

## **SEISMIC RESPONSE OF A SECONDARY SYSTEM ATTACHED TO A TORSIONALLY COUPLED PRIMARY SYSTEM UNDER BI-DIRECTIONAL GROUND MOTION**

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### **ABSTRACT**

Seismic response of primary-secondary structural system with the consideration of torsional coupling of the primary system is obtained for bi-directional earthquake excitations. Response quantities of interest are the relative displacement between the primary and the secondary system, and the absolute acceleration of the secondary system. Responses of the secondary system are obtained by the frequency domain spectral analysis using white noise input excitation in two orthogonal directions. The response behavior of the secondary system is examined under a set of important parametric variations. The parameters include the uncoupled lateral frequency of the primary system,  $\omega_p$ , the natural frequency of the secondary system,  $\omega_s$ , the ratio of uncoupled lateral to rotational frequencies,  $\omega_p/\omega_\theta$ , of the primary system, eccentricity ratios of the primary,  $e_{px}/r$  and  $e_{py}/r$ , and the secondary,  $e_{sx}/r$  and  $e_{sy}/r$ , systems in X and Y directions, damping ratios of the primary,  $\xi_p$ , and the secondary,  $\xi_s$ , systems, and the mass ratio,  $m_s/m_p$ , of the two systems.

**KEYWORDS:** Primary and Secondary Systems, Non-Classical Damping, Torsionally Coupled Systems, Frequency Domain Analysis, Bi-directional Excitation.

### **INTRODUCTION**

Seismic response analysis of light secondary systems (S-system) is important in relation to the performance of delicate equipments and suspension systems in buildings, nuclear power plants, lifeline systems and other important structures. The seismic design of such structures has attracted considerable attention in recent years (Chen and Soong 1989, Der Kiureghian et al. 1983, Jangid and Datta 1993). For integrity and serviceability of the S-system, response quantities of interest are the relative displacement between the primary and the secondary system (PS-system) and the absolute acceleration of the S-system itself. Idealization of the PS-system, with the primary system (P-system) as torsionally uncoupled, is valid only under limited cases. In realistic situations, the primary structures are torsionally coupled. Thus, the sensitivity and response analysis of a S-system attached to a torsionally coupled P-system is of practical interest for the design and performance of the S-system.

A large amount of literature is available on the seismic response analysis of the S-system mounted on the P-system which is torsionally uncoupled. Mode synthesis method for the analysis of the equipment structural system was developed by Suarez and Singh (1987) to random ground excitation, by considering interaction between the equipment and the primary structural systems. Since no assumption was made about the size of the equipment system, the mode synthesis method was intended for light as well as heavy equipment systems. Sinha and Igusa (1995) studied responses of the S-system mounted over a linear P-system under short-duration and wide-band white noise input excitation using modal analysis. Falsone et al. (1992) obtained the response of the PS-system by time domain integration technique using a step-by-step procedure for the operator of Taylor's expansion. The procedure avoided the calculation of complex eigen-properties of the system leading to the saving of the computational time. It was shown that responses of the S-system can have unusual and undesirable characteristics under such type of input excitations. Sackman and Kelly (1979), Sackman et al. (1983) and Der

Kiureghian et al. (1983) investigated responses of the light single-degree-of-freedom S-system attached to a multi-degree-of-freedom P-system to stochastic input excitation using perturbation technique. They found that interaction between the equipment and the primary structural system is significant, particularly when the equipment is tuned to one of the natural frequencies of the P-system. Singh (1988) and Chen and Soong (1988) have presented a state-of-the-art review of the PS-system.

Most of the above studies on the PS-system are strictly valid for symmetric buildings or buildings with very small eccentricity or torsionally stiff buildings under uni-directional earthquake excitation. There is a lack of study in exploring the parametric behavior of the S-system mounted over torsionally coupled P-system. Yang and Huang (1993, 1998) studied the responses of a light equipment item attached to a multi-storey building, that may be subjected to large torsional deformations due to eccentricity and uni-directional earthquake excitation. Recently, Agrawal and Datta (1997) studied the behavior of a S-system mounted on a torsionally coupled linear P-system under uni-directional random ground excitation. Agrawal and Datta (1998) studied the behavior of a S-system mounted on a nonlinear torsionally coupled P-system under uni-directional random ground excitation using both linearized frequency and time domain methods of analysis. The effect of torsional coupling in the PS-system was represented by simple 2-D model with one translational and a rotational degree-of-freedom. Since bi-directional ground motion is a more realistic representation of the seismic excitation, it is appropriate to study the effect of torsional coupling of the P-system on responses of the S-system under the simultaneous action of the two components of the ground motion. Under the bi-directional ground excitation, the P-system can be torsionally excited by varying degrees, depending upon the relative value of ground motion in the two directions, the phase difference between the two ground motions and the degree of asymmetry of the P-system. The resultant motion of the S-system is thus greatly influenced by the two components of the earthquake and its behavior is expected to be different than that observed for uni-directional ground motion.

Herein, the behavior of a S-system mounted over the 3-D model of a torsionally coupled linear P-system is studied under a number of parametric variations for bi-directional ground excitations modelled as white noise. The objectives of the present study are (i) to investigate the effect of torsional coupling of the P-system on the response behavior of the S-system; and (ii) to investigate the behavior of the S-system under different important parametric variations.

## THEORETICAL FORMULATION

### 1. System Model

Figure 1 shows the structural system considered, which is an idealized single story building model over which a cantilever type S-system is mounted. It is assumed that the cantilever rod is vertically inextensible and has the same flexural stiffness corresponding to the displacement in any direction in the horizontal plane. The total lateral stiffness of the P-system is taken to be the same in both X and Y directions. Similarly, damping of the S-system is assumed as constant in all directions. The normalized eccentricities of the P-system are varied to provide different degrees of torsional coupling to the P-system. Support of the PS-system is excited by random ground excitations in two mutually perpendicular directions (i.e., in X and Y directions). The columns of the P-system remain in the elastic range under the earthquake excitation.

Let  $K_{pi}$  ( $i = 1, \dots, 4$ ) represent the lateral stiffness of the  $i$  th resisting column. Then, the total stiffness of the P-system in X and Y directions is given by

$$K_p = \sum_{i=1}^4 K_{pi} \quad (1)$$

and the stiffness of the S-system in any direction is given by  $K_s$ .

Let  $R_i$  denote the distance of the  $i$  th column from the center of mass (CM) of the P-system. Then, the total torsional stiffness of the P-system, defined about the CM, is given by

$$K_{\theta} = \sum_{i=1}^4 K_{pi} R_i^2 \quad (2)$$

in which it is assumed that the torsional stiffness of an individual column is negligible. The eccentricities of the P-system in the two orthogonal directions, with respect to the CM of the P-system, are given by (see Figure 1).

