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AN OBJECTIVE APPROACH IN THE DETERMINATION OF THE TRAVEL TIME CURVE[1]

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Introduction

Seismic arrays have advantages over a worldwide network of single instrument stations because (1) They provide direct measurements of $dT/d\Delta$ over a limited range of Δ and enable resolution of minor perturbations in the travel time curve; (2) Enhancement of signal-to noise ratio (SNR), allows the detection of weak signals, which might be missed on single instruments and (3) The later arrivals are detected and analysed on the basis of their apparent velocities to include them in the inversion of the slowness (inverse of apparent velocity) and travel time data. In array measurements of $dT/d\Delta$ it is as necessary to allow for the effects of crust and upper mantle structure as it is in travel time studies. However, because of the reduced dimensions of the array compared to a large network of stations, the accuracy with which these effects are required to be known is much greater. Also, seismic arrays are prone to systematic errors in the determination of slowness and, if care is not taken, array data become somewhat less reliable for the determination of velocities than are travel time data, where some averaging of station anomalies may be expected to occur. Whereas travel time studies provide an average velocity structure over a large area of observations, array measurements allow derivation of velocity models which are better representatives of particular regions. Array measurements have led to the determination of the deviations of the velocity structure from the classical velocity distributions of Jeffreys and Gutenberg and confirmed some results of surface wave dispersion studies, which had demanded, for example, the existence of an S low-velocity layer, at least in some regions (Press, 1959; Takeuchi et al., 1959; Dorman et al., 1960) and two zones of rapid velocity increase at depths near 350-450 km and 600-700 km (Anderson and Toksoz, 1963 and Toksoz and Anderson, 1966; Toksoz et al., 1967). Also, surface wave dispersion results differ for paths through the oceans, shields and tectonic regions so that the concept of a spherically symmetric earth is no longer tenable (Kanamori, 1970; Dzeiwonski, 1971). However the fine structure (e.g. the number of velocity discontinuities, their depths and velocity gradients between various layers) has varied between various studies (see, for instance, Johnson, 1967; Green and Hales, 1968; Kaila et al., 1968; Helmberger and Wiggins, 1971; Masses et al., 1972 Simpson et al , 1974; Ram Datt and Muirhead, 1976, 1977; Ram and Mereu, 1977; Ram Datt, 1977; Hales et al., 1978). The causes of the observed variations include (i) regional variations (ii) lack of data in certain epicentral distance ranges and (iii) differences in the procedures of analysis and interpretation of data. For the lower mantle, though it is believed to be more homogeneous, deviation from homogeneity were reported by various authors on the basis of first arrival data (Chinnery and Toksoz, 1967; Wright, 1968; Archambeau et al., 1969; Johnson, 1969; Corbishley, 1970; Wright and Cleary, 1972). Although the interpretations regarding the velocity structure in the upper mantle have been generally, based

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on observed later arrivals (except for sufficient evidence regarding the termination of the triplication branches), those in the lower mantle remain subjective. Differences in various studies have often resulted either from the density of observations or from a choice of a $dT/d\Delta - \Delta$ curve (out of the many possible ones, with little supporting evidence). For instance, assuming that $dT/d\Delta$ is a smoothly varying function of Δ, Johnson (1969) has inferred high velocity gradients (curve a in Figure 1), whereas Chinnery and Toksoz (1967) and Wright (1968) have inferred low-velocity gradients (curve b in Figure 1). The ambiguity of these results emphasizes the need of investigations in order to provide supporting evidence in favour of the interpretations which are made while deriving the velocity structure in the interior of the earth.

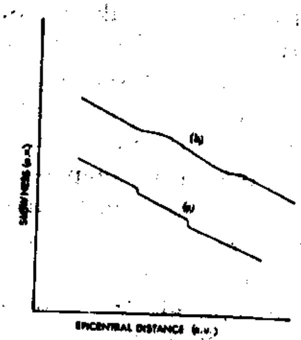

Fig. 1 Diagram to illustrate two different interpretation of the slowness-epicentral distance data

