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EXPERIMENTAL STUDY OF FRAME MODELS WITH TWIN DIAGONAL BRACES

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Synopsis

In an unbuckled state, the compression diagonal in a panel with a cross type of bracing system, also resists lateral load. An attempt has been made to determine its contribution towards spring constant, which is used for computing the frequency of vibration of a framed structure. For the experiments conducted it was found to have nearly the same contribution as tension diagonal until it buckled. Also, the simplified method of spring constant computation by assuming that braces and frame act independently and form a paralled system of shear springs was found to be approximately equal to the stiffness coefficient method. It was further observed that such analytical methods do not give the correct value of spring constat for all location of braces.

Introduction

In the domain of experimental study of braced frames, attempts have been made to determine the following :

(i) Damping[1,2]‡

(ii) Dynamic characteristics like period and mode shapes[3]

(iii) Restoring force characteristics under lateral load[1,4]

(iv) Contribution of stiffness by braces in a frame under lateral load[2]

Stiffness distribution in a multistorey framed structure influences both the period and mode shapes remarkably and axial forces present in a braced frame modify the dynamic characteristics appreciably[3]. Wakabayashi and Tsuji[4] found the braced frames to have an unstable equilibrium after the buckling of the compression bracing. With an increase in deflection amplitude, the unstable equilibrium disappears and the curve shows a spindle shape. Funahashi, Kinoshita and Saito[1] found a double bilinear hysteretic characteristics for braced frames. Gosain and Chandrasekaran[2] found experimentally that spring constant of braced frame computed by considering the braces and frame acting independently and forming a parallel system of shear springs, is not valid for all location of braces. In the

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‡ Refers to serial number of reference given at the end.

above investigation, compression braces were so designed that they buckled under a nominal load, and all observations were made for one tension diagonal active in a panel. The number of bays were also limited to two.

In order to further testify the assumptions made in computation of spring constant of braced frames, and to investigate the contribution of compression diagonal towards stiffness, another experimental study was taken up. Lateral load and free vibration tests were conducted on perspex models having one bay one storey, two bay two storey and three bay two storey frames. Experiments revealed a contiguity in the stiffness contribution by tension and compression diagonals, unless the compression diagonal buckled. It was also noted that stiffness of a brace increases significantly by increasing its end and lateral restraints.

Spring Constant of Braced Farmes

In calculating the spring constant of braced frames, it has generally been assumed that frame and braces act independently and form a parallel system of shear springs[5].

Spring constant of the columns, taking into account joint rotation, is given by :

$$K_c = \alpha \cdot \frac{12\,EI}{L^3} \cdot n$$

where
- α = $1/F^3$, a multiplication factor given by Chandrasekaran[6]
- E = Young's modulus of elasticity
- I = Moment of inertia of column
- L = Length of column
- n = number of columns

Due to an application of a lateral load, any particular storey in a multistorey frame undergoes deflections due to the following effects :

(a) Brace elongation
(b) Beam shortening
(c) Column elongation and shortening.

Figs. 1(a), (b) and (c) illustrate such deflections under a lateral load F. Deflection due to axial deformation of the brace is given by