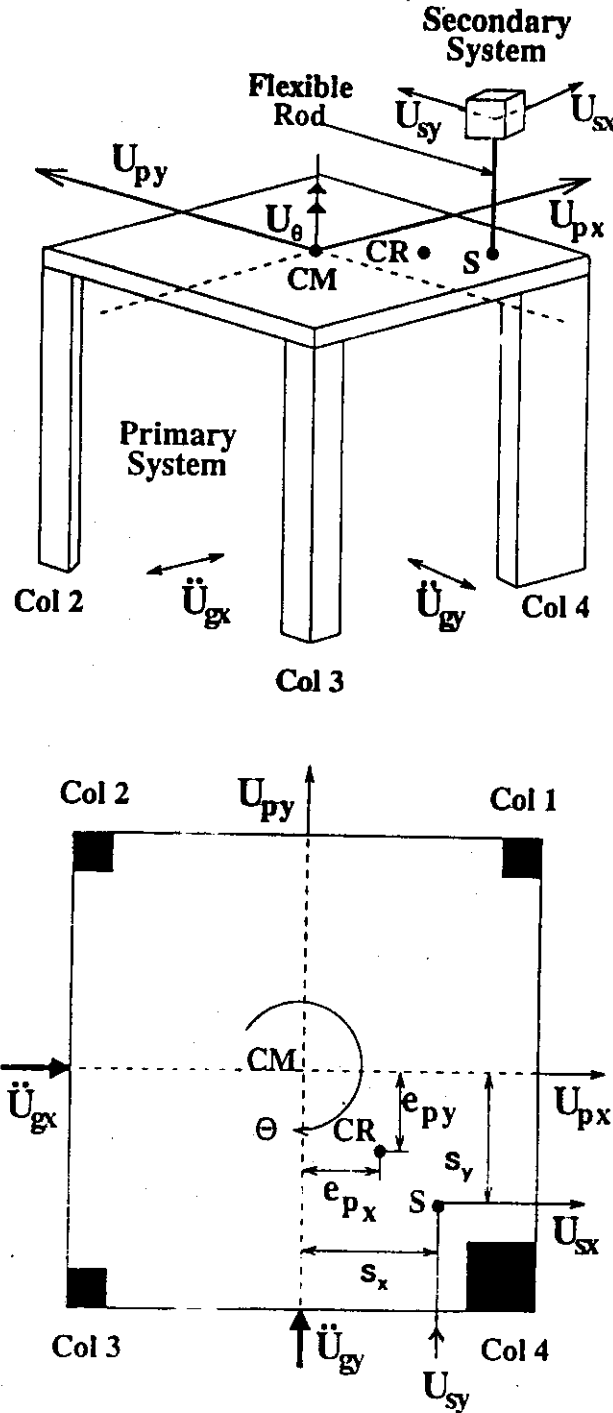


Fig. 1 Structural Model

$$e_{px} = \frac{\sum_{i=1}^4 K_{pi} x_i}{\sum_{i=1}^4 K_{pi}} \quad (3)$$

$$e_{py} = \frac{\sum_{i=1}^4 K_{pi} y_i}{\sum_{i=1}^4 K_{pi}} \quad (4)$$

The two uncoupled frequency parameters of the P-system are defined as

$$\omega_p = \sqrt{\frac{K_p}{m_p}} \quad (5)$$

and

$$\omega_\theta = \sqrt{\frac{K_\theta}{m_p r^2}} \quad (6)$$

and the natural frequency of the S-system is given by

$$\omega_s = \sqrt{\frac{K_s}{m_s}} \quad (7)$$

in which  $m_p$  and  $m_s$  are the masses of the primary and the secondary systems, respectively, and  $r$  is the radius of gyration of the primary mass about the vertical axis through the CM. The frequencies,  $\omega_p$  and  $\omega_\theta$ , may be interpreted as the natural frequencies of the P-system if they were torsionally uncoupled, i.e., a system with  $e_{px} = 0$  and  $e_{py} = 0$ , but  $m_p$ ,  $K_p$  and  $K_\theta$  are the same as in the coupled system. The mass ratio,  $\rho$ , is defined as  $\rho = m_s / m_p$ . The values of  $K_p$  and  $m_p$  of the P-system are varied to provide different values of the frequency parameters,  $\omega_p$  and  $\omega_\theta$ . These parameters are taken to be the same in both X and Y directions. The eccentricities of the S-system with respect to the CM of the P-system (Figure 1) are  $e_{sx} = s_x$  and  $e_{sy} = s_y$ , since the S-system is a stick model attached to the P-system.

## 2. Equations of Motion

Referring to Figure 1, the equation of motion for the PS-system, subjected to bi-directional excitation may be written as

$$[M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K]\{U\} = -[M][I]\{\ddot{U}_g\} = f(t) \quad (8)$$

where  $\{U\} = \{U_{px}, U_{py}, U_\theta, U_{sx}, U_{sy}\}^T$  is the displacement vector of the system model;  $U_{px}$  and  $U_{py}$  are the displacements of the CM of the P-system;  $[I]$ ,  $[M]$  and  $[C]$  are the influencing coefficient, mass and damping matrices, respectively, and  $\{\ddot{U}_g\} = \{\ddot{U}_{gx}, \ddot{U}_{gy}\}^T$  is the ground acceleration vector. The matrices  $[I]$ ,  $[M]$ ,  $[C]$  and  $[K]$  are expressed as

$$[I] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}^T \quad (9)$$

$$[M] = \text{diag} [m_p, m_p, m_p r^2, m_s, m_s] \quad (10)$$

$$[C] = \begin{bmatrix} \sum C_{pi} + C_s & 0 & \sum C_{px\theta} + C_s e_{sy} & -C_s & 0 \\ 0 & \sum C_{pi} + C_s & \sum C_{py} + C_s e_{sx} & 0 & -C_s \\ \sum C_{\theta px} + C_s e_{sy} & \sum C_{\theta py} + C_s e_{sx} & C_\theta + C_s (e_{sx}^2 + e_{sy}^2) & -C_s e_{sy} & -C_s e_{sx} \\ -C_s & 0 & -C_s e_{sy} & C_s & 0 \\ 0 & -C_s & -C_s e_{sx} & 0 & C_s \end{bmatrix} \quad (11)$$

and

$$[K] = \begin{bmatrix} \sum K_{pi} + K_s & 0 & \sum K_{pi} y_i + K_s e_{sy} & -K_s & 0 \\ 0 & \sum K_{pi} + K_s & \sum K_{pi} x_i + K_s e_{sx} & 0 & -K_s \\ \sum K_{pi} y_i + K_s e_{sy} & \sum K_{pi} x_i + K_s e_{sx} & \sum K_{pi} (x_i^2 + y_i^2) + K_s (e_{sx}^2 + e_{sy}^2) & -K_s e_{sy} & -K_s e_{sx} \\ -K_s & 0 & -K_s e_{sy} & K_s & 0 \\ 0 & -K_s & -K_s e_{sx} & 0 & K_s \end{bmatrix} \quad (12)$$

where  $\sum C_{pi}$ ,  $\sum C_{px\theta}$  etc. are the elements of the damping matrix neglecting the S-system;  $C_s (= 2\xi_s m_s \omega_s)$  is the damping for the S-system; and  $\xi_s$  is the percentage critical damping. The elements of the damping matrix, concerning the P-system, are determined by assuming that the damping matrix is proportional to its mass and stiffness matrices. Using the modal damping ratio and the first two undamped mode shapes of the P-system (only), these elements are obtained by the standard procedure (Clough and Penzien 1993, Paz 1991).

### 3. Response Analysis

The frequency response function matrix  $H(\omega)$  for the PS-system is given by

$$H(\omega) = (-\omega^2 [M] + i\omega [C] + [K])^{-1} \quad (13)$$

If  $S_{U_g}(j = x, y)$  is the power spectral density function (PSDF) of the input excitation, then the PSDF matrix of the response is given by

$$S_U(\omega) = H(\omega) S_f(\omega) H(\omega)^{*T} \quad (14)$$

in which  $S_f(\omega)$  is the power spectral density function matrix of  $f(t)$ . The symbol, \* denotes complex conjugate. Referring to Equation (8),  $f(t)$  is given by

$$f(t) = -[M][I]\ddot{U}_g = M\bar{U}_g \quad (15a)$$

in which

$$\bar{U}_g = [\ddot{u}_{gx}, \ddot{u}_{gy}, 0, \ddot{u}_{gx}, \ddot{u}_{gy}]^T \quad (15b)$$

Therefore,  $S_f(\omega)$  may be written as

$$S_f(\omega) = [M]^T S_{\bar{U}_g} [M] \quad (15c)$$

Equation (15c) may be simplified as

$$S_f(\omega) = \begin{bmatrix} m_p^2 & 0 & 0 & m_p m_s & 0 \\ 0 & m_p^2 & 0 & 0 & m_p m_s \\ 0 & 0 & 0 & 0 & 0 \\ m_p m_s & 0 & 0 & m_s^2 & 0 \\ 0 & m_p m_s & 0 & 0 & m_s^2 \end{bmatrix} S_{\bar{U}_g}(\omega) \quad (15d)$$

in which  $S_{\bar{U}_g} = S_{\ddot{u}_{gx}} = S_{\ddot{u}_{gy}}$ ;  $S_{\ddot{u}_g}$  is the PSDF of ground acceleration.

