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DEVELOPMENTS IN RESPONSE SPECTRUM -BASED STOCHASTIC RESPONSE OF STRUCTURAL SYSTEMS

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ABSTRACT

Response spectra are commonly used for the estimation of the largest peak response of a linear structural system in a seismic environment. Traditionally, this has been done through the use of appropriate modal combination rules in case of multi-degree-of-freedom systems. While these methods do not consider uncertainty in response due to phasing in seismic waves, those also do not go beyond estimating the largest peak response, and have natural limitation of being applied accurately to only a few types of structural systems. This paper considers a review of alternative methods which have been developed since mid-1970's to give probabilistic estimates of response peaks, while continuing to use the information available through response spectra. These methods have the convenience of being applied in a variety of situations, do not usually suffer from the inaccuracies associated with the use of modal combination rules, and present state-of-the-art methodology in linear seismic response analysis. The limitations of various formulations proposed under these methods are identified, and future directions of required work are suggested.

KEYWORDS: Ground Motion Process, Response Spectrum-Based Characterization, Linear MDOF Systems, Peak Stochastic Response

INTRODUCTION

1. Seismic Response Analysis

Computation of the dynamic response of structures to earthquake ground motions involves, in general, three steps: (i) formulation and selection of the physical model, (ii) mathematical formulation of the governing equations, and (iii) computation of the response. The extent of rigour in each of these steps depends on the type of application, required output, and on the desired accuracy.

Formulation and selection of the physical model also depends on the past experience, with similar and related modeling, and has to represent, as realistically as possible, the desired characteristics of response. Arriving at the final model inevitably involves simplification of the complete full-scale structure. This may require iterations, involving verification via specific measurements on similar or related structures (e.g., see Trifunac and Todorovska, 1999a; Kojic et al., 1984a, 1984b, 1993), observation of general trends in full-scale structural response (Udwadia and Trifunac, 1974a; Trifunac et al., 2001a, 2001b, 2001c, 2002) or of selected features of full-scale response (Trifunac and Lee, 1979, 1986). Model geometry and its properties may be chosen suitably to study specific aspects of one versus two versus three-dimensional characteristics of response (e.g., see Kojic and Trifunac, 1988, 1989, 1991a, 1991b), or to isolate and emphasize the effects of the selected model properties (e.g., see Kashefi and Trifunac, 1986).

Mathematical formulation of the methods for computation of response of the chosen model can result in the selection of vibrational or wave propagation types of analyses. The wave propagation approach may be advantageous, particularly for impulsive excitations; but this has become obvious and has been used in earthquake engineering only recently (e.g., see Todorovska et al., 1988, 2001a, 2001b; Todorovska and Trifunac, 1989, 1990a, 1990b; Trifunac, 1997, 2000; Trifunac and Todorovska, 1997). The wave propagation method of analysis is essential in the study of soil-structure interaction effects, in the structural models supported by flexible foundations (Hayir et al., 2001; Todorovska and Trifunac, 2001; Todorovska et al., 2001c; Trifunac and Todorovska, 1999b). This method of solution, when combined with a finite element or finite difference formulation, also offers an excellent tool for solution of many problems involving irregular geometry and non-linear material properties.

Computation of the response usually involves direct integration of the differential equations in time domain, if the time-history of the excitation is known *a priori* (Trifunac, 2003). For linear problems which can use the superposition principle, this may be performed in frequency domain also. When there is uncertainty regarding the time-history of the excitation, stochastic methods of response estimation can be used. Though these methods have traditionally centered around the most elementary discrete representation of the structures, in terms of the single-degree-of-freedom (SDOF) or multi-degree-of-freedom (MDOF) systems, those can be extended to the continuum model representation also. The stochastic methods have been particularly helpful for evaluation of the relative significance and additional contributions (i) due to torsion and rocking in strong motion (e.g., see Gupta and Trifunac, 1987a, 1989, 1990d, 1991a, 1993), (ii) due to soil-structure interaction (e.g., see Gupta and Trifunac, 1987a, 1989), and (iii) in the description of relative amplitudes of all peaks of the response (e.g., see Udwadia and Trifunac, 1973a, 1973b; Amini and Trifunac, 1981, 1984; Gupta and Trifunac, 1992, 1998d). These methods have further enabled incorporation of the response into the general framework for characterization of seismic hazard (e.g., see Todorovska et al., 1995).

2. Response Spectrum-Based Techniques

As stated above, estimating the peak response of a linear structural system with known dynamic characteristics to a given description of the ground motion forms an important part of its overall seismic safety assessment. The ground motion can be described in form of a set of smooth, expected response spectra for various damping ratios. These spectra are obtained after the statistical processing of accelerograms pertaining to similar earthquake parameters and recorded in similar site conditions. Since a given response spectrum may be consistent with many ground acceleration time-histories, the computed structural response for the given spectrum is not unique. Thus, any response analysis should account for this inherent uncertainty, while being able to use some or all of the response spectrum ordinates.

Though various consistent time-histories may look different in detail, those cannot be distinguished from each other as regards their overall statistical properties. However, it is possible to completely describe a random process like this in a compact manner by a power spectral density function (PSDF). One may then estimate statistical variations in the largest structural response by working with the PSDF of the ground motion process and by using the stationary theory of random vibrations. Since ground motion processes are highly non-stationary, such a (time-independent) PSDF, even though it can be viewed as a 'temporally averaged' PSDF, fails to estimate peak parameters of the process correctly. Further, when such PSDF is used along with time-independent transfer functions of structural responses, the non-stationarity caused by sudden application of the excitation is not accounted for. This may lead to additional response errors, unless the system is very stiff and/or highly damped. It is possible to provide corrections for these errors by directly using the response spectrum ordinates in case of MDOF systems (e.g., see Amini and Trifunac, 1985; Gupta and Trifunac, 1987c), or by using a fictitious PSDF corresponding to an 'equivalent stationary' ground motion process (e.g., see Singh and Chu, 1976; Der Kiureghian, 1981; Shrikhande and Gupta, 1997a; Gupta and Trifunac, 1998a). In the former, simplifications lead to the development of modal combination rules (e.g., see Rosenblueth and Elorduy, 1969; Wilson et al., 1981; Singh and Mehta, 1983; Der Kiureghian and Nakamura, 1993; Gupta, 1996a), wherein the response spectra are used to predict the largest peak response for each mode of the structure, and then these individual modal maxima are combined to give the expected peak response. These simple methods however do not address the inherent uncertainty in the ground motion due to random phasing. They can also not be applied with consistent accuracy to different types of structural systems. In the case of 'fictitious' PSDF, one may either have compatibility with peak ground acceleration (PGA) or with a set of response spectrum ordinates. In case of compatibility with PGA, however, it is necessary to use the transient transfer functions to account for the non-stationarity caused by sudden application of the excitation.