Array Processing

SNR improvements in the processed output of an array results when the outputs from all the sensors of the array are added with appropriate time shifts so that the signal is added in phase and the noise is added "not in phase". The improvement depends on the characteristics of the noise as well as that of the signal across the array. The basic objective of processing array data is therefore, to determine the time shifts which produce the desired SNR on the one hand and to apply them to various sensor outputs to produce records with maximum SNR, on the other. If the signal is fully correlated and the noise uncorrelated, SNR of $\sqrt{N}$ may be achieved by this simple process which is called "delayed sum process", N being the number of sensors in the array. If the signal is not fully correlated but shows an average correlation R_s across the array, and the average correlation for the noise is R_n, then the SNR improvement in the delayed sum process is given by :

$$G = \left[\frac{1 + (N-1)\,R_s}{1 + (N-1)\,R_n}\right]^{1/2}$$

so that the improvement in SNR decreases when the noise shows increased intersensor correlation. Alternatively, if the outputs from all the instruments of an array are added with time shifts for all possible velocities and azimuths, the time shifts which provide maximum SNR determine the velocity and direction of the coherent signal. Birtill and Whiteway (1965) demonstrate that the best method of optimizing the SNR, and hence of determining the signal directions in the case of the U.K.A.E.A. type arrays is to add the array output into two groups, multiply the two partial sums and average the products over an interval of time to form a time-averaged-product (TAP). This approach has been adopted at the U.K.A.E.A. arrays with some variations (Birtill and Whiteway, 1965, Weichert et al., 1967, Cleary et al., 1968, Ram Datt et al., 1969). Further improvements in SNR are possible with the use of non-linear techniques such as sign-bit processing (Melton and Karr, 1957), logarithmic processing (Weichert, 1975) and Nth root processing (Muirhead, 1968; Muirhead and Ram Datt, 1976) under certain circumstances. This extra improvement in SNR through non-linear processing results from an increase in the high frequency content of the signal. Investigations of the Nth root process using synthetic as well as actual seismic data have shown that this technique, though introduces signal distortion, emphasizes those parts of the signal where it is largest and coherent, and thus provides more accurate estimates of signal directions. Also, the detection capability of the non-linear process allows us to pick up, and measure the directions of, many weak signals, which would have been

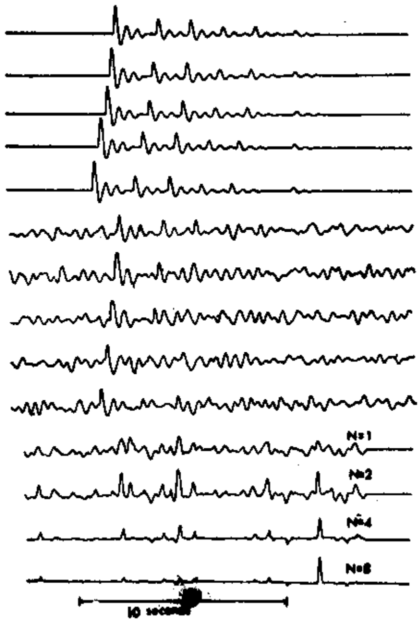

Fig. 2 Six synthetic signals simulated at WRA with different slownesses and the N^{th} root processed array outputs. The first five traces (from the top) show five representative channels with pure signals. The next five traces are the same five channels added with uniformly distributed band limited noise. The last four traces are the N^{th} root phase array sums, with the array phased to the last signals

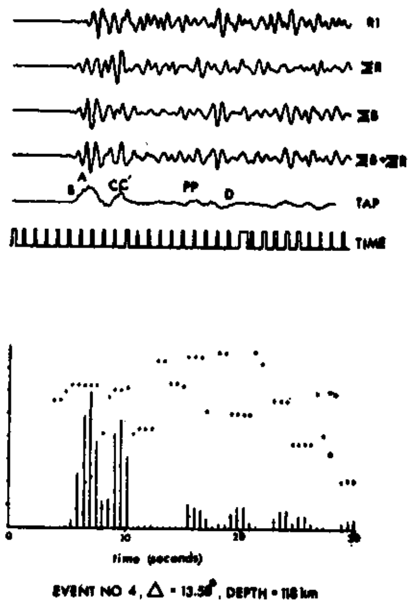

Fig. 3 A single channel output of the array (R_1) and the linear array beam comprising the two partial sums (ΣR, ΣB), the total array sum ($\Sigma B + \Sigma R$) and the time averaged product (TAP). The lower part of the figure shows the results of slowness measurements along the seismic trace (The dots show the measured values of slowness and the lengths of the lines below the dots are proportional to the energy in the correlator output).

otherwise missed in a linear process. In this process an Nth root (N = 2, 3, 4......) of the output of each seismometer is taken (its algebraic sign being preserved) before adding it to form a partial sum. The partial sums are then raised back to the Nth power (again with the sign preserved). An example of the improvement in SNR is demonstrated in Figure 2, where six synthetic signals, approaching the array at different apparent velocities, have been simulated in the presence of band limited random noise. The array is steered to the last, but the weakest signal which has been synthesized to approximate a PcP phase from an epicentral distance of about 15°.