$$\triangle H_1 = \frac{F.\,L_D\,\sec^2\phi}{A_D.\,E}$$

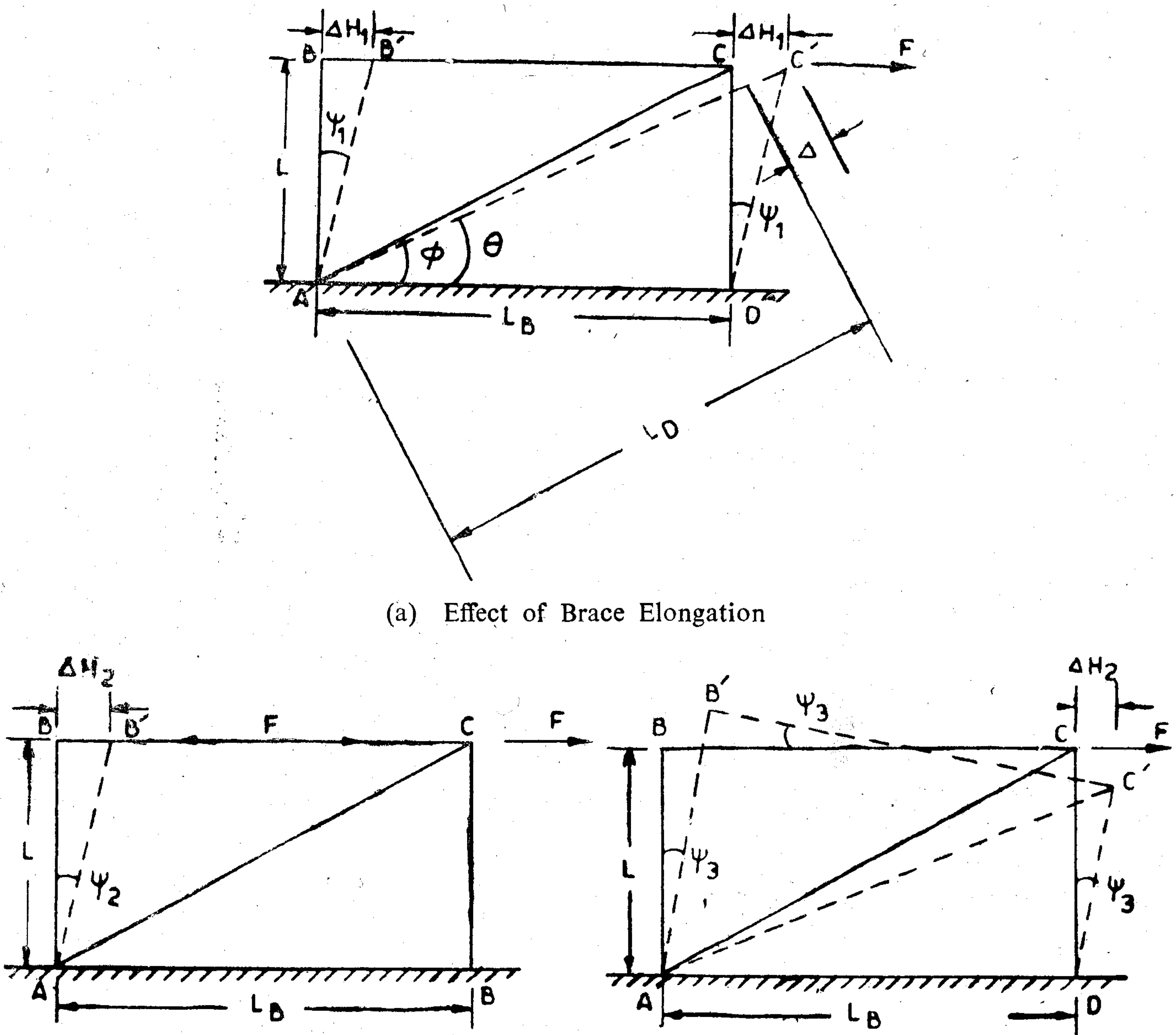
The beam will shorten due to force F by an amount $\triangle H_2$ and thus cause horizontal deflection.

$$\triangle H_2 = \frac{F\,.\,L}{A_B.\,E}$$

Changes in length due to axial forces in two columns of the braced bay also cause horizontal storey deflection $\triangle H_3$.

$$\triangle H_3 = \frac{2F\,.\,L}{A_C\,.\,E} \cdot \tan^2\phi$$

Total deflection, $\triangle H = \triangle H_1 + \triangle H_2 + \triangle H_3$.



(a) Effect of Brace Elongation

(b) Effect of Beam Shortening

(c) Effect of column Deformation

Notations :

AB′C′D = Deflected Shape of Frame ABCD

$\triangle H_1, \triangle H_2, \triangle H_3$ = Horizontal Deflections of the frame at storey level considered in the above mentioned three effects respectively

$\triangle$ = Axial deformation of brace AC

ϕ = Angle made by the brace with the horizontal

θ = Angle made by the brace with the horizontal in the deformed state

$\theta \approx \phi$

A_B, A_C, A_D = Cross sectional areas of beam, column and diagonal brace respectively

L, L_B, L_D = Length of column, beam and diagonal brace respectively

ψ_1, ψ_2, ψ_3 = Angles made by the deformed column with the vertical considered in the three effects respectively.

