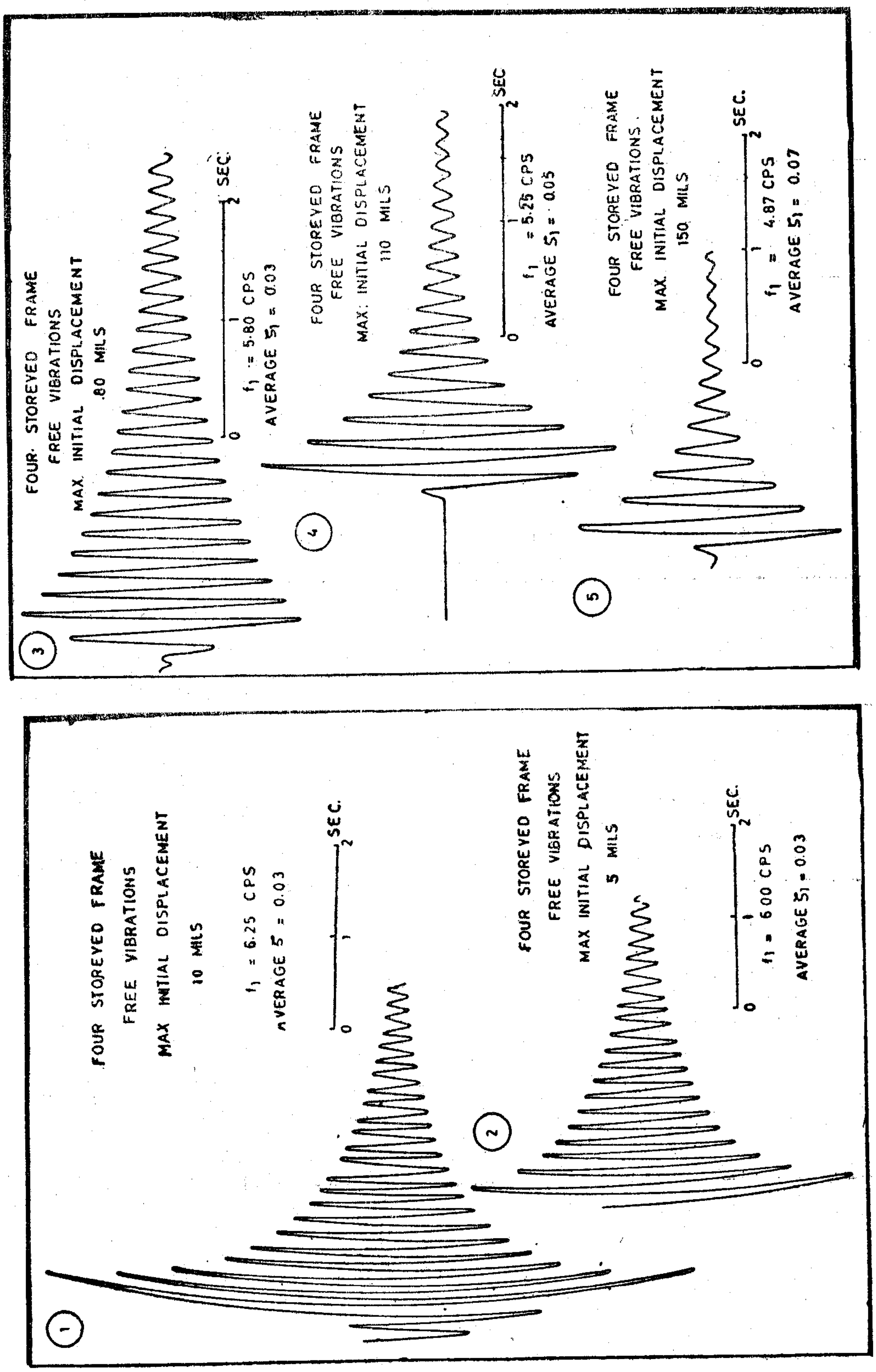
where [M] and [K] are as defined by equation 1.

Z_i = displacement of the i^{th} mass, relative to a coordinate system fixed in the base.