At the top of Figure 3 is shown a single channel record of the Warramunga array (WRA) of an earthquake from the Banda Sea region. The next three traces, in this figure, show the two linear partial sums (i.e. for N = 1) and the total array sum respectively. The fifth trace is the TAP. The phases which have been marked on the TAP trace are associated with the travel time branches of the model which was derived from WRA data (Ram Datt 1977). The sixth trace gives the time marks at one second interval. The lower part of this figure shows the results of slowness measurements using the Nth root process. Here, the dots indicate the estimated slowness along the trace (as a function of time). The vertical line below each dot represents the relative energy in the N^{th} root TAP output. Using these processed outputs it is possible to read signal onset times to an accuracy of 0.1 sec. and to measure slowness within an accuracy of $\pm$ 0.2 sec/deg for the U.K.A.E.A (although the accuracies are inferior for later arrivals).

Analysis of array data

As was mentioned earlier, array measurements of $dT/d\Delta$ are biased by the structure beneath the array (Niazi, 1966; Otsuka, 1966). Efforts were made by some investigators to derive structures beneath the array in terms of dipping interfaces in order to correct for the bias but these met only limited success. This is because :

1. the derivation of the structures assumes a travel time curve (to which the data are made to satisfy) and
2. an infinite number of structures can fit the data, i.e. any structure with a given velocity contrast and dip angle may be replaced by another structure, or a set of structures, with different values of these parameters.

However, if events are confined to a narrow azimuth range with respect to the array so, that seismic rays travel almost the same path near the array, slowness measurements are less likely to be perturbed, except for a constant or a slowly varying term. The results of slowness measurements along the seismic traces of over one hundred and fifty events are plotted on a reduced travel time plot in Figure 4. At the time of each measurement is drawn a slant line whose slope represents the measured slowness and the length of which has been made proportional to the energy in the N^{th} root TAP output a (horizontal line represents a slowness of 9.67 s/deg). The base line at each epicentral distance has been adjusted to the least square error travel time curve to overcome the difficulty caused by misalignment which may result from inaccuracies in the I.S.C. (International Seismological Centre) estimates of hypocentre and origin time. This figure shows that slowness lines deviate from the travel time curve, so that the measured values are systematically higher by about

2% (on the average). All the measured values were, therefore, reduced by this factor to approximately correct for local structure. This empirical approach has the advantage that the original character of the data is not altered while applying the correction, as far as the discontinuities in the velocity and the velocity gradient are concerned.

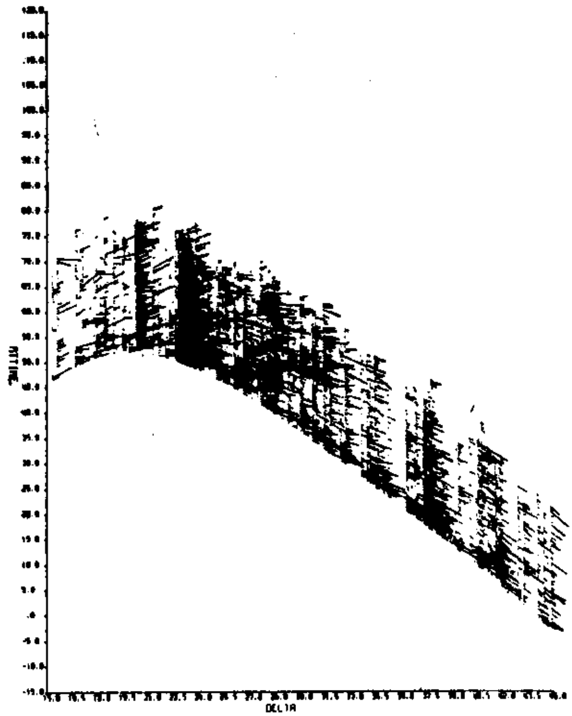

Fig. 4 Results of slowness analysis of the events. Each slant line indicates a measured value of slowness. The length of the slant line has been made proportional to the energy in the correlator output

Figure 5 shows the slowness of the first arrival plotted as a function of epicentral distance. The upper part of the figure shows the measurements for the individual events. In the lower part, average values for events, which are within 1° interval of epicentral distance, are plotted and the number of events, which have been averaged, is marked below each point. The vertical bars show the standard errors. Simple arithmatic operations on this data indicate rapid changes in the slowness of the first arrival at epicentral distances near 19.5, 20.5, 29.5, 38 and 42 degrees. If these apparent decreases in slowness are not an artifact of the averaging process, they represent triplications in the travel time curve, which could be produced by a layered structure. Later arrivals in the seismic traces should at least be, therefore, identified in order to establish these triplications. Alternatively, processed records for the relevant epicentral distances should demonstrate the absence of later arrivals if, in spite of the rapid changes in slowness of the first arrival, layered structure is to be excluded. When the slowness of the two phases are too close to each other to be resolved by the usual beam forming techniques a special method, which is illustrated in Figure 6, has been used.

