A REVIEW OF MACHINE FOUNDATION BEHAVIOUR

Shamsher Prakash* and U. K. Bhatia**

SYNOPSIS

A machine foundation differs from any other type of foundation, because of the dynamic nature of loads. Till about 1930, a machine foundation design was based upon emperical methods. These methods did not take into account the properties of the underlying soil. In the years to follow, attempts have been made to understand the problem scientifically.

Characteristics of the underlying soil strata affect the resonance of the system with the machine. The available literature on the subject is scattered and no systematic investigation, covering the present trend is available. The present investigation of the Behaviour of Machine Foundation is intended to make a systematic study of available literature and is believed to lead to a better understanding of the problems connected. The various methods to determine the resonant frequency of machine foundation are critically reviewed, compared with each other and their limitations discussed.

A simple emperical equation for determination of resonant frequency of the system is proposed. Suggestions for further research have also been made.

INTRODUCTION

General :

The function of a machine foundation, similar to any other foundation, is to transmit the imposed loads safely on to the soil on which it is placed. Its special feature, however, is that in addition to the static load, due to the weight of the machine, and the foundation, vibrating or pulsating forces varying with time have to be considered. Such forces may be of short duration, such as shock or impact forces in forging hammers or may vary periodically as in reciprocating and rotating machines. As a result, waves or steady vibrations are set up in the foundation soil. If the natural frequency of the foundation soil system happens to coincide with or lie close to the frequency of the exciting forces generated by the machines, excessive vibration amplitudes may occur, which lead to structural damage or operational failure of the machine.

Role of soil mechanics:

Problems connected with machine foundations, were not considered to be important till the advent of the thirties, when greater use of heavy industrial plants and consistent failure of machine foundations attracted attention of designers (Tschebotarioff 1951). Before that, the design of machine foundations was purely emperical, the simplest being to provide heavy foundation blocks. It was considered adequate to rest the machine on rigid foundation so as to avoid excessive amplitudes of vibration. It was believed that natural frequency of the rigid foundation would be higher than the operating frequency of the machine. But invariably the rigid foundation has to rest on the ground. The result is that rigid foundation transmits the vibrations to the ground, which is relatively flexible, so that danger of resonance is still there. The importance of this fact was realized when even after the provision of rigid foundations, excessive vibrations due to resonance were caused. With the advent of soil mechanics, the problem of machine foundation has been tackled more rationally and scientifically. The investigations, both theoretical and experimental have led to better understanding of the behaviour of machine foundations resulting in economy. At the same time, the number of failurs of machine foundations which are mostly due to excessive amplitudes of vibrations have been minimized.

* Reader in Civil Engineering, School of Research and Training in Earthquake Engineering, University of Roorkee, Roorkee, (U.P.) INDIA,

**Assistant Professor of Civil Engineering, Birla Engineering College, Pilani, INDIA.

Concept of Resonant Frequency :

Experimental studies on the phenomena of ground vibrations were first systematically conducted by Degebo (Deustche Gesellschaft für Bodenmechanik) organisation in Germany from 1928 to 1939. Their earlier experiments were with a vibrator with fixed static and dynamic forces. The results showed that soil at any site had a natural frequency or self frequancy of vibrations, depending only on the soil type (Lorenz 1934). This was further substantiated by Andrews and Crockett (1945) who independently determined the natural frequency by a study of resonance between heavy industrial plant, and the ground. At about the same time, Vios, the Institute for Engineering Foundation Research performed similar tests in Russia (Barkan 1936).

Later experimental and analytical studies (Lorenz 1953, Reissner 1936, Sung 1953, Quinlan 1953, Richart 1960, Tschebotarioff and Ward 1948) have shown that the natural frequency of soil as such is meaningless. It varies not only with the type of underlying soil but depends also on the shape and extent of the contact area and magnitude of dynamic and static loads. Hence it will be proper to use the term resonant frequency of the machine foundation-soil system, or simply the resonant frequency of the system.

The problem of determining the resonant frequency of the system has been tackled both by theoretical methods and emperical methods based on past data of resonance. The theoretical approaches are based on the assumption of (a) Soil as semi-infinite elastic solid and (b) Soil as a spring, usually elastic.

Semi-Infinite Elastic Solid :

One of the theoretical approaches is concerned with the "*Dynamic Boussineq's Problem*". This approach considers the machine foundations resting on the ground and oscillating on the surface of the semi-infinite elastic, isotropic and homogeneous medium. Prominent contributions based on this concept are those of Reissner, (1936), Quinlan (1953), Sung (1953) for vertical vibrations and those of Arnold, Bycroft and Warburton (1955), Bycroft (1959), and Hesih (1962), for other modes of vibration viz., horizontal and rotational modes.

This analysis is based on the knowledge of the dynamic response of the ground in terms of dynamic soil constants i.e. modulus of elasticity or modulus of rigidity, and Poisson's ratio. Dynamic testing of soils is done by measuring the velocities of propogation in the medium. Rayleigh (1865), Lamb (1904), Leet (1950) and others have analysed the velocity of wave propegatian in the semi infinite elastic homogeneous and isotropic medium. Bergstrom and Linderholm (1946), Bernhard and Finelli (1953), Jones (1955, 1958), Bernhard (1956), Vanderpoel (1951), Nijboer and Vanderpoel (1953) Nijboer (1959) Heukelom and Foster, (1960), and many others have given the analysis applicable to soils for determining in-situ dynamic constants.

Mass Spring System :

The other theoretical approach is to assume the ground to be a spring, with or without, damping. In initial studies (Rausch 1936), this spring was assumed to be weightless and linear. But experimental studies by Degebo (Lorenz 1934), Vios (Barkan 1936) and their subsequent analysis have shown that some soil mass also oscillates with the machine foundation. This mass of soil was found to be 4 to 10 times the vibrator mass by Degebo. Vios concluded it to be relatively insignificant and neglected it to obtain the dynamic modulus of subgrade reaction. But indirectly it was accepted that soil mass could not be neglected and that its value must lie between two third and one and a half times that of foundation. The above two statements seem to be contrary. Terzaghi (1943), reccommends the soil mass to be three times the dynamic force transmitted to the ground. Andrews and Crockett (1945), Crockett and Hammond (1948, 1949) suggested that mass of soil which vibrates with the foundation must bear some relation to the bulb of pressure which gives the stress distribution under a uniformly loaded area on an elastic medium. None of them gave any precise relationship. It was, however, regarded that this mass must vary with the area of contact and the dynamic unbalance forces. Balakrishna and Nagraj (1960) and Balakrishna Rao, (1961, 1962) further advanced the concept of pressure bulb and have suggested that the mass oscillating should be taken as the mass of the soil within the pressure bulb of the same intensity (lb./sq. ft.) as the density of the soil (lb./cu. ft.).

The value of the spring constant has been taken as the load required per unit reversible deflection (Barkan 1936, Newcomb 1951). Lorenz (1934) has obtained k' dynamic modulus of subgrade reaction and W_s the soil weight, from two vibrator tests under different loading conditions and with different areas, by assuming both k' and W_s to be constant. But this assumption is not justified, as the tests have shown that both spring constant and soil weight vary with different vibrator sizes and loading conditions even on the same type of soil.