$y(t)$ = displacement of the base, relative to a 'fixed' frame of reference.

[C] = damping matrix.

It will be assumed that [C] would be such that the undamped model columns do remain



3
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX. INITIAL DISPLACEMENT
80 MILS
0 1 2 SEC.
f_1 = 5.80 CPS
AVERAGE ξ_1 = 0.03
4
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX. INITIAL DISPLACEMENT
110 MILS
0 1 2 SEC
f_1 = 5.25 CPS
AVERAGE ξ_1 = 0.05
5
FOUR STOREYED FRAME
FREE VIBRATIONS.
MAX. INITIAL DISPLACEMENT
150 MILS
0 1 2 SEC.
f_1 = 4.87 CPS
AVERAGE ξ_1 = 0.07
1
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX INITIAL DISPLACEMENT
10 MILS
f_1 = 6.25 CPS
AVERAGE ξ = 0.03
0 1 2 SEC.
2
FOUR STOREYED FRAME
FREE VIBRATIONS
MAX INITIAL DISPLACEMENT
5 MILS
0 1 2 SEC.
f_1 = 6.00 CPS
AVERAGE ξ_1 = 0.03

Fig. 4

STATIC DEFLECTION TEST

TABLE 2

Load		Deflection in Mils							
		Mass no. 1		Mass no. 2		Mass no. 3		Mass no. 4	
Lbs.	Kgs.	Loading	Un-Loading	Loading	Un-Loading	Loading	Un-Loading	Loading	Un-Loading
0	0	0.0	0.75	0.0	0.50	0.0	0.40	0.0	0.25
5	2.27	3.0	3.75	2.25	2.75	1.5	2.0	0.75	1.00
10	4.54	6.0	7.0	4.50	5.0	3.0	3.5	1.50	1.75
15	6.80	9.0	10.0	6.75	7.25	4.5	5.0	2.25	2.50
20	9.07	12.0		9.0		6.0		3.0	

TABLE 3

Load in kgs.	Deflection in Mils							
	Mass no. 1		Mass no. 2		Mass no. 3		Mass no. 4	
	Loading	Un-Loading	Loading	Un-Loading	Loading	Un-Loading	Loading	Un-Loading
0	0.0	3.0	0.0	2.0	0.0	1.50	0.0	1.00
10	13.5	18.0	9.0	12.5	6.75	9.50	3.50	5.00
20	28.0	34.0	21.0	24.5	14.50	18.0	7.50	9.50
30	46.5	51.0	35.0	37.0	24.00	26.25	12.25	13.50
40	64.0		49.0		33.0		17.00	

valid. It has been shown (Rayleigh, 1945) that [C] then should be a linear combination of mass [M] and stiffness [K] matrices. It would be further assumed that the percentage of critical damping in each mode of vibration remains the same. It is possible to choose a damping matrix that will satisfy the above assumption.

If the ground excitation is sinusoidal and is expressed as $\ddot{y} = \ddot{y}_b \sin wt$, then the steady state solution of equation 6 is given by

$$Z_i = -\sum_{r=1}^{n} B_i^{(r)} \cdot \frac{\ddot{y}_b}{p_r^2} \cdot \mu_r \cdot \text{Sin}\ (\omega t - \theta_r) \tag{7}$$

where $B_i^{(r)}$ = mode factor in the r^{th} mode of vibration

$$= \frac{\phi_i^{(r)} \cdot \sum_{j=1}^{n} m_j \phi_j^{(r)}}{\sum_{j=1}^{n} m_j (\phi_j^{(r)})^2}$$

$\ddot{y}_b$ = maximum amplitude of ground acceleration

p_r = natural frequency of vibration in the r^{th} mode.

μ_r = a dimensionless parameter denoting the dynamic amplification factor in the r^{th} mode

$$= \frac{1}{[(1-\eta_r^2)^2 + (2\eta_r\zeta_r)^2]^{\frac{1}{2}}}$$

$\eta_r = \omega/p_r$ = frequency ratio

ζ_r = damping factor in the r^{th} mode.