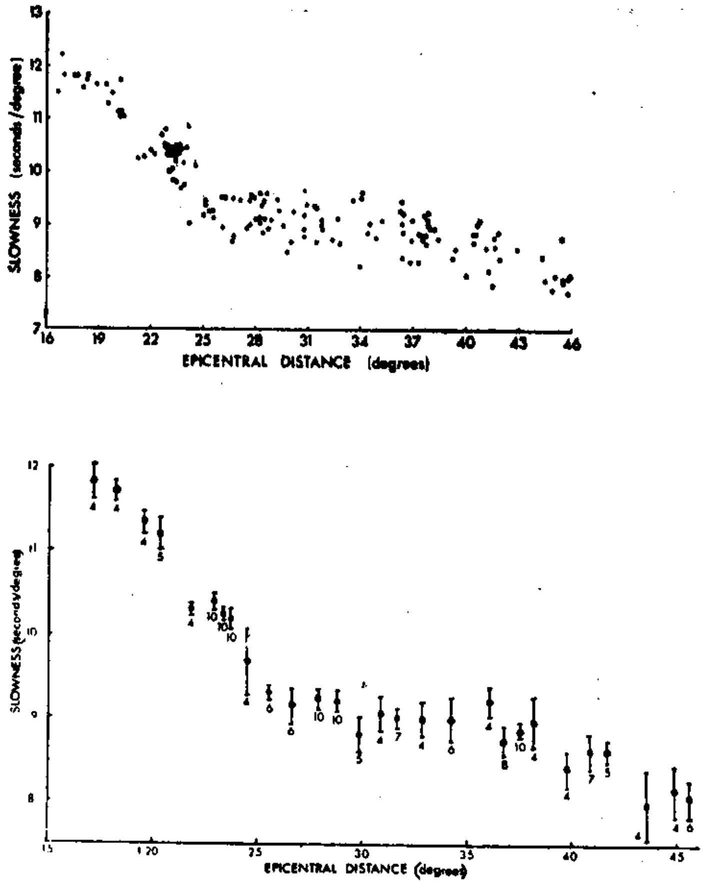

Fig. 5 First arrival slowness data (top) and averaged slowness data (bottom) in the distance range 14°-46°. Vertical bars show the standard errors. The number below each point indicates the number of observations averaged to obtain the point

The top left hand trace in this figure is the linear array sum, and represents an SNR enhanced record, when the array is steered to a slowness of 10 s/deg with a fixed azimuth (determined by the I.S.C. source location). The centre top trace is the linear TAP and the right hand top trace is the N^{th} root TAP. The traces in the subsequent lines are the same except that the slowness of steering the array has been increased every time by 0.4 s/deg. It can be seen from this figure that a later arrival C' becomes prominent as the array steering slowness is increased beyond 11.6 s/deg. The TAP pattern also demonstrates that the slowness of the C' arrival is greater than that of the C arrival. Though it is not possible to exactly measure the slowness of the C' arrival solely on the basis of this figure, determination of their relative arrival times for events which are far apart but in the same direction allows the slowness of these arrivals to be estimated relatively accurately. The triplications are identified and confirmed once the later arrivals are identified over an extended range of epicentral distance (provided the corresponding branches extend to these distances) and the slowness of the first arrivals before the crossover point is matched with that determined from the later arrivals using these techniques. If regional travel time curves are determined, these later arrivals are likely to prove helpful in relocating the sources by identifying some phases on the seismogram and determining their relative arrival times. For the deep events, if the depth phase is identified, the problem of hypocentre determination would be very much simplified.