The PSDFs of relative displacement  $U_{rx}$  and  $U_{ry}$  and absolute acceleration  $\ddot{U}_{aj}$  ( $j = x, y$ ), of the S-system are obtained in the following manner:

$$U_{rx} = U_{xx} - U_{px} - U_{\theta} e_{py} \quad (16)$$

$$U_{ry} = U_{yy} - U_{py} - U_{\theta} e_{ex} \quad (17)$$

and

$$\ddot{U}_{aj} = \ddot{U}_{sj} + \ddot{U}_{gj} \quad (18)$$

$S_{U_{rx}}$ ,  $S_{U_{ry}}$ , and  $S_{U_{aj}}$  ( $j = x, y$ ) are, therefore, given by

$$S_{U_{rx}} = S_{U_{xx}} + e_{py}^2 S_{U_{\theta}} + S_{U_{xx}} + e_{py} S_{U_{px}U_{\theta}} + e_{py} S_{U_{\theta}U_{px}} - S_{U_{px}U_{xx}} - S_{U_{xx}U_{px}} - e_{py} S_{U_{xx}U_{\theta}} - e_{py} S_{U_{\theta}U_{xx}} \quad (19)$$

$$S_{U_{ry}} = S_{U_{yy}} + e_{ex}^2 S_{U_{\theta}} + S_{U_{yy}} + e_{ex} S_{U_{py}U_{\theta}} + e_{ex} S_{U_{\theta}U_{py}} - S_{U_{py}U_{yy}} - S_{U_{yy}U_{py}} - e_{ex} S_{U_{yy}U_{\theta}} - e_{ex} S_{U_{\theta}U_{yy}} \quad (20)$$

$$S_{\ddot{U}_{aj}} = S_{\ddot{U}_{sj}} + S_{\ddot{U}_{gj}} + S_{\ddot{U}_{sj}\ddot{U}_{gj}} + S_{\ddot{U}_{gj}\ddot{U}_{sj}} \quad (21)$$

The elements of the right hand side (RHS) of Equations (19) and (20) can be directly obtained from the elements of the matrix  $S_U(\omega)$ . The elements of the RHS of the Equation (21) are derived as

$$S_{\ddot{U}_{sj}} = \omega^4 S_{U_{sj}} \quad (22)$$

$$S_{\ddot{U}_{rx}\ddot{U}_{rx}} = H(4,1) S_{\ddot{U}_{sx}} \quad (23)$$

$$S_{\ddot{U}_{ry}\ddot{U}_{ry}} = H(5,2) S_{\ddot{U}_{sy}} \quad (24)$$

$$S_{\ddot{U}_{rx}\ddot{U}_{rx}} = H(4,1)^* S_{\ddot{U}_{sx}} \quad (25)$$

$$S_{\ddot{U}_{ry}\ddot{U}_{ry}} = H(5,2)^* S_{\ddot{U}_{sy}} \quad (26)$$

where  $H(4,1)$  and  $H(5,2)$  are the elements of the  $H(\omega)$  matrix. The \* sign indicates complex conjugate. The variances of the response quantities are obtained as follows.

$$\sigma_{\ddot{U}_{sj}}^2 = \int_{-\infty}^{+\infty} S_{\ddot{U}_{sj}}(\omega) d\omega \quad (27)$$

$$\sigma_{\ddot{U}_{aj}}^2 = \int_{-\infty}^{+\infty} S_{\ddot{U}_{aj}}(\omega) d\omega \quad (28)$$

## PARAMETRIC STUDY

In the numerical study,  $e_{px}$  is taken same as  $e_{py}$ . Similarly,  $s_x = s_y$ . As a result, responses in the two directions become the same under bidirectional excitations having the same input ground motions (without any phase difference). These assumptions are made in order to reduce the number of response parameters to be studied. The responses (absolute acceleration ( $\sigma_{\ddot{x}_s}/g = \sigma_{\ddot{u}_{sx}}/g = \sigma_{\ddot{u}_{sy}}/g$ ) and the relative displacement ( $\sigma_{x_r} = \sigma_{U_{rx}} = \sigma_{U_{ry}}$ ) of the S-system are influenced by a large number of parameters. The important parameters that are considered here are the normalized eccentricities of the P-system ( $e_{px}/r$  and  $e_{py}/r$ ) and the S-system ( $e_{sx}/r$  and  $e_{sy}/r$ ) in two orthogonal directions (X and Y), the uncoupled lateral frequencies of the P-system ( $\omega_p$ ) and the S-system ( $\omega_s$ ), the damping ratios of the P-system ( $\xi_p$ ) and the S-system ( $\xi_s$ ), the ratio of uncoupled lateral to rotational frequencies ( $\omega_p/\omega_{\theta}$ ) of the P-system and the mass ratio  $m_s/m_p$  of the PS-system. Values of the other parameters (held constant throughout) are  $\omega_p = 3.0$  rad/sec,  $\xi_p = 5.0\%$ ,  $\xi_s = 2.0\%$ ,  $r = 3.0$  m. For convenience, eccentricities of the P-system in X and Y directions are taken as same. Similarly, eccentricities of the S-system, with respect to the CM of the P-system are taken to be same in both X and Y directions. The intensity of white

intensity of white noise input excitation is same in both X and Y directions and is taken to be 0.013 m<sup>2</sup>/sec/rad. Since responses of the S-system are influenced by a complex interaction of a large number of parameters, it is difficult to explain the reason for the nature of variation of responses with a particular parameter. However, the variation of responses of the S-system with some important parameters are explained here to study the behavior of the S-system attached to a torsionally coupled P-system. The parametric variation of  $(\sigma_x$  and  $\sigma_x/g)$  are shown in the same figure in order to reduce the number of figures. Note that  $\sigma_x$  is in meters while  $\sigma_x/g$  is non-dimensional.

**1. Effect of  $\omega_p/\omega_\theta$  of the Primary System**

Figure 2 shows the effect of the torsional coupling  $\omega_p/\omega_\theta$  of the P-system on the responses of the S-system for strong torsionally coupled P-systems under the tuned condition. Since the dotted lines in the figure indicate responses without torsional coupling, they are shown as invariant of  $e_{px}/r$  or  $e_{py}/r$ . It is seen from the figure that responses of the S-system are less when the torsional coupling is considered in the analysis. Further, responses decrease with the increase in the normalized eccentricity of the P-system.

Figure 3 shows the same effect for the weak torsionally coupled P-systems under the detuned condition. Responses are almost the same for both torsionally coupled and uncoupled P-systems. Also, responses are insensitive to the variation of the normalized eccentricity of the P-system.

**2. Effect of  $e_{px}/r$  and  $e_{py}/r$**

Figures 4 to 7 show the effect of the eccentricity of the P-system on the responses of the S-system, for strong and weak torsionally coupled P-system under both tuned and detuned conditions. Responses decrease with the increase in  $e_{px}/r$  or  $e_{py}/r$  for the tuned condition (see Figures 4 and 6). However, for the detuned condition ( $\omega_p/\omega_s = 1.5$ ) responses are almost insensitive to the variation of  $e_{px}/r$  or  $e_{py}/r$  (see Figures 5 and 7), and the variations of responses with  $e_{px}/r$  or  $e_{py}/r$  are nearly the same for both strong and weak torsionally coupled P-systems.

**3. Effect of  $e_{xx}/r$  and  $e_{yy}/r$**

Figures 8 and 9 show the variations of responses with  $e_{xx}/r$  and  $e_{yy}/r$  for strong torsionally coupled P-systems under both tuned and detuned conditions. It is observed that responses increase linearly with the increase in  $e_{xx}/r$  or  $e_{yy}/r$  under the tuned condition (see Figure 8). Responses decreases mildly with the change in  $e_{xx}/r$  and  $e_{yy}/r$  for the detuned condition (see Figure 9).