Uncertainty due to random phasing may also be considered by modelling the nonstationarity in ground motions more explicitly and by using the transient transfer functions to describe the time-dependent harmonic response to a suddenly applied harmonic excitation. In most of the methods in this category, the ground motions are modeled as the envelope function modulated stationary processes. Formulations based on such a modelling (e.g., see Gasparini, 1979; To, 1982; Bucher, 1988; Borino et al., 1988; Sun and Kareem, 1989) are however complicated for practical design applications, and those have also been devoid of a framework to characterize the perceived seismic hazard at a site through the parameters of proposed models. On the other hand, the wavelet-based methods in this category (e.g., see

Basu and Gupta, 1997) can account for frequency non-stationarity (associated with arrival of different types of seismic waves at different times and the dispersion in these waves), besides using the efficient characterization of seismic hazard via spectrum-compatible wavelet functionals (see Mukherjee and Gupta, 2002).

This paper provides an overview of various PSDF-based and wavelet-based methods available for estimating peak structural response of linear MDOF systems in case of response spectrum-based characterization of seismic hazard. For facilitating a convenient comparison in case of PSDF-based methods, the problem is first formulated for the response of a symmetric shear building, and then various available formulations are analyzed. Possible directions are also identified for further research work. It is hoped that together with Gupta and Trifunac (1996), this review will also provide the reader a comprehensive general summary of methods available for estimation of maximum stochastic response by using response spectrum.

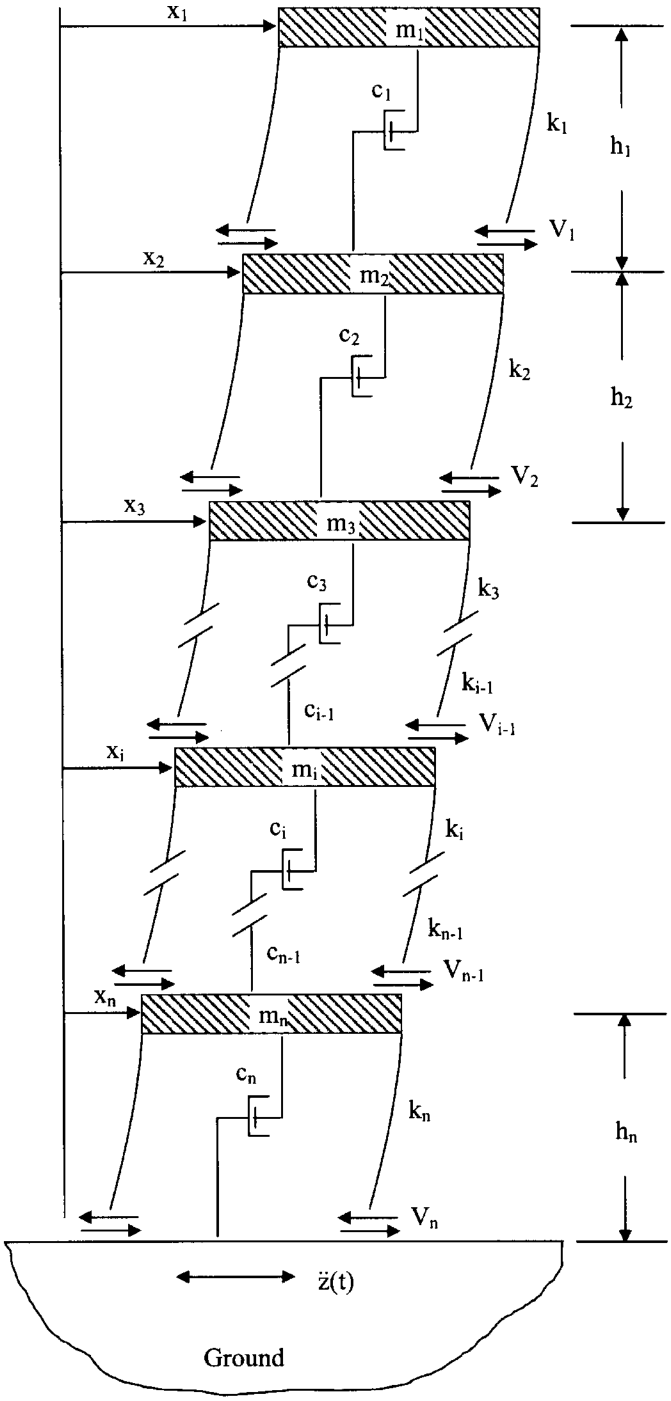

Fig. 1 Shear building model for n-storied building

FORMULATION OF PSDF-BASED METHODS

Consider a symmetric shear building, shown in Figure 1, where the lumped floor masses m_i, $i = 1, 2, ...n$ are interconnected through massless column springs of stiffnesses k_i, $i = 1, 2, ...n$, and the viscous dampers representing the interstory damping of magnitude c_i, $i = 1, 2, ...n$. The n-coupled equations of motion for this system can be written as

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = -\ddot{z}[m]\{\Gamma\} \tag{1}$$

where $[m]$, $[c]$ and $[k]$ respectively are the $n \times n$ mass, damping and stiffness matrices in terms of m_i, c_i and k_i, $i = 1, 2, ...n$; $\{\Gamma\}$ is the $n \times 1$ ground displacement influence vector; $\{x\}$ is the $n \times 1$ vector comprising of relative displacements $x_i(t)$, $i = 1, 2, ...n$ of the floor masses; and, $\{\dot{x}\}\left(= \frac{d}{dt}\{x\}\right)$, $\{\ddot{x}\}\left(= \frac{d^2}{dt^2}\{x\}\right)$ are the time derivatives of $\{x\}$. Viscous damping, $[c]$ is assumed to be of such a form that it can be diagonalized by the transformation $[A]^T[c][A]$ where $[A]$ $(= [\{\phi^{(1)}\}\{\phi^{(2)}\}....\{\phi^{(n)}\}])$ is the $n \times n$ modal matrix of eigenvectors $\{\phi^{(j)}\}$, $j = 1, 2, ..n$ obtained by solving the eigenvalue problem $\omega^2[m]\{\phi\} = [k]\{\phi\}$. The jth element of this diagonal form is denoted as $2\zeta_j\omega_j M_j$, where ω_j and ζ_j respectively are the eigenvalue and damping ratio corresponding to the eigenvector $\{\phi^{(j)}\}$, and $M_j = \{\phi^{(j)}\}^T[m]\{\phi^{(j)}\}$ is the jth modal mass. Premultiplying Equation (1) by $[A]^T$ and making normal mode decomposition of relative displacement by the transformation, $\{x(t)\} = [A]\{\xi(t)\}$, it follows that

$$[A]^T[m][A]\{\ddot{\xi}\} + [A]^T[c][A]\{\dot{\xi}\} + [A]^T[k][A]\{\xi\} = -\ddot{z}[A]^T[m]\{I\} \tag{2}$$

