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# FUNDAMENTAL PERIOD OF CONCRETE CHIMNEYS

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## SYNOPSIS

Fundamental period is an important dynamic property necessary for use in SEAOC code, Rumman's and Indian code procedures for the design of concrete chimneys subjected to earthquake effects. At present two formulae given by A C I Committee 307 and Indian code IS 1893-1983 are preferred most for computing the period. It is found that there is discrepancy in the prediction of these two equations which makes it necessary to examine the problem further. In this paper two rational methods based on Rayleigh's minimum principal and computer analysis are presented for evaluating the period. Finally based on computer solution two equations are proposed for reckoning the period expeditiously in the design office.

## INTRODUCTION

The current trend in the industrial construction all over the world is to go in for very tall chimneys as high as four hundred metres to prevent air pollution. Consequently the slender dimensions and tapering geometry of tall chimneys cause structural problems due to wind gust and earthquake. At present three simplified procedures in lieu of a rigorous computer oriented analysis are preferred for preliminary design of chimneys subjected to earthquake effects. They are

1. SEAOC code procedure[1]
   (Structural Engineer's Association of California)
2. Rumman's method[2]
3. Is 1983-1984 recommendations[3]

The above methods are based on the phenomenon of base shear induced at the junction of the superstructure and the foundation of the chimney during earthquake. The base shear in the above procedure is distributed up through the height as horizontal force from which design shear force and bending moment at any section are reckoned. For assessing the base shear and overturning moment during earthquake, the important dynamic property of the chimney, viz, the fundamental period is necessary.

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## RESEARCH SIGNIFICANCE

Among the many solutions available in literature, at present the two formulae given by A C I Committee 307[1] and IS 1983-1984[3] are preferred most for computing the period. While the A C I formula is an empirical one the provenance of Indian code equation appear to be the formula for the period of a cantilever with uniform section. By applying these two equations to nineteen chimneys of varying heights, it is found that there is discrepancy in the prediction of these two solutions. Consequently, there is a need to search into the problem in a realistic manner. With this objective in view this paper address itself to advancing two rational solutions based on classic Rayleigh's minimum principle and rigorus computer solution. The results of the computer solution are compared with the answers of ACI equation and the Indian code formula. It is found that the computer solution lies in between the above two results in the majority of the cases. Finally bassd on the computer results, two equations are proposed for rapidly computing the period in the design office.

## RAYLEIGH's MINIMUM PRINCIPLE

Rayleigh's principle is a powerful concept for finding the fundamental frequency of an undamped freely vibrating system with satisfactory results. This concept leads to an upper bound solution to the frequency. For a system in which the mass is continuously distributed, the fundamental frequency can be found using the quotient

$$p^2 = \frac{Eg \int I_x \left( \frac{d^2y}{dx^2} \right)^2 dx}{\int w_x \; y^2 \; dx} \quad \ldots\ldots (1)$$

where

$p$ = fundamental frequency in radians per sec,
$y$ = shape runction,
$I_x$ = moment of inertia function,
$w_x$ = self weight distribution function.

The period T is then given by the expression

$$T = \frac{2\pi}{p} \quad \ldots\ldots (2)$$

where T = period in seconds.

Knowing the deflected shape of the chimney due to inertia forces, Eq. [1] can be evaluated. For a true dynamic deflection curve, Eq. [1] will lead to an exact answer. For other functions, Eq. [1] will result in upper bound solution to the frequency. For the shape function, any reasonable curve resembling the dynamic deflection shape may be assumed, Even if the boundary cohditions are not fully satisfied, the probability of getting a realistic solution is not unpledged. Herein lies the efficacy of Rayleigh's concept.

In the present study, the following two shape functions which fairly represent the dynamic deflection profile of a tapered chimney are made use. of for finding the frequency using Eq. [1].

(1) $y = c\,x^2$ ...... (3)

(2) $y = c[x^4 - 6h^2 x^2]$ ...... (4)

where h = height of the chimney,
c = a constant,
x = height of any section from the base of the chimney.

It may be seen that the kinematic and most of the force boundary conditions are met by these two shape functions, Therefore, it is only natural that these polynomials will provide reasonable upperbound to the frequency in accordance with Rayleigh's principle.