Pauw (1953) by assuming soil as truncated spring, has developed expressions for spring factors and mass factors for different modes of vibration.

Use of Experimental Behaviour :

Another approach uses the past records of resonant frequencies observed. Emperical relations have been developed. Tschebotarioff and Ward (1948) and Tschebotarioff (1951, 1953) have obtained logarithmic relationship between reduced natural frequency and the contact area. Newcomb (1951) observed a linear relationship between the resonant frequency and the static pressures.

Another approach which considers soil as sublinear spring, uses the resonance curves obtained from the test vibrator to plot the sublinear characteristic of soil and has been put forward by Lorenz (1953) and Alpan (1961).

Scope of Study :

The problem of machine foundation has been receiving importance since the thirties of this century. The importance of underlying soil strata has been realized in respect of the resonance phenomena. The available literature is scattered and no systematic investigation covering the present trend is available. It is felt that this investigation of the behaviour and design of machine foundation, based on a systematic study of available literature, will lead to better understanding of the problem connected. The nature of the problem consists of a system to be analyzed (Alpan 1961). This system consists of the machine, the foundation and the soil and involves the following procedure :

(a) Weight and operating frequency of the machine and the magnitude of the dynamic forces is given.

(b) The properties and the dynamic response of the foundation soil are to be determined, or assumed.

(c) The foundation is designed based on soil properties. The general shape and dimensions of which may be assumed for preliminary design.

The type of the foundation considered in this paper is massive block resting directly on the ground. Isolators and shock absorbers are not considered.

Review of the available literature on resonant frequency leads to an interesting observation. Spring constant in all the approaches can be expressed as a simple multiple of G r_0, while the mass factor a simple multiple of ρ r_0^3,

where G is the modulus of ridiity of underlying soil,

ρ is the mass density of underlying soil and

r_0 is the radius of the foundation base in contact with soil.

Out of the available approaches to resonant frequency determination, Pauw's (1953) analysis is recommended. Balakrishna Rao's (1962) modification is recommended in connection with vertical vibrations only.

Almost all the experimental investigations reported in literature have been carried out for vertical vibrations. The approaches already existing should be verified for other modes of vibrations as well. Pressure distribution under machine foundation have not been investigated. Information on effect of dynamic load on bearing capacity is not available. Based on these observations, suggestions for further research have been made.

It is felt that the dynamic behaviour of soil be evaluated by observing the resonant frequency of a test vibrator under different loading conditions and contact areas. The information obtained then can be used for determination of resonant frequencies of the foundation soil system.

BEHAVIOUR OF MACHINE FOUNDATIONS

General :

Machine foundations are important substructures. For the safety of operation of every factory, dependable foundations of its machines are essential. If the foundation is not properly designed, not only the machine gets damaged but the adjoining structures may also be damaged. In addition, for proper working conditions in a factory, the vibrations produced by the machine should be such as not to interfer with the worker's comfort. For a proper design of machine foundation it is essential that its behaviour be understood. The discussion to follow has been prepared from evidence reported in literature from time to time. It is essential to know the magnitude of the dynamic forces and its frequency. Based on its behaviour, the requirements of machine foundation are also discussed.

Behaviour of a Machine Foundation :

A machine foundation is different from other foundations, mainly because it is subjected to a dynamic load, which is usually periodic. Under the influence of this load, the foundation starts vibrating. For every system, there is a natural frequency, which is defined as the frequency with which it will vibrate, when subjected to free vibrations. For a body with spring stiffness as k and mass m_0, the natural frequency ω_0 is given by (neglecting damping).

$$\omega_0 = \sqrt{k/m_0} \quad \ldots \text{(1a)}$$

$$= \sqrt{kg/W_0} \quad \ldots \text{(1b)}$$

Under forced vibrations, as in machine foundations the phenomena of resonance occurs, if the operating frequency coincides with this natural frequency. For no damping the amplitude at resonance tends to infinity. If damping is included in the system, the amplitude of vibration is still maximum close to resonance, though of finite value. The ratio of actual amplitude to free amplitude (the static deflection of spring, due to dynamic load) is called magnification factor N_1 (Denhartog 1947). At frequency ratio (the ratio of operating frequency to the natural frequency) of 1.0, the magnification factor is maximum.

The transmissiblity is defined as the ratio of force transmitted to the foundation and dynamic force. For small damping, transmissibility is maximum at frequency ratio of unity. For machines having dynamic loads independent of frequency, it is maximum at frequency ratio of 1.0, for large damping as well. If the dynamic load is proportional to the square of the frequency (which is true for rotating and reciprocating machines) transmissibility may be maximum for higher frequency ratios (Mykelstad 1956). In such cases it is preferable to keep frequency ratio lower than 1.0.

In addition, at resonance, the power required to keep the system oscillating is maximum. This has been observed experimentally by Lorenz (1934), Crockett and Hammond (1948) and analytically by Reissner (1936), Sung (1953) and Quinlan (1953). Thus it is seen that at resonance, the amplitude of vibration, the force transmitted, and the power input requirement of the machine, are maximum. Hence resonance has to be avoided.

In an attempt to avoid resonance, the foundation was made rigid and firm. The natural frequency of such a rigid body like mass concrete foundation is very high. The equivalent spring constant (k) is (Timoshenko 1937):

$$k \propto \frac{E}{1-\nu^2} \quad \ldots \text{(2)}$$

where E is modulus of elasticity and

ν is the Poisson's ratio.

Values of E and ν for the concrete are of the order of 2 to 5×10^6 psi and 0.15 respectively. This would give a very high natural frequency of the foundation, with hardly any chance of resonance with machine's operating frequency. But it has been observed that even these massive foundations start vibrating and sometimes the amplitudes become quite large. The answer lies in the fact, that though the foundation is rigid in itself, it is resting on the ground. The ground is not as rigid. The value of E for soils is of the order of 10 to 15×10^3 psi and Poisson's ratio of the order of 0.3 to 0.4. This means that soil and foundation are in series (two springs in series), with the soil characteristics predominating. That is why phenomenon of resonance can be noticed even after providing a rigid foundation.

Consider a rigid, concrete foundation block, which supports a steam engine with speed of 250 r.p.m. resting on ground. Assume that resonant frequency of the system is 300 r.p.m. This will lead to fairly excessive amplitude of vibration, as the frequency ratio is close to unity. If this foundation is made 'stronger' by adding more concrete mass to the foundation block, the value of m_o (equation 1 a) increases. This will lead to decreased resonant frequency of the system, and the frequency ratio approaches closer to unity leading to still more severe vibration amplitudes. This example shows that the quality of the foundation does not necessarily improve with the mass of the foundation- block.

Forced vibrations are transmitted through the ground. Even at a distance, if some adjoining foundation has a natural frequency close to the frequency of transmitted vibrations, resonance may occur leading to damage. For this reason, the foundation under heavy machines and forging hammers are isolated, and shock absorbers are used.

Classification of Dynamic Loads :

The nature of vibrations and dynamic forces associated with a machine can, in general, be classified as :—

(a) Shock loads occuring at regular intervals e.g. vertical loads as in punching press, forging hammers and horizontal shock loads, as in looms.