Fig. 1

Since spring constant is defined as load per unit deflection,

$$K_B = F/\triangle H$$
$$= \frac{F}{\triangle H_1 + \triangle H_2 + \triangle H_3}$$
$$= \frac{1}{\frac{L_D \sec^2 \phi}{A_D.E} + \frac{L}{A_B.E} + \frac{2L}{A_C.E} \cdot \tan^2 \phi}$$

where K_B = spring constant due to axial strains in brace, beam and columns.

Test Model

The bay width and storey height of each panel of the perspex model was kept as 30 cm. Cross section of both beams and columns were kept as 2.5 × 0.635 cm., thereby giving a ratio of moment of inertia per unit length of beam to that of column, i.e. $S_b/S_c = 1$. Beams were cemented to the columns by araldite. The enlarged portion of the column was clamped in a fixing frame, thus providing fixity at the base. For fixing diagonal braces (cross section 0.635 × 0.157 cm) gusset plates in the form of quadrants of circular perspex plates 7 cm dia. and 3 mm thick, were fixed at each joint of the frame by means of araldite. These gusset plates were fixed exactly at mid depth of the beams and columns.

In order to reduce the frequency of vibration of the model to a recordable value, flat iron strips having a weight of 1.46 kgm were fixed rigidly to all beams of the frame.

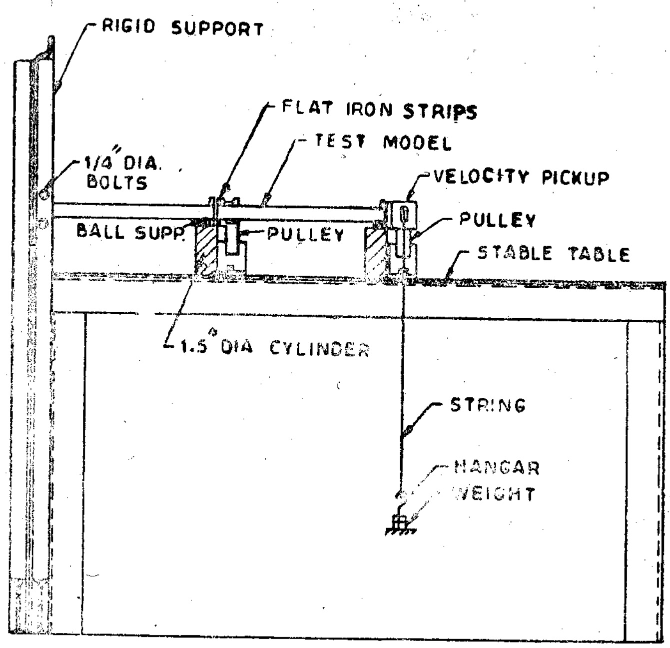


Fig. 2. Experimental Set-up for Static Testing

The model was kept in a horizontal position supported at the storey level by ball supports free to move in all directions. As seen in Fig. 2, the balls were placed on cyclindrical uprights with machined surface for free movement of the balls, and hence the frame. Such an orientation of the frame was adopted to prevent buckling in a direction perpendicular to the direction of application of load. This also eliminates any torsional effect during free vibration test.

In all, thirty three arrangements of bays, storeys and braces have been studied. Their distribution is as follows :

(a) Three arrangements of braces in single bay, single storey model.

(b) Five arrangements of braces in single bay, two storey model.

(c) Nine arrangements of braces in two bay, two storey model.

(d) Sixteen arrangements of braces in three bay, two storey model.