where $\{\xi(t)\}$ is the $n \times 1$ vector of normal coordinates. Since, $\{\phi^{(j)}\}^T[k]\{\phi^{(j)}\} = M_j\omega_j^2$, and the eigenvectors $\{\phi^{(j)}\}$, $j = 1, 2, ...n$ are mutually orthogonal with respect to the matrices $[m]$ and $[k]$, following n decoupled equations can be obtained from Equation (2), each describing the motion in a specific mode,

$$\ddot{\xi}_j + 2\zeta_j\omega_j\dot{\xi}_j + \omega_j^2\xi_j = -\alpha_j\ddot{z}; \qquad j = 1, 2, ..., n \tag{3}$$

where, $\alpha_j = \{\phi^{(j)}\}^T[m]\{\Gamma\}/M_j$ is the modal participation factor in the jth mode. Fourier transformation of Equation (3) into the frequency domain leads to

$$-\omega^2\Xi_j(\omega) + 2\zeta_j\omega_j\Xi_j(\omega)i + \omega_j^2\Xi_j(\omega) = -\alpha_j Z(\omega) \tag{4}$$

where $i = \sqrt{-1}$, $\Xi_j(\omega) = \mathcal{F}(\xi_j(t))$ and $Z(\omega) = \mathcal{F}(\ddot{z}(t))$, the operator, $\mathcal{F}$ representing the Fourier transformation of time-dependent variable into its frequency domain counterpart. Solving for $\Xi_j(\omega)$ from Equation (4) and using the relation $\{x(t)\} = [A]\{\xi(t)\}$ in the frequency domain, i.e. $\{X(\omega)\} = [A]\{\Xi(\omega)\}$ where $\{X(\omega)\} = \mathcal{F}(\{x(t)\})$, the relative displacement of the ith floor, $X_i(\omega)$, can be expressed as

$$X_i(\omega) = \sum_{j=1}^{n} A_{ij}\alpha_j H_j(\omega)Z(\omega)$$

$$= \sum_{j=1}^{n} \phi_i^{(j)}\alpha_j H_j(\omega)Z(\omega) \tag{5}$$

where

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + 2i\zeta_j\omega_j\omega} \tag{6}$$

is the transfer function relating the relative displacement of the equivalent SDOF oscillator in the jth mode to the input base excitation, and $\phi_i^{(j)}$ is the ith element of the jth mode shape vector, $\{\phi^{(j)}\}$. Thus, on assuming stationarity in the excitation and the response, the PSDF, $S_{x_i}(\omega)$, of displacement of the ith floor, may be expressed as

$$S_{x_i}(\omega) = S_{\ddot{z}}(\omega)\sum_{j=1}^{n}[(\phi_i^{(j)})^2\alpha_j^2 \,|\, H_j(\omega)\,|^2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k \mathrm{Re}(H_j(\omega)H_k^*(\omega))] \tag{7}$$

where, $S_{\ddot{z}}(\omega)$ is the PSDF of the input base excitation, $\ddot{z}(t)$. The double summation term in the above equation can be seen to represent the effects of interaction of the jth mode with the other $n-1$ modes of vibration. This can be further simplified by using the partial fractions for $\mathrm{Re}(H_j(\omega)H_k^*(\omega))$ (as in Gupta and Trifunac, 1990b) as

$$S_{x_i}(\omega) = S_{\ddot{z}}(\omega)\sum_{j=1}^{n}|\, H_j(\omega)\,|^2\,[(\phi_i^{(j)})^2\alpha_j^2 + \sum_{k=1,k\neq j}^{n} \phi_i^{(j)}\phi_i^{(k)}\alpha_j\alpha_k\{C_{jk} + (1-\frac{\omega^2}{\omega_j^2})D_{jk}\}], \tag{8}$$

where, C_{jk} and D_{jk} are the coefficients given in terms of ζ_j, ζ_k and $\varrho = \omega_k/\omega_j$ as

$$C_{jk} = \frac{1}{B_{jk}}\left[8\zeta_j(\zeta_j + \zeta_k\varrho)\{(1-\varrho^2)^2 - 4\varrho(\zeta_j - \zeta_k\varrho)(\zeta_k - \zeta_j\varrho)\}\right] \tag{9}$$

$$D_{jk} = \frac{1}{B_{jk}}\left[2(1-\varrho^2)\{4\varrho(\zeta_j - \zeta_k\varrho)(\zeta_k - \zeta_j\varrho) - (1-\varrho^2)^2\}\right] \tag{10}$$

and

$$B_{jk} = 8\varrho^2\left[(\zeta_j^2 + \zeta_k^2)(1-\varrho^2)^2 - 2(\zeta_k^2 - \zeta_j^2\varrho^2)(\zeta_j^2 - \zeta_k^2\varrho^2)\right] + (1-\varrho^2)^4 \tag{11}$$

Generalizing Equation (8) for any response, $r(t)$ of a linear system, such that $r(t) = \sum_{j=1}^{n}\rho_j\xi_j(t)$, one can write

$$S_r(\omega) = S_{\ddot{z}}(\omega)\sum_{j=1}^{n}[\{\rho_j^2\alpha_j^2 + \sum_{k=1,k\neq j}^{n}\rho_j\rho_k\alpha_j\alpha_k(C_{jk}+D_{jk})\}\,|\,H_j(\omega)\,|^2 - \frac{|\,\omega H_j(\omega)\,|^2}{\omega_j^2}\sum_{k=1,k\neq j}^{n}\rho_j\rho_k\alpha_j\alpha_k D_{jk}] \tag{12}$$

as the spectral density function of $r(t)$. Here, ρ_j is the normalized amplitude of response, $r(t)$, in the jth mode of vibration, and is expressed as a linear combination of the elements of the jth mode shape, $\{\phi^{(j)}\}$. For example, it is equal to $\phi_i^{(j)}$ for the displacement response, and to $\sum_{l=1}^{i} m_l\omega_j^2\phi_l^{(j)}$ for the shear response at the ith floor level.