(b) Vibratory loads, which repeat after a particular period and are cyclic in nature. These may include the vibrations caused in any of the six degrees of freedom (for a single mass) by three translatory loads and three rotational torques.

Translatory vibrations include vertical, longitudinal and lateral movements along three coordinate axes, x, y, z. Rotary motion about vertical axis (z axis) is called "ya wing" while about longitudinal y-axis is called "rotation" and pitching about lateral axis (x-axis). For symmetrical foundations, vertical vibrations and ya-wing can exist independently, but rocking is associated with longitudinal vibrations.

In most of the machine foundations, the vibrations occur in vertical direction or in rocking.

Requirments of Machine Foundations :

A properly designed foundation for a machine must first of all meet the general requirements for all foundations for the particular load transmitted to the ground. These are as follows (Tschebotarioff 1951):—

1. The loads of the structure should be transferred to soil layers capable of supporting them without a shear failure.
2. The deformation of the soil layers underlying the foundation should be compatible with those which the foundation, the super-structure, as well as adjoining existing structures can safely undergo.

In addition, the machine foundation must meet the following additional requirements, which are characteristics of dynamic loading:—

(a) Vibrational Amplitude:

It is not possible to eliminate the oscillating motion completely from a foundation which is subjected to significant dynamic impulses. The designer can only attempt to reduce the foundation vibration to a magnitude which is tolerable at the operating frequency.

In the absence of the design specifications for limiting vibrations, either of the following recommendations may be used as a guide. One of those is originally suggested by Rausch, (1936), and reported in English by Converse (1962). According to this, the permissible amplitude in inches is given by:

$$\text{Permissible amplitude} = \frac{9.54}{f} \text{ for frequencies less than 1800 r.p.m.} \quad \ldots (3a)$$

Lorenz (1934) and Balakrishna Rao (1961) observed experimentally that the resonant frequency of the system decreases with increase in dynamic load. This may be explained on the assumption that in such a case, the pressure tends to become more intense near the centre of the oscillator resulting in smaller effective radius. However, no quantative explanation is available.

Arnold, Bycroft and Warburton (1957) and Bycroft (1959) have extended the analysis to the other vibratory modes. They have considered circular vibrator with rigid base distribution. As the circular vibrator is the case of axial symmetry, in reality the vibrating mass has four degrees of freedom, i.e., translation horizontally and vertically and rotation about horizontal and vertical axes.

The work done by Reissner, Quinlan, Sung and Arnold Bycroft and Warburton, dealt with one of the six modes of vibration at a time and, therefore, is limited to the case, where the six modes exist independently. It is possible to evaluate the equations of motion directly from the above theories. An interesting transformation suggested by Hseih (1962) makes it possible to find the equations of motion.

But if the machine foundation is symmetrical about contact base, the vertical translation and rotation about vertical axis exist independently while the horizontal translation is coupled with rotation about horizontal axis. If the centroid of contact area and c.g. of the machine foundation coincide all the six degrees of freedom are decoupled. This is a hypothetical case and is not possible in practice. The same conclusions are reached by Pauw (1953) by considering the equations of motion, which are obtained by the soil spring analogy.

This method offers a correlation between the two theoretical approaches viz., soil as elastic solid, and soil as spring.

Ford and Haddow (1960) have obtained the natural frequency of machine foundation based on Rayleigh's principle for rigid foundations. For a conservative system, according to Rayleigh's principle, the maximum strain energy is equal to the maximum kinetic energy. It is based mainly on the following assumptions:

(a) Vertical Vibration:

1. The system may be considered as conservative in order to determine the natural frequency.
2. Dynamic pressure is transmitted through soil contained in a solid formed by the base of foundation and a soil surface.
3. The dynamic stress at depth z is uniformly distributed over a section parallel to the base of the foundation.

The last assumption is inaccurate, but is useful for the development of expressions.

(b) Horizontal Vibration:

The same assumption as for vertical vibrations are made with additional assumption that the dynamic shearing stress is uniformly distributed over a section of the solid parallel to x, y plane.

Equating the kinetic energy of the soil and machine foundation to the maximum strain energy of the soil, the author has obtained the natural frequencies in vertical and horizontal modes of vibration.

Discussion

Reissner's analysis (1936) forms the basis of the subsequent analysis given by Quinlan (1958), Sung (1953), Hseih (1962) Richart (1953, 1960), Bycroft (1959), and Arnold, Bycroft and Warburton (1955). It forms a sound basis as long as soil can be assumed homogeneous, elastic and isotropic solid. Reissner's analysis assumes the distribution under the circular base as uniform, which obviously is not the case. It is evidenced from the experiments that the resonant frequency decreases and maximum amplitude of vibration increases with the increase in the *exciter forces* as indicated by Lorenz (1934, 1953, 1959) and Balakrishna Rao (1961). Reissner's analysis does not give varying frequency of resonance for change in the exciter force and as such deviates from the experimental data.

The modifications by Sung (1953) and Quinlan (1953) for the different load distribution (parabolic, uniform, rigid) show that as the pressures tend to concentrate near the centre of circular base, the resonant frequency decreases and amplitude of vibration increases. The increase in dynamic force, may qualitatively be assumed to be associated with the change in pressure distribution or the decrease in "effective radius" corresponding to Richart's (1953) suggestion. This tries to bridge the gap between theory and experimental evidence. However, no quantitative explanation as to the change in pressure distribution, or the change in effective radius, with the change in dynamic forces is available. Sung (1953) and Quinlan (1953) observed the natural frequency of vertical vibrations on the test vibrators and compared the results with theory and found them in good agreement. Richart (1960) has applied this theory to the design of machine foundations.

Bycroft (1953) Arnold, Bycroft and Warburton (1955) have extended the above theory for other modes of vibrations as well. But they consider only the rigid base distribution. The expressions obtained by them are of the similar nature and form as in case of vertical vibrations. Now this can be applied only if the vibration in a particular mode can exist independently, which is true only for motion in vertical direction when the machine foundation is symmetrical about the contact base. The theoretical analysis of horizontal and rotational modes of oscillation have not been experimentally verified.

The equation of motion for six degrees of freedom cannot be directly obtaind from this theory. A transformation suggested by Hseih (1962) makes it possible to obtain the six second order differential equations of motion which may be solved by means of analogue computors. For the case of machine foundation symmetrical about contact base (which is usually the case) the horizontal translatory vibration is coupled with rotary motion about horizontal, perpendicular to the translatory vibrations. This has been independently confirmed by Pauw (1953) by truncated spring analogy. Only for the case when centre of gravity of machine foundation coincides with the centroid of the contact area, all motions are decoupled (This is not feasible, practically).

Ford and Haddow (1960) obviously have given a basically different approach to the problem by considering the machine foundation soil as a conservatives ystem. But this gives the resonant frequency which is independent of the dynamic load, which certainly is not correct as shown by the experiments of Lorenz (1934, 1953, 1959) and Balakrishna Rao (1961).

For the foundations other than circular ones, which have been considered in the theoretical analyses, following modifications have been suggested by various investigators.