**4. Effect of  $m_s/m_p$  Ratio**

Figures 10 to 13 show the effect of the  $m_s/m_p$  ratio on the absolute acceleration of the S-system. It is seen that the increase in the  $m_s/m_p$  ratio decreases the response of the S-system for all cases (i.e., tuned and detuned conditions). Under the tuned condition, responses decrease with the increase in  $e_{px}/r$  or  $e_{py}/r$ .

**5. Effect of Damping Ratio of the S-System**

Figures 14 and 15 show the variations of responses with the damping ratio of the S-system ( $\xi_s$ ) for the strong torsionally coupled P-system, under both tuned and detuned conditions. Responses decrease with the increase in  $\xi_s$ , as would be expected. For the detuned condition, the rate of decrease in response with the increase in  $\xi_s$  becomes very less at the higher values of  $\xi_s$ .

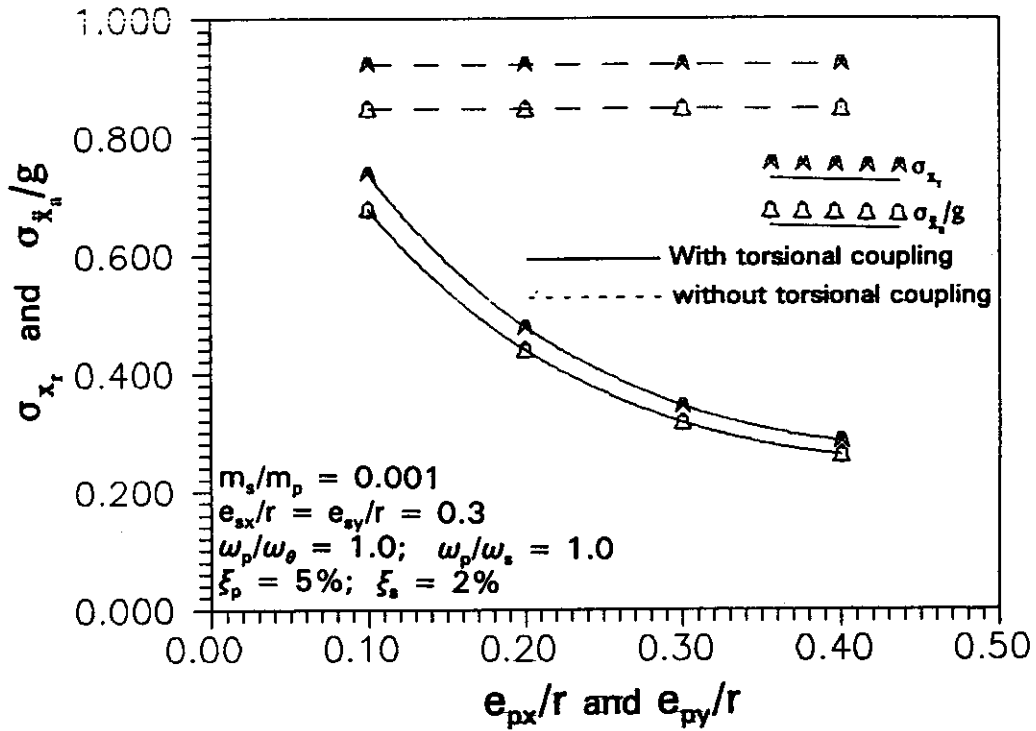


Fig. 2 Variation of  $\sigma_x/g$  and  $\sigma_{x_r}$  with  $e_{px}/r$  and  $e_{py}/r$ , for  $\omega_p/\omega_\theta = 1.0$ ,  $\omega_p/\omega_s = 1.0$ , and for torsionally coupled and torsionally uncoupled P-systems

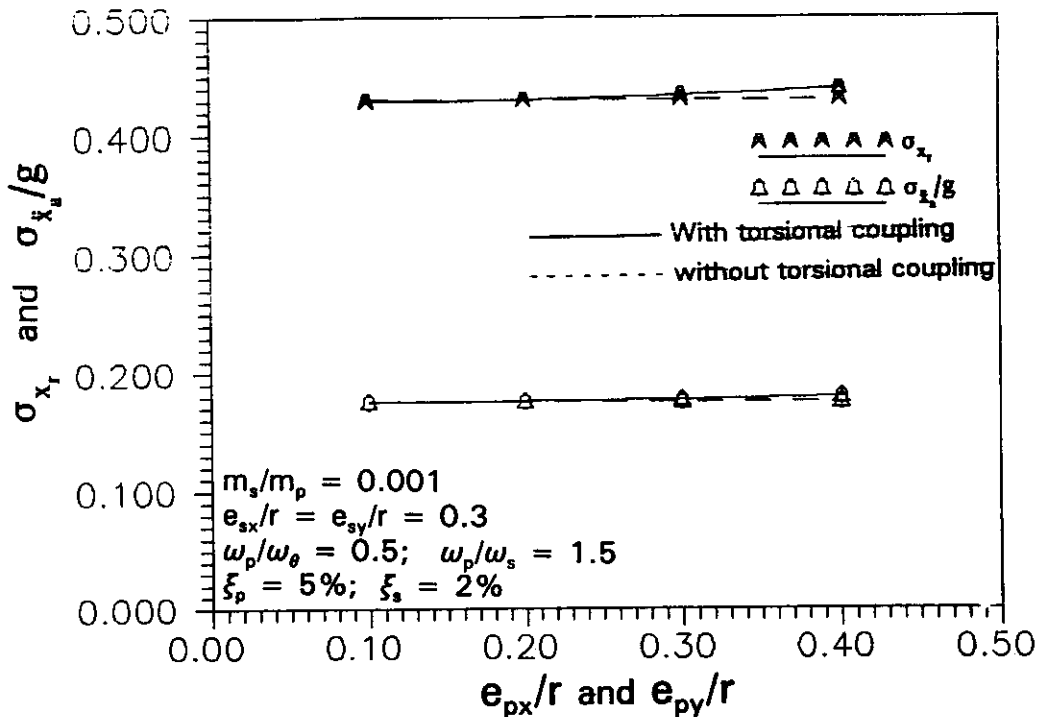


Fig. 3 Variation of  $\sigma_x/g$  and  $\sigma_{x_r}$  with  $e_{px}/r$  and  $e_{py}/r$ ,  $\omega_p/\omega_\theta = 0.5$ ,  $\omega_p/\omega_s = 1.5$ , and for torsionally coupled and torsionally uncoupled P-systems