Taking moments of $S_r(\omega)$ about the origin leads to

$$\lambda_p = \int_0^\infty \omega^p S_r(\omega)\,\mathrm{d}\omega = \sum_{j=1}^{n}[\{\rho_j^2\alpha_j^2 + \sum_{k=1,k\neq j}^{n}\rho_j\rho_k\alpha_j\alpha_k(C_{jk}+D_{jk})\}\lambda_{p,j}^D - \frac{\lambda_{p,j}^V}{\omega_j^2}\sum_{k=1,k\neq j}^{n}\rho_j\rho_k\alpha_j\alpha_k D_{jk}], \quad p = 0,\ 1,\ 2,... \tag{13}$$

where

$$\lambda_{p,j}^{D} = \int_0^{\infty} S_{\ddot{z}}(\omega) \left| H_j(\omega) \right|^2 \omega^p \, \mathrm{d}\omega, \quad p = 0,1,2,...$$

is the pth moment of the PSDF of displacement response of a SDOF oscillator with ω_j frequency and ζ_j damping ratio and subjected to the base acceleration, $\ddot{z}(t)$, and

$$\lambda_{p,j}^{V} = \lambda_{p+2,j}^{D} = \int_0^{\infty} S_{\ddot{z}}(\omega) \left| \omega H_j(\omega) \right|^2 \omega^p \, \mathrm{d}\omega, \quad p = 0,1,2,...$$

is the pth moment of the PSDF for relative velocity response of this oscillator. Equation (13) can alternatively be put in the following form,

$$\lambda_p = \sum_{j=1}^{n} \rho_j^2 \alpha_j^2 \lambda_{p,j}^{D} (1 + \delta_{p,j}), \quad p = 0,1,2,... \tag{14}$$

where,

$$\delta_{p,j} = \sum_{k=1, k \neq j}^{n} \frac{\rho_k \alpha_k}{\rho_j \alpha_j} (C_{jk} + D_{jk} \gamma_{p,j}) \tag{15}$$

is the correction term accounting for the cross-correlation of the jth mode with the remaining $n-1$ modes. $\gamma_{p,j}$ is the multiplying factor for D_{jk}

$$\gamma_{p,j} = 1 - \frac{\lambda_{p,j}^{V}}{\omega_j^2 \lambda_{p,j}^{D}}. \tag{16}$$

For p = 0, this factor is a measure of the deviation of the rate of zero crossings of the displacement response of the SDOF system from what it would have been, had this system been subjected to an ideal white noise. For most excitation processes, this continuously decreases with the increasing natural period of the oscillator, remaining negative when the SDOF system is more flexible compared to the excitation process, and positive when the SDOF system is stiffer. It becomes zero near the dominant frequency of the ground motion. It may be noteworthy that the terms, 'C_{jk}' and '$D_{jk}\gamma_{p,j}$', in the expression for $\delta_{p,j}$ contribute to different types of cross-correlation. While 'C_{jk}' is important in case of closer spacing of jth and kth modes, '$D_{jk}\gamma_{p,j}$', becomes important when ω_j is not close to the dominant frequency of the ground motion. In case of the excitation being like a white noise over the frequencies of interest, the second term may be ignored without introducing significant error in $\delta_{p,j}$.

By calculating the required moments from Equation (13) or (14) for p = 0, 2 and 4, and then by multiplying the root-mean-square (r.m.s.) value with the stationary peak factor, the peak amplitude of the response, $r(t)$, of desired order and level of confidence can be determined (see Appendix A). The response process is however non-stationary, partly because the excitation process is non-stationary and partly because the excitation suddenly starts at zero time with finite operating time. This may significantly affect all three moments, λ_0, λ_2 and λ_4, and since the calculation of the r.m.s. value, r_{rms}, of $r(t)$ depends only on λ_0, r_{rms} needs to be corrected for nonstationarity by multiplication with a 'non-stationarity factor'. However, it may not be necessary to revise the peak factors, because the input parameters depend on the ratios of these moments, and it may be reasonable to assume that all the three moments are affected by nonstationarity to the same extent.

For better understanding, one can express r_{rms} as a SRSS combination of its contributions from different modes,

$$r_{\mathrm{rms}}^2 = \sum_{j=1}^{n} r_{j,\mathrm{rms}}^2 \tag{17}$$

where,

$$r_{j,\text{rms}} = \rho_j \alpha_j \sqrt{\lambda_{0,j}^D (1+\delta_{0,j})} \tag{18}$$

is the r.m.s. value of $r(t)$, if the contribution of the jth mode only is considered. It may be noted that the term, $\sqrt{\lambda_{0,j}^D}$, represents the r.m.s. value of relative displacement response of the equivalent SDOF oscillator corresponding to the jth mode. This may alternatively be represented as $\text{SD}_j \big/ \beta_j \eta_j^{(1)}$, where SD_j is the spectral displacement corresponding to the natural frequency ω_j and the damping ratio ζ_j, β_j is the 'non-stationarity factor' by which the r.m.s. value in the jth mode is modified, and $\eta_j^{(1)}$ is the (stationary) peak factor for the largest peak displacement response of the jth mode oscillator. This may be obtained from the knowledge of the moments in the jth mode and by using the expressions given in Appendix A. Then, one can modify the r.m.s. value to $\overline{r}_{\text{rms}}$ by multiplying it with a factor, β. Thus, the ith peak value of $r(t)$ response, i.e. $\overline{r}_{\text{peak}}^{(i)}$, is given by

$$\begin{aligned}\overline{r}_{\text{peak}}^{(i)^2} &= \eta^{(i)^2} \overline{r}_{\text{rms}}^2 \\ &= \beta^2 \eta^{(i)^2} r_{\text{rms}}^2 \\ &= \sum_{j=1}^{n} \frac{\beta^2}{\beta_j^2} \frac{\eta^{(i)^2}}{\eta_j^{(1)^2}} \rho_j^2 \alpha_j^2 \text{SD}_j^2 \left(1+\delta_{0,j}\right)\end{aligned} \tag{19}$$

where, $\eta^{(i)}$ is the (stationary) peak factor corresponding to the ith ordered peak of response, $r(t)$. Let $\overline{r}_{j,\max} = \rho_j \alpha_j \text{SD}_j$ represent the maximum value of the response $r(t)$ in the jth mode, as obtained from the response spectrum curve. Then, $\overline{r}_{\text{peak}}^{(i)}$ may be expressed as

$$\overline{r}_{\text{peak}}^{(i)^2} = \sum_{j=1}^{n} \frac{\beta^2}{\beta_j^2} \frac{\eta^{(i)^2}}{\eta_j^{(1)^2}} \overline{r}_{j,\max}^2 \left(1+\delta_{0,j}\right) \tag{20}$$

Alternatively,

$$\overline{r}_{\text{peak}}^{(i)^2} = \sum_{j=1}^{n} \overline{r}_{j,\text{peak}}^{(i)^2} \tag{21}$$

becomes the familiar SRSS form for the peak response, with

$$\overline{r}_{j,\text{peak}}^{(i)} = \frac{\beta}{\beta_j} \frac{\eta^{(i)}}{\eta_j^{(1)}} \overline{r}_{j,\max} \left(1+\delta_{0,j}\right)^{\frac{1}{2}} \tag{22}$$

representing the contribution of the jth mode to the ith peak response.

DISCUSSION OF PSDF-BASED METHODS

The above PSDF-based formulation for the ith peak response, in essence, has formed the basis of most response spectrum-based probabilistic approaches presented over the last two decades. There are three broad categories of these approaches.