1. For translatory motion, use an equivalent radius which gives the area of circle equal to that of the contact area of the foundation with ground (Sung 1953, Richart 1960, Hseih 1962).
2. For rotational motion use an equivalent radius which gives the moment of inertia of the circle equal to that of the contact area of the foundation about the axis of rotation (Hseih 1962).
3. Bycroft (1959) has suggested that if Z is the amplitude for equivalent circular base and Z_r for the rectangular base, then $Z_r = m \times Z$, where m is a shape factor and may be taken as for static case given by Timoshenko (1937).
4. As the distribution assumed by Bycroft (1959) is the rigid base distribution, the concept of effective radius is suggested to find the values of frequency and amplitude for other type of distribution.

In all the above theoretical methods based on the homogeneous semi-infinite, isotropic, elastic solid, certain values of G (modulus of rigidity) and ν (Poisson's ratio), have to be estimated. The value of modulus of rigidity G vareis with depth and so does the Poisson's ratio ν. The test on oscillator, will no doubt give certain value of G and ν, but it is valid only for the depth, which may be taken approximately equal to three times the base width of test vibrator. With increase in prototype area, the value of G and ν should be valid for depth upto three times the foundation width, and these values will obviously be different from those in test vibrator.

Another short-coming is the pressure distribution to be assumed. No data is available from field records, regarding the actual distribution and change in distribution with increase in dynamic loads. It has further been found from the survey of available literature that while the ratio of dynamic to static force in the prototype is about 4% to 5%, its value in the model vibrator is any where from 25% to 90%. The distribution in two cases may be different, but no quantative information is available.

Soil as Elastic Spring:

The first known approach to analyse the foundation vibrations considered the vibrating system to behave as a single mass supported by a weightless spring and subjected to viscous damping (Lorenz 1934, and Barkan 1936).

For undamped forced vibrations, resonance occurs at $f/f_0 = 1.0$, where f is frequncy of forced vibrations and f_0 is natural frequency of the system. For small values of damping the peak amplitude occurs at frequency ratio so close to unity that the difference is usually negligible.

Another possible variable with frequency ratio is the work per unit of time required to operate the vibrator. This work consists of two parts (Terzaghi*1943). One part is used up in overcoming the friction in bearings and other resistances within the mechanism. It has been found that this part increases approximately in direct proportion to the square of frequency. The second part is consumed by the viscous resistance of soil against periodic deformation. The rate of work for operation is maximum at resonance frequency.

From 1930 onwards, the work was carried by Degebo (Lorenz 1934). Their standard experimental set up consisted of a vibrator, weighing 2700 kgm with base area 1 sq. meter.

Experiments were conducted on different sites using the above vibrator. Amplitude of vibration, the phase angle between the exciting force and the resulting vibrations and the power requirement of the vibrator were determined at various frequencies. The reasonant frequency is determined where maximum amplitude occurs and checked with the frequency where maximum power is required and also where phase difference ψ is $\pi/2$. Comparison of the experimental plots with the theoretical plots gives the value for damping factor.

The frequencies which correspond to individual soil type and hence to bearing capacity according to Lorenz (1934) indicate that the higher the natural frequency, the higher the safe soil pressure. λ the damping factor was found to have the following significance. A value in excess of about 3 to 4 sec^{-1} combined with an important settlement of the base was considered an indication of high compressibility and sensitivity to vibrations.

During these experiments it was found that W in equation 1b is not the weight of vibrator alone, but also includes the weight of the soil vibrating with it. Expression for natural frequency then becomes;

$$\omega_0 = \sqrt{\frac{K.g}{W_0+W_s}} \quad \dots (3)$$

where W_s *is equivalent soil weight which is assumed to be concentrated at the c.g. of foundation mass.*

In order to determine W_s, the weight of vibrator was increased by means of surcharge, and the test repeated. The natural circular frequency of system decreases from ω_0 to ω_0'. Assuming, for the sake of simiplification, that the increase in weight of the vibrator has no effect on W_s, two equations are obtained which make it possible to determine W_s.

Another suggested method is to increase the area of the vibrator base, keeping weight of the vibrator same. Replacing k by k'.A in equation (3) ω_0 is obtained as;

$$\omega_0 = \sqrt{\frac{k'.\,A.g.}{W_0+W_s}} \quad \dots (4)$$

where A is the area of base plate, and
k' is the modulus of dynamic subgrade reaction.

Assuming value of k' to be same from one test to another, value of W_s can be determined.

By increasing the weight of vibrator from 1.8 to 3.4 metric tons, the value W_s was found to be 12.5 tons (Lorenz 1934). Similarly keeping the vibrator weight at 2700 kgm (2.7 metric tons), and changing the area from 1/4 sq. meter to 1 sq. meter, the value of W_s for the same site was found again to be 12.5 tons.

In another series of the tests, W_s was equal to 1 metric ton, when the weight of vibrator was increased from 2060 kgm to 2700 kgm. These results indicate that value of W_s is likely to vary between wide limits. For change in eccentricity (increase in dynamic loads), the natural frequency was found to decrease.

At about the same time, the independent tests were carried out in Russia which are reported by Barkan (1936, 1963). For the vabrations so produced as to give both gyration (rotation) and translatory d splacement the system has two degrees of freedom, the resonant frequencies are coupled, and two resonant frequencies were noted.

The experimental foundations weighed upto 30 tons and had an area at bottom upto 8 sq.m.. Value of $k=k'.A$ was determined by the statical tests (reversible displacement x k' = normal stress). k' was determined for areas 2, 4 and 8 sq.m. From the determined values of k', the frequencies of the vertical vibrations only were calculated. The foundation was subjected to forced vertical vibration with the aid of vibrating machine and resonance recorded. In nearly all the cases, frequencies differed but little from the theoretically calculated ones. In the analysis the weight of the soil participating has been neglected. Barkan (1936) also observed the resonant frequency of 11 cps for the eccentricities of 22.5 mm, 17.5 mm, 6.5 mm. This seems erronous as the resonant frequency has been found to decrease with increase in eccentricity or the dynamic loading (Lorenz 1934, Crockett and Hammond 1948, 1949, Lorenz 1953).

Later experiments in Sweden (Bergstrom and Linderholm 1946) have shown that for large base plates (of the order of 3 m. radius), the value of subgrade reaction k' corresponds to the values obtained from wave velocity measurements.

Andrews and Crockett (1945), and Crockett and Hammond (1948. 1949, 1958) also measured natural frequencies using a vibrograph to pick up the oscillations in the vicinity of large hammers. These frequencies are roughly the same as those reported by Degebo (Lorenz 1934). Crockett and Hammond (1948) also obtained the same natural frequency irrespective of the size of the foundation for any particular type of ground. The largest foundation tested had an area of 2500 sq. ft.

However, they suggested that mass of soil which vibrates with the foundation must bear some relation to bulb of stress, which gives the stress distribution under a uniformally loaded area on an elastic medium. The active ground weight is assumed to be within a certain bulb of pressure, but no relationship had been indicated.

Pauw (1953) has given an analytical procedure whereby the dynamic soil constants required for the prediction of natural frequencies of a foundation soil system may be determined. The foundation soil system is treated by considering the foundation to be supported by a truncated pyramid of "soil springs".