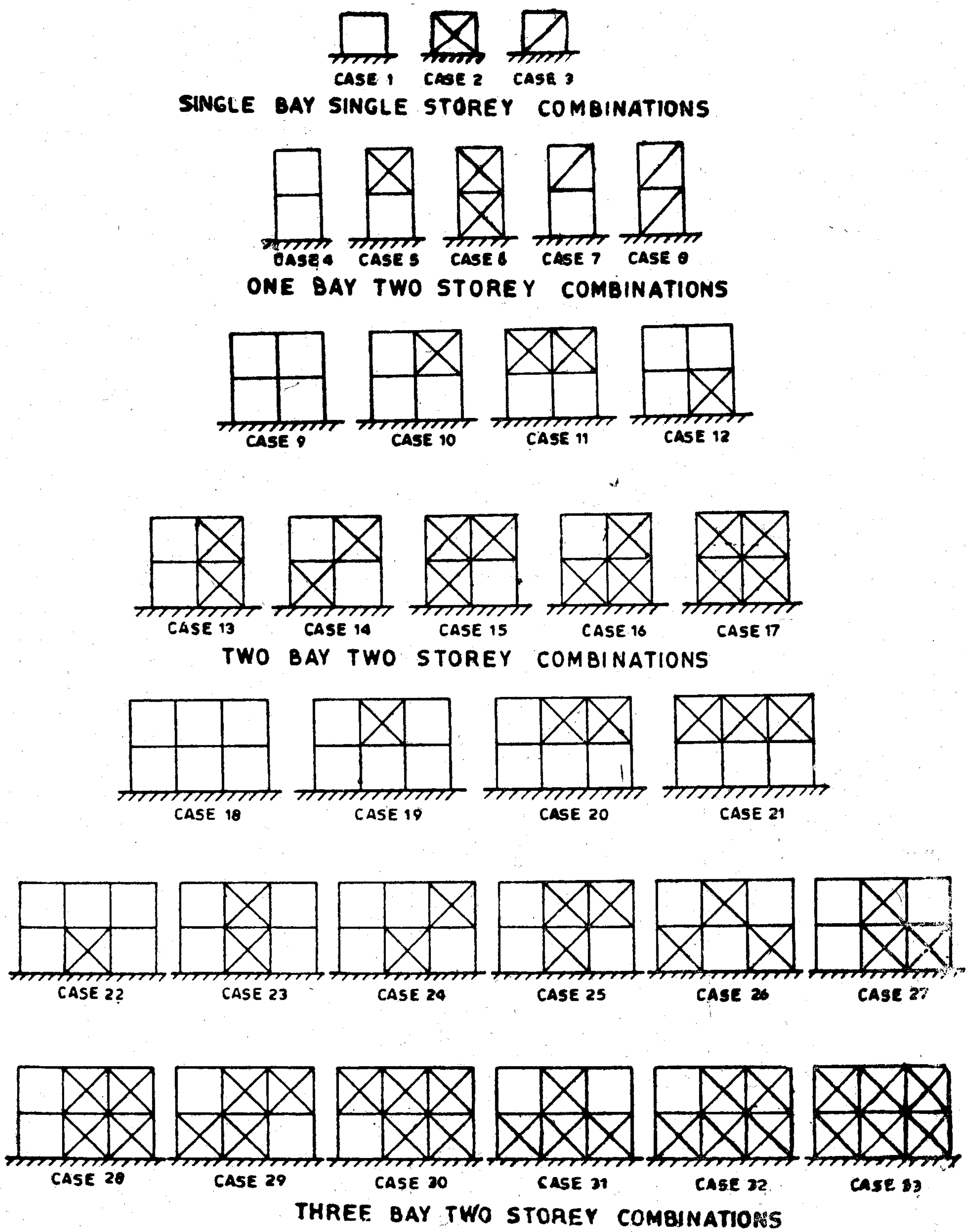

CASE 1
CASE 2
CASE 3
SINGLE BAY SINGLE STOREY COMBINATIONS
CASE 4
CASE 5
CASE 6
CASE 7
CASE 8
ONE BAY TWO STOREY COMBINATIONS
CASE 9
CASE 10
CASE 11
CASE 12
CASE 13
CASE 14
CASE 15
CASE 16
CASE 17
TWO BAY TWO STOREY COMBINATIONS
CASE 18
CASE 19
CASE 20
CASE 21
CASE 22
CASE 23
CASE 24
CASE 25
CASE 26
CASE 27
CASE 28
CASE 29
CASE 30
CASE 31
CASE 32
CASE 33
THREE BAY TWO STOREY COMBINATIONS

Fig. 3

All these cases are shown in Fig. 3. Only single bay single storey, and single bay double storey frames were tested with and without compression diagonals. Rest of the frames were tested with twin diagonals.