## COMPUTER ANALYSIS

During earthquake, the chimney is subjected principally to horizontal motion. Between these two, the horizontal motion causes severe damage to the structure and hence the theory described in the paper is based on translation in the horizontal direction. Further more, the effect of damping and the chimney lining on the free vibration of the structure is disregarded.

For an n degree of freedom undamped lumped mass discrete system vibrating freely, the differential equation of motion can be written in terms of translational flexibility matrix δ, as

$$\delta\, m\, q + \ddot{q} = 0 \qquad \text{...(5)}$$

where δ = Lateral flexibility matrix (a square positive definite symmetric matrix),
m = mass matrix (a diagonal matrix),
$\ddot{q}$ = acceleration vector (a column matrix), and
q = displacement vector (a column matrix).

Eq [5] represents a eigen value problem. It has a distinct advantage as the frequency analysis on the SYSTEM or DYNAMIC MATRIX [δ m] will straight way yield first the fundamental frequency in the matrix iteration (STODOLA's METHOD) which is described in reference [4]. For formulating the system matrix [δ m], the lateral flexibility matrix δ must be known besides the mass matrix m which can be easily set up. The formulation of the δ matrix is described below in a compendious manner.

## PROPOSED METHOD FOR THE FORMULATION OF δ MATRIX IN THE COMPUTER.

For formulating the lateral flexibility matrix, δ in the computer, the moment area method was made use of. The given chimney was discretized into forty two sections of equal height to suite the size of memory available in the computer. Unit load was applied at different sections in succession and the flexibility influence coefficients were generated. Thus a 42×42 translational flexibility matrix was set up. The lumped mass matrix was set up. The lumped mass matrix m was then formulated. Multiplication of the above two matrices [δ m] resulted in the SYSTEM MATRIX.

## FREQUENCY ANALYSIS

Nineteen chimneys with height varying from 90 metres to 420 metres were selected. The properties of the chimneys are listed in Table 1.

**(a) Rayleigh's Method :**

With the help of Eq. [3], Eq. [4] and Eq. [1], frequencies were computed for the nineteen chimneys manually. The periods thus obtained are listed in Table 2.

(b) Computer Analysis (STODOLA'S METHOD)

Using the SYSTEM MATRIX and matrix iteration (Stodola's method), the fundamental frequency and tha corresponding mode shape were obtained in the computer. The periods of the computer solution for the nineteen chimneys are shown in Table 2.

## CURRENTLY AVAILABLE FORMULAE

The two formulae given by A C I committee 307 and IS 1893-1984 for computing the period of concrete chimney are as follows :

1. A C I fotmula :

$$T = \frac{0.49\ H^2}{(3D-d)}\sqrt{E_0} \qquad \ldots\ldots (6)$$

where

$T$ = period in seconds,
$H$ = height of the chimney in metres,
$D$ = bottom outer diameter in metres,
$d$ = top outer diameter in metres, and
$E_c$ = Young's modulus of concrete in MPA.

2. Indian code formula : [IS 1893—1984]

Period of free vibration T, of chimney when fixed at base is computed as follows :

$$T = C_t \sqrt{\frac{W_t h'}{E_s A g}} \qquad \text{...... (7)}$$

where

$C_t$ = coefficient depending upon slenderness ratio of the chimney as suggested in the code,
$W_t$ = total weight of structure,
$h'$ = height of the chimney,
$E_s$ = Young's modulus of concrete
$A$ = area of cross section at the base of structural shell, and
$g$ = acceleration due to gravity.

## DISCUSSION OF THE VARIOUS PREDICTIONS

In Table 2. the prediction of the A C T equation and the Indian code formula is shown along with the solutions of computer, Eq. [3] and Eq. [4] From a persual of the results in in Table 2, it is seen that toe A C I equation predicts lesser value than the Indian code equation.

The discrepancy is conspicuous in the case of taller chimneys. The computer solution of the present study falls in between the above two predictions in the majority of the cases. However in the case of very tall chimneys, there is only a slight difference between the computer solution and the Indian code prediction.