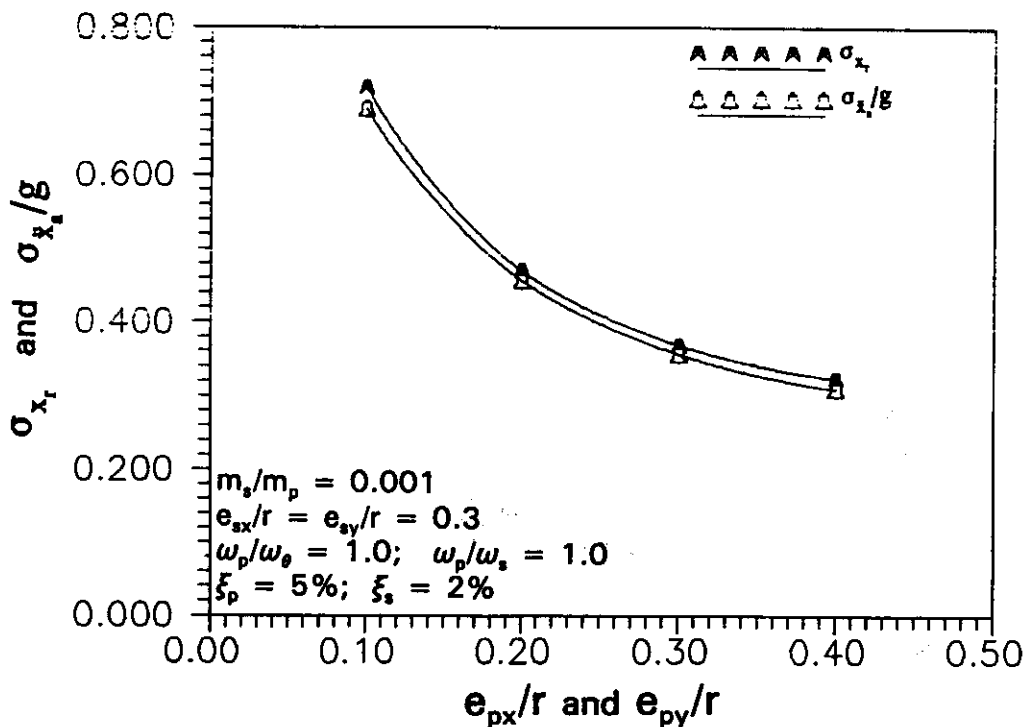


Fig. 4 Variation of  $\sigma_{x_i}/g$  and  $\sigma_{x_r}$  with  $e_{p_x}/r$  and  $e_{p_y}/r$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.0$ .

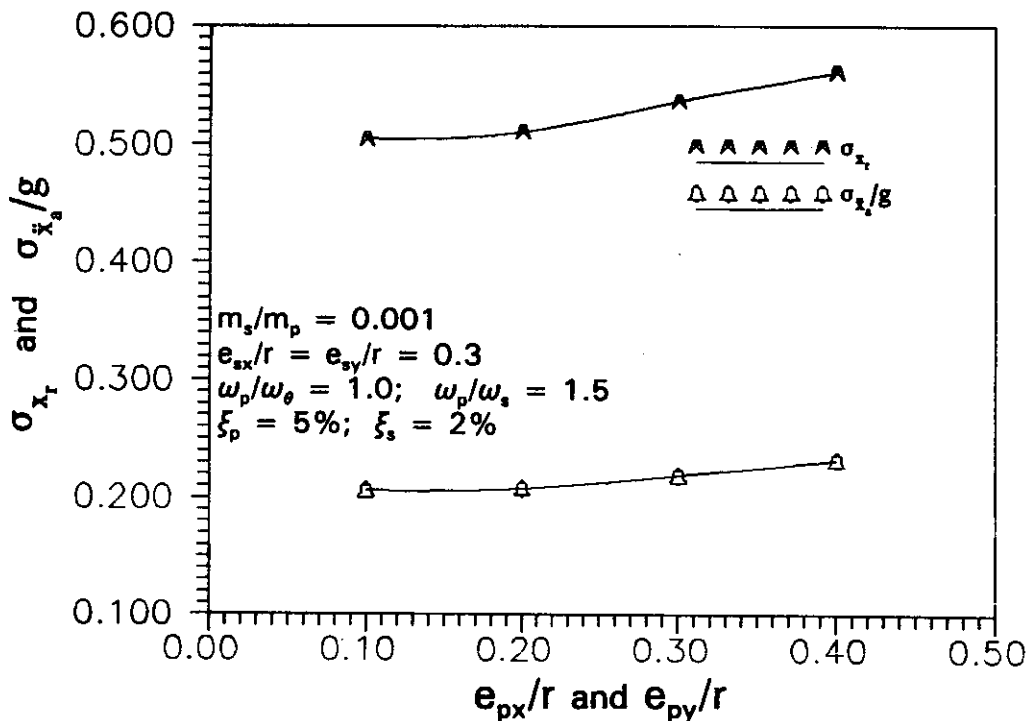


Fig. 5 Variation of  $\sigma_{x_i}/g$  and  $\sigma_{x_r}$  with  $e_{p_x}/r$  and  $e_{p_y}/r$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.5$ .

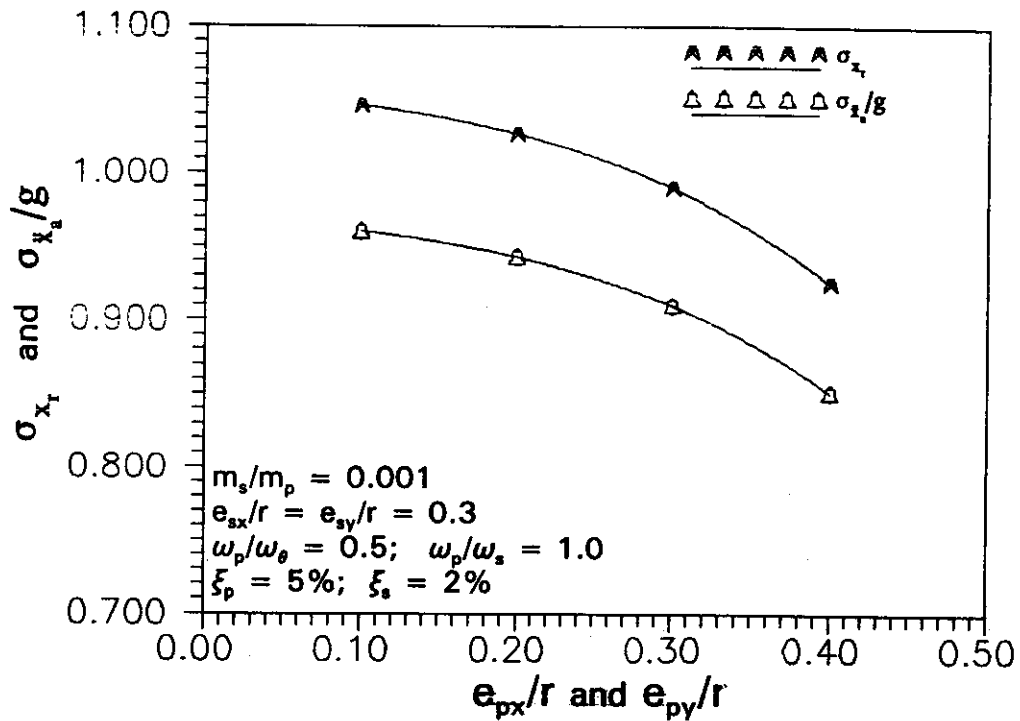


Fig. 6 Variation of  $\sigma_{x_r}/g$  and  $\sigma_{x_r}$  with  $e_{px}/r$  and  $e_{py}/r$ , for  $\omega_p/\omega_\theta = 0.5$  and  $\omega_p/\omega_s = 1.0$ .