1. Category I Methods

In this category, $S_{\ddot{z}}(\omega)$ is obtained from the Fourier transform $Z(\omega)$ of $\ddot{z}(t)$ by calculating the temporal PSDF, for several samples, with the knowledge of the ground motion duration, and then by averaging over the temporal PSDFs for these samples. Here, the ground motion duration, T_s, is taken as the average length of the stationary segment of the sample time histories of excitation process. Thus, $S_{\ddot{z}}(\omega)$ may be expressed as

$$S_{\ddot{z}}(\omega) = \frac{E\left[\left|Z(\omega)\right|^2\right]}{\pi T_s} \tag{23}$$

where, $E[.]$ represents the expectation operator. Due to this idealization of input PSDF, the 'non-stationarity factors' account for both types of non-stationarity: (i) inherent non-stationarity in the excitation process, and (ii) that acquired by sudden application of the excitation process.

Amini and Trifunac (1985) assumed the cross-correlation between various modes to be negligible (i.e., $\delta_{0,j}$ = 0), as in the case of the SRSS method (Goodman et al., 1953), and β_j to be same as β for all modes. This resulted in the following contribution of the jth mode,

$$\overline{r}_{j,\text{peak}}^{(i)} = \frac{\eta^{(i)}}{\eta_j^{(1)}} \overline{r}_{j,\max} \tag{24}$$

This formulation overestimates the higher order peak responses (for $i \geq 2$), and therefore, Gupta and Trifunac (1987c) proposed an improved estimation of $\eta^{(i)}$, using the concept of order statistics. They also proposed to include the effects of modal correlation in the calculation of $\eta^{(i)}$ and $\eta_j^{(i)}$, while assuming $\gamma_{p,j}$ to be zero. The formulation of Gupta and Trifunac (1987c) for negligible modal cross-correlation has been extended further as follows. Gupta and Trifunac (1987d) extended this to find the resultant response caused by the simultaneous excitation of all three translational components of ground motion. Gupta and Trifunac (1987b) obtained the response of simple symmetric buildings to torsional component of ground motion, and Gupta and Trifunac (1988a) obtained the response to simultaneous action of translational and rocking components of ground motion, while assuming those to be in phase. Gupta and Trifunac (1990a) extended the formulation of Gupta and Trifunac (1988a) to include the effects of relative deformations of foundation with respect to the surrounding soil, while assuming those to be in phase with the excitations, in case of significant soil-structure interaction effects.

Gupta and Trifunac (1990b) included the modal cross-correlation completely, while assuming the factor, β, to be same as in the case of negligible cross-correlation, i.e. by taking

$$\beta = \left[\frac{\sum_{j=1}^{n} (\overline{r}_{j,\max}/\eta_j^{(1)})^2}{\sum_{j=1}^{n} (\overline{r}_{j,\max}/\beta_j \eta_j^{(1)})^2}\right]^{1/2} \tag{25}$$

in Equation (22). They extended this approach to apply in case of simultaneous excitation of translational and rocking components of ground motion, with the two components assumed to be at a constant phase difference. Gupta and Trifunac (1990c) extended this to apply in case of simultaneous action of (i) translational and torsional, and (ii) translational, rocking and torsional components of ground motion for simple symmetric buildings, when constant phase differences may be assumed between the translational and rotational components. Gupta and Trifunac (1991b) extended the formulation of Gupta and Trifunac (1990b) to include the effects of dynamic soil-structure interaction (i) by modifying the Fourier spectra of and phase difference between the two components, and (ii) by estimating β from 'non-stationarity factors' associated with the fixed-base building response and soil rocking response to the translational component of motion.

It is implicitly assumed in Equation (25) that the extent to which non-stationarity affects the response in any mode is not influenced by the extent to which other modes correlate with this mode, and that this extent is same for all modes. The degree of non-stationarity may however be significantly different in various modal responses due to the different modal frequencies and damping ratios, unless the excitation process is very long, with a higher mode associated with reduced effect of non-stationarity. In view of this, Agarwal and Gupta (1995) used the assumption of $\beta_j = \beta$, as in Equation (24), for determining more accurate lateral-torsional response of torsionally coupled buildings. For simplicity, they assumed the response process in each mode to be truly narrow-banded, and thus considered $\gamma_{0,j}$ to be zero for all modes (as in the case of the CQC method (Wilson et al., 1981)). This resulted in the following contribution of the jth mode,

$$\bar{r}_{j,\text{peak}}^{(i)} = \frac{\eta^{(i)}}{\eta_j^{(1)}} \bar{r}_{j,\max} \left(1 + \sum_{k=1,k\neq j}^{n} \frac{\rho_k \alpha_k}{\rho_j \alpha_j} C_{jk} \right)^{\frac{1}{2}} \tag{26}$$

The formulation of Agarwal and Gupta (1995) is thus expected to work better when the modal cross-correlation is primarily due to the closeness of interacting modes. By including $\gamma_{0,j}$ in Equation (26), this formulation can be generalized to other situations also.

It may be observed that the crucial element of the Category I methods is how β is estimated. β depends on the values of β_j, where β_j is expected to increase with the value of j. It may thus be expected that β lies somewhere in between β_1 and β_n, with a value closer to β_1, due to the dominance of lower modes in the total response. Further studies are however needed to explore whether a weighted sum of various β_j's or some other alternative will lead to further improvement in the approach of Agarwal and Gupta (1995). It is also noted that the methods in this category require the specification of Fourier spectrum along with the response spectra, which makes those unattractive without any obvious advantages in comparison to other PSDF-based methods.

2. Category II Methods

In the second category, the input PSDF is obtained in the same way as in the case of the first category of methods, but the 'non-stationarity factors' account only for the inherent non-stationarity in the ground motion process. The acquired non-stationarity in the response process is decoupled from the inherent non-stationarity, and is accounted for by the use of transient transfer function (instead of steady-state transfer function, as in Equation (6)). These methods require only the response spectrum ordinate at zero-period, i.e. PGA. On the other hand, those require more computationally-intensive non-stationary peak factors for the evolutionary processes, instead of the stationary peak factors applicable for stationary processes (see Appendix A for details).

Shrikhande and Gupta (1997a) proposed to replace the given non-stationary ground acceleration process by an equivalent stationary process. The PSDF of this equivalent process is obtained by scaling-up the PSDF described by Equation (23), so as to correspond to a specified value of PGA. They also proposed to use the transient transfer function, $\tilde{H}_j(\omega,t)$, in place of the steady-state transfer function, $H_j(\omega)$, in Equation (7). In view of this, the (evolutionary) spectral density of $r(t)$ becomes

$$S_r(\omega,t) = S_{\ddot{z}}(\omega) \sum_{j=1}^{n} [\rho_j^2 \alpha_j^2 \,|\, \tilde{H}_j(\omega,t)|^2 + \sum_{k=1,k\neq j}^{n} \rho_j \rho_k \alpha_j \alpha_k \text{Re}(\tilde{H}_j(\omega,t) \tilde{H}_k^*(\omega,t))] \tag{27}$$

To estimate the ordered peak response from $S_r(\omega,t)$, the order statistics approach of peak factors for stationary processes has been proposed to be extended to evolutionary processes in a simple, approximate, but computationally more intensive manner. Later, Shrikhande and Gupta (1999) have used this approach in formulating the seismic response of suspension bridges in case of significant soil-structure interaction and spatially varying ground motions.