## PROPOSED EQUATION FOR THE PERIOD

The Rayleigh method of computing the period is a tedious process The shear volume of computation involved in the procedure prohibits its use in the design office where as computer solution is time consuming. In practice a reliable, tested, sound and time saving formula is often useful for estimating the period. Such a formula should not only be simple but have all the influential parameters as well. Keeping this maximum in mind, a simple intuitive equation is proposed as follows :

$$T = \frac{0.1813\,H^2}{D\sqrt{E_c}} + \frac{H\,(H\,d)^{0.5}}{(35300-11H)} \qquad \text{... (8)}$$

where T = period in seconds,

H = height of the chimney in metres,

D = outer base diameter in metres,

d = outer top diametet in metres

$E_c$ = Young's modulus of concrete in MPa.

The prediction of Eq. [8] for the nineteen chimneys is shown in Table 3. It is seen that reasonable agreement prevails between the prediction of this equation and the computer solution.

## APPROXIMATE EQUATION

A good structural engineer continually makes approximate calculations to check himself. The habit of making an estimate of the answer to a problem before the start of the detailed calculations is very much stressed in profession. Such a mental exercise will be invaluable in developing the judgement and intuition of the analyst for assessing the structural behaviour of the system. With this aim and to assist the structural engineer the following equation was obtained by plotting the results. During inceptive stages of the project, the only data available will be the height of the chimney. Knowing the height, the approximate period can be estimated with the aid of the simple formula.

$$T = 0.015\,H \qquad \text{...... (9)}$$

where T = period in seconds,

H = height of the chimney in metres.

The prediction of the Eq. [9] is compared with the computer solution in Table 3.

## RECAPITULATION

Fundamental period of a tapered concrete chimney can be obtained using the Rayleigh principal and computer solution. For reckoning the period in the design office, an equation is proposed. Prediction of the equation is in fair agreement with the computer solution. For rapidly computing the period during inceptive stages an approximate equation is suggested. It is believed that this equation will be quite useful in preliminary stages of the project.

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## DEDICATION

This paper is reverentially dedicated to the patriot Mahakavi C. Subramania Bharathi whose stellar and salutary influence remains as a source of inspiration to the senior author for prosecuting the task of this kind with zeal.

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**Table :—1 Properties of the Chimney**

| Sl. No. | Height in metres | Outer diameter at bottm in metres | Outer diameter at top in metres | Thickness at bottom in metres | Thickness at top in metres | Young's Modulus in M Pax $10^4$ |
|---|---|---|---|---|---|---|
| 1. | 91.34 | 8 13 | 3.02 | 0.61 | 0.15 | 2.434 |
| 2. | 104.71 | 10.06 | 5,03 | 0.46 | 0.23 | 2.399 |
| 3. | 107.44 | 8.36 | 5.23 | 0.61 | 0.15 | 2.405 |
| 4, | 127.10 | 11,45 | 6.45 | 0.46 | 0.15 | 2.599 |
| 5. | 137.16 | 12.19 | 5,41 | 0.61 | 0.18 | 2.144 |
| 6. | 162.76 | 10.68 | 5.69 | 0.91 | 0.23 | 2.420 |
| 7. | 180.00 | 15.00 | 6.00 | 0.40 | 0.22 | 2.720 |
| 8. | 200.00 | 16.66 | 7.00 | 0.45 | 0.25 | 2.720 |
| 9. | 220 00 | 18.00 | 7.50 | 0.50 | 0.27 | 2.720 |
| 10. | 240 00 | 20.00 | 8.00 | 0.55 | 0.30 | 2.720 |
| 11. | 260.00 | 21.50 | 9.00 | 0.60 | 0.35 | 2.720 |
| 12. | 280.00 | 23.00 | 10.50 | 0.65 | 0.40 | 2.720 |
| 13. | 300 00 | 25.00 | 11.00 | 0.70 | 0.45 | 2.720 |
| 14. | 320.00 | 27.00 | 12.00 | 0.80 | 0.50 | 2 720 |
| 15, | 340.00 | 30.00 | 15.00 | 0.90 | 0.60 | 2.720 |
| 16, | 360.00 | 34.00 | 18.00 | 1.00 | 0.75 | 2.720 |
| 17. | 380.00 | 36.00 | 20.00 | 1.10 | 0.90 | 2.720 |
| 18. | 400 00 | 38.00 | 21.50 | 1.25 | 1.00 | 2.720 |
| 19. | 420.00 | 40.00 | 23.00 | 1.40 | 1.10 | 2 720 |