Computation of Spring Constant of Test Model

(a) *By Considering Brace and Frame to be Acting in Parallel*

For the single storey model and $S_b/S_c = 1$, $F = 1.126$

$$K_c = \frac{1}{(1.126)^3} \frac{12 \times 30 \times 10^3 \times 0.0533}{(30)^3} \text{ (for one column)}$$

$$= 0.500 \text{ kgm/cm}$$

For the two storey model and $S_b/S_c = 1$, $F = 1\ 248$

$$K_c = \frac{1}{(1.248)^3} \times \frac{12 \times 30 \times 10^3 \times 0.0533}{(30)^3} \text{ (for one column)}$$

$$= 0.326 \text{ kgm/cm.}$$

(From some preliminary experiments it was found that the value of E of perspex varied with the thickness of the material. Therefore, the value used for theoretical computation, i.e. $E = 30 \times 10^3$ kgm/cm^2, is an average value).

Also

$$K_B = \frac{E}{\dfrac{L_D \sec^2\phi}{A_D} + \dfrac{L}{A_B} + \dfrac{2L}{A_C} \tan^2\phi}$$

$$= \frac{30 \times 10^3}{\dfrac{42.42 \times 2}{0\ 063} + \dfrac{30}{1\ 588} + \dfrac{2 \times 30}{1\ 588} \times 1}$$

$$= 21.5 \text{ kgm/cm}$$

If the effect of beam and column strain is reglected,

$$K_B = \frac{A_D E}{L_D \sec^2\phi} = \frac{0.063 \times 30 \times 10^3}{42.42 \times 2}$$

$$= 22.25 \text{ kgm/cm}$$

Thus the beam and column strains reduce the stiffness by 3.38% in the test model.

(b) *Spring Constant by Stiffness Matrix Method*

Once the area of cross section and moment of inertias of beams, column and braces are known, the spring constant can be obtained by the stiffness Matrix method. A digital computer program[7] was used to solve a few typical cases. The analysis was carried out by considering the compression diagonal also to be effective. Stiffness matrices of cases 1 to 4, 13, 27 and 33 obtained by the above program, are given in Table 1, along with the stiffness matrices obtained by considering the brace and frame to be acting in parallel. There is a fairly good correspondence in the values obtained by the two methods.

Experimental Determination of Spring Constant

In order to get the influence coefficient matrix for the test model, load was applied at the top storey level (1) to give g_{11} and g_{12}, and at the bottom storey level (2) to give g_{22} and g_{21}. Load was applied directly by weights and deflections were measured both during loading and unloading by dial gauges having a least count of 0.01 mm.

Fig. 4 shows representative load—deflection curves. These curves are approximately bilinear. Upto a certain load, the compression diagonal also participates actively in taking the shear, but after it buckles, only the tension diagonal remains effective. Such an observation was also made by Funahashi, Kinoshita and Saito[1] Influence coefficients have been determined by using the slope of the line joining the origin to the peak of the curve. These are termed as secant slope values. These influence coefficients are then inverted to get the stiffness matrix.

The spring constant of the brace, K_B is obtained by deducting the experimental spring constant of the skeleton frame K_c from the experimental value K_T for different cases[2], i.e.

$$[K_B] = [K^T] - [K_c]$$

Tables 2, 3, 4 and 5 give a comparison between experimental and theoretical spring constant values for single bay single storey frames, one bay two storey frames, two bay two storey frames and three bay two storey frames respectively.