The formulation of Shrikhande and Gupta (1997a) considered the use of a simpler form of $\tilde{H}_j(\omega,t)$ through the use of time-dependent damping ratio, $\zeta_j(t) = \zeta_j/[1-e^{-2\omega_j\zeta_j t}]$, in place of ζ_j. Gupta and Trifunac (2000) have identified situations where such a simplification may lead to large errors in response estimation, and thus, it is preferable to use the following exact form of $\tilde{H}_j(\omega,t)$ for more accurate results in all situations:

$$\tilde{H}_j(\omega,t) = H_j(\omega) \left[e^{-i\omega t} - e^{-\zeta_j \omega_j t} \left\{ \cos\omega_j t\sqrt{1-\zeta_j^2} + \frac{\zeta_j \omega_j - i\omega}{\omega_j \sqrt{1-\zeta_j^2}} \sin\omega_j t\sqrt{1-\zeta_j^2} \right\} \right] \tag{28}$$

Further, it will be useful to consider a more realistic energy distribution of the ‘equivalent stationary’ ground motion (as against the ‘time-averaged’ energy distribution), perhaps that based on the (truncated) Fourier transform of the stationary phase only (Udwadia and Trifunac, 1974b). It is obvious that if the PSDF of the ‘equivalent stationary’ motion does not correctly describe the instantaneous PSDF of the excitation in the neighbourhood of the instant when maximum response peak occurs, the resulting inaccuracy in the peak estimation may be substantial. This condition is usually difficult to satisfy, and therefore, the Category II methods are useful only when the non-stationarity due to finite operating time dominates over the inherent non-stationarity. This may happen when either the structural system is very flexible or the ground motion has a long stationary phase.

3. Category III Methods

In the third category of PSDF-based methods, PSDF of the ground excitation is taken to be spectrum-compatible PSDF, i.e. the PSDF that is compatible with the given response spectrum (see, for example, Kaul, 1978; Unruh and Kana, 1981; Christian, 1989; Gupta and Trifunac, 1998b). Alternatively, as shown by Gupta and Trifunac (1998b), strong motion duration in Equation (23) is taken to be frequency-dependent. Since a spectrum-compatible PSDF incorporates all effects of inherent and response non-stationarities for the responses of a set of SDOF oscillators of certain damping ratio, these methods do not require the use of ‘non-stationarity factors’. Thus, contribution of the jth mode to the ith peak response becomes

$$\overline{r}_{j,\text{peak}}^{(i)} = \rho_j \alpha_j \eta^{(i)} \sqrt{\lambda_{0,j}^{D}(1+\delta_{0,j})} \tag{29}$$

Singh and Chu (1976) proposed the estimation of the largest peak only (i = 1), and used this approach for generating floor response spectra of a building. Der Kiureghian (1981) also proposed the estimation of largest peak only, but without requiring to calculate $S_{\ddot{z}}(\omega)$ from the given response spectrum. Instead, various moments involving $S_{\ddot{z}}(\omega)$ were directly expressed in form of response spectrum ordinates to calculate $\eta^{(1)}$ and $\lambda_{0,j}^{D}$. Further, $\gamma_{0,j}$ was taken as zero for all modes, thus including the cross-correlation only due to the closeness of interacting modes.

Gupta (1996b, 1997) used the spectrum-compatible PSDF-based approach for generating the floor response spectra. Dey and Gupta (1998, 1999), and Ray Chaudhuri and Gupta (2002a, 2002b) used this approach for calculating the seismic response of multiply-supported secondary systems. Gupta and Trifunac (1998a) proposed the use of the transient transfer function at $t = T$, i.e. $\tilde{H}_j(\omega,T)$, where T is the total duration of the excitation, in place of the steady-state transfer function, i.e. $H_j(\omega)$ ($\equiv \tilde{H}_j(\omega,\infty)$). This is due to the fact that the spectrum-compatible PSDF, they propose to use in their formulation, is obtained by using the transient transfer function at $t = T$ (Gupta and Trifunac, 1998b). If the spectrum-compatible PSDF is obtained by using the steady-state transfer function, as is the case with most other formulations, it becomes necessary to use the steady-state form for the modal transfer functions also (with no change in the assumed value of total duration). Using the spectrum-compatible PSDF of Gupta and Trifunac (1998b), Gupta and Trifunac (1999) have also proposed to estimate seismic response of non-classically damped MDOF systems.

Even though the Category III methods are simpler to use and are quite accurate, as regards the largest peak response, those require *a priori* the iterative computation of spectrum-compatible PSDF, corresponding to a particular response spectrum curve. Since the response spectrum curves for other damping ratios are not used in this process, the so-obtained PSDF is usually not compatible with the entire set of response spectrum curves. One may perhaps use the envelope PSDF (as used by Dey and Gupta, 1998) for conservative response estimates. One may also obtain PSDFs for spectra of different damping ratios through the use of transient transfer functions, and since these PSDFs differ little from each other, one may find an ‘average’ PSDF (see Shrikhande and Gupta, 1996). Such a PSDF will be useful in case of Category II methods, because of the need to use transient modal transfer functions in response calculations.

WAVELET-BASED METHODS

Wavelet-based methods using the response spectrum-based characterization of seismic hazard involve the calculation of spectrum-compatible wavelet functionals (as described in Mukherjee and Gupta, 2002), just as spectrum-compatible PSDF is calculated in the case of Category III PSDF-based methods. The difference here is that these functionals are made more realistic through an additional input of desired structure of non-stationarity (depending on the existing conditions of earthquake source mechanism, wave propagation, and local site effects) in form of the wavelet coefficients of a recorded accelerogram. Once the ground motion process is characterized in form of the statistical functionals of wavelet coefficients, instantaneous PSDF of the response process is obtained in terms of system properties and input functionals, and the ordered peak response is estimated as in the case of the Category II PSDF-based methods.