θ_r = phase angle in the r^{th} mode.

$$= \tan^{-1} \frac{2\eta_r \zeta_r}{1-\eta_r^2}$$

The steady state acceleration $\ddot{Z}_i$ will be given by, differentiating equation (7),

$$\ddot{Z}_i = \sum_{r=1}^{n} B_i^{(r)} \cdot \ddot{y}_b \cdot \eta_r^2 \, \mu_r \cdot \text{Sin}\ (\omega t - \theta_r) \tag{8}$$

The absolute acceleration $\ddot{x}_i$ of any mass i, is equal to $(\ddot{Z} + \ddot{y})$

$$\ddot{x}_i = \ddot{y}_b \left[\sin \omega t + \sum_{r=1}^{n} B_i^{(r)} \cdot \eta_r^2 \cdot \mu_r \cdot \sin\ (\omega t - \theta_r) \right] \tag{9}$$

$$\frac{(\ddot{x}_i)_{max}}{\ddot{y}_b} = \left[\left\{ 1 + \left(\sum_{r=1}^{n} B_i^{(r)} \eta_r^2 \, \mu_r \cos \theta_r \right) \right\}^2 + \left\{ \sum_{r=1}^{n} B_i^{(r)} \cdot \eta_r^2 \cdot \mu_r \cdot \sin \theta_r \right\}^2 \right]^{\frac{1}{2}} \tag{10}$$

The values of $B_i^{(r)}$ for the four storeyed frame is given in Table 4 below. (These have been obtained by making use of Table 1)

TABLE 4

Mass i	Mode Factor $B_i^{(r)}$			
	1st Mode	2nd Mode	3rd Mode	4th Mode
1	1.2411	—0 3333	0.1199	—0.0277
2	1.0914	0.0000	—0.1615	0.0700
3	0.8101	0.3333	—0.0638	—0.0797
4	0.4310	0.3333	0.1836	0.0520

Figure 5 shows a theoretical relationship between $(\ddot{x}_1)_{max.}/\ddot{y}_b$ Vs. η_1 assuming that ζ_r remains constant for all r's and equal to 0.10.