Based on the concept that the modulus of elasticity is approximately proportional to the shearing strength, Pauw made the following assumptions :—

1. For Cohesionless soils the modulus of elasticity is proportional to the effective depth which equals the actual depth plus equivalent surcharge.
2. For cohesive soils the modulus of elasticity is constant. Intermediate soil conditions may be interpolated on the basis of Coloumb's law.
3. The distribution of stress takes place within a truncated pyramid.
4. The soil pressure below the foundation and also at any depth is uniform.

Spring factor is defined as the force or moment exerted on a system when it is displaced a unit distance or rotated through a unit angle, from the equilibrium position. For a foundation with six degrees of freedom, six spring constants are required for each surface in contact with soil. Apparent mass of soil vibrating with the foundation is estimated by equating the kinetic energy of an equivalent concentrated mass at the surface to the total kinetic energy in the effective zone. He has given these factors for horizontal and vertical surface.

The integral for mass factor in case of translatory vibrations for cohesive soils does not yeild to a converging solution. He has also considered the equations of motion for a symmetrical foundation (c.g. of machine foundation directly above centroid of the contact surface) and found that only vertical vibrations and rotation about vertical axis exist independently. The horizontal translatory motion is coupled with rotation about horizontal axis. Balakrishna Rao and Nagraj (1960) and Balakrishna Rao (1961, 1962) have developed further the concept of oscillation of Bulb of pressure as advanced by Crockett and Hammond (1948, 1949). This has been modified to the density pressure bulb concept.

The weight of the soil mass participating in vibration (equation 3) is estimated by taking the weight of the soil contained in a definite pressure bulb. This pressure bulb is obtained by considering the sum of static and maximum positive dynamic load of the machine and the foundation block to act as concentrated load at the mass centre of the foundation block. The reason advanced for adding the dynamic load is that the additional static stresses are developed by dynamic load (Nagraj and Balakrishna Rao 1959). The boundary of this pressure bulb is supposed to be given by pressure intensity of $|\gamma|$ lbs per sq. ft. where γ is the density of the soil mass in lbs per cft.

The above authors have suggested to take the value of K or spring constant according to that given by Pauw, or for that matter from the dynamic soil tests. Knowing the value of K and W_s, the natural frequency can be determined.

Balakrishna Rao has checked the resonant frequency as calculated by pressure bulb concept with the published results of Converse (1953) and Eastwood (1953).

Barkan (1963) has developed an analysis on the analogy that soil acts as spring. For horizontal forces, the foundation soil system has been considered to be a two degree of freedom system.

Discussion

The approach is based on the assumption that soil acts as a linear spring, and for the purpose of resonant frequency it is sufficiently accurate to state that resonance occurs at the frequency ratio of unity. Barkan (1936) has verified this for the assumption that no soil mass is participating in the vibration, but Lorenz (1934, 1953 b), Crockett and Hammond (1948), Eastwood (1953), have shown that this soil mass cannot be neglected, though it is an uncertain factor. Hence it seems that neglecting the soil mass in denominator may have been compensated by the fact that Barkan has taken the value of spring constant from the static deflection test. (He has defined the spring constant as the load required to produce the unit reversible settlement).

It seems only proper that while giving W_s a certain value, the spring constant also must be considered. It is both W_s and K that vary rather than any single factor. Hence to suggest that Degebo obtained W_s as 4 to 10 times the static weight and Vios got W_s as 2/3 to 1.5 times the static weight is an indefinite statement.

Observations of Crocket and Hammond (1948) that natural frequency of foundation soil system is independent of the size of the contact area is inconsistent with recent tests of Eastwood (1953).

Pauw (1953) has given the spring factors, and mass factors for different modes of vibration based on theoretical derivation which assumes soil as truncated pyramid spring. But these spring and mass factors seem to be alright for the soils where Young's modulus increases linearly, with depth. If the Young's modulus is a constant, the approach does not give definite results, and a modification in surcharge as suggested by Converse (1962) may be used.

Andrews and Crockett (1945) Crockett and Hammond (1948, 1949) have suggested that the mass of soil oscillating may be taken as that contained within a pressure bulb, which they have not specified. Also as stated earlier, specifying W_s is not significant unless the spring constant is also defined.

Balakrishna Rao (1960. 1961, 1962) considers the pressure bulb for the combined static plus dynamic load, and has given a specific value to this pressure bulb. This explains the phenomenon of decreased resonant frequency for higher dynamic load as it assumes that the soil mass will increase. The pressure bulb for distributed load based on equivalent sphere did not give appreciable difference. He recommended the use of spring constants according to Pauw's approach. Thus W_s is calculated on the assumption of soil as uniform, homogeneous, elastic medium (having same Young's modulus at various depths), the spring constant K is calculated on the assumption that Young's modulus increases with depth (Pauw's approach). That is, the value of W_s and K are calculated on the basis of contrary assumptions. But it offers a good emperical means to evaluate the effect of changed dynamic force.

It is important therefore, that unless and untill both values of K and W_s are specified, the approach is not bound to be rational. In latter experiments with road subgrades Heukelom (1959) and Heukelom and Foster (1960) have kept the spring constant same, and embodied the deviation solely in the mass of vibrating soil.

Miscellaneous Methods :

Emperical approaches to the problem of determining resonant frequency will be discussed under this section :—

Tschebotarioff and Ward (1948) and Tschebotarioff (1951, 1953) have suggested a logarithmic relation between the area of foundation and the reduced natural frequency. The resonant frequency is given by ;

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k'.A.g}{W_o+W_s}} \qquad \ldots (5a)$$

$$= \frac{1}{2\pi}\sqrt{\frac{k'.g}{1+W_s/W_o}}\cdot\sqrt{\frac{A}{W_o}} \qquad \ldots (5b)$$

W_o/A is equal to the static load on foundation per unit area (σ st). Thus at unit static pressure, the frequency would be ;

$$f_{nr} = \frac{1}{2\pi}\sqrt{\frac{k'.g}{1+W_s/W_o}}\cdot\sqrt{1} \qquad \ldots (5e)$$

This is termed as the reduced natural frequency.

$$\text{or} \quad f_{nr} = f_o\sqrt{\sigma\, st} \qquad \ldots (6)$$

Tschebotarioff worked out values of reduced natural frequency from the published data of Degebo, Vios, Newcomb and other records and also the result of some tests which he simself carried out in a C.B.R. test mould and found that a straight line relationship existed between area of contact and reduced natural frequency on a log log plot for one soil type. For different soils, the straight lines were parallel and can be represented by an equation,

$$A = j_1\,(f_{nr})^t \qquad \ldots (7a)$$

where A is the contact area,

j_1 and t are constants for the plot.

Substitute for f_{nr} in terms of f_o. we get ;

$$A = j_1\left(\sqrt{\frac{W_o}{A}}\,.\;f_o\right)^t \qquad \ldots (7b)$$

$$\text{whence } f_o = j_1^{-1/t}.\;\frac{A^{\left(\frac{1}{t}+\frac{1}{2}\right)}}{\sqrt{W_o}} \qquad \ldots (7c)$$

Goyal and Alam Singh (1960) have expressed the emperical equation of the straight line of above plot for sands and clays. Transferring their value to the form of equation (7c), the exponent of A for sands is 0.242 and for clays is 0.248.