Let the ground acceleration process be assumed to be zero mean, locally Gaussian, and be characterized through the (spectrum-compatible) statistical functionals (Mukherjee and Gupta, 2000), $E[W_\psi^2 \ddot{z}(a_p, b_q)]$, where

$$W_\psi \ddot{z}(a_p, b_q) = \int_{-\infty}^{\infty} \ddot{z}(t)\psi_{a_p,b_q}(t)\mathrm{d}t \tag{30}$$

denotes the wavelet coefficient of $\ddot{z}(t)$ at scale parameter, a_p, and translation parameter, b_q, and

$$\psi_{a_p,b_q}(t) = \frac{1}{\sqrt{a_p}}\psi\left(\frac{t-b_q}{a_p}\right) \tag{31}$$

is the dilated and translated form of the mother wavelet, $\psi(t)$. As suggested by Basu and Gupta (1998), let $a_p = \sigma^p$ with $\sigma = 2^{1/4}$, $b_q = (q-1)\Delta b$ (Δb being the digitization interval), and

$$\psi(t) = \frac{1}{\pi\sqrt{(\sigma-1)}} \frac{\sin\sigma\pi t - \sin\pi t}{t} \tag{32}$$

with Fourier transform

$$\begin{aligned} \hat{\psi}(\omega) &= \frac{1}{\sqrt{2(\sigma-1)\pi}}, \quad \pi \le |\omega| \le \sigma\pi \\ &= 0 \qquad\qquad \text{otherwise} \end{aligned} \tag{33}$$

The instantaneous PSDF in this case may be expressed as (Basu and Gupta, 1997)

$$\begin{aligned} S_r(\omega,t)|_{t=b_q} = \sum_p \frac{K}{a_p} E[W_\psi^2 \ddot{z}(a_p,b_q)] \sum_{j=1}^{n} \Bigg[& \{\rho_j^2\alpha_j^2 + \sum_{k=1,k\neq j}^{n} \rho_j\rho_k\alpha_j\alpha_k(C_{jk}+D_{jk})\} |H_j(\omega)|^2 \\ & - \frac{|\omega H_j(\omega)|^2}{\omega_j^2} \sum_{k=1,k\neq j}^{n} \rho_j\rho_k\alpha_j\alpha_k D_{jk} \Bigg] |\hat{\psi}_{a_p,b_q}(\omega)|^2 \end{aligned} \tag{34}$$

where,

$$K = \frac{1}{4\pi C_\psi}\left(\sigma - \frac{1}{\sigma}\right) \tag{35}$$

and

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|}\mathrm{d}\omega \tag{36}$$

Accordingly, the instantaneous value of the sth moment may be obtained as

$$\lambda_s(t)|_{t=b_q} = \sum_p \frac{K}{(\sigma-1)\pi} E[W_\psi^2 \ddot{z}(a_p,b_q)] \sum_{j=1}^{n} \rho_j^2 \alpha_j^2 \lambda_{s,j}^{p,D} (1+\delta_{s,j}^p), \quad s=0,1,2,... \tag{37}$$

where,

$$\delta_{s,j}^p = \sum_{k=1,k\neq j}^{n} \frac{\rho_k \alpha_k}{\rho_j \alpha_j} \left[C_{jk} + D_{jk} \left(1 - \frac{\lambda_{s,j}^{p,V}}{\omega_j^2 \lambda_{s,j}^{p,D}} \right) \right] \tag{38}$$

is the term accounting for the instantaneous cross-correlation of the jth mode with the remaining $n-1$ modes; and $\lambda_{s,j}^{p,D}$ and $\lambda_{s,j}^{p,V}$ respectively are the sth moments for the displacement and velocity responses of the jth mode oscillator for the pth band of energy (see Basu and Gupta, 1997, for the closed form expressions for these moments). The simplification achieved by ignoring $\delta_{s,j}^p$ is more accurate than that achieved by ignoring $\delta_{p,j}$ in Equation (14). In fact, this simplification leads to reasonably accurate results even when the natural frequencies of the structural system are closely spaced (Basu and Gupta, 1997). The wavelet-based approach has been extended to include the effects of soil-structure interaction in case of tanks (Chatterjee and Basu, 2001) and to the use of equivalent linearization in case of non-linear systems (Basu and Gupta, 1999a, 1999b, 2000, 2001).

CONCLUSIONS

It has been seen that there are various response spectrum-based formulations available for the stochastic estimation of the response of a MDOF system, and that each one of those caters to a different type of need. This need is usually dictated by the available input data, and by the balance the user wishes to adopt between the desired levels of accuracy and simplicity. On one hand, there are Category III PSDF-based methods that just require the response spectrum ordinates, while on the other, the Category I PSDF-based methods require specification of Fourier spectrum also, and the wavelet-based methods require the specification of a recorded time-history. Since all PSDF-based methods are based on the concept of a fictitious ground motion process, their utility is limited to finding the first few largest response peaks. For other response statistics, a more detailed description of excitation process is required. In such situations, though being computationally more intensive, the wavelet-based methods can be more useful. There is however a need to further reduce the number of input data points in the specification of the (spectrum-compatible) input process, and to apply wavelet-based approach to different types of structural systems. Also, specification of the input process in case of two or more response spectra (of different damping ratios) needs to be addressed.

To the extent that one may be interested in response peak amplitudes only, the Category III PSDF-based methods appear to be more accurate than the Category I methods (in tackling the effects of non-stationarity) and most convenient, amongst all methods discussed in this paper. Those however suffer from the limitation of being based on the response spectrum of a single value of damping ratio. Except for the idea of envelope PSDF used by some investigators, this problem has not received much attention so far. Further, since the use of transient transfer function may not usually be a computationally viable option, it needs to be seen whether the use of transfer function, $\overline{H}_j(\omega,T)$, is indeed a better option than $H_j(\omega)$, or there should be some other value of t at which the transient transfer function is to be evaluated. When the non-stationarity in response due to the short length of the excitation process is significant, compared to that due to the stationary phase being too short in the excitation process, it may be desirable to consider the transient transfer function. Then, the Category II PSDF-based methods requiring the input of Fourier spectrum and PGA only may be more convenient.

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systems through the use of response spectra and on extending these analyses to estimate amplitudes of second largest and higher order peaks.

APPENDIX A: STATIONARY AND NON-STATIONARY PEAK FACTORS

1. Stationary Peak Factors

Consider a stationary, zero mean, and Gaussian process, $X(t)$, with PSDF $S_X(\omega)$ and duration T. Peak amplitude of desired order and level of confidence in this process may be estimated by computing the root-mean-square (r.m.s.) value as the square-root of the area under $S_X(\omega)$, and by multiplying this with the corresponding peak factor. The peak factor for the ith peak may be determined by using its probability density and cumulative probability functions given by (Gupta and Trifunac, 1988b)

$$p^{(i)}(\eta) = \frac{N!}{(N-i)!(i-1)!}\left[P(\eta)\right]^{i-1}\left[1-P(\eta)\right]^{N-i} p(\eta) \tag{A.1}$$

and

$$P^{(i)}(\eta) = \int_{\eta}^{\infty} p^{(i)}(u)\mathrm{d}u = \sum_{r=i}^{N!} \frac{N!}{(N-r)!r!}\left[P(\eta)\right]^{r}\left[1-P(\eta)\right]^{N-r} \tag{A.2}$$