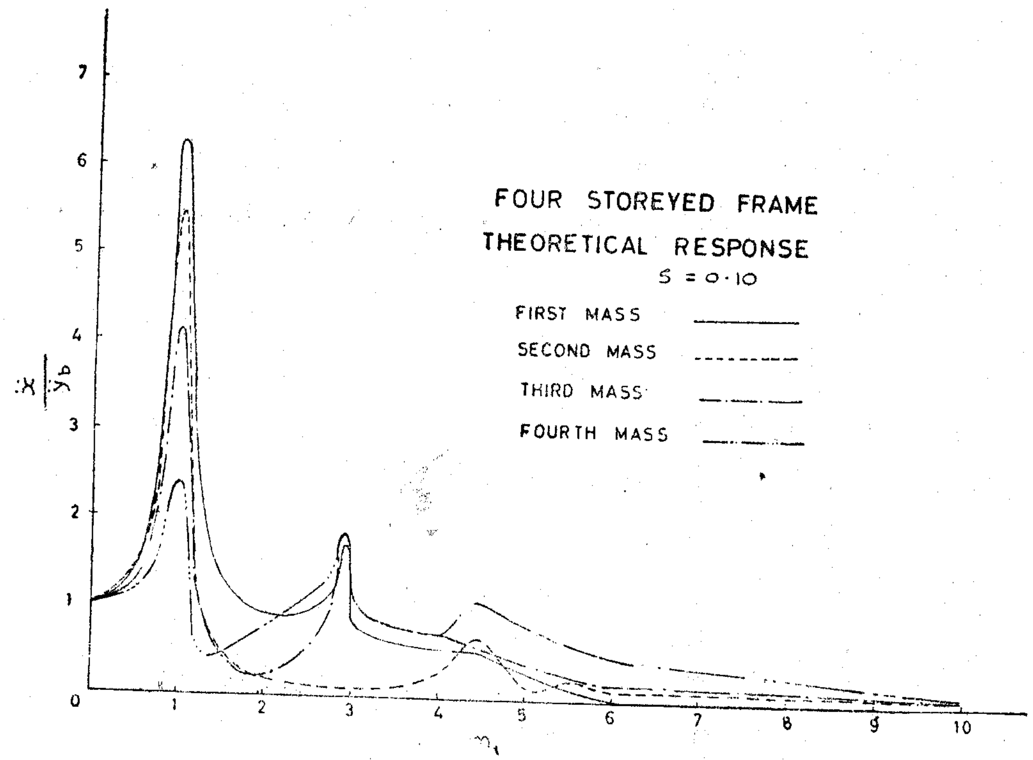


Fig. 5

Experimental Set Up

The four storeyed frame was cast in situ on a platform which rests on rollers such that it was free to move in a horizontal direction. Lazan Mechanical Oscillator was firmly attached to the platform. The oscillator was driven by a Graham's variable speed transmission drive. This set up is capable of giving a steady state sinusoidal excitation to the table.

The oscillator utilises the centrifugal force of unbalanced masses to generate a variable alternating force. It consists of two eccentrices (unbalanced masses) mounted in such a way that they produce a harmonic force in one direction only. There is also an arrangement to vary the amplitude of excitation by adjusting the position of eccentrices.

The frequency of excitation is varied by means of the transmisssion drive which use the compound planetary gear system except that the non-rotating member is a traction ring which engages tapered rollers at varying diameters . The traction ring is moved lengthwise of the transmission to change the speed.

The acceleration of the base (platform) and that of the various floors of the frame were measured by Miller accelerometers, which are variable air gap inductance pick-ups. The pickup is connected to a bridge circuit and the output of the bridge is amplified by a Brush Universal Amplifier and recorded on a Brush ink writing oscillograph.

An experimental run consists of measuring the acceleration of the base and that of a floor for various exciting frequencies. Acceleration of all the floors were measured.

Discussion of Results

Table 5 and Figure 6 give the response of various floors, obtained experimentally. The

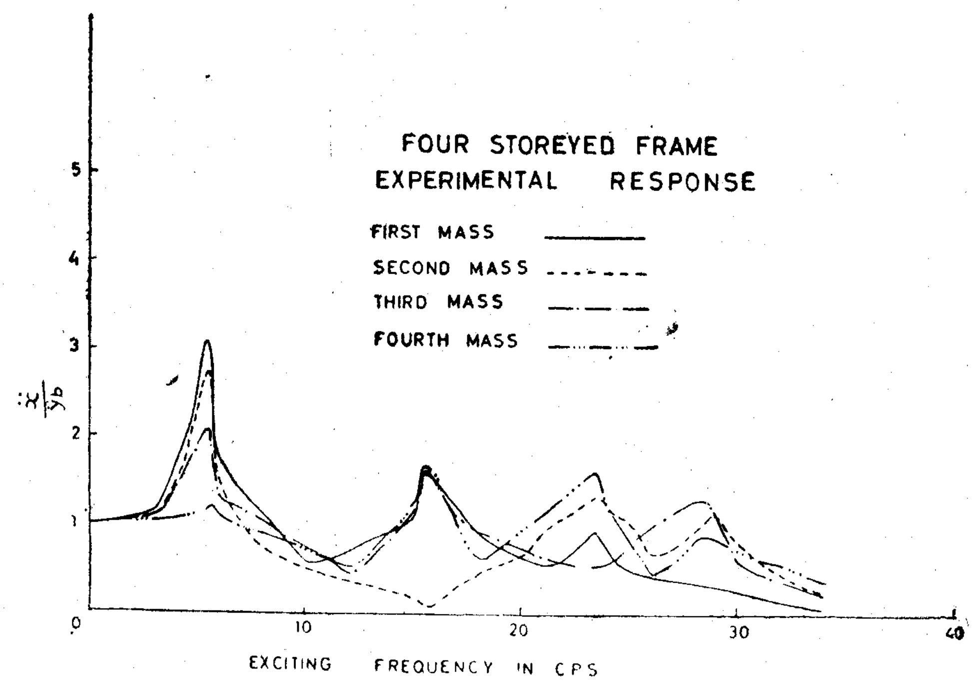


Fig. 6

resonant frequencies correspond to that of the natural frequencies as obtained theoretically for a four degree freedom system. However, the experimental response curves (Fig. 6) do not resemble the theoretical response curves. The theoretical response curves were evaluated for a constant damping factor in all modes. In practice, this is not the case.