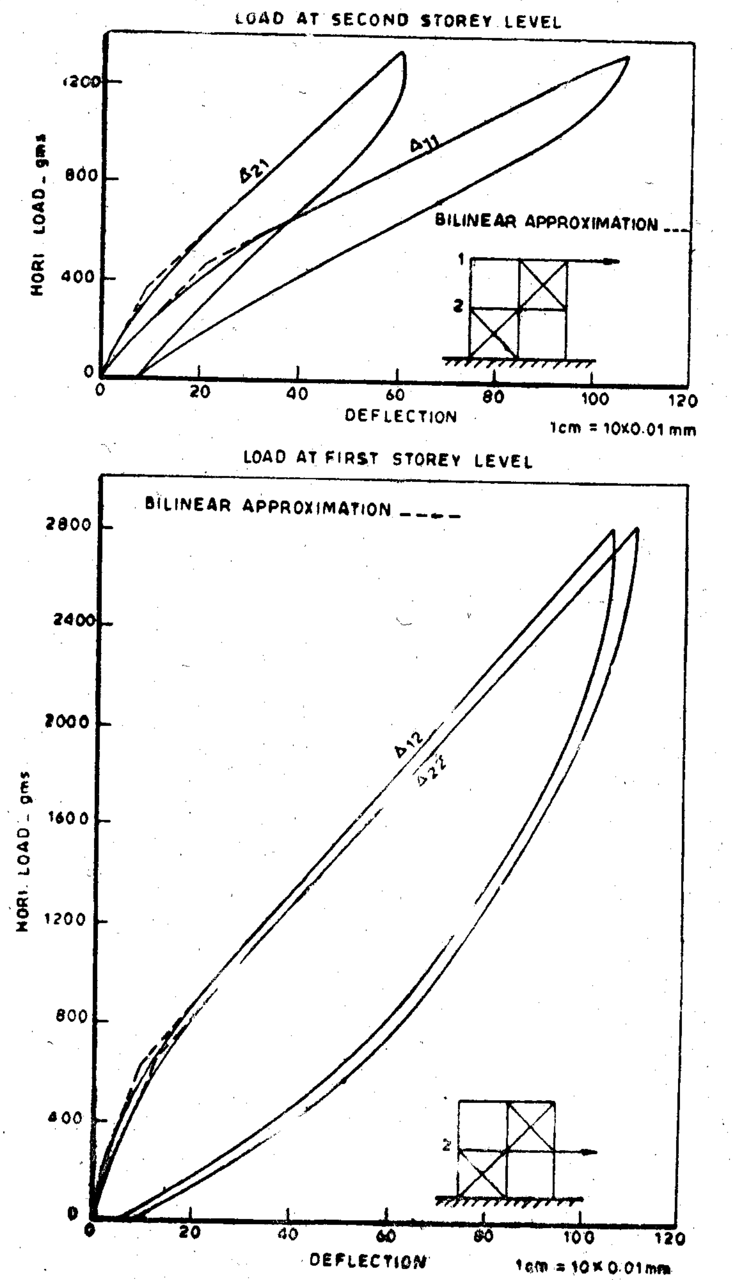


Fig. 4. Load deflection curves for case 14

Free Vibration Test

For determining the natural frequencies of vibration of the models, a self generating type velocity pick-up was fixed at the upper storey level as shown in Fig, 2. The signals were amplified by a high gain D. C. Amplifier and these were recorded by an Ink Writing Oscillograph.

Vibrations were imparted by pulling the model by a certain extent and then letting it go. The experimental values of frequencies are tabulated in Table 6 along with the

Table 1
Comparison of Spring Constants by Stiffness Matrix and Parallel Spring Methods

Case No.	Spring Constant by Stiffness Matrix $[K_T]$ kg/cm	Spring Constant $[K_T]$ by consideration of parallel springs kg/cm
1	1.0	1.0
2	44.6	44.0
3	22.8	22.5
4	$\begin{bmatrix} 0.8 & -1.05 \\ -1.05 & 2.42 \end{bmatrix}$	$\begin{bmatrix} 0.73 & -0.73 \\ -0.73 & 1.46 \end{bmatrix}$
13	$\begin{bmatrix} 43.2 & -44.6 \\ -44.6 & 91.5 \end{bmatrix}$	$\begin{bmatrix} 44.1 & -44.1 \\ -44.1 & 88.2 \end{bmatrix}$
27	$\begin{bmatrix} 40.8 & -38.2 \\ -38.2 & 122.0 \end{bmatrix}$	$\begin{bmatrix} 44.5 & -44.5 \\ -44.5 & 130.5 \end{bmatrix}$
33	$\begin{bmatrix} 115.0 & -108.0 \\ -108.0 & 226.0 \end{bmatrix}$	$\begin{bmatrix} 130.5 & -130.5 \\ -130.5 & 261.0 \end{bmatrix}$

Table 2
Comparison Between Theoretical and Experimental Spring Constant of Braces
Single Bay Single Storey Frame

Case No.	Position of braces*	Stiffness (K_T) kg/cm	Stiffness contribution by brace (K_B) kg/cm	Difference between theoretical and exp. values of (K_B)%
1	—	0.81	—	—
2	X	20.60	19.79	8.0
3	/	16.65	15.84	26.4

* *Note* – Locations of braces have been given in the second columns of these tables. The signs denote the following

— denotes no brace,

X denotes a two diagonal brace

/ denotes a single diagonal brace

For example, $\begin{matrix}-X\\X-\end{matrix}$ (Fig 3, case 14) denotes a two bay two storey frame having no brace in left panel of upper storey and right panel of lower storey and has twin diagonal brace in right panel of upper storey and left panel of lower storey. Similarly $\begin{matrix}-XX\\XXX\end{matrix}$ (case 32) denotes a three bay two storey frame in which all panels are provided with diagonal braces except the extreme left panel in the upper storey.