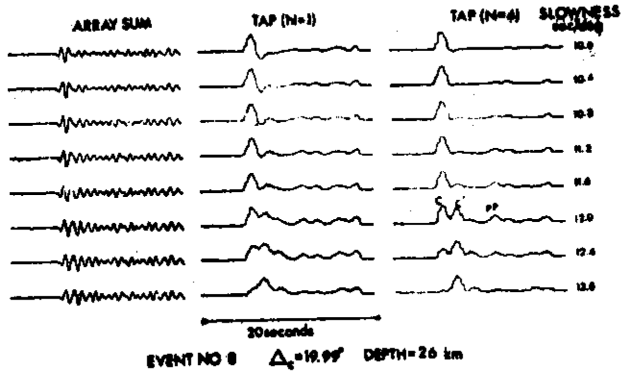

Fig. 6 Diagram illustrating the beam forming technique used for resolving closely spaced (both in time and slowness) phases

Discussion

Variations in the measured slowness of the first arrival can occur because of many reasons such as bending of the seismic ray while refracting through structures near either the source or the receiver, abnormalities in the transmission path or layering in the interior of the earth. It is very unlikely, though may not be impossible, that later arrivals observed over large epicentral distance ranges are produced by structures near the source, because of the required magnitude of such structures. For example, if the later arrivals are due to structures in the source region, then rays having different slowness can be projected back from the receiver to the source (Fig. 7) In such a case a ray, which could have reached at another point from the source must travel a sizeable distance before they get bent sharply by the structures to reach the receiver. A tight constraint on such arrivals is that these rays are required to have travel time consistent with those observed for the first arrivals. It is also difficult to postulate structures near the array which will produce both first and second arrivals as we have observed for a large number of events distributed over large epicentral distance range. Apart from structures under the source and the receiver, anomalies in the

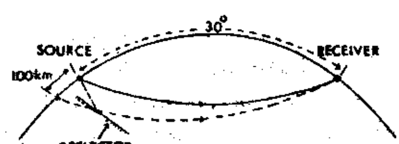

Fig. 7 A hypothetical structure near the source that can cause multiple arrivals at a receiver

travel times could be produced in regions where the rays bottom. To investigate this possibility, it is required to compare the observations with similar data from other regions, so that similarities as well as dissimilarities in these data are separated out.

A difficulty, which has been encountered in modelling the data to produce a velocity structure has arisen from the slowly decaying amplitudes of the triplication branches extending to very large distances, as a result of which it has been difficult to determine the end points of the triplication branches. If the observed later arrivals at such distances are interpreted simply in terms of the refraction branches they require negative though very small velocity gradients. Negative velocity gradients in the interior of the earth in many layers at depths below 400 km raise serious questions about the temperature and pressure behaviour of the elastic moduli, particularly at those depths, and also the temperature distribution in the interior of the earth. It was also contemplated to examine these extensions in terms of reflections produced by the underside of discontinuities (Ram Datt and Muirhead, 1977). For both these possibilities the velocity discontinuities have to be sharp and the velocity gradient within the layer have to be small. This kind of situation does not allow one to really resolve the refraction branch and the underside reflection branch. (See Fig. 8). An extensive set of data with improved processing techniques only will allow to resolve these branches and, once it is done, it will provide very tight constraints on the velocity structure in the interior of the earth.

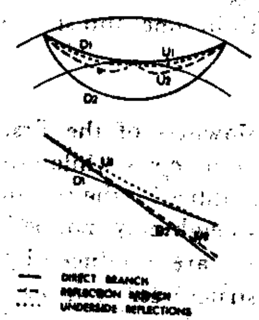

Fig. 8 Diagram illustrating underside reflection and the corresponding refraction branches

In view of the varied topographical and tectonic features, the existence of the Himalayan region on the one side, the mid-oceanic ridges on the other and the earthquake belts, for instance, the Indian subcontinent acquires significant importance in seismological studies. This is particularly so when it is viewed on the basis of the theory of continental drift. If it is assumed that it has moved northwards and collided with the Asian continent regional differences are not ruled out. There are not many published studies, particularly at depths below 300 km. Kaila et. al. (1968) have analysed travel time data from some events having their epicentres in the Himalayan, Tibet, Chinese and USSR regions and have inferred velocity discontinuities at depths (below the crust) of 380 ± 20, 580 ± 50 and 1000 ± 120 km. Many features of the data analysed at N.G.R.I. have resemblance, particularly with respect the existence and the depths of discontinuities at depths below 650 km, with those of the data referred above (Dr. P.R. Ready, Personal communication July 1977). However, some results of Ram and Mereu (1977) from the Gauribidanur array data have indicate the absence of any major velocity discontinuity (at least the 650 km one) in the Himalayan region, where according to these authors the 400 km discontinuity is followed by a broad high velocity gradient zone. There are also differences in the velocity estimates at various depths.

It is clear from the above results that a consolidated effort is needed to determine detailed and complete regionalized travel time curves. For this, it is necessary to collect all the published and unpublished data (both from arrays and single instruments stations) and carry out regional studies by dividing the entire region into a suitable number of azimuth ranges with respect to an array (GBA, at the moment). In such a collrection data, GBA can provide the much needed confirmation of the later arrivals in seismograms.

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