TABLE 5

Four Storeyed Frame

Forced Vibration by Steady state Sinusoidal Excitation

RESPONSE OF MASSES

Exciting Frequency 'f' in C.P.S.	RESPONSE			
	$\ddot{x}_1 / \ddot{y}_b$	$\ddot{x}_2 / \ddot{y}_b$	$\ddot{x}_3 / \ddot{y}_b$	$\ddot{x}_4 / \ddot{y}_b$
3.0	1.15	1.10	1.10	1.05
4.5	1.96	1.85	1.70	1.10
5.5	3.10	2.75	2.08	1.20
6.0	1.80	1.50	1.30	1.10
7.5	1.26	0.86	1.12	0.94
8.5	1.05	0.67	0.95	0.88
10.0	0.60	0.52	0.76	0.74
12.0	0.72	0.38	0.46	0.55
14.0	0.94	0.27	0.87	0.95
15.0	1.10	0.18	1.15	1.25
15.5	1.60	0.10	1.65	1.70
17.0	1.20	0.32	1.12	1.05
18.0	0.95	0.47	0.95	0.64
20.0	0.65	0.67	0.77	0.98
21.0	0.57	0.95	0.64	1.13
22.0	0.64	1.14	0.58	1.35
23.5	0.98	1.35	0.55	1.64
24.0	0.73	1.22	0.60	1.24
25.0	0.52	1.08	0.80	0.83
26.0	0.48	0.70	1.00	0.48
27.0	0.42	0.78	1.15	0.60
28.5	0.37	1.05	1.31	0.92
30.0	0.28	0.86	0.61	0.73
32.0	0.12	0.47	0.42	0.60
34.0	0.08	0.29	0.21	0.40

Note: Subscripts 1, 2, 3 and 4 refers to masses, numbered serially from top downwards. $\ddot{y}_b$ represents acceleration of base.

At resonance, the amplification is essentially due to vibration in the mode correspon-ding to the resonent frequency. By considering the amplification at resonant frequencies, the damping factor works out as follows :

Resonant Frequency Corresponds to	*Damping Factor.*
1st mode.	0.20
2nd mode.	0.10
3rd mode.	0.05
4th mode.	0.03

It appears as though the damping as obtained from steady state vibration records is somewhat higher than that of those from free vibration records. This might be due to inade-quacy of the speed control unit which cannot give very small increments to forcing frequency. There is a possibility that the structure has not been excited at true resonance. Nielsen (1964) has reported that a very sensitive speed control unit is required to excite a structure resonance.

CONCLUSIONS

Free vibration tests indicate that the experimental values of fundamental frequency is in accordance with the theory. The resonant frequencies as obtained from forced vibration tests had a good correspondance with the various natural frequencies calculated by theory. Damping depends on the amplitude of vibration, larger the amplitude more the damping. An average value of damping, for this structure, is 5%. Steady state forced vibration tests indicate that damping corresponding to forced vibration is more than that obtained from free vibration tests. This might be due to the speed control unit being not very sensitive. It was observed that damping was maximum in the first mode and it decreases in higher modes.

ACKNOWLEDGEMENT

The author is thankful to the staff of Earthquake Research School, University of Roorkee for assistance rendered in the experimental work and to Dr. Jai Krishna for his valuable guidance in this research project.

REFERENCES

Nielsen, N.N. (1964), "Dynamic Response of Multistorey Buildings", Earthquake Engineering Research Laboratory, Pasadena, California.

Rayleigh B. (1945), "The Theory of Sound", Vol. 1, Dover Publications, New York, N.Y.