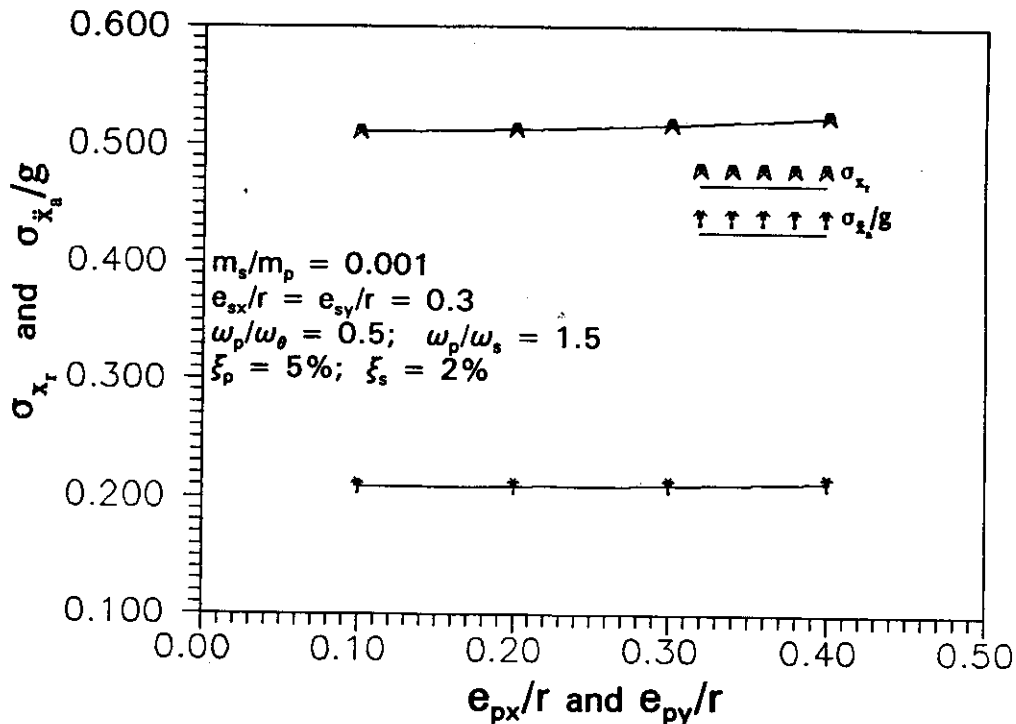


Fig. 7 Variation of  $\sigma_{x_r}/g$  and  $\sigma_{x_r}$  with  $e_{px}/r$  and  $e_{py}/r$ , for  $\omega_p/\omega_\theta = 0.5$  and  $\omega_p/\omega_s = 1.5$ .

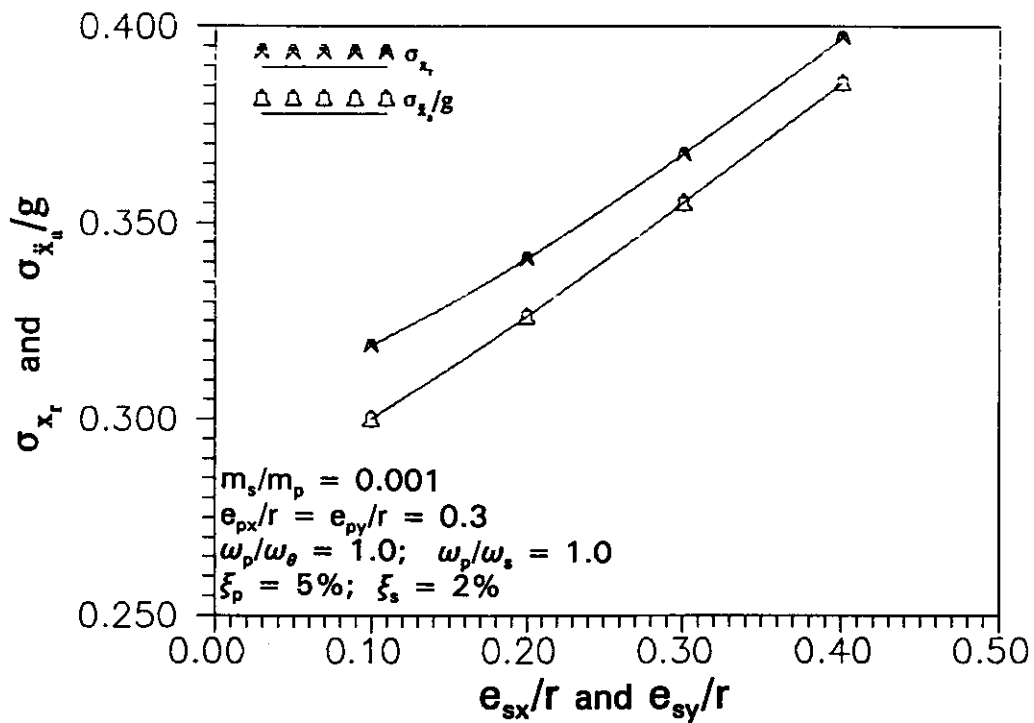


Fig. 8 Variation of  $\sigma_{x_i}/g$  and  $\sigma_{x_r}$  with  $e_{sx}/r$  and  $e_{sy}/r$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.0$ .

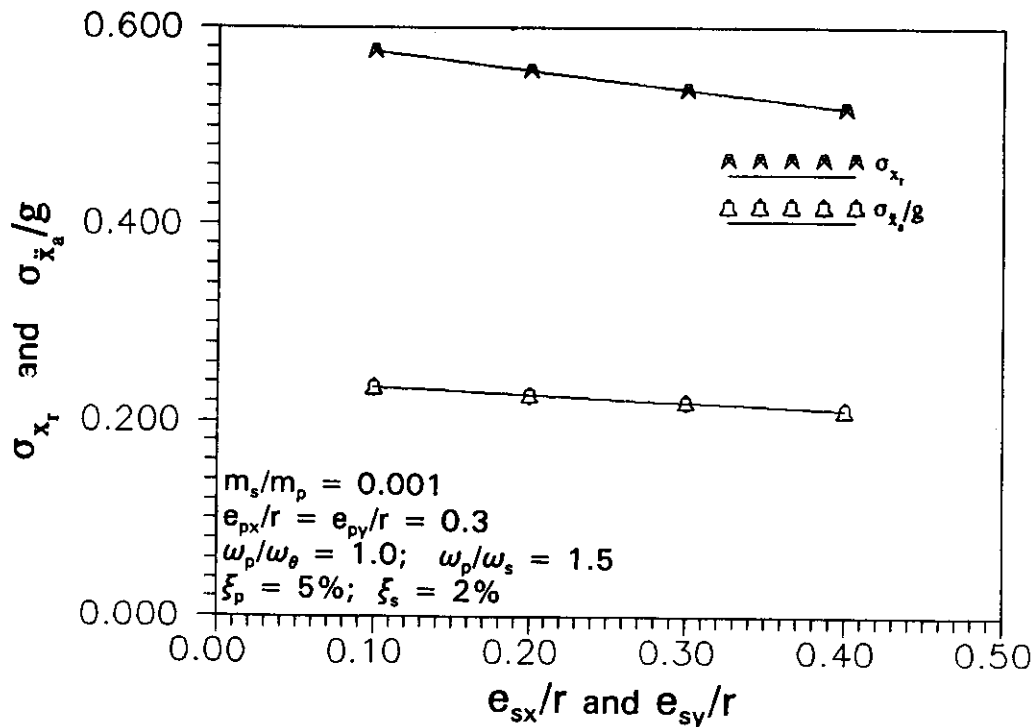


Fig. 9 Variation of  $\sigma_{x_i}/g$  and  $\sigma_{x_r}$  with  $e_{sx}/r$  and  $e_{sy}/r$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.5$ .

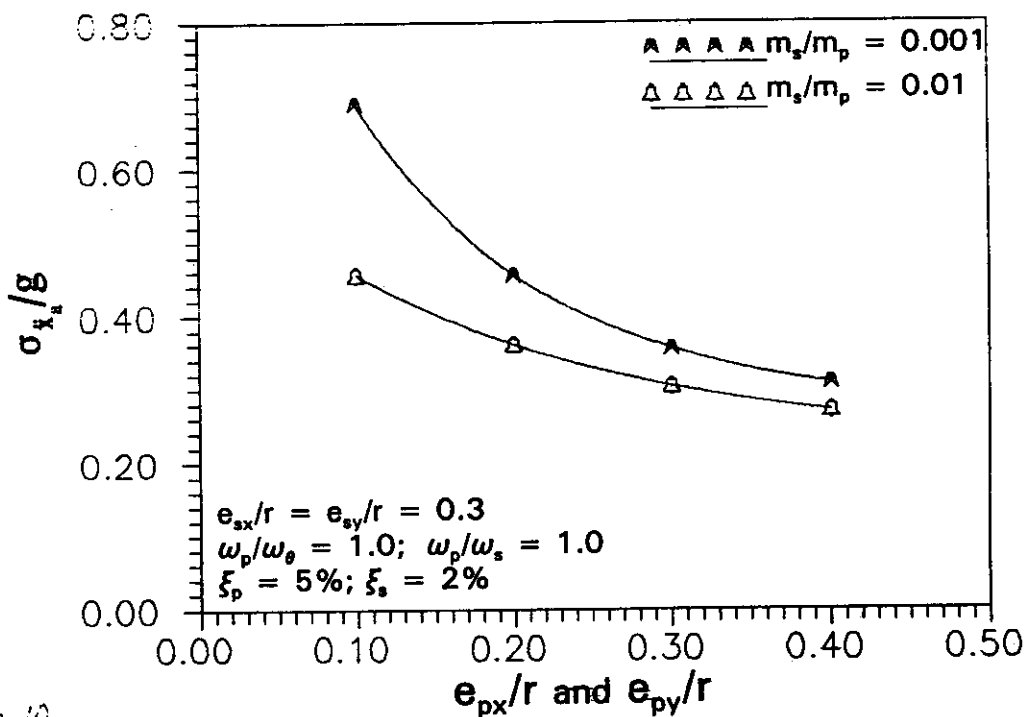


Fig.10 Variation of  $\sigma_{x_s}/g$  with  $e_{px}/r$  and  $e_{py}/r$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.0$ .