Alpan (1961) found the value of t in the above equation as – 3.98 and hence, the exponent of A is 0.248 for all type of soils. Equation (7c) can then be written is ;

$$f_o = \frac{J'}{\sqrt{W_o}}\,A^{0.248} \qquad \ldots (8)$$

where J' is a constant.

The contact area in Tschebotarioff's diagram ranges from 1 to 1000 sq. meters (10 to 10000 sq ft). For smaller areas, results of laboratory experiments with model footings are available (Eastwood 1953). These tests were made to investigate the factors influencing the resonant frequency on dry and inundated sand. The oscillations were generated by impact. The sand employed in his tests had a dry density of 1.74 g/c.c. and a void ratio of 0.525.

Alpan (1961) plotted the results of Eastwood in terms of reduced natural frequency f_{nr} versus area A on log-loge scales and compared it with extrapolated values as obtained by Tschebotarioff's plot for sands. The lines obtained by Alpan are quite different from those of Tschebotarioff. Actually the points are anything upto 100% high or 50% low, the errors being masked by the log-log scale (Eastwood's comments 1953). From the data which Tschebotarioff has used for his plot (Tschebotarioff 1953) it will be seen that the resonant frequency was obtained by forced vibration test and shock or impact and exciting force was either vertical or horizontal and vertical or, only horizontal. But it is a known fact that the nature of the vibrations and the method by which they are induced materially affect the frequency response of the system. Hence Tschebotarrioff's plot is not the true picture of frequencies.

Eastwood's (1953) tests show that for the same applied load per unit area, the natural frequency of a 12" × 3" model footing is the same as that for a 24" × 3" model footing. Thus they will also have the same reduced natural frequency even though area of one is twice that of the other. He has suggested a possible relation between reduced natural frequency and the least dimension of footing.

To obtain same values of f_{nr} (for same area), whatever be the applied load expression $\sqrt{k'.g/1 + W_s/W_o}$ in quation (5b) has to be constant for different areas. This means that either W_s must increase at exactly the same ratio as W_o or alternatively W_s is always negligible compared to W_o. The latter is impossible and the former extremely unlikely (Eastwood 1953).

Alpan (1961) has made an attempt to analyze from first principles, the relation between frequency and area and obtained an expression for natural frequency as;

$$f_o = \frac{\text{constant}}{\sqrt{m}} \cdot \frac{A^{0.25}}{\sqrt{W_o}} \qquad \dots (9)$$

where m is shape factor. Equation (9) differs from equation (8) in only that a shape factor m is involved, and that exponential power of A is 0.25 instead of 0.248.

Now shape factor is not only dependent upon the length width ratio, but may depend also on the type of the load distribution. This probably may be able to remove the discrepancies in the plot of Tschebotarioff. For example, Eastwood (1953) obtained the same natural frequency f_o, for 24"×3" and 12"×3" foundation models, for the same static load intensity though the area is twice. This may be explained by introduction of shape factor. Further work has to be done along these lines.

From the results of the field testing programme mainly conducted to note the effect of various parameters, on the compaction of sand by vibration in a test pit 6 feet deep, 10 feet square, an empirical equation (Converse 1953) for resonant frequency of a vibrator sand mass was developed as follows:

$$f_o = \frac{\sqrt{g}}{2\pi} \sqrt{840 \frac{\gamma}{G} (1.64 - F_o/W_o) + 0.55\, Gr_o/W_o} \qquad \dots (10)$$

$$o < \frac{F_o}{W_o} < 1.$$

where f_o is frequency in cps

G is modulus of rigidity of soil, psi.

F_o is maximum dynamic force, lbs.

W_o is static weight of vibrator lbs. and

r_o is radius of vibrator, in. and

γ is unit weight of soil, lb. per cuft.

Converse verified the resonant frequency based on the above formula with that of the field test results obtained with base plates 15.7, 19.2, 24.0 and 45.0 inch in diameter.

The development of the emperical equation is significant, as it involves not only the soil constants, but also the vibrator dimensions (r_o), weight (W_o) and dynamic force (F_o) still, the equation is developed only for one type of soil and it is only reasonable to expect that this will vary with the type of soil and such an equation is not universal in nature.

Blessing has presented curves for determination of resonant frequency and amplitudes of compressor or engine fondations. These are based partly on elastic analysis and on the experience gained from their installations. These curves have been adopted in the Indian Standard on Design of Machine Foundations.

BASIC SIMILARITY OF VARIOUS APPROACHES.

In all the methods for predicting resonant frequency which assume soil to be homogeneous, an interesting similarity is pointed out. Based on this an empirical formula for resonant freqnency is suggested.

For static loads, the value of spring constant for various contact pressure distributions are reported by Jones (1958) and Heuklom (1959).

$$k = 4\ Gro/\ (1 - \nu) \qquad \text{... (11a)}$$

For rigid plate condition. (Sneddon 1951)

$$k = \pi\ Gro/\ 1 - \nu \qquad \text{... (11b)}$$

for uniform stress distribution, (Boussinesq 1885).

$$k = \frac{3\pi}{4}\ \frac{Gro}{1-\nu} \qquad \text{... (11c)}$$

for parabolic stress distribution. (Frohlich 1934).

Equations (11) show that spring constant is multiple of Gro. Now consider the theory of vibration of elastic homogeneous, semi-infinite, isotropic medium (soil). In this analysis, mass ratio 'b' and dimensionless frequency term 'a' are used, where

$$b = \frac{m_o}{\rho ro^3} \text{ and } a_o = \omega_o r_o \sqrt{\rho/G} \qquad \text{... (12a)}$$

$$\therefore\ ba_o^2 = \frac{m_o}{\rho ro^3}\ \omega_o^2\ r_o^2\ \frac{\rho}{G} \qquad \text{... (12b)}$$

which after re-arranging gives,

$$\omega_o^2 = \frac{Gro}{m_o}\ (ba_o^2) \qquad \text{... (12c)}$$

Value of equivalent spring constant is, therefore, (assuming $\omega_o^2 = k/m_o$).

$$k = Gro.\ (ba_o^2) \qquad \text{... (12d)}$$

(ba_o^2) is, a function of load distribution Poisson's ratio and 'a'.

Hseih's transformation (1962) gives spring constant as

$$k = Gro\ F_1 \qquad \text{... (13)}$$

where F_1 has been evaluated in terms of 'a' for rigid distribution and different Poisson's ratio

$$\therefore\ \omega_o^2 = \frac{Gro\ F_1}{m_o} \qquad \text{... (14a)}$$

Putting value of F_1 as given by him, we get (for $\nu = 1/2$),

$$\omega_o^2 = \frac{Gro\ (8-2.0\ a_o^2)}{m_o} \qquad \text{... (14b)}$$

Substituting value of 'a_o' $=\omega_o ro\ \sqrt{\rho/G}$ and rearranging we obtain,

$$\omega_o^2 = \frac{8\ Gro}{m_o+2\ \rho\ ro^3} \qquad \text{... (15a)}$$

Similarly for $\nu = 1/4$,

$$\omega_o^2 = \frac{5.3\ Gro}{m_o+\rho ro^3} \qquad \text{... (15b)}$$

For $\nu = 0$,

$$\omega_o^2 = \frac{4.0\ Gro}{m_o+0.5\ \rho ro^3} \qquad \text{... (15c)}$$

Values of spring constant from equation (11a) for rigid base distribution is 4 Gro/1 — ν, which for ν = 1/2, 1/4,0 is 8 Gro, 5.33 Gro and 4.0 Gro respectively. This tallies with spring constants in equations (15).