Table 3
Comparison Between Theoretical and Experimental Spring Constants of Braces
One Bay Two Storey Frame

Case No.	Location of brace	Stiffness matrix of braces only [K_B] kg/cm	Stiffness contributed by		Difference between theoretical and experimental stiffness values of braces	
			each upper storey brace kg/cm	each lower storey brace kg/cm	Upper storey brace kg/cm	Lower storey brace kg/cm
5	X —	$\begin{bmatrix} 17.61 & -17.89 \\ -18.77 & 19.00 \end{bmatrix}$	18.32	—	−6.68	—
6	X X	$\begin{bmatrix} 22.84 & -23.02 \\ -23.47 & 51.78 \end{bmatrix}$	23.11	28.67	−1.89	3.67
7	/ —	$\begin{bmatrix} 16.61 & -16.79 \\ -16.97 & 17.16 \end{bmatrix}$	16.88	—	−8.12	—
8	/ /	$\begin{bmatrix} 22.31 & -22.49 \\ -22.67 & 43.88 \end{bmatrix}$	22.49	21.39	−2.51	−3.61

Note : The negative sign indicates that the theoretical value of the stiffness of a brace [K_B] is greater than its experimental value.

frequencies calculated from influence coefficients. In general, it may be noted that the measured frequencies are higher than the computed frequencies. This may be due to the initial slope of the load deflection curves governing the free vibrations[8].

Conclusions

1. Spring constant computation by stiffness matrix method and by assuming the frame and brace to be acting in parallel give a fairly good correspondence.

2. The assumption that only the tension diagonal brace resists the lateral load is limited to cases in which the compression diagonal buckles completely. The stiffness contributed by an unbuckled compression diagonal brace may be as much as that by a tension diagonal brace.

3. With the increase in end restraint of the braces, which may be due to the addition of another bay, storey or an adjoining braced panel, the stiffness contributed by each brace increases significantly.

4. A change in relative location of an isolated brace in a three bay two storey frame model, does not have a significant effect on the spring constant of the brace.

5. Measured frequencies are higher than the frequencies computed by using the influence coefficients obtained from the secant slope of the load-deflection curves. This is so because theoretical studies have indicated that the frequencies are more or less dependent on the initial slope of the bi-linear load deflection curves.

Table 4

Comparison between Theoretical and Experimental Spring Constant of Braces
Two Bay Two Storey Frame

Case No.	Location of braces	Stiffness Matrix of braces only $[K_B]$ kg/cm	Stiffness contributed by		Difference between theoretical and experimental stiffness values of braces	
			each upper storey brace kg/cm	each lower storey brace kg/cm	Upper storey brace kg/cm	Lower storey brace kg/cm
10	— X — —	$\begin{bmatrix} 20.53 & -20.46 \\ -23.70 & 23.67 \end{bmatrix}$	22.1	—	−2.9	—
11	X X — —	$\begin{bmatrix} 42.08 & -42.01 \\ -44.50 & 63.92 \end{bmatrix}$	24.06	—	−0.94	—
12	— — — X	$\begin{bmatrix} 0 & 0 \\ 0 & 22.24 \end{bmatrix}$	—	22.24	—	−2.76
13	— X — X	$\begin{bmatrix} 22.33 & -22.26 \\ -28.25 & 53.82 \end{bmatrix}$	24.28	29.54	−0.72	4.54
14	— X X —	$\begin{bmatrix} 22.53 & -22.46 \\ -27.40 & 52.12 \end{bmatrix}$	24.13	27.99	−0.87	2.99
15	X X — X	$\begin{bmatrix} 53.77 & -53.70 \\ -55.20 & 82.50 \end{bmatrix}$	27.11	28.29	2.11	3.29
16	— X X X	$\begin{bmatrix} 26.83 & -24.91 \\ -28.85 & 86.12 \end{bmatrix}$	26.86	29.63	1.86	4.63
17	X X X X	$\begin{bmatrix} 53.38 & -53.31 \\ -57.70 & 114.42 \end{bmatrix}$	27.4	29.81	2.40	4.81