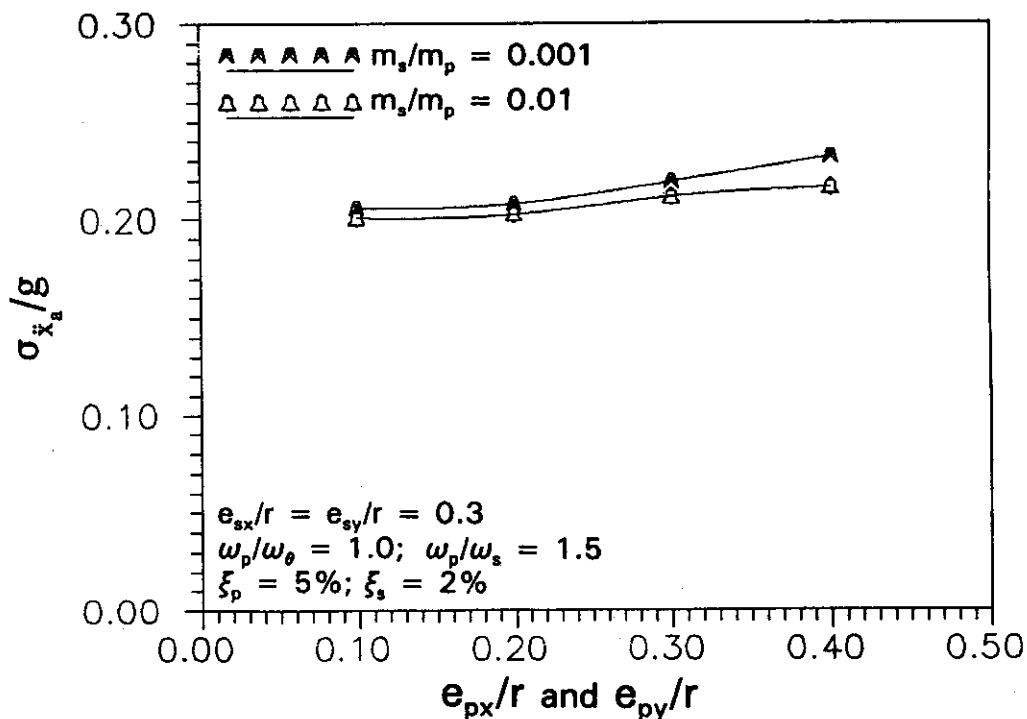


Fig. 11 Variation of  $\sigma_{x_s}/g$  with  $e_{px}/r$  and  $e_{py}/r$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.5$ .

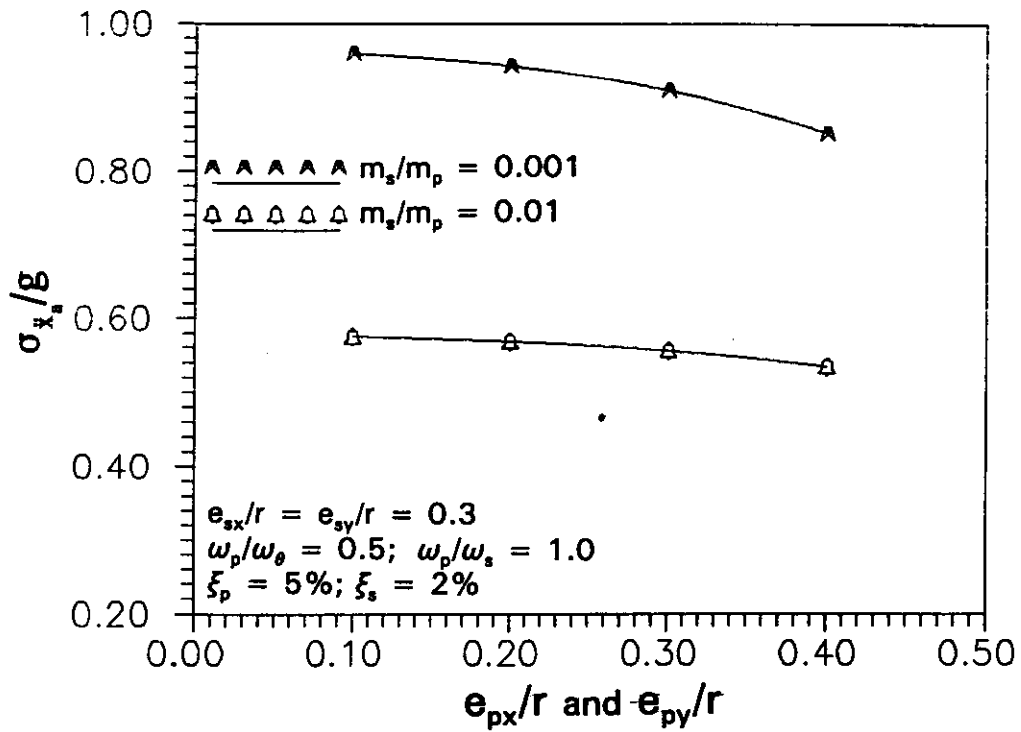


Fig. 12 Variation of  $\sigma_{x_i}/g$  with  $e_{px}/r$  and  $e_{py}/r$ , for  $\omega_p/\omega_\theta = 0.5$  and  $\omega_p/\omega_s = 1.0$ .

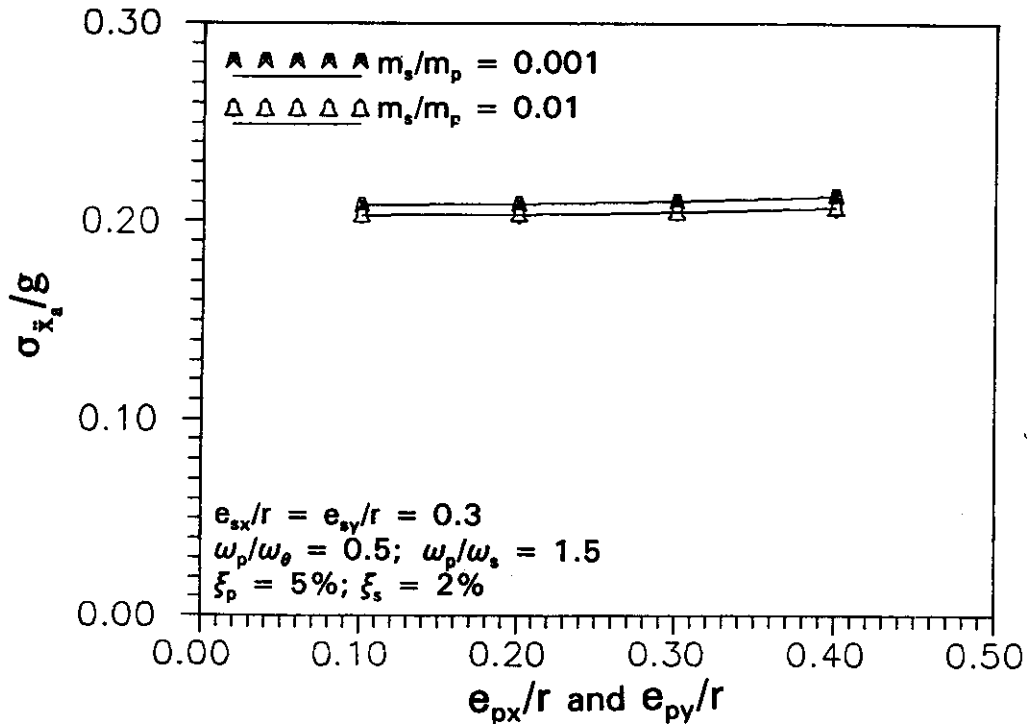


Fig. 13 Variation of  $\sigma_{x_i}/g$  with  $e_{px}/r$  and  $e_{py}/r$ , for  $\omega_p/\omega_\theta = 0.5$  and  $\omega_p/\omega_s = 1.5$ .