In Ford and Haddow's analysis (1960) it can be shown (Bhatia 1963) that the equivalent spring constant is a multiple of Gro, and the mass of soil participating in vibration is a multiple of $\rho\ ro^3$.

Now consider the emperical plot of Tschebotarioff (1948, 1951, 1953). The equation of the plot is

$$f_o = \frac{J'}{\sqrt{W_o}}\ A^{0.248} \qquad \text{... (8)}$$

Substituting $A = \pi\ ro^2$ in equation (8), we get

$$f_o = \text{const} \times \frac{r_o^{0.496}}{\sqrt{W_o}} \qquad \text{... (16a)}$$

$$= \text{constant} \times \sqrt{\frac{r_o^{0.992}}{W_o}} \qquad \text{... (16b)}$$

where the constant depends upon soil type (and hence on value of G)

Therefore, spring constant may be taken as a multiple of Gro. Note very small power difference between 1.0 and 0.992.

Converse (1953) has given an emperical equation for resonant frequency of sand vibrator system (Eq. 10).

For large values of G, first term under the root is small and may be neglected and this reduces to,

$$f_o = \sqrt{\frac{Gro}{W_o}} \times \text{a constant} \quad \dots (17)$$

Pauw (1953) has evaluated the spring constants for the cohesive soils for which values of E or G can be assumed to be constant. Though it was not possible to evaluate the mass factor, it will be of interest to see nature of spring constant by given him. Expression for vertical vibration is ;

$$k = E\, \alpha\, B\, \gamma_z \quad \dots (18a)$$

where $\tan^{-1} \alpha/2$ denotes the angle of pressure distribution and γ_z is a factor depending upon L/B ratio.

For circular vibrator B = 2 ro, and $\gamma_z = 1.0$, (Pauw 1953).

$$\therefore \quad k = E.\alpha\,(2\,ro)\,.1 = 2\,G\,(1 + \nu).\ (2ro).$$
$$= Gro\,\{4\,(1 + \nu)\,\} \quad \dots (18b)$$

i.e. the spring constant is a multiple of Gro.

Experiments by Nijboer (1953, 1959) Vander Poel (1951, 1953) Heukelom (1959) Heukelom and Foster (1960), obtained the value of spring constant as approximately 7.6 to 7.7 Gro, in their dynamic tests.

i.e. k = 7.6 Gro ... (19)

From consideration of equations (11), (12), (14a), (15), (16b), (17), (18) and (19) a simplified form of the natural frequency expression is suggested viz.,

$$\omega_o^2 = \frac{\lambda_1 \times Gro}{m_o + \lambda_2\, \rho ro^3} \quad \dots (20)$$

Introducing the shape factor 'm' in equation (20) we obtain,

$$\omega_o^2 = (\lambda_1/m).\frac{Gro}{m_o + \lambda_2\, \rho\, ro^3} \quad \dots (21)$$

where λ_1, λ_2 are constants for the system and m is the shape factor depending upon L/B ratio

The effect of change in dynamic load upon λ_1 and λ_2 will have to be investigated experimentally.

DESIGN OF A MACHINE FOUNDATION.

The general method of designing a foundation is to determine the natural frequency of the system based on analytical mothods discussed in this report and ascertain the amplitude of oscillation. The various steps in these methods are :

1 The dynamic unbalanced forces, and their frequencies of operation are calculated, or these may be supplied by the manufacturers.

2 The dimensions of the foundation block are assumed, taking care that allowable soil pressures (which are less than in case of static loads only) are not exceeded.

3 The soil type is analysed by borings, and sampling. To analyse behaviour of the soil, in-stitu vibrator tests, should be conducted. In these tests, either the resonant frequency and amplitude of vibration are determined for different combination of static and dynamic weights, or the velocity of wave propogation is determined. The value of β (rate of increase of Young's modulus with depth) can also be calculated with the help of Pauw's analysis. The values of decay factor or B_1, a constant in Ford and Haddow's analysis can be calculated. It is recommended that tests with at least 3 different areas of vibrator be performed. Balakrishna Rao (1962) has suggested a linear variation of β with area A, based on analysis of data of Ford and Haddow. Pauw (1953) has however, assumed β to be constant.

4 Knowing the above soil properties, the resonant frequency of the actual machine foundation soil system can be calculated by any one of the methods described previously.

5 Resonant frequency is then checked with operational frequency of the machine, and if the frequency ratio is within safe limits (less then 0.5, or more then 2.0), the design may be checked for amplitude of vibrations.

6 Usually the amplitude of vibrations, can be determined with sufficient accuracy by assumption of a simple spring, in which the damping value may be neglected or a reasonable value usually 0.25 may be assumed.

7 If this is found to be within permissible limits, which can be tolerated by the machine and the structure, then the design is safe.

8 If not, assume another preliminary design and repeat the above steps.

Usually vertical vibrations exist independently. If the vibrations occur in more then one direction the frequencies will be coupled as shown by Pauw (1953), and, Hseih (1962). In that case Pauw's method is recommended: Pauw's method is applicable only for cohesionless soils.

For fairly homogeneous soils, either modification in surcharge as suggested by Converse (1962) may be used, or Barkan's (1963) method may be used.

SUGGESTIONS FOR FOURTHER RESEARCH.

In view of the study made, the following suggestions for further research are made :

1 A simple equation for the natural frequency of the system is evolved (Equation 21). The factors affecting λ_1 and λ_2 need be studied systematically.

2 Shape factors (m) i.e. the effect of shape of base area (characterized by L/B ratio) needs to be investigated experimentally.

3 Balakrishna Rao (1960, 61, 62) has evolved density pressure bulb concept. It accounts for the change in the natural frequency of system with change in dynamic loads. This approach has given good results for vertical vibrations when Pauw's spring factor for cohesionless soils (E increasing with depth) is used. The procedure needs experimental verification for cohesive soils.

4 Experimntal investigations of bearing capacity of soil under dynamic loads are being undertaken (A.S. T.M. 1961). A systematic study of the problem will prove useful to the designer of machine foundations.

5 The problem of machine foundations on piles has not been tackled at all. Experimental and analytical investigations of this problem need be made.

6 No data from the actual machine foundations in India is available. It is suggested that a questionaire be prepared and sent to various industries, designers, and research workers dealing with machine foundations. The purpose will be to undertake a systematic analysis of field data regarding actual behaviour of machine foundations. This will help in co-ordinating the efforts of various workers and will consequently lead to a standard practice for the design of machine foundations.