**Table :—2 Prediction of Period of Chimneys by Differant Formulae in Seconds.**

| Sl. No. | Chimney height in metres | A C I formula Sec. | Indian code equation Sec. | Computer solution Sec. | Rayleigh's principle | |
|---|---|---|---|---|---|---|
| | | | | | Eq. (4) Sec. | Eq. (3) Sec. |
| 1. | 91.34 | 1.23 | 1.39 | 1.12 | 1.11 | 1.13 |
| 2. | 104 71 | 1.38 | 1.55 | 1.40 | 1.41 | 1.33 |
| 3. | 107.44 | 1,84 | 1 93 | 1.59 | 1.61 | 1.56 |
| 4. | 127.10 | 1.76 | 1.87 | 1.60 | 1.64 | 1.57 |
| 5. | 137.16 | 2.02 | 2.21 | 1.75 | 1.85 | 1,85 |
| 6. | 162.76 | 3.17 | 3 36 | 2.14 | 2.87 | 2.83 |
| 7. | 180.00 | 2.47 | 2.78 | 2.57 | 2,58 | 1.57 |
| 8. | 200.00 | 2.77 | 3.11 | 2.88 | 2.90 | 2 88 |
| 9. | 220.00 | 3.09 | 3.47 | 3.20 | 3.24 | 3.22 |
| 10. | 240,00 | 3.29 | 3.71 | 3.42 | 3.45 | 3.53 |
| 11. | 260 00 | 3.62 | 4.09 | 3.83 | 3.81 | 3.79 |
| 12. | 280.00 | 3.98 | 4.50 | 4,27 | 4.29 | 4,22 |
| 13. | 300.00 | 4.18 | 4.76 | 4.54 | 4.56 | 4.49 |
| 14. | 320.00 | 4.41 | 5.02 | 4.76 | 4.78 | 4.71 |
| 15. | 340.00 | 4.58 | 5.21 | 5,01 | 5.02 | 4.88 |
| 16. | 360.00 | 4.51 | 5:29 | 5,16 | 5.17 | 4.97 |
| 17. | 380.00 | 4.88 | 5.68 | 5.61 | 5,61 | 5 34 |
| 18. | 400.00 | 5.14 | 5.97 | 5.88 | 5.90 | 5.59 |
| 19. | 420.00 | 5.40 | 6.27 | 6.15 | 6,23 | 5.91 |

**Table :—3 Prediction of Proposed Eq. (9) and Comparison.**

| Sl. No. | Chimney height m | Computer solution sec. | Proposed Eq. (8) sce. | Approximate Eq. (9) sec. |
|---|---|---|---|---|
| 1. | 91.34 | 1.12 | 1.23 | 1.37 |
| 2. | 104.71 | 1.40 | 1.37 | 1.57 |
| 3. | 107.44 | 1.59 | 1.68 | 1.61 |
| 4, | 127.10 | 1.60 | 1.30 | 1.91 |
| 5. | 137.16 | 1.75 | 2.07 | 2.06 |
| 6. | 162.76 | 2.84 | 3.04 | 2.44 |
| 7. | 180.00 | 2.57 | 2.55 | 2.70 |
| 8. | 200.00 | 2.88 | 2.87 | 3.00 |
| 9. | 220 00 | 3.20 | 3.23 | 3.30 |
| 10. | 240 00 | 3.42 | 3.49 | 3.60 |
| 11. | 260.00 | 3.83 | 3.85 | 3.90 |
| 12. | 280.00 | 4.27 | 4.22 | 4.20 |
| 13. | 300.00 | 4.54 | 4.50 | 4.50 |
| 14. | 320.00 | 4.76 | 4 79 | 4.80 |
| 15. | 340.00 | 5.01 | 5.01 | 5.10 |
| 16. | 360.00 | 5.16 | 5.12 | 5.40 |
| 17. | 380 00 | 5.61 | 5.48 | 5.70 |