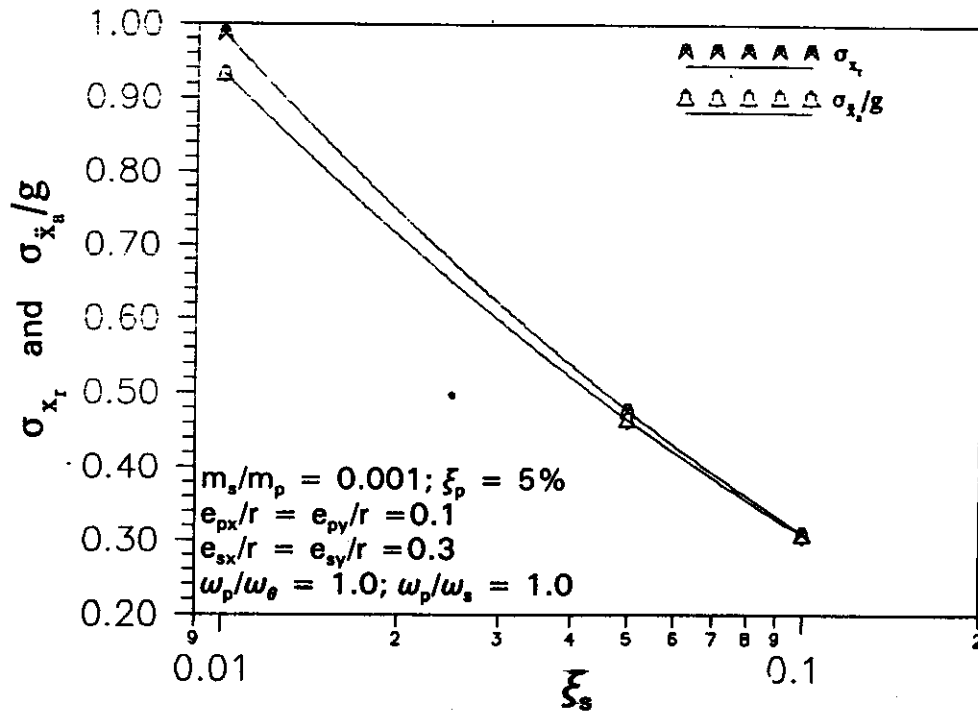


Fig. 14 Variation of  $\sigma_{x_1}/g$  and  $\sigma_{x_1}$  with  $\xi_s$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.0$ .

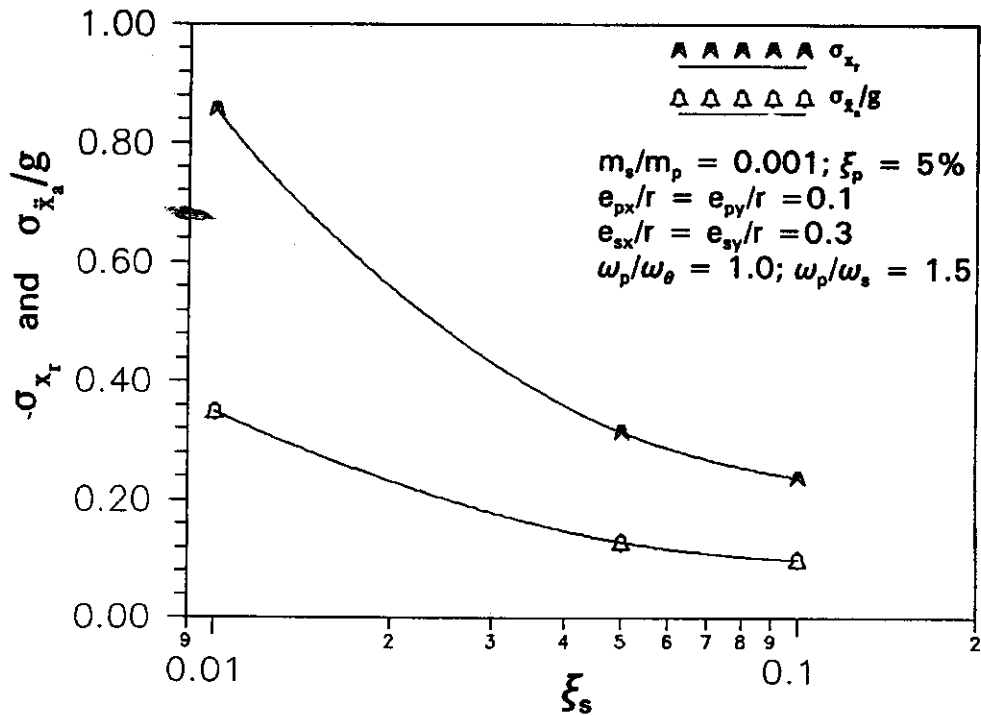


Fig. 15 Variation of  $\sigma_{x_1}/g$  and  $\sigma_{x_1}$  with  $\xi_s$ , for  $\omega_p/\omega_\theta = 1.0$  and  $\omega_p/\omega_s = 1.5$ .

## CONCLUSIONS

The seismic behavior of a S-system mounted on a torsionally coupled linear P-system has been investigated for bi-directional earthquake excitations modelled as a white noise. The response quantities of interest are the standard deviation of the relative displacement and the absolute acceleration of the S-system. Responses are obtained by the frequency domain spectral analysis. Observations made from the parametric study are as follows:

- (1) For the strong torsionally coupled P-system under the tuned condition, responses obtained by considering the torsional coupling in the P-system are less than those for the uncoupled P-system. For the weak torsionally coupled P-system under the detuned condition, responses are almost the same for both torsionally coupled and uncoupled P-systems.
- (2) Under the tuned condition, responses of the S-system decrease with the increase in  $e_{px}/r$  or  $e_{py}/r$ . However, for the detuned condition, responses are almost insensitive to the change in  $e_{px}/r$  or  $e_{py}/r$ .
- (3) Responses increase with the increase in  $e_{rx}/r$  and  $e_{ry}/r$  for the strong torsionally coupled P-system under the tuned condition. However, an opposite trend is observed for the strong torsionally coupled P-system under the detuned condition.
- (4) For higher values of the  $m_s/m_p$  ratio, responses of the S-system are reduced.
- (5) Responses decrease with the increase in  $\xi_s$ . For the detuned condition, the rate of decrease of response is significantly reduced for higher values of  $\xi_s$ .

Thus, the design of S-system mounted on a P-system such as building structures, should duly consider the degree of torsional coupling of the P-system. Under the tuned condition, neglecting the effect of torsional coupling may lead to considerable error in the computation of stresses in the S-system. However, for highly detuned condition, the effect of torsional coupling may be ignored for design of the S-system. For very light equipments, the condition of tuning should be avoided as far as possible.

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