Whereas the distribution function here can be used iteratively to obtain the peak factor for any desired confidence level, the density function can be used to find the peak factors for the expected and the most probable peak amplitudes. In Equations (A.1) and (A.2), $p(\eta)$ is the probability density function of the (unordered) peaks in $|X(t)|$ given by (Cartwright and Longuet-Higgins, 1956; Gupta, 1994)

$$p(\eta) = \frac{\sqrt{2}}{\sqrt{\pi}(1+\sqrt{1-\varepsilon^2})}\left[\varepsilon e^{-\eta^2/2\varepsilon^2} + (1-\varepsilon^2)^{1/2}\eta e^{-\eta^2/2}\int_{-\infty}^{\eta(1-\varepsilon^2)^{1/2}/\varepsilon} e^{-x^2/2}\mathrm{d}x\right];\ \eta \ge 0 \tag{A.3}$$

$P(\eta) = \int_{\eta}^{\infty} p(u)\mathrm{d}u$ is the cumulative probability function of these peaks, and N is the total number of these peaks given by

$$N = \frac{T}{2\pi}(1+\sqrt{1-\varepsilon^2})\left[\frac{\lambda_4}{\lambda_2}\right]^{1/2} \tag{A.4}$$

In Equation (A.3), ε is the band-width parameter of $S_X(\omega)$, and is given by

$$\varepsilon = \left[\frac{\lambda_0\lambda_4 - \lambda_2^2}{\lambda_0\lambda_4}\right]^{1/2} \tag{A.5}$$

where, in general, the nth moment, λ_n of the PSDF is defined by

$$\lambda_n = \int_0^{\infty} \omega^n S_X(\omega)\mathrm{d}\omega \qquad (n = 0,1,2,...) \tag{A.6}$$

For the 'expected' ith order peak amplitude, the peak factor, $\overline{\eta}^{(i)}$, is given by

$$\overline{\eta}^{(i)} = \int_{-\infty}^{\infty} u p^{(i)}(u)\mathrm{d}u \tag{A.7}$$

For an approximate and direct evaluation scheme using the numerical integration, Equation (A.7) may be written alternatively as

$$\overline{\eta}^{(i)} = \int_0^{\infty} P^{(i)}(u)\mathrm{d}u - \int_0^{\infty} \mathrm{d}(uP^{(i)}(u)) \tag{A.8}$$

and thus, $\overline{\eta}^{(i)}$ approximately becomes equal to [area under the $P^{(i)}(\eta)$ curve between η = 0 and $\eta_{\lim}$] minus [$\eta_{\lim}P^{(i)}(\eta)|_{\eta=\eta_{\lim}}$]. Here, $\eta_{\lim}$ is the value of η at which $P^{(i)}(\eta)$ becomes very small. This value may safely be taken as 9.0 for all orders of peaks (Gupta, 1994).

It may be noted that Equations (A.1) and (A.2) are for the N peaks (distributed as in Equation (A.3)) assumed to be statistically independent. As shown by Basu et al. (1996) and Gupta and Trifunac (1998c), this assumption gives reasonably good results for the expected values of first few ordered peaks.

2. Non-stationary Peak Factors

If the process, $X(t)$, is non-stationary with the PSDF, $S_X(\omega,t)$, we can obtain the instantaneous values of moments, band-width parameter, and number of peaks, as

$$\lambda_n(t) = \int_0^\infty \omega^n S_X(\omega,t)\mathrm{d}\omega \qquad (n = 0,1,2,...) \tag{A.9}$$

$$\varepsilon(t) = \left[\frac{\lambda_0(t)\lambda_4(t) - \lambda_2^2(t)}{\lambda_0(t)\lambda_4(t)}\right]^{1/2} \tag{A.10}$$

$$N(t) = \frac{T}{2\pi}\left(1+\sqrt{1-\varepsilon^2(t)}\right)\left[\frac{\lambda_4(t)}{\lambda_2(t)}\right]^{1/2} \tag{A.11}$$

Using these parameters, the ith largest peak value may be expressed, in case of evolutionary response processes, as (Shrikhande and Gupta, 1997a, 1997b)

$$X_{\text{peak}}^{(i)} = \left[\frac{1}{T}\int_0^T (\eta^{(i)}(t))^2 \lambda_0(t)\mathrm{d}t\right]^{1/2} \tag{A.12}$$

where, $\eta^{(i)}(t)$ is the peak factor for the ith largest peak value in case of a fictitious stationary process having duration, T, and the band-width parameter and number of peaks respectively as $\varepsilon(t)$ and $N(t)$. $\eta^{(i)}(t)$ is to be evaluated by using Equation (A.1) or (A.2) for the same level of confidence, for which $X_{\text{peak}}^{(i)}$ is desired to be obtained. The integral in Equation (A.12) may be evaluated efficiently by means of any standard quadrature routine.

In a more generalized situation, as proposed by Basu and Gupta (1998), $X_{\text{peak}}^{(i)}$ may be estimated by estimating the largest peak amplitude via the first passage formulation (Vanmarcke, 1975) and by scaling this down to the higher order peak amplitudes via the order statistics formulation (Gupta and Trifunac, 1988b). For the largest peak, we use the expression of the probability that the process, $|X(t)|$, remains below the level, x, during the time interval, ($0,T$), as

$$P_T(x) = \exp\left[-\int_0^T \alpha(t)\mathrm{d}t\right] \tag{A.13}$$

where

$$\alpha(t) = \frac{\Omega(t)}{\pi}\frac{1-\exp(-\sqrt{\pi/2}\,\mu^{1.2}(t)\frac{x}{\sigma(t)})}{1-\exp(-x^2/2\sigma^2(t))}e^{-x^2/2\sigma^2(t)} \tag{A.14}$$

with

$$\sigma(t) = \sqrt{\lambda_0(t)} \tag{A.15}$$

$$\Omega(t) = \sqrt{\frac{\lambda_2(t)}{\lambda_0(t)}} \tag{A.16}$$

$$\mu(t) = \sqrt{1 - \frac{\lambda_1^2(t)}{\lambda_0(t)\lambda_2(t)}} \quad \text{(A.17)}$$

For the ith largest peak (i > 1), we consider an equivalent stationary process which has the duration as

$$T^* = \frac{\int_0^T \Omega(t)\mathrm{d}t}{\Omega(t)\,|_{t=t_m}} \quad \text{(A.18)}$$

and the band-width parameter and number of peaks respectively as $\varepsilon(t)\,|_{t=t_m}$ and $N(t)\,|_{t=t_m}$, with t_m denoting the time-instant of occurrence of the largest mean-square response in the process. Corresponding to this process, (stationary) peak factors, $\eta^{(1)}(t)\,|_{t=t_m}$ and $\eta^{(i)}(t)\,|_{t=t_m}$, are determined for the largest peak and the ith largest peak respectively, and then, the largest peak amplitude estimated via Equation (A.13) is scaled down in the ratio, $\eta^{(i)}(t)\,|_{t=t_m} / \eta^{(1)}(t)\,|_{t=t_m}$.

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