Table 5

Comparison between Theoretical and Experimental Spring Constant of Braces Three Bay Two Storey Frame

Case No.	Location of braces	Stiffness Matrix of braces only [K_B] kg/cm	Stiffness contributed by		Difference between theoretical and experimental stiffness values of braces	
			each upper storey brace kg/cm	each lower storey brace kg/cm	Upper storey brace kg/cm	Lower storey brace kg/cm
19	–X– –––	$\begin{bmatrix} 28.68 & -28.38 \\ -28.79 & 28.70 \end{bmatrix}$	28.64	—	3.64	—
20	–X X –––	$\begin{bmatrix} 55.28 & -54.98 \\ -55.49 & 53.20 \end{bmatrix}$	27.37	—	2.37	—
21	X X X –––	$\begin{bmatrix} 95.98 & -95.68 \\ -97.19 & 96.80 \end{bmatrix}$	32.13	—	7.13	—
22	––– –X–	$\begin{bmatrix} 0 & -0 \\ 0 & 33.25 \end{bmatrix}$	—	33.25	—	8.25
23	–X– –X–	$\begin{bmatrix} 40.98 & -40.68 \\ -42.69 & 79.40 \end{bmatrix}$	41.78	37.62	16.78	12.62
24	––X –X–	$\begin{bmatrix} 40.38 & -37.08 \\ -39.29 & 70.30 \end{bmatrix}$	38.92	31.38	13.92	6.38
25	–X X –X–	$\begin{bmatrix} 80.88 & -80.58 \\ -82.09 & 115.60 \end{bmatrix}$	40.59	34.42	15.59	9.42
26	–X– X–X	$\begin{bmatrix} 35.28 & -35.82 \\ -37.29 & 111.60 \end{bmatrix}$	36.13	37.73	11.13	12.73
27	–X– –X X	$\begin{bmatrix} 35.78 & -35.20 \\ -37.39 & 100.30 \end{bmatrix}$	36.12	32.09	11.12	7.09
28	–X X –X X	$\begin{bmatrix} 78.48 & -77.28 \\ -84.19 & 147.00 \end{bmatrix}$	39.99	33.51	14.99	8.51
29	–X X X X–	$\begin{bmatrix} 79.38 & -77.78 \\ -85.29 & 147.80 \end{bmatrix}$	40.78	33.51	15.78	8.51
30	X X X –X X	$\begin{bmatrix} 151.58 & -150.38 \\ -153.39 & 223.50 \end{bmatrix}$	50.59	35.86	25.59	10.86
31	–X– X X X	$\begin{bmatrix} 36.08 & -34.23 \\ -36.99 & 141.70 \end{bmatrix}$	35.77	35.31	10.77	10.31
32	–X X X X X	$\begin{bmatrix} 39.03 & -84.78 \\ -99.39 & 219.30 \end{bmatrix}$	43.86	43.86	18.86	18.86
33	X X X X X X	$\begin{bmatrix} 169.98 & -124.18 \\ -186.29 & 334.80 \end{bmatrix}$	57.83	53.77	32.83	28.77

Table 6

Frequencies

Case No.	Location of braces	Measured frequency c/s	Computed frequency based on experimental influence coefficients c/s	Percentage difference between measured and computed frequency
1	2	3	4	5
1	—	5.44	3.56	34.60
2	X	17.85	17.95	—0.56
3	/	16.00	16.15	—0.93
4	— —	3.10	3.497	—12.80
5	X —	3.76	3.132	16.70
6	X X	19.22	12.727	33.75
7	/ —	3.65	3.103	15.00
8	//	11.47	11.527	—0.49
9	—— ——	2.55	1.988	22.00
10	—X ——	3.39	2.495	26.40
11	XX ——	3.48	2.650	23.85
12	—— —X	6.57	3.096	52.90
13	—X —X	15.25	8.364	45.10
14	—X X—	16.00	8.611	46.10
15	XX —X	16.45	9.346	43.20
16	—X XX	18.65	11.564	38.00