COCNLUSIONS

From the study of the available literature the the following conclusions could be drawn :—

1. Resonance phenomena cannot be ignored in machine foundations and design should account for it.
2. For soils in which the value of E increases linearly with depth (for sand, and normally loaded clays), Pauw's method is recommended. For vertical vibrations only, Balakrishna Rao's (1962) method to calculate apparent mass factor should be used.
3. For soils in which value of E, can be assumed to be uniform, Pauw's method cannot be applied as such. Modification in surcharge as suggested by Converse (1962) is recommended.
4. For other modes of vibration, Pauw's (1953) analysis can be used.
5. In most cases the amplitude of vibration can be determined with sufficient accuracy by simple mass spring analogy.
6. A simple equation, based on the general similarity of analytical approach is developed, for determination of the natural frequency of system.

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COMPUTING PERIODS OF VIBRATION OF MULTISTOREYED BUILDING FRAMES

A. S. Arya*

SYNOPSIS

It is emphasised in this paper that in the dynamic analysis of multistoreyed buildings, the flexibility of girders must be considered along with the stiffness columns because assuming the girders rigid may lead to large errors in the calculated periods of vibration. A simplified method is suggested for computing the natural periods and the corresponding mode shapes taking the girder flexibility into account.

INTRODUCTION

Dynamic analysis of any structure subjected to impulsive or vibrating forces requires the knowledge of its stiffness and the damping characteristics. Stiffness determines the natural periods of vibration and the corresponding mode shapes almost exclusively since the effect of damping on natural periods is small and negligible. Damping has the influence of reducing the amplification factors under forced vibration. The effect is more pronounced when imposed period of vibration is close to the natural period, otherwise when forcing frequency is considerably less or more than the natural frequency, the effect of damping is small and negligible. The amplification of displacement and consequently of forces produced in the structure depends upon the ratios of forcing periods to the natural periods and the corresponding mode shapes. Therefore, determination of natural periods of vibration of a structure constitutes the most important step in the process of its dynamic analysis.

Multistoreyed buildings are very complex structures from the point of view of structural analysis especially under horizontal loads causing sidesway. This fact alone has resulted in the introduction of a large number of approximate methods of analysis for sidesway problems under static loads. Under dynamic loads the problem is all the more complicated because the natural periods of the building are found to be influenced to a great extent by the method of construction, floor slabs, wall panels, partitions, etc., which are otherwise assumed not to contribute to the stiffness of the building. How far they influence the stiffness and whether such contribution is of a permanent nature or may be destroyed in the very first shock given to the building is not known. It has been found (1)† that for existing multistoreyed buildings with steel frames, the measured fundamental period comes close to that computed by assuming building floors to be rigid, that is, having infinite moment of inertia as compared with the columns But for a bare frame or if the contribution to stiffness of filling material (which is not designed to carry forces due to shocks) is destroyed the stiffness is bound to decrease and actual girder stiffness must be considered for computing natural periods. Whereas assumption of rigid girders greatly simplifies the problem of analysis and much literature exists on this procedure, consideration of flexibility of girders complicates the problem because of consequent joint rotations. But this must be taken into account since the influence of girder flexibility on the natural periods is considerable. For example, for a particular 19-storey steel-frame building, the fundamental period was 1.3 sec. when girders were assumed rigid but 3.4 sec. when girder flexibility was considered(2). Figure (1) further shows the influence of girder flexibility on the fundamental periods of single bay multistoreyed frames (1). In the case of actual buildings, where beams are stiffened by the floor slabs, the margin between the two assumptions decreases considerably.

From the above discussion it follows that in dynamic computations stiffness properties of beams and

* Reader in Civil Engineering, University of Roorkee, Roorkee.

† Numbers in parentheses refer to the list of References at the end of the paper.

columns must be taken into account. Where a digital computer is available, such calculations may be done easily. But where such a facility is not present, this computation involves too much work. It is the aim of this paper to present a simplified method for calculating the fundamental period by Rayleigh's procedure, or all the periods and mode shapes if desired by the flexibility equations.

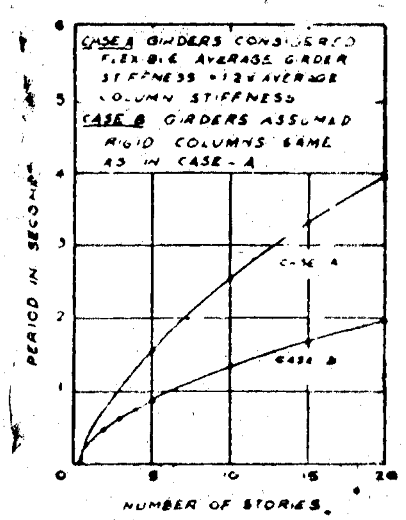

Fig. 1

METHOD OF ANALYSIS

It has been shown in (3), (4) and (5) that any general multistoreyed frame can be represented by a modified frame with single column. The stiffnesses in the single-column frame are modified in such a way that for the horizontal loads producing sideway of the original frame, the modified frame could be analysed as a non-sway frame. The sidesway deflections and moments in the modified frame are thus determined in a direct manner. From these, the moments in the original frame are then obtained by moment distribution done on the original frame. For determining the time-periods and corresponding mode shapes, the horizontal displacements are the only quantities required which can be obtained from the modified frame itself. Therefore no further reference to the original frame is necessary and hence the simplification. The modified frames for a few cases are shown in Fig. 2. In case (a) a two—column symmetrical frame is shown. The modified frame has its column twice as stiff as the individual columns of the original frame and the beams are twelve times as stiff as the corresponding beams. In this case the equivalence of the modified frame is "exact". The resulting deformations—slopes and displaments—of the modified frame will be exactly equal to those of the original frame without any approximation. In case (b) a frame obtained on the principle of multiples is shown together with the component one-bay frames which may be assumed to be making it. If the applied horizontal load is proportioned in the two component frames in the ratio 1:m, it is easily seen that the rotations and displacements of joints in the two component one-bay frames will be the same. Thus if the two adjacent columns of the one-bay frames are combined in one. neither compatibility nor equilibrium is disturbed. Therefore, for this frame also the column stiffness of modified frame is the sum of corresponding column stiffnesses of the original frame and the beam stiffness of the single column frame is 12 times the sum of all corresponding beam stiffnesses. The equivalence is again exact since the rotations of all the joints at any floor of such a frame are exactly equal. In case (c) a general frame is shown with irregular stiffness and nonuniform heights in different bays. It is well known that the joint rotations at any floor of this frame will in general be unequal. If the average value of all such rotations at any floor is considered, the general frame can be represented by a single column frame shown in Fig. 2 (c). In this case also the single column will have an aggregated stiffness of all columns in any storey but the factor with beam stiffnesses will be different from 12. A study of large number of frames (3) indicates that a factor 'A' may be used for the floors and 'A'' for the roof. Factors A and A' are in general different from 12, but near to it. For certain frames their values may be taken from Table 1.

If the bases of the columns are hinged, the stiffness of single column of the modified frame becomes zero in the bottom storey as shown in Fig. 2 (d). The modified frame in cases (c) and (d) is not exactly equivalent to the original frame as its joint rotations are approximately equal to the average of joint rotations at any floor